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Neutron to proton mass difference, parton distribution functions and baryon resonances from dynamics on the Lie group u(3)

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Abstract

We present a hamiltonian structure on the Lie group u(3) to describe the baryon spectrum. The ground state is identified with the proton. From this single fit we calculate approximately the relative neutron to proton mass shift to within half a percentage of the experimental value. From the same fit we calculate the nucleon and delta resonance spectrum. For specific spin eigenfunctions we calculate the delta to nucleon mass ratio to within one percent.

We derive parton distribution functions. The distributions are generated by projecting the proton state to space via the exterior derivative on u(3). We predict scarce neutral flavour singlets which should be visible in neutron diffraction dissociation experiments or in invariant mass spectra of protons and negative pions in B-decays and in photoproduction on neutrons. The presence of such singlet states distinguishes experimentally the present model from the standard model as does the prediction of the neutron to proton mass splitting. Conceptually the Hamiltonian may describe an effective phenomenology or more radically describe interior dynamics implying quarks and gluons as projections from u(3) which we then call allospace.

The allospatial hypothesis

We wish to generate projection fields transforming under the SU(3) algebra with the fields possibly being electrically charged. This points to a configuration space containing both su(3) and u(1). Thus we choose the Lie group u(3) as configuration space and assume the following Hamiltonian

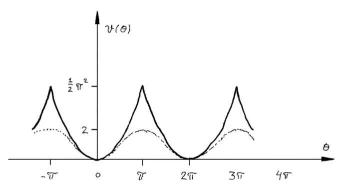
$$\frac{\hbar c}{a}[-\tfrac{1}{2}\Delta+\tfrac{1}{2}d^2(e,u)]\Psi(u)=E\Psi(u) \qquad u=e^{i\alpha_k T_k} \qquad (1)$$

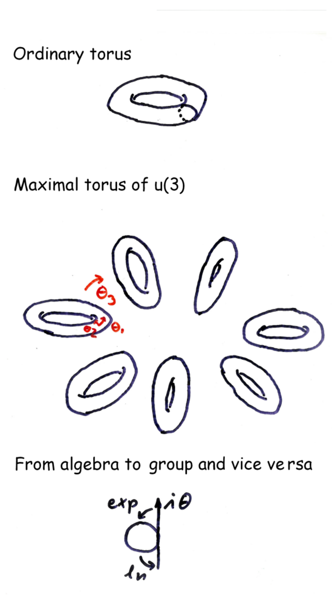
It is the hypothesis of the present work, that the eigenstates of the above Schrödinger equation describe the baryon spectrum with u being the configuration variable of a sole baryonic entity and a is a scale. We find exact solutions for alleged N-states and approximate solutions for both alleged N-states and Δ-states. It should be mentioned that, when unfolded, the structure of (1) leaves room for degrees of freedom to mimic both spin, hypercharge and isospin.

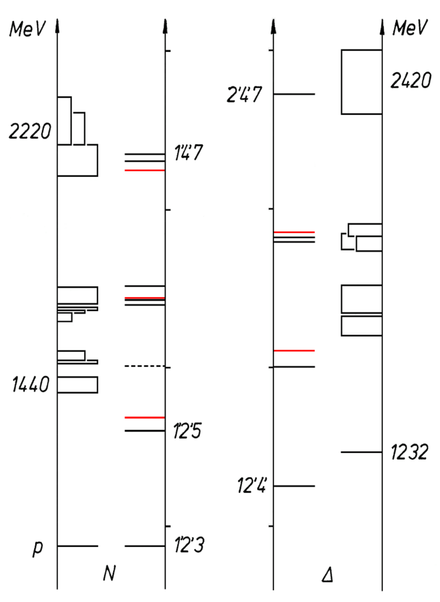
The potential is half the squared geodetic distance from the 'point' u to the 'origo' e

$$d^2(e,u)=\theta_1^2+\theta_2^2+\theta_3^2, \quad -\pi\le\theta_j\le\pi \qquad (2)$$

where $e^{i\theta_j}$ are the eigenvalues of u.







