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# Enhanced least squares Monte Carlo method for real-time decision optimizations for evolving natural hazards

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**ABSTRACT:** The present paper aims at enhancing a solution approach proposed by Anders & Nishijima (2011) to real-time decision problems in civil engineering. The approach takes basis in the Least Squares Monte Carlo method (LSM) originally proposed by Longstaff & Schwartz (2001) for computing American option prices. In Anders & Nishijima (2011) the LSM is adapted for a real-time operational decision problem; however it is found that further improvement is required in regard to the computational efficiency, in order to facilitate it for practice. This is the focus in the present paper. The idea behind the improvement of the computational efficiency is to “best utilize” the least squares method; i.e. least squares method is applied for estimating the expected utility for terminal decisions, conditional on realizations of underlying random phenomena at respective times in a parametric way. The implementation and efficiency of the enhancement is shown with an example on evacuation in an avalanche risk situation.

## 1 INTRODUCTION

Real-time decision optimization has become an interesting and challenging topic with the progress of real-time information processing technology. Relevant applications in civil engineering include situations where operational decisions have to be made in response to real-time information on evolving natural hazard events. In these situations, all real-time information available can and should be best utilized to find the optimal decisions at respective times; taking into account not only possible future outcomes, but also opportunities to make decisions in future times. This type of decision problem is generally described within the framework of the pre-posterior/sequential decision analysis, see Nishijima et al. (2009); however, the development of efficient solution schemes to the formulated decision problems has remained a technical challenge.

An efficient solution scheme is proposed by Anders & Nishijima (2011), taking basis in the Least Squares Monte Carlo method (hereafter, abbreviated as LSM), which is developed originally by Longstaff & Schwartz (2001) for American option pricing. In Anders & Nishijima (2011) the original LSM is extended and applied to an example for a real-time operational decision problem for shut-down of the operation of a technical facility in the face of an approaching typhoon. However, due to multiple evaluations of the expected consequences for different possible future states of the typhoon by means of Monte Carlo simulation (MCS), the solution scheme becomes less efficient, if the computational time required for MCS becomes dominant. The present paper proposes an enhanced solution scheme, which overcomes this drawback.

The present paper is organized as follows. Section 2 formulates the real-time decision problems in consideration within the framework presented in Nishijima & Anders (2012). Section 3 provides a brief introduction to the extensions of the LSM. Thereafter, the proposed enhancement to the extended LSM is introduced. Section 4 presents an application example, which illustrates the performance of the enhanced LSM (eLSM). Section 5 concludes the presented work.

## 2 REAL-TIME DECISION FRAMEWORK

### 2.1 Problem setting

The decision situation considered in the present work is characterized by the following characteristics, see Nishijima et al. (2009): (a) The hazard process evolves relatively slowly and allows for reactive decision making; (b) information relevant to predict the severity of the evolving hazard event can be obtained prior to its impact; (c) the decision making is subject to uncertainties, part of which might be reduced at a cost; (d) decision makers have options for risk reducing activities which may be commenced at any time, supported by the information available up to the time. Here, “waiting” to commence the risk reducing measures implies the reduction of uncertainty but might also reduce available time to complete the risk reducing activities; (e) and on top of all, the decisions must be made fast, in near-real time. The decision makers are then required to make decisions whether they commence one of the risk reducing activities which at the same time terminates the decision process (hence, hereafter these are called terminal decisions) or they postpone making a terminal decision.

### 2.2 Formulation of decision problem

The decision problem characterized above can be formulated in accordance with Nishijima & Anders (2012). Denote by  $A_t$  the decision set consisting of possible decision alternatives at time  $t$ . Here, time is discretized. It is assumed that the decisions must be terminated before or at time  $n$ ; hence,  $t = \{0, 1, 2, \dots, n\}$ . The decision set  $A_t$  generally depends on the decisions made before time  $t$ . If a decision maker decides to terminate the decision process, no decision alternative is available at later decision times. It is thus convenient to divide the decision set into two mutually exclusive subsets; i.e.  $A_t = A_t^{(c)} \cup A_t^{(s)}$ ,  $A_t^{(c)} \cap A_t^{(s)} = \emptyset$  where  $A_t^{(c)}$  consists of one decision alternative  $a_t^{(c)}$  “waiting” (i.e.  $A_t^{(c)} = \{a_t^{(c)}\}$ ) and  $A_t^{(s)}$  is the set consisting of risk reducing decisions available. Let  $\mathbf{E}_t$  be a set of variables representing possible information available at time  $t$  on the states of the evolving natural hazard event in consideration.

