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Energy Efficient Refrigeration and Flexible Power Consumption in a Smart Grid

Tobias Gybel Hovgaard, Rasmus Halvgaard, Lars F. S. Larsen and John Bagterp Jørgensen

Abstract

Refrigeration and heating systems consume substantial amounts of energy worldwide. However, due to the thermal capacity there is a potential for storing "coldness" or heat in the system. This feature allows for implementation of different load shifting and shedding strategies in order to optimize the operation energywise, but without compromising the original cooling and indoor climate quality. In this work we investigate the potential of such a strategy and its ability to significantly lower the cost related to operating systems such as supermarket refrigeration and heat pumps for residential houses. With modern Economic Model Predictive Control (MPC) methods we make use of weather forecasts and predictions of varying electricity prices to apply more load to the system when the thermodynamic cycle is most efficient, and to consume larger shares of the electricity when the demand and thereby the prices are low. The ability to adjust power consumption according to the demands on the power grid is a highly wanted feature in a future Smart Grid. Efficient utilization of greater amounts of renewable energy calls for solutions to control the power consumption such that it increases when an energy surplus is available and decreases when there is a shortage. This should happen almost instantly to accommodate intermittent energy sources as e.g. wind turbines. We expect our power management solution to render systems with thermal storage capabilities suitable for flexible power consumption. The aggregation of several units will contribute significantly to the shedding of total electricity demand. Using small case studies we demonstrate the potential for utilizing daily variations to deliver a power efficient cooling or heating and for the implementation of Virtual Power Plants in Smart Grid scenarios.

1 Introduction

The energy policies in the Nordic countries stipulate that 50% of the energy consumed by 2025 should come from renewable and CO_2-free energy sources. By 2050 the Nordic countries should be independent of fossil fuels. This transformation of the energy system is needed to reduce CO_2 emissions and global warming as well as to protect the Nordic economies from the consequences of sharply rising prices of fossil fuels due to an increasing world population and depletion of fossil fuel resources [1]. To obtain an increasing amount of electricity from intermittent energy sources such as solar and wind, we must not only control the production of electricity but also the consumption of electricity in an efficient, agile and proactive manner. In contrast to the current rather centralized power generation system, the future electricity grid is going to be a network of a very large number of independent power generators. The Smart Grid is the future intelligent electricity grid and is intended to be the smart electrical infrastructure required to increase the amount of green energy significantly. The Danish transmission system operator (TSO) has the following definition of Smart Grids which we adopt in this work: "Intelligent electrical systems that can integrate the behavior and actions of all connected users - those who produce, those who consume and those who do both - in order to provide a sustainable, economical and reliable electricity supply efficiently" [2]. In this paper we utilize the flexibility of the refrigeration system to offer ancillary demand response to the power grid as regulating power. Different means of utilizing demand response have been investigated in an increasing number of publications e.g. [3]–[6] for plug-in electrical vehicles and heat pumps and in general concerning price elasticity in [7].

In this paper we consider two utilizations of a vapor compression cycle. One for supermarket refrigeration and one for heating residential buildings using heat pumps. Buildings account for approximately 40% of the total energy use in the Nordic countries and in Denmark around 4500 supermarkets consume more than 550 GWh annually. As heat pumps are

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driven by electricity and connected to house floors with large thermal capacities, they have a large potential to shift the electricity consumption and adapt to the stochastic electricity production from wind turbines. The same holds for refrigeration systems where the thermal capacity in the refrigerated goods can be used to store "coldness" and thereby shift the load in time while keeping the temperatures within certain bounds. These bounds are chosen such that they have no impact on food quality and indoor comfort. We exploit that the dynamics of the temperature in the cold room and residential buildings are rather slow while the power consumption can be changed rapidly.

Simple weather conditions such as outdoor temperature and solar radiation are included in our simulation models. By including forecasts of prices and weather conditions energy consumption is made flexible. It is thus possible to predict where to place the energy consumption and minimize the electricity cost of operating the systems without violating the constraints such as indoor temperature comfort intervals and food storage conditions. The thermal capacities determine how much of the electricity consumption that can be shifted to times with cheap electricity. We investigate different scenarios spanning from a case study with total collaboration between producers and consumers to decoupling through price signals from the NordPool electricity spot market. Utilizing load shifting capabilities to reduce total energy consumption has also been described in e.g. [8], [9].

Our proposed control strategy is an economic optimizing model predictive controller, Economic MPC. Predictive control for constrained systems has emerged during the last 30 years as one of the most successful methodologies for control of industrial processes [10] and is increasingly being considered for both refrigeration and power systems control [11], [12]. MPC based on optimizing economic objectives has only recently emerged as a general methodology with efficient numerical implementations and provable stability properties [13].

This paper is organized as follows. In section 2 we discuss different control architectures for enabling flexible consumption from many distributed units in a Smart Grid. Section 3 describes the models used in our case studies as well as the formulation of our Economic MPC strategy. In section 4 the simulations and results for three cases are illustrated. The first two cases demonstrate direct control and price signal based control with rather simple models while the third case uses more advanced nonlinear models verified with data from real supermarkets and a combination of price and frequency based control. In section 5 means for handling uncertainties in the framework of our proposed strategy are presented. We give conclusions in 6.