The theory unfolded

The Laplacian in (1) contains off-toroidal derivatives which are represented by the off-diagonal Gell-Mann matrices. We choose three of these to represent spin and group them into $\mathbf{K}=(K_1,K_2,K_3)$. This interpretation is supported by their commutation relations as body fixed angular momentum. The relation between space and allospace is like the relation in nuclear physics between fixed coordinate systems and intrinsic body fixed coordinate systems for the description of rotational degrees of freedom. The remaining three are grouped into $\mathbf{M}=(M_1,M_2,M_3)$, which is related to hypercharge and isospin. They connect the algebra by commuting into the subspace of $\mathbf{K}$. The fully parametrized Laplacian in polar decompostion reads

$$\Delta=\sum_{j=1}^{3}\frac{1}{J}\frac{\partial^2}{\partial\theta_j^2}J+2-\sum_{\substack{i<j\\ k\neq i,j}}^{3}\frac{K_k^2+M_k^2}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)}$$

The constant term is interpreted as a curvature potential and the off-torus term is analogous to the centrifugal term in the usual treatment of the radial wave function for the hydrogen atom

$$-\frac{\hbar}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}-\frac{1}{r^2}\mathbf{L}^2\right]\psi(r,\theta,\varphi)+V(r)\psi(r,\theta,\varphi)=E\psi(r,\theta,\varphi) \qquad \psi(r,\theta,\varphi)=R(r)Y(\theta,\varphi)$$

With the periodic potential in (2) our complete Schrödinger equation reads with $\mathrm{E}=E/\Lambda$ and $\Lambda\equiv\hbar c/a=210\,\mathrm{MeV}$

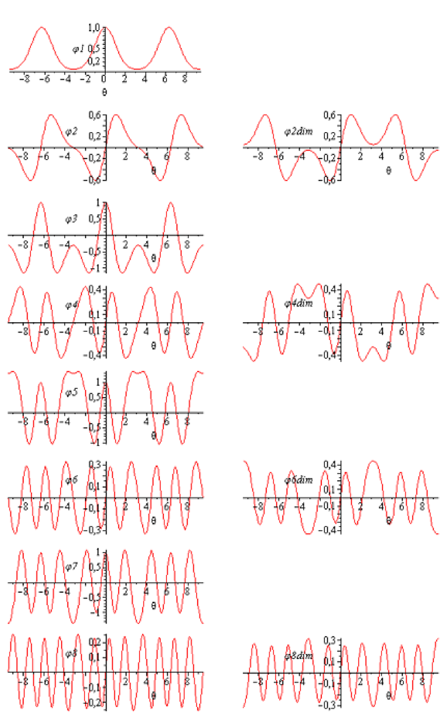
$$[-\tfrac{1}{2}(\sum_{j=1}^{3}\frac{1}{J}\frac{\partial^2}{\partial\theta_j^2}J+2-\sum_{\substack{i<j\\ k\neq i,j}}^{3}\frac{K_k^2+M_k^2}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)})+v(\theta_1)+v(\theta_2)+v(\theta_3)]\Psi(u)=\mathrm{E}\Psi(u) \qquad (3)$$

And a similar factorization of $\Psi(u)=\tau(\theta_1,\theta_2,\theta_3)\cdot\Upsilon_{KM}(\alpha_4,\alpha_5,\alpha_6,\alpha_7,\alpha_8,\alpha_9)$ gives for $\Phi(u)=R(\boldsymbol{\theta})\cdot\Upsilon_{KM}$ with $R(\boldsymbol{\theta})=J(\boldsymbol{\theta})\cdot\tau(\boldsymbol{\theta})$

$$[-\Delta_e+V]R_{KM}(\theta_1,\theta_2,\theta_3)=2\mathrm{E}\,R_{KM}(\theta_1,\theta_2,\theta_3)$$

where $\Delta_e=\sum_{j=1}^{3}\frac{\partial^2}{\partial\theta_j^2}$ and $V=-2+\frac{1}{3}(K(K+1)+M^2)\sum_{i<j}^{3}\frac{1}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)}+2(v(\theta_1)+v(\theta_2)+v(\theta_3))$. Now R can be expanded on Slater determinants constructed from parametric eigenstates

$$R_{lmn}(\boldsymbol{\theta})=\begin{vmatrix}\varphi_l(\theta_1) & \varphi_l(\theta_2) & \varphi_l(\theta_3)\\ \varphi_m(\theta_1) & \varphi_m(\theta_2) & \varphi_m(\theta_3)\\ \varphi_n(\theta_1) & \varphi_n(\theta_2) & \varphi_n(\theta_3)\end{vmatrix}$$



The figure shows parametric eigenstates with periodicity 2π to the left and periodicity 4π for diminished states in the right column.
We can couple a diminishing period doubling in level two with an augmenting period doubling in level one. We interpret these coupled period doublings as representing the transformation from a neutral state (e.g. the neutron) to a charged state (e.g. the proton).