Given that no terminal decision is made up to time  $t$ , the optimal decision  $a_t^*$  at time  $t$  is identified as the one that maximizes the expected utility at time  $t$  conditional on the collection of the information up to time  $t$ :

$$E[U_t(\mathbf{Z}, a_t^*) | \underline{\mathbf{e}}_t] = \begin{cases} \max_{a_t \in A_t} E[U_t(\mathbf{Z}, a_t) | \underline{\mathbf{e}}_t], & \text{for } t = 0, 1, \dots, n-1 \\ \max_{a_t \in A_t^{(s)}} E[U_t(\mathbf{Z}, a_t) | \underline{\mathbf{e}}_t], & \text{for } t = n \end{cases} \quad (1)$$

where, for  $t = 0, 1, \dots, n-1$  and  $a_t^{(c)}$ ,

$$E[U_t(\mathbf{Z}, a_t^{(c)}) | \underline{\mathbf{e}}_t] = \int E[U_{t+1}(\mathbf{Z}, a_{t+1}^*) | a_t^{(c)}, \underline{\mathbf{e}}_{t+1}] f(\mathbf{e}_{t+1} | \underline{\mathbf{e}}_t) d\mathbf{e}_{t+1}. \quad (2)$$

Here,  $U_t(\mathbf{z}, a_t)$  is the utility, which is a function of the decision alternative  $a_t$  and the realization  $\mathbf{z}$  of the hazard index  $\mathbf{Z}$  relevant for the decision problem. The hazard index  $\mathbf{Z}$  is defined through the underlying random sequence  $\{\mathbf{Y}_t\}_{t=0}^n$ , representing the evolution of the natural hazard event.  $\underline{\mathbf{e}}_t = (\mathbf{e}_0, \mathbf{e}_1, \dots, \mathbf{e}_t)$  is the collection of the information available up to time  $t$ . Here, it is assumed that  $\mathbf{y}_t = \mathbf{e}_t$ , ( $t = 0, 1, \dots, n$ ); namely, the state of the event relevant to the decision problem is known to the decision maker without uncertainty. Thus, the symbols  $\mathbf{y}_t$  and  $\mathbf{e}_t$  are utilized interchangeably in the following.  $f_t(\cdot | \underline{\mathbf{e}}_t)$  is the conditional probability density/mass function of information  $\mathbf{E}_{t+1}$  given  $\mathbf{E}_t = \underline{\mathbf{e}}_t$ . From Equation 2 it is seen that for the decision  $a_t^{(c)}$  at time  $t$  the optimization requires to know all optimal decisions at future times,  $t+1, t+2, \dots, n$ ; hence, backward induction is required. Equation 1 can be rewritten as:

$$q_t(\underline{\mathbf{e}}_t) = \begin{cases} \max \{h_t(\underline{\mathbf{e}}_t), c_t(\underline{\mathbf{e}}_t)\}, & \text{for } t = 0, 1, \dots, n-1 \\ h_t(\underline{\mathbf{e}}_t), & \text{for } t = n. \end{cases} \quad (3)$$

Here,

$$q_t(\underline{\mathbf{e}}_t) = E[U_t(\mathbf{Z}, a_t^*) | \underline{\mathbf{e}}_t] \quad (4)$$

$$h_t(\underline{\mathbf{e}}_t) = \max_{a_t \in A_t^{(s)}} l_t(a_t, \underline{\mathbf{e}}_t) \quad (5)$$

$$l_t(a_t, \underline{\mathbf{e}}_t) = E[U_t(\mathbf{Z}, a_t) | \underline{\mathbf{e}}_t], \quad a_t \in A_t^{(s)} \quad (6)$$

$$c_t(\underline{\mathbf{e}}_t) = E[q_{t+1}(\underline{\mathbf{e}}_t, \mathbf{E}_{t+1}) | \underline{\mathbf{e}}_t]. \quad (7)$$

The function  $q_t(\underline{\mathbf{e}}_t)$ ,  $t=0,1,\dots,n$ , is the maximized expected utility, hereafter abbreviated as MEU. The functions  $h_t(\underline{\mathbf{e}}_t)$  and  $c_t(\underline{\mathbf{e}}_t)$  are named stopping value function (SVF) and continuing value function (CVF), respectively. Note that, whereas the evaluation of the SVF is straightforward in the sense that it does not require backward induction, the evaluation of CVF requires backward induction. However, no matter how complex the structure of the decision optimization problem may seem,  $c_t(\underline{\mathbf{e}}_t)$  is only a function of  $\underline{\mathbf{e}}_t$ . Furthermore, if the underlying random sequence  $\{\mathbf{Y}_t\}_{t=0}^n$  follows  $s^{\text{th}}$ -order Markov sequence,  $c_t(\underline{\mathbf{e}}_t)$  is a function effectively of the last  $s$  information,  $\mathbf{e}_{t-s+1}, \mathbf{e}_{t-s+2}, \dots, \mathbf{e}_t$ .