2 Control Architectures for Virtual Power Plants

A way of creating flexibility in the power grid is Virtual Power Plants (VPP). The concept pools several, otherwise too small, production and consumption units, such as multiple smaller power plants, wind turbines and heat pumps, and make them behave as one unit. The VPP concept enables a huge amount of possibilities for load balancing since it allows active control of the consumer [14].

In this paper we consider individual thermal storage units such as refrigeration systems and houses with heat pumps that are to be aggregated in the VPP framework (see Fig. 1). The control within the aggregated units can rely on different strategies. The following control strategies have been suggested for load balancing and load shifting in electrical grids [15].

1) **Direct control**. In this case producers and consumers are assumed to collaborate on a common goal of minimizing the total cost of operation. Given that the communication infrastructure required for sending control signals to the consumers to raise or reduce the demand is established, the producers are allowed a more direct control of the demand. Furthermore, it allows the controller in the consumer to be quite simple as it only sends information and receives commands from the VPP. The drawback of course is that the VPP must solve large-scale optimization problems to coordinate a large number of units. The result might not be optimal for the consumer alone and the VPP decides the consumption schedule entirely.
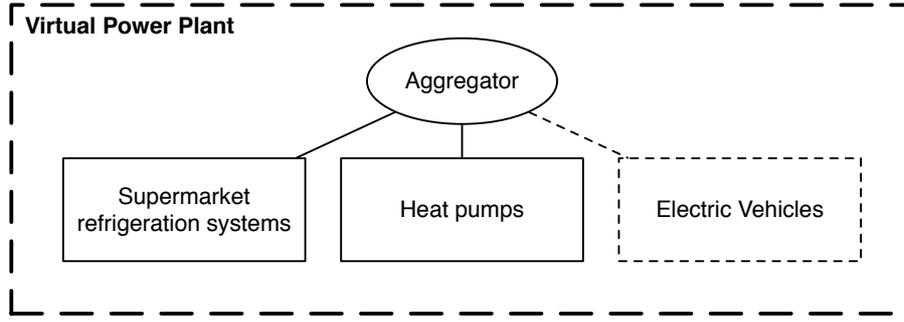
Fig. 1. Sketch illustrating a Virtual Power Plant (VPP).

2) **Price based control**. The individual units compute a schedule for the consumption based on dynamic price information given by the VPP. This enables the consumer to shift its load to times with low electricity price. It requires a communication infrastructure between VPP and heat pump/supermarket systems. The drawback of this control strategy is that it is relatively complex and that the VPP does not have any control of the actual load response but merely sends out a guideline in the form of price signals. How to calculate optimal price signals is still an open research issue. In this paper we use the electricity spot price to illustrate such a signal.

3) **Frequency based control**. The consumer measures the grid frequency, which in Europe has the nominal value of 50 Hz. When demand exceeds supply, the frequency falls. When supply exceeds demand, the frequency increases. One way of using frequency activation to change demand is demonstrated for residential fridges in [16]. The advantage of this type of control is the low price of the controller, because no communication is necessary. However the consumer does not have any economical incentives to do this unless the enabling of flexible consumption can make the fridge less expensive to buy. Another possibility of frequency activation is to sign a contract where each player participates with a power amount (MW) specified on an hourly basis. The consumer is paid for being at the disposal of the grid (DKK/MW) regardless of the actual activation. Activation is automatic and linearly frequency dependent in the range 50 Hz ± 200 mHz.

The authors have previously demonstrated some of these strategies in suitable scenarios [17]–[19]. Results and comparisons are provided in this paper.

Note that the two consumers studied in this paper both act as thermal storage to the VPP and the consumed electric energy can not be retrieved as electric energy again. It can only be consumed at the right times. On the contrary electric vehicles are in reality electric batteries able to store electricity when connected to the grid and can thus be used as both a consumer and a producer to the grid.

3 Models and Economic MPC formulation

3.1 Supermarket Refrigeration Systems

The supermarket refrigeration systems we consider utilize a vapor compression cycle where a refrigerant is circulated in a closed loop consisting of a compressor, an expansion valve and two heat exchangers, an evaporator in the cold storage room and a condenser/gas cooler located in the surroundings. When the refrigerant evaporates it absorbs heat from the cold reservoir which is rejected to the hot reservoir. The setup is sketched in Fig. 2 with one cold storage room and one frost room connected to the system. Usually several cold storage rooms, e.g. display cases, are connected to a common compressor rack and condensing unit.

The dynamics in the cold room can be described by the simple energy balance:

$$mc_p\frac{dT_{cr}}{dt} = \dot{Q}_{load} - \dot{Q}_e \quad , \text{with:} \quad \begin{cases} \dot{Q}_{load} = (UA)_{amb-cr} \cdot (T_{amb} - T_{cr}) \\ \dot{Q}_e = (UA)_{cr-e} \cdot (T_{cr} - T_e) \end{cases} \tag{1}$$

where UA is the heat transfer coefficient and m and c_p are the mass and the specific heat capacity of the refrigerated goods, respectively. T_{amb} is the temperature of the ambient air
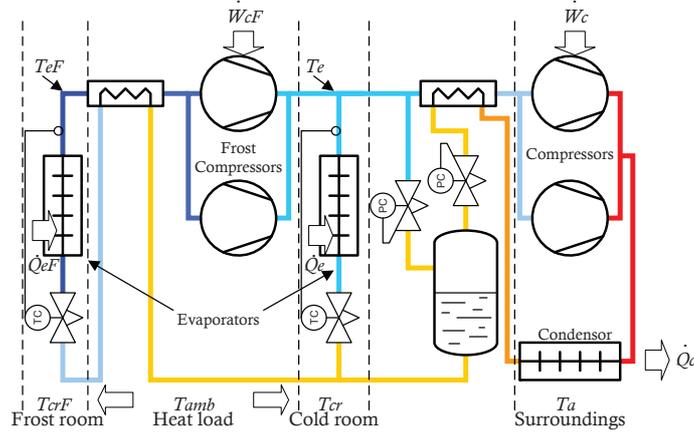
Fig. 2.   Schematic layout of basic refrigeration system.