$n \to p$

$$R_{1'2'3}(\boldsymbol{\theta})=\begin{vmatrix}e^{-i\frac{1}{2}\theta_1}g_{1'}(\theta_1) & e^{-i\frac{1}{2}\theta_2}g_{1'}(\theta_2) & e^{-i\frac{1}{2}\theta_3}g_{1'}(\theta_3)\\ e^{i\frac{1}{2}\theta_1}g_{2'}(\theta_1) & e^{i\frac{1}{2}\theta_2}g_{2'}(\theta_2) & e^{i\frac{1}{2}\theta_3}g_{2'}(\theta_3)\\ \varphi_3(\theta_1) & \varphi_3(\theta_2) & \varphi_3(\theta_3)\end{vmatrix}$$

$$\frac{E-E''}{E''}=0.13847\,\%\approx0.13784\%=\frac{m_n-m_p}{m_p}$$

R is similar to spin rotation functions $D^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta_1,\theta_2,\theta_3)=e^{-i\frac{1}{2}\theta_1}d^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta_2)e^{i\frac{1}{2}\theta_3}=-e^{-i\frac{1}{2}\theta_1}\sin(\tfrac{1}{2}\theta_2)e^{i\frac{1}{2}\theta_3}$

Conclusions

The allospatial Hamiltonian in (1) or (3) may be seen as an effective phenomenology or interpreted more radically in a conceptual interpretation where we see

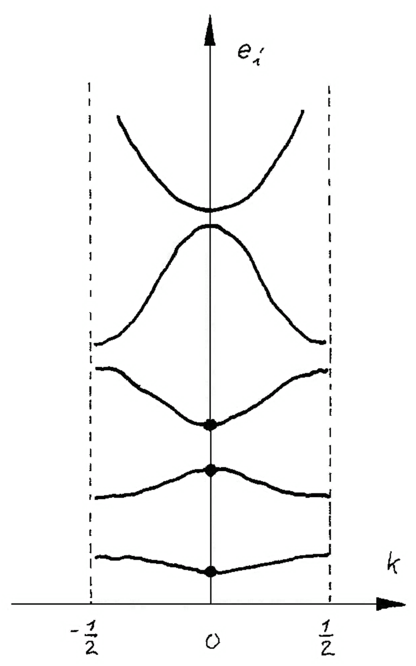
Resonances - **from space**: The impact momentum as introtangling operators generate the maximal torus of u(3).
Decay, fragmentation, confinement – **from allospace**: The momentum form induces quark an gluon fields.

The model has no fitting parameters except the scale $\Lambda\equiv\hbar c/a=210\,\mathrm{MeV}$.

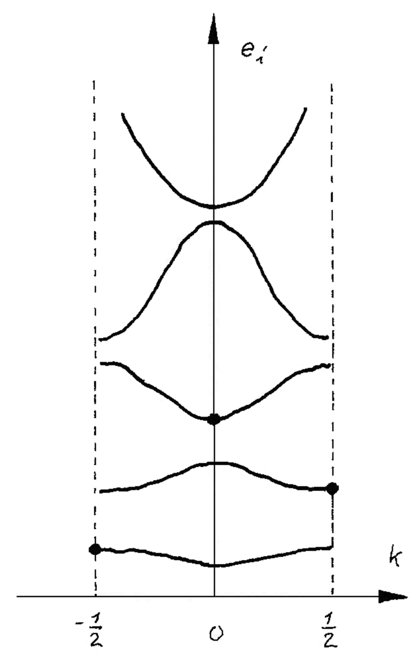
A quite accurate prediction of the relative neutron to proton mass shift 0.138 % follows from approximate solutions to the Schrödinger equation. A projection of states to space is given via the exterior derivative. This projection has shown to yield parton distribution functions that compares rather well with those of the proton valence quark distributions already in a first order approximation. A kinematic parametrization for the projection gives a natural transition between a confinement domain where the dynamics unfolds in the global group space and an asymptotic free domain where the algebra approximates the group. A promising ratio between the Δ(1232) and N(939) masses has been calculated based on specific *D*-functions. We expect the allospatial energy eigenstate spectrum to project into partial wave amplitude resonances of specific spin and parity via expansions on specific combinations of *D*-functions. Singlet neutral flavour resonances are predicted above the free charm threshold of $\Sigma_c^+(2455)D^-$.

Periodic potential and reduced zone scheme

Approximate energy levels for baryonic states are found by combinations of three parametric eigenstates of the three torus angles. These eigenstates originally have the same periodicity as the potential. However a coupled period doubling can decrease the total energy.



$n \to p$



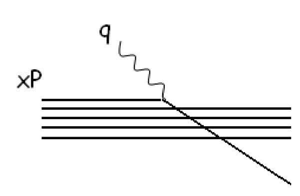
We interpret the period doublings as related to the creation of the proton charge in the neutron decay. Similar states all with one even label give the N resonances.