### 3 ENHANCEMENT OF THE EXTENDED LSM

#### 3.1 Extended LSM

The main technical challenge of the optimization problem formulated in Section 2.2 is the evaluation of the CVF. The CVF can in principle be evaluated by calculating the expected utility for each combination of all possible discretized future states and possible decision opportunities. However, in practice this is not computationally feasible, since the total number of the possible combinations increases exponentially as a function of the number  $n$ . The LSM circumvents this by employing the least squares method. The idea behind the LSM is that any regular function can be represented by a linear combination of an appropriate set of basis functions; therefore, the CVF is approximated as such, for details see Longstaff & Schwartz (2001). In the context of American option pricing, this means that if the price of a stock follows a first order Markov sequence, the price of its American option is a function only of the current stock price. Consequently the CVF is approximated as a superposition of basis functions whose argument is only the current stock price. The way on how this idea is implemented in the optimization is explained along with the extended version of the LSM (called extended LSM) in the following.

In Anders & Nishijima (2011), it is demonstrated that the idea behind the LSM can be applied for the case where the underlying random sequence follows an inhomogeneous higher-order Markov sequence. Therein, two extensions are made: (1) the assumptions on the underlying random sequence is relaxed from stationary first-order Markov sequence to non-stationary higher-order Markov sequence, and (2) the SVF is evaluated by MCS. Note that in many engineering applications the SVF cannot be evaluated analytically, unlike the case when executing American options. Moreover, the MCS in the second extension is computationally expensive and the computational effort increases proportional to  $n$ . In the following, the steps of the extended LSM are presented:

Step 1: A set of  $b$  independent realizations (paths) of the random sequence  $\mathbf{Y}_t$  is generated by MCS according to the Markov transition density  $f_t(\mathbf{y}_{t+1} | \underline{\mathbf{y}}_t)$ ,  $t=0,1,\dots,n-1$  with the initial condition  $\mathbf{Y}_0 = \mathbf{y}_0$ , where  $\underline{\mathbf{y}}_t = (\mathbf{y}_0, \mathbf{y}_1, \dots, \mathbf{y}_t)$ . These paths are denoted by  $\mathbf{y}^i = (\mathbf{y}_0^i, \mathbf{y}_1^i, \dots, \mathbf{y}_n^i)$ ,  $i=1,2,\dots,b$ , where  $\mathbf{y}_0^i = \mathbf{y}_0$  for all paths, see Figure 1 (a).

Step 2: The SVF for all realizations  $\{\mathbf{y}_t^i\}_{t=0}^n$ ,  $i=1,2,\dots,b$ , are estimated by additional MCS.

Step 3: Starting at the time horizon  $n$  as illustrated in Figure 1 (a), for each path  $i$  the value of the MEU  $q_n(\underline{\mathbf{y}}_{n-1}, \mathbf{Y}_n)$  is identified by equating  $q_n(\underline{\mathbf{y}}_n^i) = h_n(\mathbf{y}_n^i)$  according to Equation 3.

Step 4: Moving to time  $n-1$  the CVF is approximated. This begins by relating each MEU  $q_n(\underline{\mathbf{y}}_n^i)$  to  $\underline{\mathbf{y}}_{n-1}^i$ , to obtain the dataset  $(\underline{\mathbf{y}}_{n-1}^i, q_n(\underline{\mathbf{y}}_n^i))$ ,  $i=1,2,\dots,b$ , see the dots in Figure 1 (b). This dataset is utilized to approximate the CVF  $c_{n-1}(\underline{\mathbf{y}}_{n-1})$  with the least squares method. The

approximated CVF is illustrated by the curve in Figure 1 (b). See Nishijima & Anders (2012) for details. The approximated CVF is denoted by  $\hat{c}_{n-1}(\underline{\mathbf{y}}_{n-1})$ .

Step 5: Having obtained  $\hat{c}_{n-1}(\underline{\mathbf{y}}_{n-1})$  for time  $t = n-1$ , the realizations of  $q_{n-1}((\underline{\mathbf{y}}_{n-2}, \mathbf{Y}_{n-1}))$ , i.e.  $q_{n-1}(\underline{\mathbf{y}}_{n-1}^i)$ ,  $i = 1, 2, \dots, b$ , are determined as follows:

$$q_{n-1}(\underline{\mathbf{y}}_{n-1}^i) = \begin{cases} h_{n-1}(\underline{\mathbf{y}}_{n-1}^i), & \text{if } h_{n-1}(\underline{\mathbf{y}}_{n-1}^i) > \hat{c}_{n-1}(\underline{\mathbf{y}}_{n-1}^i) \\ q_n(\underline{\mathbf{y}}_{n-1}^i), & \text{otherwise.} \end{cases} \quad (8)$$

The procedure is repeated backwards in time until  $t = 1$ , hence  $q_1(\underline{\mathbf{y}}_1^i)$  is obtained for all paths.