which puts the heat load on the refrigeration system. The states and control variables of the system are limited by the following constraints:

$$T_{cr,min} \leq T_{cr} \leq T_{cr,max} \tag{2a}$$

$$0 \leq T_{cr} - T_e \leq \infty \tag{2b}$$

$$0 \leq \dot{Q}_e \leq (UA)_{cr-e,max} \cdot (T_{cr} - T_e) \tag{2c}$$

The work done by the compressor dominates the power consumption in the system and can be expressed by the mass flow of refrigerant (m_{ref}) and the change in energy content of the refrigerant. Energy content is described by enthalpy of the refrigerant at the inlet and at the outlet of the compressor (h_{ic} and h_{oc} respectively). Hereby the expression in Eq. (3) is given.

$$\dot{W}_c = \frac{m_{ref} \cdot (h_{oc}(T_e, P_c) - h_{ic}(T_e))}{\eta_{is}(P_c/P_e)} \tag{3}$$

where the enthalpies depend on the evaporation temperature and the condensing pressure as stated. The mass flow can be determined as the ratio between cooling capacity and change of enthalpy over the evaporator:

$$m_{ref} = \frac{\dot{Q}_e}{h_{oe}(T_e) - h_{ie}(P_c)} \tag{4}$$

All the enthalpies given here as functions of (T_e, P_c) or both are non-linear refrigerant dependent functions which can be calculated e.g. by the software package "RefEqns" [20].

For the studies in section 4.1 we have assumed that the work done in the compressor is directly proportional with the delivered cooling capacity while we in section 4.3 use the real non-linear description of $\dot{W}_c$ described in [21] where polynomials are fitted for the enthalpy differences. For the latter we have furthermore collected data from several supermarkets in real operation in Denmark. From these data typical parameters such as time constants, heat loads, temperature ranges and capacities in both individual display cases and for the overall system have been estimated. The running compressor capacity have been monitored and from the data sheets the relation to energy consumption has been found.

3.2 Building with Heat Pump

Heat dynamics of a building

In this section, we develop a model of the heat dynamics of a house floor heating system connected to a geothermal heat pump. The system is illustrated in Fig. 3. The model is based on the energy balances for the air in the room, the floor and the water in the floor heating pipes and condenser water tank. The house is considered to be one big room with the following simplifying assumptions: 1) One uniform air temperature, 2) no ventilation, 3) no influence from humidity of the air, 4) no influence from the heat released from people in the room, 5) no influence from wind. In our model two heat accumulating media are included
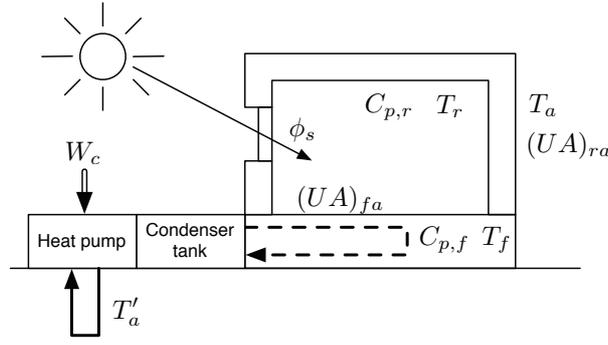
Fig. 3. House and heat pump floor heating system and its thermal properties. The dashed line represents the floor heating pipes that has heat conductivity $(UA)_{wf}$ and contains water with heat capacity $C_{p,w}$.

with heat capacities $C_{p,r}$ and $C_{p,f}$, to capture the short-term and long-term variations of the room air and floor heat dynamics [22]. The resulting energy balances are

$$C_{p,r}\dot{T}_r = Q_{fr} - Q_{ra} + (1-p)\phi_s \qquad (5)$$
$$C_{p,f}\dot{T}_f = Q_{wf} - Q_{fr} + p\,\phi_s \qquad (6)$$

The disturbances influencing the room air and floor temperature, T_r and T_f, are the ambient temperature, T_a, and the solar radiation, ϕ_s, through a window with fraction p of the incident radiation on the floor. The energy balance for the water circulating in the floor heating pipes can be stated as

$$C_{p,w}\dot{T}_w = Q_c - Q_{wf} \qquad (7)$$

in which Q_c is the heat transferred to the water from the condenser in the heat pump. Q_{wf} is the heat transferred from the water to the floor. The conductive heat transfer rates are

$$Q_{ra} = (UA)_{ra}(T_r - T_a), \quad Q_{fr} = (UA)_{fr}(T_f - T_r), \quad Q_{wf} = (UA)_{wf}(T_w - T_f) \qquad (8)$$

Q_{ra} is the heat transferred from the air in the room to the surroundings, Q_{fr} is the heat transferred from the floor to the air in the room, and Q_{wf} is the heat transferred from the water in the floor heating pipes to the floor. The term $U \cdot A$ is a product of the heat conductivity and the surface area of the layer between two heat exchanging media. Its reciprocal value $R = 1/(UA)$ is often used since it can be interpreted as a resistance against heat flow.

