



## RMS slope of exponentially correlated surface roughness for radar applications

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$$c_1 = 32a^2b^2(x_{j1}^2 + x_{j2}^2) + 64b^4v_{i1}(v_{i1} - x_{j1}) + 64a^4v_{i2}(v_{i2} - x_{j2}) + 32a^2b^2(b^2 - a^2 - v_{i1}^2 - v_{i2}^2) \quad (4)$$

$$c_2 = 16a^2b^2(b^2x_{j1}^2 - a^2x_{j2}^2 + a^4 + b^4 - b^2v_{i2}^2) + 80a^2b^2(a^2v_{i2}^2 - b^2v_{i1}^2) - 80a^6v_{i2}^2 + 16b^2v_{i1}^2 - 32b^2(b^4x_{j1}v_{i1} - a^4x_{j2}v_{i2}) + 64b^4a^4 + 80b^6v_{i1}^2 \quad (5)$$

$$c_3 = 32a^4b^4(b^2 - a^2) \quad (6)$$

$$c_4 = 16a^6b^6 \quad (7)$$

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## RMS Slope of Exponentially Correlated Surface Roughness for Radar Applications

Wolfgang Dierking

**Abstract**—In radar signature analysis, the root mean square (RMS) surface slope is utilized to assess the relative contribution of multiple scattering effects. For an exponentially correlated surface, an effective RMS slope can be determined by truncating the high frequency tail of the roughness spectrum. The choice of the cutoff frequency and the effect on surface scattering simulations are discussed.

**Index Terms**—Electromagnetic scattering, radar, rough surfaces.

## I. INTRODUCTION

In theoretical modeling of microwave surface scattering, the root mean square (RMS) slope is regarded as useful in order to examine which scattering model can be applied for a particular type of surface roughness and to assess whether the contribution of multiple scattering to the backscattered signal can be neglected (e.g., [1, p. 231]; [2, Ch. 12]). The ACF's of many natural surfaces such as soil or sea ice can often be well fitted by an exponential function or combinations of exponential and Gaussian functions [1], [3]–[5]. The simple exponential function  $R(x) = s^2 \exp(-|x|/l)$  (where  $s$  is the RMS height, and  $l$  is the correlation length) is common in surface scattering simulations, since it has an analytical form for its spectrum. However, for an exponentially correlated roughness, the RMS slope is infinite (e.g., [1, p. 119]). Considering the fact that the radar is not affected by components of the surface roughness spectrum with length scales much smaller than the radar wavelength, an "effective" RMS slope of an exponentially correlated surface can be evaluated by truncating the roughness spectrum at higher spatial frequencies. This effective RMS slope may then be applied to assess the applicability of a scattering model and to separate single and multiple scattering regimes. Details are discussed in the following sections.

## II. DETERMINATION OF THE RMS-SLOPE AND RADIUS OF CURVATURE

The ACF  $R(x)$  and the spectral density of surface roughness  $S(f_x)$  are related by [6, p. 123]

$$R(x) = \int_0^\infty S(f_x) \cos(2\pi f_x x) df_x \quad (1a)$$

$$S(f_x) = 4 \int_0^\infty R(x) \cos(2\pi f_x x) dx \quad (1b)$$

where  $f_x$  is the spatial frequency, and  $S(f_x)$  is the one-sided roughness spectrum ( $f_x \geq 0$ ). For the exponential ACF, the pair  $R(x) \leftrightarrow S(f_x)$  is given by

$$R(x) = s^2 \exp(-|x|/l) \leftrightarrow S(f_x) = \frac{4s^2l}{(1 + 4\pi^2l^2f_x^2)} \quad (2)$$

where  $s$  is the RMS height, and  $l$  is the correlation length. The spectrum of an exponentially correlated surface is depicted in Fig. 1 in comparison with a Gaussian spectrum  $S(f_x) = 2ls^2\sqrt{\pi} \exp(-\pi^2l^2f_x^2)$ . At large spatial frequencies  $lf_x > 1$ , the exponential spectrum approaches

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a power law of the form  $f_x^{-2}$ , whereas in the case of the Gaussian spectrum, the spectral densities  $S(f_x)$  of roughness elements with  $lf_x \geq 1$  are smaller than the exponential spectrum by orders of magnitude. The RMS slope  $m$  of one-dimensional (1-D) profiles can be obtained from the second derivative of the ACF at the origin  $x = 0$  [6, p. 158]

$$m^2 = - \left. \frac{d^2 R}{dx^2} \right|_{x=0} = \int_0^\infty 4\pi^2 f_x^2 S(f_x) df_x. \quad (3)$$

For the Gaussian ACF,  $m = \sqrt{2}s/l$ . For the exponential ACF,  $m$  is infinite.