Step 6: At  $t = 0$  the estimate  $\hat{c}_0 = \hat{c}_0(\mathbf{y}_0)$  is defined as the average of the realizations  $q_1(\underline{\mathbf{y}}_1^i)$ ,  $i = 1, 2, \dots, b$ . Finally  $q_0(\mathbf{y}_0)$  is obtained as the maximum of  $\hat{c}_0(\mathbf{y}_0)$  and  $h_0(\mathbf{y}_0)$ . The optimal decision is the one that corresponds to the maximum.

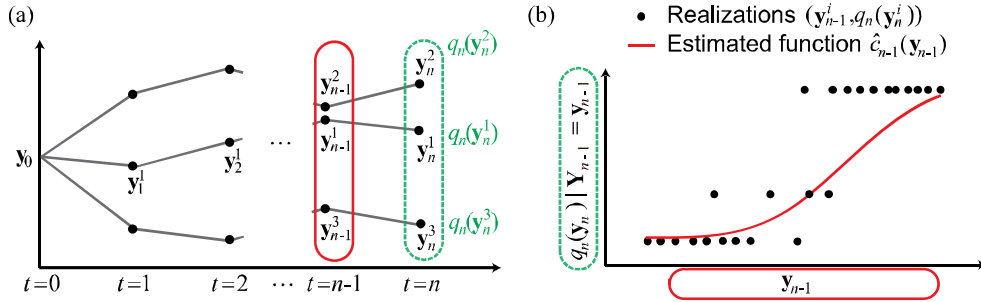


Figure 1. Illustration of (a) three paths of an underlying random sequence with corresponding values  $q_n(\underline{\mathbf{y}}_{n-1}^i)$  ( $i = 1, 2, 3$ ) at time  $n$  and (b) the estimation of the CVF using the sets  $(\underline{\mathbf{y}}_{n-1}^i, q_n(\underline{\mathbf{y}}_{n-1}^i))$ .

### 3.2 Enhancement of the extended LSM

As seen in Section 3.1, additional MCS are required in Step 2 to estimate the SVF in the extended LSM. The enhanced LSM (eLSM) circumvents this by applying the least squares method for the estimation of the SVF. The general idea is explained in the following.

Analogous to Equation 5 the SVF  $h_{t,\text{eLSM}}(\underline{\mathbf{y}}_t)$  of the eLSM is defined as maximum of the conditional expected utilities  $l_{t,\text{eLSM}}(a_t^{(j)}, \underline{\mathbf{y}}_t)$  with respect to the terminal decisions  $a_t^{(j)} \in A_t^{(s)}$ . Here, the functions  $l_{t,\text{eLSM}}(a_t^{(j)}, \underline{\mathbf{y}}_t)$  are estimated with the least squares method using the realizations  $\{\underline{\mathbf{y}}_t^i\}_{i=1}^b$ , similar to the estimation of the CVF described in Section 3.1; i.e. by linear combination of basis functions  $\{L_{t,k}(\cdot)\}_{k=1}^K$  with unknown coefficients  $r_{t,k}^{(j)}$

$$l_{t,\text{eLSM}}(a_t^{(j)}, \underline{\mathbf{y}}_t) \cong \sum_{k=1}^K L_{t,k}(\underline{\mathbf{y}}_t) r_{t,k}^{(j)}. \quad (9)$$

Therein the least squares method is utilized to estimate the coefficients  $\mathbf{r}_t^{(j)} = (r_{t,1}^{(j)}, r_{t,2}^{(j)}, \dots, r_{t,K}^{(j)})^T$  by minimizing the sum of the squared distances between the observed realizations of the dependent variable  $l_{t,\text{eLSM}}(a_t^{(j)}, \underline{\mathbf{y}}_t)$  in the dataset and their fitted values; in the matrix form this is expressed by

$$\mathbf{r}_t^{(j)} = \arg \min_{\mathbf{r}} \|\mathbf{u}_t^{(j)} - \mathbf{L}_t \mathbf{r}\|_2^2 \quad (10)$$

where  $\|\cdot\|_2$  denotes the Euclidian norm,  $\mathbf{L}_t$  is a  $b \times K$  matrix consisting of values of basis functions  $\{L_{t,k}(\cdot)\}_{k=1}^K$  which are functions of realizations of  $\underline{\mathbf{y}}_t$  and  $\mathbf{u}_t^{(j)}$  the  $b \times 1$  vector of observed future utilities  $u_t(\mathbf{z}^i, a_t^{(j)})$ ,  $i = 1, 2, \dots, b$ , given the realization  $\mathbf{z}^i$  of the hazard index related to the path  $\underline{\mathbf{y}}_t^i$  and decision  $a_t^{(j)}$  is made at time  $t$ . Note that  $u_t(\mathbf{z}^i, a_t^{(j)})$  is a realization of  $l_{t,\text{eLSM}}(a_t^{(j)}, \underline{\mathbf{y}}_t^i)$ . Furthermore, to avoid a bias introduced by the least squares estimation within the determination of the MEU, Equation 8 is changed to:

$$q_{t,\text{eLSM}}(\underline{\mathbf{y}}_t^i) = \begin{cases} u_t^*(\mathbf{z}^i, a_t^*), & \text{if } \hat{h}_{t,\text{eLSM}}(\underline{\mathbf{y}}_t^i) > \hat{c}_{t,\text{eLSM}}(\underline{\mathbf{y}}_t^i) \\ q_{t+1,\text{eLSM}}(\underline{\mathbf{y}}_{t+1}^i), & \text{otherwise} \end{cases} \quad (11)$$

where  $u_t^*(\mathbf{z}^i, a_t^*)$  is the observed future utility of path  $i$  for the optimal terminal decision  $a_t^*$ .