Heat Pump

A heat pump is a device that transfers heat from a low temperature zone to a higher temperature zone using mechanical work. Heat pumps normally draw heat from the air or from the ground and uses a vapor compression refrigeration cycle. This cycle is also used in the supermarket refrigeration system studied in section 3.1. In order to take advantage of the heat produced in the cycle instead of the cooling, the condenser and evaporator functions are switched such that the condenser is inside the house. As the heat pump dynamics is much faster than the thermodynamics of the building, we can assume a static model for the heat pump. The amount of heat transferred from the condenser to the water, Q_{cw}, is related to the work of the compressor, W_c, using the coefficient of performance

$$Q_{cw} = \eta W_c \qquad (9)$$

The coefficient of performance η for heat pumps varies with type, outdoor ground temperature, and the condenser temperature. As these two temperatures are approximately constant, we can assume that the coefficient of performance is also constant.

3.3 Economic Optimizing MPC

Our systems are influenced by a number of disturbances that can be predicted to some degree of certainty over a time horizon into the future. These must to be handled by the controller that also has to obey certain constraints for the systems while minimizing the cost of operation. Thus, we find it reasonable to aim at formulating our controller as an economic

optimizing MPC problem. Whereas the cost function in MPC traditionally penalizes a deviation from a set-point our proposed economic MPC directly reflects the actual costs of operating the plant. This formulation is tractable for refrigeration and heating systems where we are interested in keeping the outputs (temperatures) within certain ranges while minimizing the cost of doing so.

The models described in the previous sections are converted to their discrete-time state space formulations using zero-order-hold sampling of the input signals

$$x_{k+1} = Ax_k + Bu_k + Ed_k \tag{10a}$$
$$y_k = Cx_k + Du_k + Fd_k \tag{10b}$$

defining x as the states, the manipulable variable u, disturbances d and outputs y. Using this model to predict the future outputs, we may formulate a linear program that minimizes the electricity cost for operating the system while keeping the temperatures within prespecified intervals

$$\min_{\{x,u,y\}} \quad \phi = \sum_{k \in \mathcal{N}} c_{y,k}^T y_k + c_{u,k}^T u_k + \rho_v^T v_k \tag{11a}$$

$$s.t. \quad x_{k+1} = Ax_k + Bu_k + Ed_k \qquad k \in \mathcal{N} \tag{11b}$$
$$y_k = Cx_k + Du_k + Fd_k \qquad k \in \mathcal{N} \tag{11c}$$
$$u_{\min} \le u_k \le u_{\max} \qquad k \in \mathcal{N} \tag{11d}$$
$$\Delta u_{\min} \le \Delta u_k \le \Delta u_{\max} \qquad k \in \mathcal{N} \tag{11e}$$
$$y_{\min} \le y_k + v_k \qquad k \in \mathcal{N} \tag{11f}$$
$$y_{\max} \ge y_k - v_k \qquad k \in \mathcal{N} \tag{11g}$$
$$v_k \ge 0 \qquad k \in \mathcal{N} \tag{11h}$$

$\mathcal{N} \in \{0, 1, \ldots, N\}$ and N is the prediction horizon. The electricity prices enter the optimization problem as the cost coefficients $c_{u,k}$. The output cost on temperature is zero, $c_{y,k} = 0$. It may not always be possible to meet the temperature demand. Therefore, the MPC problem is relaxed by introduction of a slack variable v_k and the associated penalty cost ρ_v. The penalties can be set sufficiently large, such that the output constraints are met whenever possible. The Economic MPC also contains bound constraints and rate-of-movement constraints on the control variables. The prediction horizon, N, is normally selected large to avoid discrepancies between open-loop and closed-loop profiles. At each sampling time, we solve the linear program (11) to obtain $\{u_k^*\}_{k=0}^{N-1}$. We implement u_0^* on the process. As new information becomes available at the next sampling time, we redo the process of solving the linear program using a moving horizon and implementing the first part, u_0^*, of the solution. The electricity prices, $\{c_{u,k}\}_{k=0}^{N-1}$, as well as the disturbances, $\{d_k\}_{k=0}^{N-1}$, must be forecasted. In this paper, we assume that the forecasts are perfect.

4 Results and Discussions

4.1 Direct Control of Cold Room

The Economic MPC has been implemented in Matlab and simulations are presented in this section [17]. We have included two conventional power generators and one large cooling house (or an aggregation of several supermarkets). Direct control, i.e. total collaboration and communication between power producers and consumers is assumed. The production by the power generators, $y_{1,k} + y_{2,k}$, must exceed the demand for power by the cooling house and the demand from the external signal r_k

$$y_{1,k} + y_{2,k} \ge y_{3,k} + r_k \quad k \in \mathcal{N} \tag{12}$$

We model farms of wind turbines as instantaneously changing systems and include the effect of their power production together with all non-controllable power consumers in the exogenous net power demand signal, r_k.