For the use in radar signature analysis,  $m$  can be determined from a limited range of the roughness spectrum, excluding the high frequency components, which do not affect the radar waves. In certain cases, it may also be necessary to truncate low frequency surface components that are much larger than the radar wavelength (e.g., if the radar signatures are measured over surfaces with a large scale topography). Large scale undulations are sensed by the radar as effective variations of the incidence angle relative to the surface normal. In this short communication, large scale topographic effects are not considered, and it is assumed that the radar incidence angle on the surface is constant.

By substituting  $S(f_x)$  given by (2) into (3), and evaluating the integral in the interval  $[0, f_{x, \max}]$ , one obtains the RMS slope for a roughness spectrum truncated at the high frequency end

$$m^2(f_{x, \max}) = \frac{4s^2}{l} \left( f_{x, \max} - \frac{1}{2\pi l} \arctan(2\pi l f_{x, \max}) \right). \quad (4)$$

Here,  $s$  and  $l$  are the values valid for the whole spectral range  $[0, \infty]$ . The contributions of roughness components with  $f > f_{x, \max}$  to the scattered radar signal can be neglected, and the RMS slope "seen" by the radar is  $m(f_{x, \max})$ . The remaining problem is to find the appropriate value for  $f_{x, \max}$ . The height variation of a horizontal profile segment is negligible if the segment length is equal to or smaller than one tenth of the radar wavelength (i.e., a profile should be sampled with a spacing  $\Delta x \leq 0.1\lambda_{\text{radar}}$  [2, p. 823]). Considering the Nyquist Theorem, the highest spatial frequency of the roughness spectrum that can be determined with a sampling interval of  $\Delta x = 0.1\lambda_{\text{radar}}$  is  $f_{x, \max} = 5/\lambda_{\text{radar}} = 5k/(2\pi)$  ( $k$ —radar wavenumber). Equation (4) then reads

$$m = \sqrt{\frac{2}{\pi}} \frac{s}{l} \sqrt{5kl - \arctan(5kl)}. \quad (5)$$

By truncating the high frequency part of the roughness spectrum, the value of the RMS height  $s$  is reduced according to

$$s_{rd}^2 = \int_0^{f_{x, \max}} S(f_x) df_x = \frac{2}{\pi} s^2 \arctan(2\pi l f_{x, \max}). \quad (6)$$

The ACF of the second derivative of a roughness profile [which is marked as  $z''$ , where  $z(x)$  is the elevation along the profile] is given by [6, p. 158]

$$R_{z''z''}(0) = - \left. \frac{d^4 R}{dx^4} \right|_{x=0} = \int_0^\infty 16\pi^4 f_x^4 S(f_x) df_x. \quad (7)$$

For the spectrum of an exponentially correlated roughness,  $R_{z''z''}(x = 0)$  evaluated in the interval  $[0, f_{x, \max}]$ , is

$$R_{z''z''}(0, f_{x, \max}) = \frac{4s^2}{l^3} \left[ \left( \frac{4}{3} \pi^2 l^2 f_{x, \max}^2 - 1 \right) f_{x, \max} + \frac{1}{2\pi l} \arctan(2\pi l f_{x, \max}) \right]. \quad (8)$$