## 4 EXAMPLE

The aim of this section is to demonstrate how the eLSM can be applied to an engineering decision problem and to compare its performance to that of the extended LSM. For this purpose, a decision situation of the evacuation of people in the face of an avalanche event is considered.

### 4.1 Problem setting

Consider a village located nearby a mountain slope having a critical angle for snow avalanches. Given prevailing winter conditions and critical snow heights, a decision has to be made whether to evacuate people from the village. Assume that the occurrence of a severe avalanche, causing significant damages to the village, depends only on the additional snow height  $S_t$ ; i.e.  $S_t$  is the hazard index. Further, if  $S_t$  exceeds the threshold  $\tilde{s}$  ( $= 800$  [mm]) a severe avalanche occurs. Weather forecast by a meteorological agency predicts that snowfall can occur within the next hours, which increases the likelihood of the occurrence of the avalanche. However, the duration and the intensity of the snowfall are uncertain. New information becomes available every 8 hours from the meteorological agency; i.e. the time interval between the subsequent decision phases is set to 8 hours ( $dt=8$ ). At each decision phase a decision is made according to information available. Three decision alternatives are assumed; i.e. to evacuate the people  $a^{(1)}$ , not to evacuate  $a^{(2)}$ , and to wait  $a^{(c)}$ . It is assumed that the evacuation takes 16 hours to complete.

### 4.2 Consequence model

The consequences are postulated as follows, see also Table 1: The consequence is equal to  $C_{Ev}=1$  in two cases: (1) when the evacuation has been initiated but the avalanche does not occur, and (2) when the evacuation is completed before the avalanche occurs. A consequence of  $C_D=10$  is incurred if the avalanche occurs and the people are not evacuated or the evacuation was initiated but not completed. No consequence is incurred only in the case when no evacuation is initiated and no avalanche occurs.

Table 1. Conditions and associated consequences postulated in the consequence model.

People	Additional snow height in the time period $[0, t]$	
	$S_t > \tilde{s} = 800$ [mm]	$S_t \leq \tilde{s} = 800$ [mm]
Not evacuated	$C_D = 10$	0
Evacuated	$C_{Ev} = 1$	$C_{Ev} = 1$

### 4.3 Probabilistic snowfall model

A hypothetical probabilistic snowfall model is assumed, which is adapted from a rainfall model developed by Hyndman & Grunwald (2000). Let  $X_t$  denote the random sequence representing the amount of snowfall in the time period  $(t-dt, t]$ . Hereafter, this time period is denoted by  $(t-1, t]$  (i.e. the time unit is  $dt=8$ ) and thus  $\{X_t\}_{t=0}^n$  for simplicity. The distribution of  $X_t$  is a mixture comprising a discrete component concentrated at  $x_t=0$  and a continuous component for  $x_t>0$ . The discrete component of  $X_t$  represents the non-occurrence of snowfall and is characterized by the Bernoulli sequence  $J_t$ , whose conditional probability function is:

$$\pi_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}) = P(J_t = 1 | \mathbf{Y}_{t-1} = \mathbf{y}_{t-1}, \mathbf{Y}_{t-2} = \mathbf{y}_{t-2}) = l(\mu_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2})) \quad (12)$$

where  $\mathbf{Y}_t = (J_t, X_t)$  and  $l(\cdot)$  denotes the logit function which is defined as  $l(\mu) = \exp(\mu) / (1 + \exp(\mu))$  if  $\mu > 0$  and  $l(\mu) = 0$  otherwise, and

$$\mu_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}) = \alpha_0 + \alpha_1 j_{t-1} + \alpha_2 j_{t-2} + \alpha_3 \log(x_{t-1} + c_1) + \alpha_4 \log(x_{t-2} + c_2) + \alpha_5 t^2. \quad (13)$$

The continuous component of  $X_t$  is strictly positive and characterizes the intensity of the snowfall. If  $J_t = 1$ ,  $X_t$  is described by the continuous conditional density  $g_t(x | \mathbf{y}_{t-1})$ ,  $x > 0$ .  $g_t(\cdot | \cdot)$  follows the Gamma distribution with shape parameter  $\kappa$  and mean  $\nu_t(\mathbf{y}_{t-1})$ , where

$$\log(\nu_t(\mathbf{y}_{t-1})) = \beta_0 + \beta_1 j_{t-1} + \beta_2 \log(x_{t-1} + c_3) + \beta_3 t^2. \quad (14)$$