Fig. 4 visualizes a simulation. In this scenario, the power demand from all other consumers than the cold room increases slowly, then stays at a steady level and eventually drops
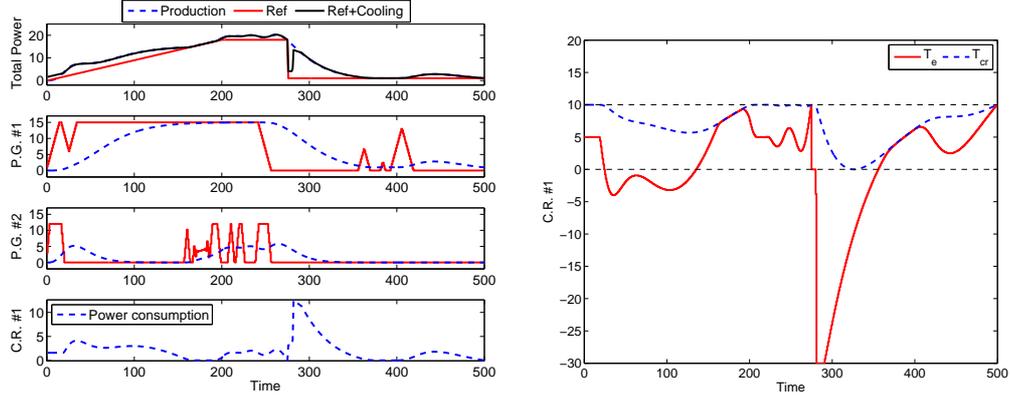
(a) P.G. #1 and 2 show the power productions from the two power plants (dotted blue) and their power set-points (solid red). C.R. #1 is power consumption in the cold room and "Total Power" shows total power production (dotted blue) versus the reference consumption (with (solid black) and without (solid red) the consumption for refrigeration.)

(b) Temperature in the cold room T_{cr} and the control signal for the refrigeration system T_e. $T_{cr,min}$ and $T_{cr,max}$ are shown with dotted black. The ambient temperature is assumed to be constant in this scenario.

Fig. 4. Simulation of Power Generation problem with two conventional power generators and on cold room with direct control.

significantly. This sudden drop could for instance be seen as an increase in wind speed that changes the demand to the power generators drastically.

If the cold room was a non-controllable load then, intuitively, the evaporation temperature T_e would stabilize at a level sufficient for keeping the temperature T_{cr} just below the upper constraint. Thus, with a constant load on the refrigeration system the power demand W_C that should be added to the reference r would simply be a constant over the entire scenario. The result is that a great amount of surplus electricity is produced after the sudden drop in demand. However, when the cold room is considered a controllable consumer it is able to absorb the majority of this otherwise redundant energy, as seen in Fig. 4. This causes the temperature in the cold room to decrease from the upper constraint to the lowest feasible level. Due to the thermal capacity in the refrigerated goods this "pre-cooling" makes it possible to entirely shut down the cooling and thereby limit power consumption at a time where the production cost has increased.

4.2 Price responsive heat pump

A building with a water based floor heating system connected to a geothermal heat pump was modeled in section 3.2. Parameters for a representative building are provided in [23] and includes values for building heat capacities and thermal conductivities.

To illustrate the potential of the Economic MPC for controlling heat pumps, we simulate scenarios using hourly electricity prices from Nordpool, the Nordic power exchange market [19]. The outdoor temperature, T_a, is modeled as diurnal cycles with added noise [24]. The sun radiation disturbance ϕ_s is not included in these simulations. We aim to minimize the total electricity cost in a given period while keeping the indoor temperature, T_r, in predefined intervals. In the case studied, we assume that the forecasts are perfect, i.e. with no uncertainty. We use long horizons ($N = 6$ days $= 144$ hours) and assume perfect model predictions.

Fig. 5 illustrates the optimal compressor schedule and the predicted indoor temperature for a six day horizon. The lower plot shows the outdoor temperature, T_a. The outdoor temperature reflects a cold climate, i.e. the outdoor temperature is lower than the indoor temperature. The middle plot shows the actual electricity prices in Western Denmark. The middle plot also contains the computed optimal heat pump power input, W_c. The upper plot shows the predicted indoor temperature along with the predefined time varying constraints. The constraints indicate that during night time the temperature is allowed to be lower than at day time. The figure reveals clearly that the power consumption is moved to periods with low electricity prices and that the thermal capacity of the house floor is able to store enough energy such that the heat pump can be left off during day time. This demonstrates that the slow heat dynamics of the floor can be used to shift the energy consumption to periods
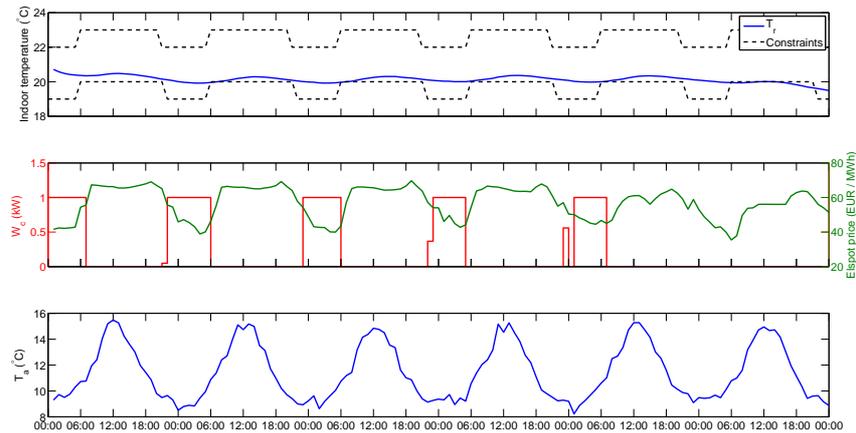
Fig. 5. Temperature in a house with time varying soft constraints, time varying electricity prices, and time varying outdoor temperatures. The time is the time since 18 JAN 2011 00:00. The upper figure shows the indoor temperature, the middle figure contains the electricity spot price and the optimal schedule for the heat pump, and the lower figure contains the ambient temperature. The compressor is on when the electricity spot price is low.