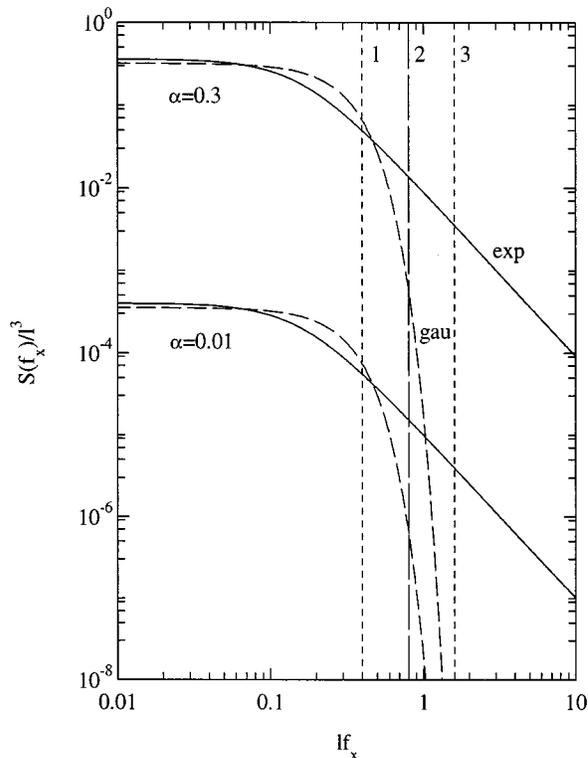


Fig. 1. Roughness spectra for surfaces with Gaussian ("gau") and exponential ("exp") ACF. The spectra are normalized by a factor of  $l^{-3}$ , whereby it is assumed that  $s = \alpha l$ . Here,  $l$  is correlation length,  $s$  is RMS height, and  $\alpha$  is a constant. The vertical lines indicate the position of the cutoff frequency at  $f_{x, \max} = \beta/\lambda_{\text{radar}}$  with  $\beta = 2.5, 5, \text{ and } 10$  (positions 1–3 from left to right).

Scattering models based on the Kirchhoff Theory require that the average radius of curvature is larger than the radar wavelength [2, Ch. 12]. According to Ulaby *et al.* [2, pp. 1011–1013], the average radius of curvature for RMS slopes  $\ll 1$  is

$$\Gamma_c = \left( \frac{2}{\pi} R_{z''z''}(0) \right)^{-1/2} \quad (9)$$

which following the above given arguments is to be evaluated at  $f_{x, \max} = 5/\lambda_{\text{radar}}$ .

### III. UTILIZATION OF TRUNCATED SPECTRA

In general, the intensity of multiple scattering depends on the surface characteristics and on the radar frequency and incidence angle. As a rule of thumb, multiple scattering contributions to the received radar signal are neglected if the RMS slope of a surface is smaller than  $m = 0.4$  [1, p. 231]. The resulting separation line in a  $kl$ – $ks$  diagram is shown in Fig. 2 for a Gaussian and an exponential ACF, using (5) in the latter case. For a given correlation length, multiple scattering effects on an exponentially correlated surface occur at significantly lower values of the RMS height than in the case of a Gaussian roughness spectrum, provided that the threshold of  $m = 0.4$  is valid independent of the shape of the ACF and of the radar frequency.

In Fig. 1, the spectra are shown as functions of  $lf_x$ . For smaller values of  $lf_x$ , a part of the roughness spectrum that is too large might be truncated. Hence, for a fixed value of  $f_{x, \max}$ , there is also a lower limit of the correlation length  $l$  for the application of (4). Following again the argument that the radar is not sensitive to height variations of the surface elevation within intervals shorter than one tenth of the radar wavelength [2], the radar "recognizes" a correlation between surface elements only if  $l > 0.1\lambda_{\text{radar}}$  ( $kl > 0.6$ ). This is in agreement with