Then the transition probability density function of  $X_t$  is defined as (see Figure 2):

$$f_t(x_t | \mathbf{y}_{t-1}, \mathbf{y}_{t-2}) = (1 - \pi_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}))\delta_0(x_t) + \pi_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2})g_t(x_t | \mathbf{y}_{t-1}) \quad (15)$$

where  $\delta_0$  is the Dirac delta function. The additional snow height is obtained by multiplying the snow intensity by the factor  $F_s$ , which accounts for the density of the snow; i.e.

$$S_t = S_t(\underline{\mathbf{y}}_t) = \sum_{s=0}^t F_s x_t 1_{\{j_s=1\}} = S_{t-1} + F_t x_t 1_{\{j_t=1\}}. \quad (16)$$

Hence,  $S_t$  (the hazard index) at time  $t$  is characterized by the index  $S_{t-1}$  at time  $t-1$  and a stochastic process composed of a second- and a first-order Markov process (the second term in the rightmost equation). The values of the parameters of the model are summarized in Table 2. The time frame is set to three days; i.e.  $n=9$ .

Table 2. Parameters of the probabilistic snowfall model.

Parameter	Value	Parameter	Value
$j_{-1}, j_0, S_0$	0, 0, 0	$\mathbf{c} = (c_1, c_2, c_3)$	(0.15, 0.3, 0.5)
$\mathbf{a} = (\alpha_0, \alpha_1, \dots, \alpha_5)$	(4.5, 0.26, 0.1, 0.5, 0.05, -0.2)	$\kappa$	1.5
$\mathbf{\beta} = (\beta_0, \beta_1, \beta_2, \beta_3)$	(1.95, -0.2, 0.25, -0.04)	$F_s$	10

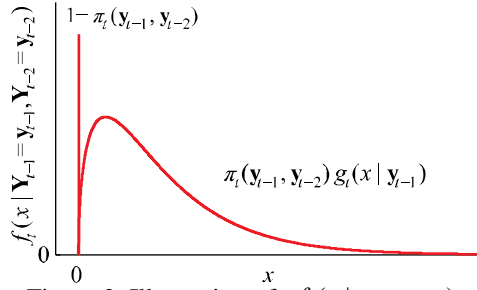


Figure 2. Illustration of  $f_t(x | \mathbf{y}_{t-1}, \mathbf{y}_{t-2})$ .

#### 4.4 Solution with the eLSM

Here, the MEU in Equation 3 is defined by the expected consequence; i.e. the minimum operator is used and the inequality sign of Equation 8 is turned. The steps in Section 3.1 are executed with the extended LSM and the eLSM to obtain the optimal decision.

Step 1: By MCS, generate  $b$  independent realizations of  $\{\mathbf{Y}_t\}_{t=1}^n$  and  $\mathbf{S}^i = (S_0^i, S_1^i, \dots, S_n^i)$ ,  $i=1, 2, \dots, b$ , where  $S_t^i = S_t^i(\mathbf{y}_t^i)$  and  $\mathbf{y}_t^i = (j_t^i, x_t^i)$ . The realizations  $\mathbf{y}_1^i, \mathbf{y}_2^i, \dots, \mathbf{y}_n^i$  are simulated according to the probability density functions in Equations 12 and 15; the paths are denoted by  $\mathbf{y}^i = (\mathbf{y}_{-1}^i, \mathbf{y}_0^i, \dots, \mathbf{y}_n^i)$ , where  $\mathbf{y}_{-1}^i = \mathbf{y}_{-1}$ ,  $\mathbf{y}_0^i = \mathbf{y}_0$  and  $i=1, 2, \dots, b$ .

Step 2: For each  $\mathbf{y}_t^i$  the value  $h_t^i = h_t(\mathbf{y}_t^i, \mathbf{y}_{t-1}^i)$  of the SVF is estimated. At time  $n=9$  the consequence related to each realization and decision is assumed to be known; i.e. either  $s_n^i$  exceeds the threshold  $\tilde{s}$  or not, thus  $h_{n,MC}^i = h_{n,eLSM}^i$  for all  $i$ . Further, for  $t=1, 2, \dots, n-1$