with low electricity prices and still maintain acceptable indoor temperatures. Notice that the constraints on room temperature are soft in this case.

We also conducted a simulation with constant electricity prices. In this case, the heat pump is now turned on just to keep the indoor temperature at its lower limit. This implies that there is no load shifting from the heat pump in this case. By comparing the case with varying electricity price to the case with constant electricity price, we observe economic savings around 33%. Using a simulation study with hard constraints on the temperature limits, the savings by load shifting were 26%.

Using actual electricity prices and weather conditions, we demonstrated that the Economic MPC is able to shift the electrical load to periods with low prices. As the Nordic Electricity spot prices reflect the amount of wind power in the system, the large thermal capacity of the house floor can essentially be used to store cheap electricity from renewable energy sources such as wind turbines. We also observed that the Economic MPC is able to shift the load and reduce the total cost of operating a heat pump to meet certain indoor temperature requirements.

4.3 Price and Frequency Controlled Supermarket Refrigeration

In this section we present the simulation of a realistic scenario with the supermarket refrigeration system in a setting where predictions of electricity prices, regulating power prices as well as outdoor temperatures exist. We have chosen a supermarket refrigeration system with three very different units attached. A shelving unit, a chest display case and a frost room. The units have different demands to temperature namely $[2;4]°$C for the shelving unit, $[1;5]°$C for the chest display case and $[-25;-15]°$C for the frost room. The models are validated with running supermarkets in Denmark, January 2011. Electricity prices have been downloaded from NordPool's hourly el-spot price for a period of one month. The same is done with the availability payment for regulating power. A sinusoidal approximation is used for a typical diurnal temperature curve.

We divide our simulations into two scenarios. One that illustrates the effect of variations in electricity prices and temperatures and one that shows how regulating power services can be offered. Simulations are performed over at least 24 hours. For the regulating power scenario the frequency dependent primary reserve is accounted for by including the availability payment in the cost function such that the controller is able to deviate from the elsewise optimal trajectory if the payment for the reserve obtained by doing so can counteract the increased cost. Thus, we are not showing the actual activation of the reserve by frequency deviations but merely how the system can prepare for such an activation and benefit from

the availability payment.

Simulation

Fig. 6 shows the simulated refrigeration system using the predicted outdoor temperature and electricity price to optimize the cost. The amplitude of the electricity price has been multiplied by four to better illustrate the effect. Today the dominating part of the price paid for electricity consist of taxes and connection fees which are all paid as flat rate charges per MWh. Hence, the simulation shown with 4 times amplitude on the el-spot price is an attempt to model a situation where the tax and other fees are charged as a percentage of the actual el-spot price. This would result in a magnification instead of a smoothening of the market signals. In this case the cost savings amount to more than 30%. If the original electricity price is used less change in cold room temperatures can be observed and the cost savings amount to 9%. If we are only exploiting the variations on outdoor temperature the economic MPC control scheme saves around 2% on the energy consumption. From the results illustrated in Fig. 6 we can conclude that the proposed economic MPC scheme has a positive effect on the cost related to operating the supermarket. Variations in outdoor temperature are utilized to minimize power consumption whereas exploiting variations in electricity prices tend to increase overall power consumption but at a lower cost.

In Fig. 7 the effect of participating in the power balancing market is simulated for a selected scenario of availability payments. In this simulation the outdoor temperature is assumed constant in order to illustrate the effect of availability payments for regulation power versus the electricity spot price as clearly as possible. This simulation reveals an additional saving of up to 70% compared to the case where only electricity spot price is used for optimization. Participating in the balancing power market seems to be beneficial for both the power system and the supermarkets if we consider the simulation in Fig. 7. At least at the time of the year/day where extra capacity is available and availability payment is sufficiently high. A large potential saving is found meaning that there is room for deviations from the simulated scenario without ruining the business case of participating with regulating power. Furthermore it is estimated from the simulations that a supermarket can offer at least 20% of its capacity as regulating power (except at the peak load days of the year). Currently in Denmark the peak demand for primary reserves is around 60 MW. With an average supermarket offering about 20% of its capacity approximately 75% of the total needs for primary reserves could be made up by supermarkets. A single supermarket is not able to participate with sufficient capacities to place bids on the balancing market however aggregation of e.g. chains of shops would be an obvious solution.