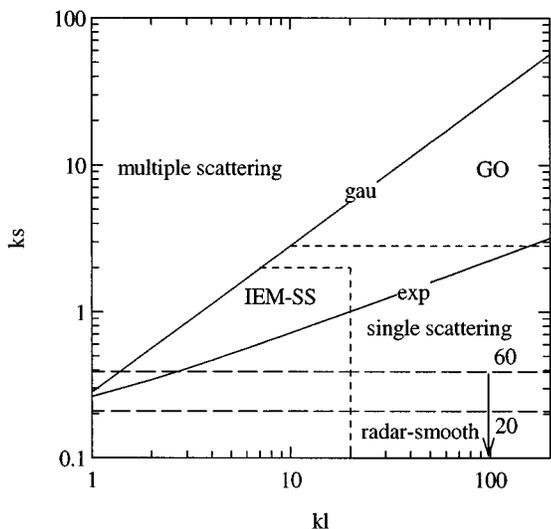


Fig. 2. Separation of single and multiple scattering regime for surfaces with Gaussian (“gau”) and exponential (“exp”) ACF, using a cutoff frequency of  $f_{x,max} = 5/\lambda_{radar}$ . Solid lines mark the borders between both regions, respectively. Long dashed lines indicate values of  $ks$  for which the surface can be regarded as smooth at incidence angles of  $20^\circ$  and  $60^\circ$  (Fraunhofer criterion). Short dashed lines outline validity regions of the single scattering integral equation model (IEM-SS) and of the geometrical optics approximation (GO).

a numerical study by Ogilvy and Foster [7], who showed that the sampling interval of a profile must be at least as small as one tenth of the surface correlation length for the inherent exponential nature of the surface to be measured. In addition, one might consider that the amplitudes of the roughness elements have to be large enough to be detected by the radar. A surface can be regarded as specular (completely smooth) at values of  $ks$  smaller than between 0.2–0.4, dependent on the incidence angle (Fraunhofer criterion [2, p. 827]), which is indicated in Fig. 2. For a given value of  $m$ , the corresponding values of  $kl$  can be determined by applying (5). If, for example, the RMS slope is  $m = 0.4$ , one obtains a value of  $kl = 2.85$  for  $ks = 0.4$ . For  $kl = 0.6$  (as the lowest threshold for  $l$ ), the value of  $ks$  is 0.23. Hence, only for surfaces with larger RMS slopes might it be necessary to deal with correlation lengths as small as the threshold of  $l \approx 0.1\lambda_{radar}$ . Considering this, a reasonable and conservative lower limit of the correlation length  $l$  for the truncation of the high frequency tail is  $l = 1/k$ , which gives  $lf_{x,max} = 0.8$  for  $f_{x,max} = 5/\lambda_{radar}$  ( $\Delta x = 0.1\lambda_{radar}$ ). Different values of  $lf_{x,max}$  are shown as vertical lines in Fig. 1. They indicate that a surface sampling rate of  $\Delta x = 0.2\lambda_{radar}$  ( $f_{x,max} = 2.5/\lambda_{radar}$ ) is not sufficient, since even in the case of the Gaussian spectrum, high frequency roughness elements with spectral densities smaller by only a factor of approximately five compared to the low frequency spectral densities are truncated.

In the  $ks$ - $kl$  diagram shown in Fig. 3, the curves of  $ks$  ( $kl, m = 0.4$ ) obtained with  $f_{x,max} = 2.5/\lambda_{radar}$  and  $f_{x,max} = 10/\lambda_{radar}$  are compared to the “standard” cutoff of  $f_{x,max} = 5/\lambda_{radar}$ . With increasing cutoff frequencies, the value of  $s$  for a given value of  $l$  at  $m = 0.4$  decreases. It is clear that a definition of the RMS slope for exponentially correlated surfaces has to be based on a fixed value of  $f_{x,max}$  in order to be of practical use. In the case of exponential and Gaussian spectra truncated at  $f_{x,max} = 2.5/\lambda_{radar}$ , it is recognized that for small values of  $kl (<2)$ , the value of  $s$  required to give an RMS slope of  $m = 0.4$  for a given correlation length is larger than the RMS height  $s = ml/\sqrt{2}$  resulting from an ideal Gaussian roughness spectrum. This is because the cutoff frequency of the roughness spectrum is too low.