(1) with the extended LSM: Simulation of additional  $M$  paths  $\mathbf{y}_n^{i,m} = (\mathbf{y}_{-1}^i, \dots, \mathbf{y}_t^i, \mathbf{y}_{t+1}^{i,m}, \dots, \mathbf{y}_n^{i,m})$ ,  $m=1, 2, \dots, M$ , for which the observed consequences  $u_t(\mathbf{s}^m, a_t^{(j)})$ ,  $j=1, 2$ , are determined. Here  $\mathbf{s}^m$  is the realization of the additional snow height related to the path realization  $\mathbf{y}_n^{i,m}$ . Define  $\hat{l}_{t,MC}^i(a_t^{(j)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i) = \sum_{m=1}^M u_t(\mathbf{s}^m, a_t^{(j)}) / M$ , then

$$\hat{h}_{t,MC}^i = \min\{\hat{l}_{t,MC}^i(a_t^{(1)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i), \hat{l}_{t,MC}^i(a_t^{(2)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i)\} \quad (17)$$

(2) with the eLSM as explained in Section 3.2: Define

$$\hat{h}_{t,eLSM}^i = \min\{\hat{l}_{t,eLSM}^i(a_t^{(1)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i), \hat{l}_{t,eLSM}^i(a_t^{(2)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i)\} \quad (18)$$

where  $\hat{l}_{t,eLSM}^i(a_t^{(j)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i) = \mathbf{L}_t^i \cdot \mathbf{r}_t^{(j)}$ ,  $j=1, 2$ . The vector  $\mathbf{r}_t^{(j)}$  of the coefficients related to  $a_t^{(j)}$  is computed by Equation 10.  $\mathbf{L}_t^i$  denotes the  $i^{\text{th}}$  row of matrix  $\mathbf{L}_t$ ;  $\mathbf{L}_t$  consists of values of basis functions with arguments  $\mathbf{y}_t$ ,  $\mathbf{y}_{t-1}$  and  $S_t$ ; e.g. for 1<sup>st</sup> order linear basis functions

$$\mathbf{L}_t = \begin{bmatrix} 1 & x_t^1 & x_{t-1}^1 & s_t^1 \\ 1 & x_t^2 & x_{t-1}^2 & s_t^2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_t^b & x_{t-1}^b & s_t^b \end{bmatrix}. \quad (19)$$

For  $t=0$  set  $\hat{l}_0^{(j)} = \hat{l}_{0,\text{MC}}(a_0^{(j)}, \mathbf{y}_0, \mathbf{y}_{-1}) = \hat{l}_{0,\text{eLSM}}(a_0^{(j)}, \mathbf{y}_0, \mathbf{y}_{-1}) = \sum_{i=1}^b u_0(\mathbf{s}^i, a_0^{(j)}) / b$ ,  $j=1,2$ .

Step 3: Starting at time  $n$ , for both LSM approaches, the values of  $q_{n,\text{MC}}^i = q_{n,\text{MC}}(\mathbf{y}_n, \mathbf{y}_{n-1})$  and  $q_{n,\text{eLSM}}^i$  are set equal to  $h_{n,\text{MC}}^i$  and  $h_{n,\text{eLSM}}^i$  respectively, for all  $i$ .

Step 4: Moving to time  $n-1$  the values of  $c_{n-1}(\mathbf{y}_{n-1}, \mathbf{y}_{n-2})$  are similarly estimated for both approaches using the least squares method as described in Section 3.1.

Step 5: Then, for each path  $i$  determine the values of  $q_{n-1}(\mathbf{y}_{n-1}, \mathbf{y}_{n-2})$ :

(1) for the extended LSM with the estimate  $\hat{h}_{i,\text{MC}}^i$  obtained by means of MCS:

$$q_{n-1,\text{MC}}^i = \begin{cases} \hat{h}_{n-1,\text{MC}}^i, & \text{if } \hat{h}_{n-1,\text{MC}}^i < \hat{c}_{n-1,\text{MC}}^i \\ q_{n,\text{MC}}^i, & \text{otherwise} \end{cases} \quad (20)$$

(2) for eLSM with the estimate  $\hat{h}_{i,\text{eLSM}}^i$  obtained by means of the least squares method:

$$q_{n-1,\text{eLSM}}^i = \begin{cases} u_{n-1}^{*,i}, & \text{if } \hat{h}_{n-1,\text{eLSM}}^i < \hat{c}_{n-1,\text{eLSM}}^i \\ q_{n,\text{eLSM}}^i, & \text{otherwise} \end{cases} \quad (21)$$

where  $u_{n-1}^{*,i}$  denotes the observed future consequence in path  $i$  for the optimal terminal decision  $a_{n-1}^*$ . As in Section 3.1, moving another time step back the same procedure is repeated. This is continued until time  $t=1$  and for each path  $q_{1,\text{MC}}^i$  and  $q_{1,\text{eLSM}}^i$  are determined.

Step 6: Execute Step 6 of Section 3.1.

#### 4.5 Results

To evaluate the performance of the eLSM compared to the extended LSM, both methods are applied to solve the decision problem of the example. The optimal decision at the initial time is obtained by estimating the expected consequences for the three decisions alternatives. Various types and degrees of basis functions are implemented; e.g. linear, Legendre and Chebyshev polynomials. Applying these basis functions, it is found that the results do not significantly differ. Thus, only the results obtained with linear basis functions are presented.