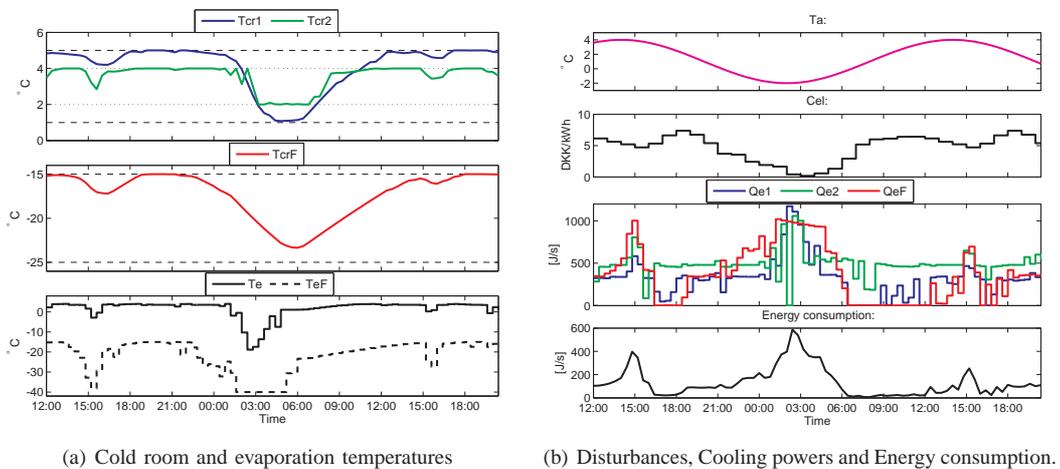(a) Cold room and evaporation temperatures   (b) Disturbances, Cooling powers and Energy consumption.

Fig. 6.   Simulation showing how variations in outdoor temperature and electricity prices are exploited by utilization of thermal storage.

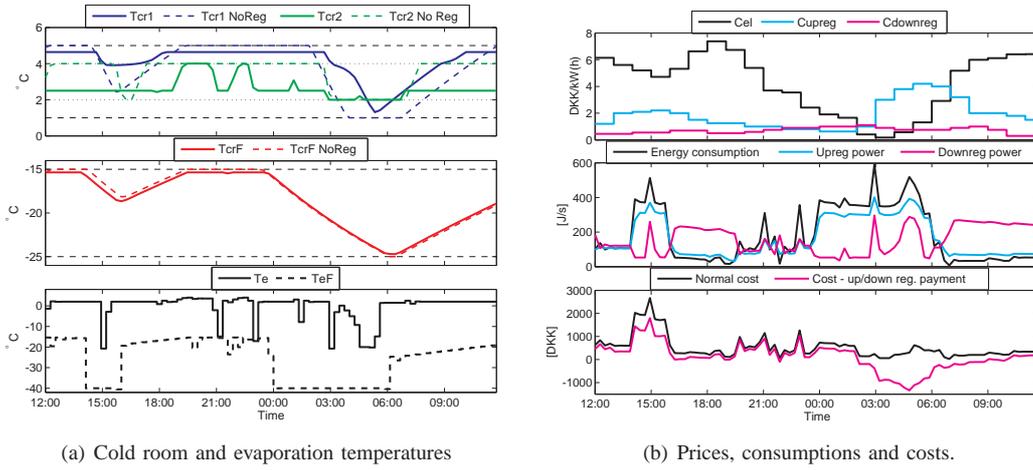(a) Cold room and evaporation temperatures

(b) Prices, consumptions and costs.

Fig. 7. Simulation showing how the flexible consumption is utilized for offering regulating power to the balancing market. The cold room temperatures for an optimization utilizing only the electricity spot price over the same period are shown to illustrate the difference.

5 Handling Uncertainties

So far we have assumed perfect models and deterministic predictions. In this section we illustrate what happens when introducing uncertainties in a more realistic scenario [25]. An optimal solution to a deterministic linear program is not always optimal, nor feasible, in the stochastic case. Therefore we describe means to handle the uncertainties in both forecasts and in the models of the system. We are using assumptions of the uncertainty belonging to certain distribution functions such that the uncertain parameters are normally Gaussian distributed. Furthermore we define the confidence level (probability) at which the constraints are satisfied. The probabilistic constraints are then reformulated as their deterministic counterparts.

First we define the system model on Finite Impulse Response (FIR) form:

$$y_k = b_k + \sum_{i=0}^{k} H_i u_{k-i}, \; H_i = \begin{cases} D & for \; i = 0 \\ CA^{i-1}B & for \; i > 0 \end{cases} \tag{13}$$

where b_k is a bias term generated by the estimator. Next, the stochastic optimization problem is defined as:

$$\min E \left\{ \sum_{k=0}^{N} \mathbf{c_k}' u_k \right\} \tag{14a}$$

$$s.t.$$

$$u_{min} \le u_k \le u_{max} \tag{14b}$$

$$Prob\{\mathbf{y_k} \ge \mathbf{r_k}\} \ge 1 - \alpha, \quad \alpha \in [0;1] \tag{14c}$$

$$\mathbf{y_k} = b_k + \sum_{i=1}^{k} \mathbf{H_i} u_{k-i} + \sum_{i=1}^{k} \mathbf{H_{D,i}} \mathbf{d_{k-i}} \tag{14d}$$

where $\mathbf{r}$ is a reference trajectory, $\mathbf{d}$ a disturbance, $1 - \alpha$ the confidence level for the constraint.

$$\begin{array}{ll} 1) & \mathbf{c_k} \sim N(\bar{c}_k, \sigma_c^2) \quad 2) \; \mathbf{r_k} \sim N(\bar{r}_k, \sigma_r^2) \\ 3) & \mathbf{H_i} \sim N(\bar{H}_i, \Sigma_H^2) \quad 4) \; \mathbf{H_{D,i}} \sim N(\bar{H}_{D,i}, \Sigma_H^2) \\ 5) & \mathbf{d_k} \sim N(\bar{d}_k, \sigma_d^2) \end{array} \tag{15}$$

1) and 2) are forecast uncertainties, 3) and 4) describe model uncertainties while 5) is uncertainty in the disturbances.