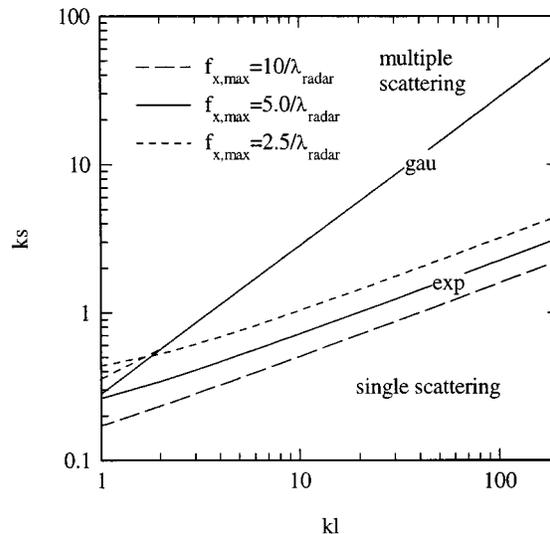


Fig. 3. Separation between single and multiple scattering regime for different cutoff frequencies. For  $kl < 2$  and  $f_{x,max} = 2.5/\lambda_{radar}$ , the separation line for a Gaussian correlated surface is also significantly affected by a high frequency truncation of the roughness spectrum (lower short dashed line).

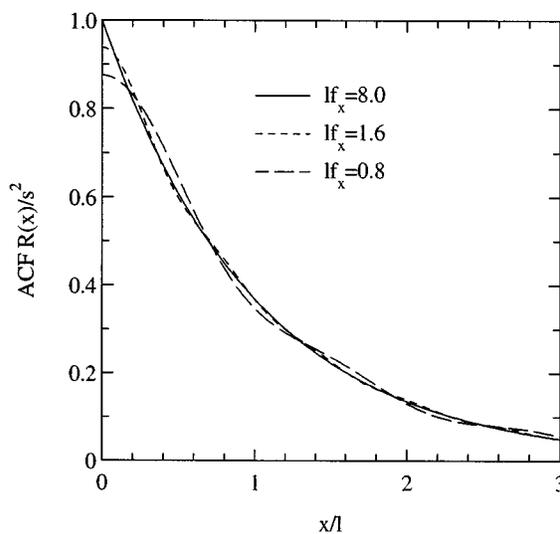


Fig. 4. Autocovariance functions of an exponential roughness spectrum with sharp cutoff, shown for different values of  $lf_x$ . The ACF’s are normalized to the height variance  $s^2$  over the spatial frequency range  $[0, \infty]$ . The case  $lf_x = 0.8$  corresponds to  $kl = 1, f_{x,max} = 5/\lambda_{radar}$ , and  $lf_x = 1.6$  corresponds to  $kl = 1, f_{x,max} = 10/\lambda_{radar}$ . If  $lf_x = 8.0$ , possible values are  $kl = 10$  and  $f_{x,max} = 5/\lambda_{radar}$ , or  $kl = 1$  and  $f_{x,max} = 50/\lambda_{radar}$ , for example.

The ACF’s of an exponential spectrum truncated at different values of  $lf_{x,max}$  are depicted in Fig. 4. Since the high frequency part of the spectrum is lacking, the form of the ACF near the origin becomes quadratic. For  $lf_{x,max} \geq 8$ , this change is restricted to a very narrow region around the origin and is hardly visible in the figure. The spectral density of high frequency roughness elements, which are neglected in this case, are smaller by more than two orders of magnitude compared to the spectral densities at low frequencies (Fig. 1). Because of the truncation, the effective RMS heights are  $s_{rd} = 0.87s$  for  $lf_{x,max} = 0.8$ ,  $s_{rd} = 0.94s$  for  $lf_{x,max} = 1.6$ , and  $s_{rd} = 0.99s$  for  $lf_{x,max} = 8.0$ .

“Real” surfaces can often well be approximated by a self-affine structure of the fractal form  $S(f_x) = c/f_x^\alpha$ , where  $S(f_x)$  is assumed to be band-limited [8]. Also in this case, the evaluation of RMS slope and average radius of curvature from a roughness spectrum with a

high frequency cutoff (dependent on the radar wavelength) might be a useful concept.