For three even labels the complex phases factorize out and the states may contribute to neutral states.

Two even labels give possibilities of double charges which we interpret as Δ resonances.

The black dots in the figures show the Bloch wave number choices for the neutron (left) and the proton state (right).

Parton distributions



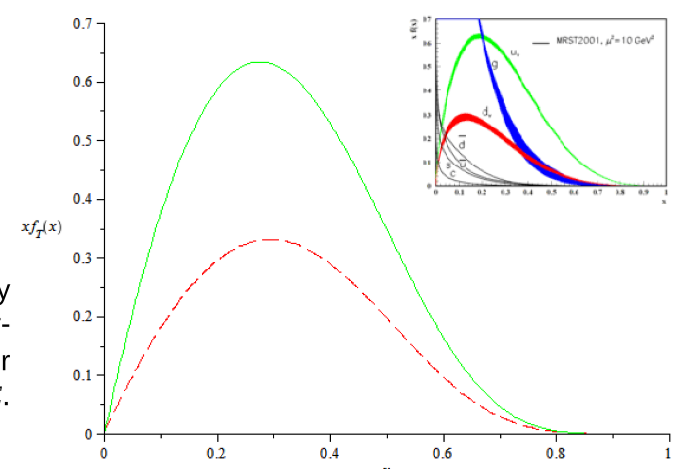
We boost a proton from rest to energy E by impacting upon it a massless four-momentum q to hit a parton xP. After impact the parton carries a mass xE. Thus $(xP_\mu+q_\mu)\cdot(xP^\mu+q^\mu)=x^2E^2$ which yields for the parton fraction

$$x=\frac{2E_0}{E+E_0} \quad \text{and boost parameter}$$

$$\xi\equiv\frac{E-E_0}{E}=\frac{2-2x}{2-x}$$

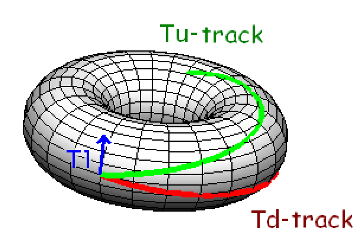
We scale the boost with different toroidal generators

$$T_u=\begin{Bmatrix}2/3 & & \\ & 0 & \\ & & -1\end{Bmatrix} \quad \text{and} \quad T_d=\begin{Bmatrix}-1/3 & & \\ & 0 & \\ & & -1\end{Bmatrix}$$



$$f_T(x)\cdot dx=(dR_{u=\exp(\theta iT)})^2\cdot d\theta, \quad \text{where} \quad \theta=\pi\xi$$

$$xf_u(x)=x\left[D(\pi\frac{2-2x}{2-x}\cdot\frac{2}{3},\pi\frac{2-2x}{2-x}\cdot0,\pi\frac{2-2x}{2-x}\cdot(-1))\right]^2\cdot\frac{\pi\cdot2}{(2-x)^2}$$



We project from a state constructed from trigonometric functions to mimic the period doublings implied in the decay to the proton state

$$b_{0,\frac{1}{2},1}(\theta_1,\theta_2,\theta_3)=\frac{1}{N}\begin{vmatrix}1 & 1 & 1\\ \sin\frac{1}{2}\theta_1 & \sin\frac{1}{2}\theta_2 & \sin\frac{1}{2}\theta_3\\ \cos\theta_1 & \cos\theta_2 & \cos\theta_3\end{vmatrix}$$

The projection involves the exterior derivative db operating on all three colour generators. The result we denote as D (directional derivative)

$$ND(\theta_1,\theta_2,\theta_3)=-\frac{1}{2}\cos\frac{\theta_1}{2}\cdot(\cos\theta_3-\cos\theta_2)-\sin\theta_1\cdot(\sin\frac{\theta_3}{2}-\sin\frac{\theta_2}{2})$$
$$+\frac{1}{2}\cos\frac{\theta_2}{2}\cdot(\cos\theta_3-\cos\theta_1)+\sin\theta_2\cdot(\sin\frac{\theta_3}{2}-\sin\frac{\theta_1}{2})$$
$$-\frac{1}{2}\cos\frac{\theta_3}{2}\cdot(\cos\theta_2-\cos\theta_1)-\sin\theta_3\cdot(\sin\frac{\theta_2}{2}-\sin\frac{\theta_1}{2})$$

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See also: O. L. Trinhammer, *Baryons from quantum mechanics on the Lie group u(3),* arXiv:1109.4732 [hep-th] (2011).

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