As we have shown in [25] we are able to reformulate the probabilistic constraints as deterministic constraints on the form

$$Prob\{\mathbf{y_k} \ge \mathbf{r_k}\} \ge 1 - \alpha \; \Leftrightarrow \; \Phi^{-1}(\alpha) \left\| \Sigma_{(\cdot)}^{1/2} \begin{bmatrix} (\cdot)_{past} \\ (\cdot) \end{bmatrix} \right\|_2 + \bar{y}_k \ge \bar{r}_k \tag{16}$$

The constraint in Eq. (16) has the form of a second order cone and the solution to the optimization problem constrained by Eq. (16) can be computed using Second Order Cone Programming (SOCP).

Simulation

Using Yalmip [26] we have simulated the scenario from section 4.1 but with the addition of uncertainties. The constraints on the cold room temperature and on balancing supply and demand are formulated as probability constraints and implemented with SOCP. A simulation scenario is provided in Fig. 8. From the figure we note the confidence intervals shown as shaded areas around each of the trajectories. The solid lines are the expected outcome while the shaded areas are created by 10,000 simulations with random instances of the noise descriptions. The 95% percentile was used both in the SOCP formulation and for plotting the shaded areas. It is easily seen how the amount of back-off from the boundaries is just enough to account for the 95% confidence interval of the uncertainty descriptions for the system. Particular this can be seen in Fig. 8(b) where the total production is above the total consumption, T_{cr} stays within the boundaries specified and $T_e \leq T_{cr}$ is satisfied. All with 95% probability. The solution here is less optimal compared to the one where we are allowed to go strictly to the boundaries but as this solution handles the always present uncertainties it is crucial for real life implementation.

6 Conclusion

To enable more use of renewable energy flexible consumption must be established. Using Economic Optimizing MPC schemes for systems with thermal storage capabilities we demonstrated both cost savings and the ability to deliver crucial services to the Smart Grid. Significant savings have been revealed e.g. 33% for heat pumps and around 10% in supermarket refrigeration. For the investigation of regulating power our perspective was seen from the refrigeration system but as it was demonstrated the involvement in the balancing market can be economically beneficial for the system itself while delivering crucial services to the Smart Grid. Different strategies for controlling the loads are available and starting with simplified setups we have demonstrated their ability to efficient control of a VPP setting. The cases have been expanded with realistic scenarios and inclusion of prediction and model uncertainties. The results showed how variations in outdoor temperature are utilized to minimize power consumption whereas exploiting variations in electricity prices tend to increase overall power consumption but at a lower cost. Hence, the goal is not solely to minimize energy consumption but merely to use energy when it is available. Future
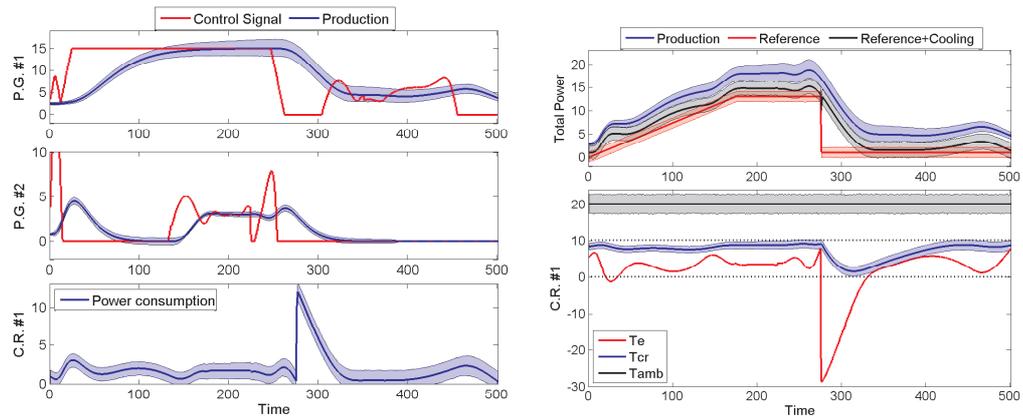
(a) Power productions / consumption. P.G. #1 and 2 show the power productions from the two power plants (blue) and their power set-points (red). C.R. #1 is power consumption in the cold room.

(b) "Total Power" shows total power production (blue) versus the reference consumption (with (black) and without (red) the refrigeration). C.R. #1 shows the temperature in the cold room T_{cr} and the control signal T_e.

Fig. 8. Simulation of Power Management scenario. $\alpha = 0.5$, $H_i \sim N(\bar{H}_i, 0.0055^2)$, $r_k \sim N(\bar{r}_k, 0.7071^2)$, $T_{amb} \sim N(\bar{T}_{amb}, 1.7321^2)$. The shaded bands show the 95% confidence interval from 10,000 random instances.

work includes simulation of more units with realistic forecasts in a VPP framework and implementation of the methods on real supermarket controllers.

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