#### IV. EFFECT ON SCATTERING SIMULATIONS

Surface scattering models are sensitive to changes in the shape of the ACF. In the low frequency region (small roughness elements), the scattering coefficients depend on a wide range of the roughness spectrum. Model simulations for the backscattering coefficient as a function of incidence angle (between 20–60°) were carried out using the single scattering model of the IEM [1] with exponential ACF's and the ACF's of the corresponding truncated roughness spectra such as the ones shown in Fig. 4. In the latter case, the required Fourier transforms of the  $n$ th power of the surface correlation functions were evaluated numerically. In the simulations,  $ks$  was gradually changed from 0.3 to 0.9, and  $kl$  from 1.3 to 3.8, with corresponding values of  $lf_{x,\max}$  between 1.0–3.0. The Gaussian RMS slope was fixed to 0.35, whereas the RMS slope according to (5) varies between 0.4–0.8 (which means that the single scattering approximation may not be sufficient). The differences between simulations with the full and the truncated roughness spectra are smaller than 0.3 dB. In the low frequency scattering region, the changes of the shape of the exponential ACF caused by neglecting the high frequency roughness not detected by the radar are very small even for  $lf_{x,\max} = 0.8$  (where the changes of the shape of the ACF are largest) and do not have to be considered in practical applications. In the high frequency region (geometrical optics approximation, GO, which requires  $ks \cos \theta > 1.6$  and  $kl > 6$ , where  $\theta$  is the radar incidence angle relative to the surface normal [2, Ch. 12]), the scattering coefficients are determined by the shape of the ACF at the origin ( $x = 0$ ). For an exponentially correlated surface, the GO cannot be applied, since the ACF is not differentiable at  $x = 0$ . Although for ACF's of truncated roughness spectra the derivative at the origin exists, the GO model is nevertheless not applicable for the cutoff frequency proposed here. Formally, this can be seen by evaluating the radius of curvature [using (8) and (9)], which is much smaller than the radar wavelength for  $kl > 6$ , whereas  $\Gamma_c > \lambda_{\text{radar}}$  is required for the GO. The high frequency roughness (in the region  $f_x < f_{x,\max}$ , to which the radar is sensitive and which is not truncated by the proposed method) is considerably larger relative to the low frequency undulations for an exponentially correlated surface than for a surface with a Gaussian ACF. This means that the existence of specular facets on the surface cannot be assumed.

#### V. CONCLUSIONS

For surfaces for which the RMS slope does not exist theoretically, it is suggested to determine an effective RMS slope by truncating the high frequency tail of the roughness spectrum to which the radar is not sensitive. A cutoff frequency of  $f_{x,\max} = 5/\lambda_{\text{radar}}$ , which corresponds to a surface sampling interval of  $\Delta x = 0.1\lambda_{\text{radar}}$  is a reasonable choice ( $\lambda_{\text{radar}}$ —radar wavelength). This sampling interval is viewed as the largest acceptable for radar applications [2, p. 823]. With the proposed cutoff frequency, the effective RMS slope  $m$  of a surface with a Gaussian ACF can be well approximated by  $m = \sqrt{2}s/l$  ( $s$ —RMS height,  $l$ —correlation length). It was shown that the effective RMS slope of an exponentially correlated surface is  $m = cs/l$ , where  $c$  is a function of  $kl$  ( $k$ —radar wavenumber). The evaluation of the effective RMS slope requires that the correlation length of the surface must be about

$l = 1/k$  or larger, otherwise, a part of the roughness spectrum that is too large is truncated.

In scattering simulations in which the IEM is applied to exponentially correlated surfaces with small to moderate roughness, the scattering coefficients for a roughness spectrum truncated at  $f_{x,\max} = 5/\lambda_{\text{radar}}$  are only slightly affected in comparison to calculations considering the whole spectrum. The differences can be neglected in practice. As a result of the truncation, the exponential ACF becomes differentiable at the origin. Nevertheless, the GO approximation cannot be applied because of the high frequency roughness elements, which remain after the truncation.

It has still to be investigated in experiments or by means of scattering models whether an effective RMS slope of  $m = 0.4$  is a sufficient criterion for separating single- and multiple scattering regimes. It has also to be confirmed that the value of  $m = 0.4$  is valid not only for the Gaussian ACF but also for the exponential ACF or, for example, for ACF's of surfaces with a band limited self-affine roughness.

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