Figure 3 illustrates the findings for different parameter settings of the LSM. Therein, Figure 3 (a) shows for increasing number  $b$  of paths,  $b = \{10^2, 3 \cdot 10^2, 10^3, 3 \cdot 10^3, 10^4, 3 \cdot 10^4, 10^5\}$ , the convergence of the consequence estimates for the three decisions. For each  $b$  the estimates are calculated by the average of 100 computations of the indicated method. To be able to compare the results 100 different yet fixed sets of random numbers are used to generate the paths in Step 1. Hence, the estimates for the terminal decisions are identical for all methods; they are presented by solid lines with circles. The following results are obtained for  $b=10^5$ :  $\hat{l}_0^{(1)} = 1.0192$ ,  $\hat{l}_0^{(2)} = 0.8969$  and e.g.  $\hat{c}_{0,\text{eLSM}} = 0.8055$  with the eLSM. The optimal decision is  $a_0^{(c)}$  which is independent of the type of LSM; see Figure 3 (a). Further, the figure shows that the estimate  $\hat{c}_0$  obtained by the extended LSM with  $M=10$  is biased. Therefore it is not considered in Figure 3 (b) which illustrates the convergence rate in terms of the coefficient of variation (COV) of the estimates  $\hat{c}_0$  as a function of the computational time [sec]. The figure shows a significant improvement with the eLSM in terms of computational time; a reduction by the factor of 100.

An application of the proposed approach in practice is presented in Figure 4. Figure 4 (a) illustrates a hypothetical time series of the additional snow height  $\{S_t\}_{t=0}^6$  where the threshold  $\tilde{s}$  is exceeded within the time interval (3,4]. Applying the eLSM subsequently for each time step it is found that the optimal decision at time  $t=0$  is  $a^{(c)}$  whereas at time  $t=1$  it is found to be  $a^{(1)}$  given that the snow height at time  $t=1$  in the figure is realized.



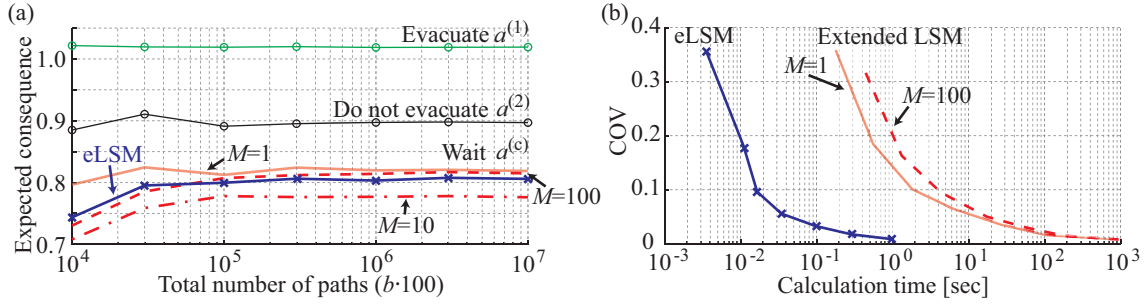


Figure 3. Comparison of the results of the extended LSM (with various numbers  $M$  of additional MCS) and eLSM. (a) Convergence of the average expected consequences with increasing total number of paths. (b) Illustration of the decreasing COV of  $\hat{c}_0$  related to the increasing calculation time for one LSM computation as the number  $b$  of paths increases.

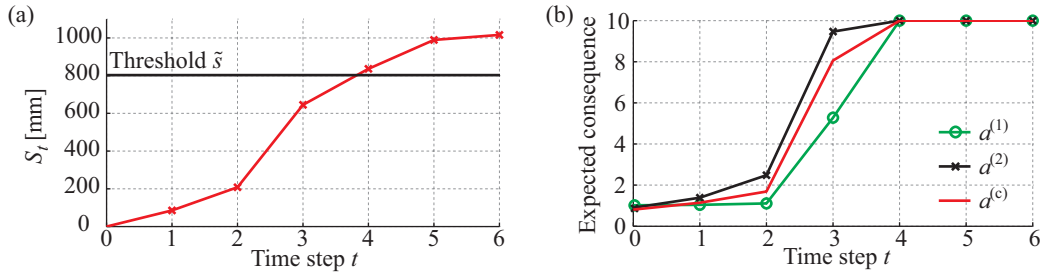


Figure 4. Illustration of (a) a hypothetical time series of  $S_t$  and (b) the corresponding time series of the estimated expected consequence of the three decision alternatives calculated with the eLSM and  $b = 10^5$ .

## 5 CONCLUSION

The present paper proposes an enhancement of the extended LSM in the context of real-time operational decision problems for evacuation in the face of emerging natural hazards. The proposed approach (eLSM) is applied to an example and it is found that the eLSM significantly improves the computational efficiency; by the factor up to 100.

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