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Influence of Bulk Dielectric Polarization upon Partial Discharge Transients

Effect of Heterogeneous Dielectric Geometry

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ABSTRACT

A physically valid theory of partial discharge (PD) transients is based upon the concept of the charge induced upon the detecting electrode by the PD. This induced charge consists of two components. One is associated with the actual space charge in the void, while the other is related to changes in the polarization of the bulk dielectric. These changes are brought about by the field produced by the space charge. The magnitude of the induced charge and its components are examined for several heterogeneous dielectric systems. It is demonstrated that, in relation to a homogeneous dielectric system, the magnitude of the induced charge either increases or decreases depending on the ratio of the dielectric permittivities and within which dielectric the void is located. It is shown that this behavior is directly related to the magnitude and polarity of the polarization component of the induced charge. Furthermore, we demonstrate that the geometry of the dielectric system and the physical dimensions of the different dielectrics influence in a similar manner the magnitude of the induced charge, although to a lesser degree.

1 INTRODUCTION

THE occurrence of partial discharge (PD) in a gaseous void leads not only to charge being induced on the detecting electrode, but also to a change in the polarization $\delta\vec{P}$ of the bulk dielectric. Pedersen et al. have shown that this change influences the magnitude of the charge induced upon the detecting electrode [1, 2].

The charge induced on a detecting electrode can be evaluated using either the λ function, or the ϕ function [1, 2]. These functions take account of the dielectric polarization either implicitly (λ) or explicitly (ϕ). Thus by using the two functions it becomes possible to identify the influence which changes in dielectric polarization have upon induced charge. Changes in polarization $\delta\vec{P}$ are related directly to changes in electric field $\delta\vec{E}$. In PD, the source of $\delta\vec{E}$ is the PD space charge which is confined to voids located within the bulk dielectric.

In the present paper, the influence of the system geometry upon $\delta\vec{P}$ is examined for a two-layer dielectric system. It is shown that the component of the induced charge due to $\delta\vec{P}$ may increase or decrease depending upon the ratio of the dielectric permittivities and within which dielectric the void is located. The magnitude of this increase/decrease is also dependent upon both the dimensions of the different dielectrics

and the non-uniformity of the system geometry. As such behavior is succinctly described in terms of the Poissonian induced charge [3], we will first introduce this concept.

2 POISSONIAN INDUCED CHARGE

Induced charge can be described in terms of either a Poissonian or a Laplacian component [3]. The Laplacian induced charge, which is equal in magnitude but opposite in polarity to the Poissonian component, is associated with the change in potential of the detecting electrode. Consequently, the Laplacian charge is the source of any PD transient signal.

The Poissonian induced charge is that component of the induced charge which is rigidly linked to the space charge source, and which together with this source gives rise to the 'basic Poisson field' [2]. This field is generated by the PD.

Mathematically, the final value of the Poissonian induced charge q due to a PD can be expressed as the sum of two components

$$q = q_\mu + q_p \tag{1}$$

where q_μ is the induced charge directly associated with the space charge in the void, and q_p represents the induced charge related to the change in dielectric polarization $\delta\vec{P}$ due to the presence of this space charge [2]. With reference to the detecting electrode and the induced charge, the effect of the void wall charges can be equated to the effect of an electric dipole of moment $\vec{\mu}$ located within the void [1].

The total Poissonian induced charge arising from a dipole is given by

$$q = -\vec{\mu} \cdot \vec{\nabla}\lambda \qquad (2)$$

where λ represents the proportionality factor between the charge in the void and the induced charge on the detecting electrode. The λ function is a solution of the *general* Laplace equation [2]

$$\vec{\nabla} \cdot (\varepsilon\vec{\nabla}\lambda) = 0 \qquad (3)$$

in which ε denotes permittivity.

The component of the Poissonian induced charge related to the void space charge alone may be obtained via another proportionality factor ϕ; viz.

$$q_\mu = -\vec{\mu} \cdot \vec{\nabla}\phi \qquad (4)$$

where the ϕ function is a solution of the *reduced* Laplace's equation [2]

$$\nabla^2\phi = 0 \qquad (5)$$

Hence, from (1), (2) and (4), the polarization component q_p of the Poissonian induced charge may be expressed as

$$q_p = -\vec{\mu} \cdot (\vec{\nabla}\lambda - \vec{\nabla}\phi) \qquad (6)$$

3   THE λ_0 FUNCTION

If the dimensions of the void are such that $\vec{\nabla}\lambda$ may be assumed constant within the void, then we can introduce another function λ_0, which represents the unperturbed λ function. That is, the system is considered void free. As λ is a solution of Laplace's equation, then by mathematical analogy with electrostatic fields, the relationship between the λ and λ_0 functions is given by

$$\vec{\nabla}\lambda = h\vec{\nabla}\lambda_0 \qquad (7)$$

For the type of void under consideration, the parameter h will be a scalar which depends on the void geometry and the relative permittivity of the associated bulk medium. Following the introduction of λ_0, the total Poissonian induced charge on the detecting electrode may be expressed as

$$q = -h\vec{\mu} \cdot \vec{\nabla}\lambda_0 \qquad (8)$$

Furthermore, using (6) and (7) we re-express q_p as

$$q_p = -\vec{\mu} \cdot (h\vec{\nabla}\lambda_0 - \vec{\nabla}\phi) \qquad (9)$$

For a homogeneous bulk dielectric system, λ_0 is a solution of the reduced Laplace equation, and hence in such situations λ_0 and ϕ are synonymous. As a consequence (9) reduces to

$$q_p = -(h - 1)\vec{\mu} \cdot \vec{\nabla}\lambda_0 \qquad (10)$$

Combining (8) and (10), we arrive at the expression

$$\frac{q_p}{q} = \frac{h - 1}{h} \qquad (11)$$

For such a homogeneous situation, the effect of void geometry upon the behavior of q_p has been reported; see [2].

In practice however, nearly all equipment will consist of a heterogeneous dielectric system. Nevertheless for a very restricted class of dielectric geometries, e.g. a coaxial disc spacer, λ_0 remains a solution of the reduced Laplace equation, and thus λ_0 and ϕ are again synonymous.

To examine the situation in which λ_0 is a solution of the general Laplace equation, we consider several specific heterogeneous dielectric systems as a means of examining the behavior of q_p. However, as indicated by (9), three independent vectors are involved in the evaluation of q_p. Hence, for an initial investigation of this behavior, we will limit our study to situations for which the three vectors are parallel.
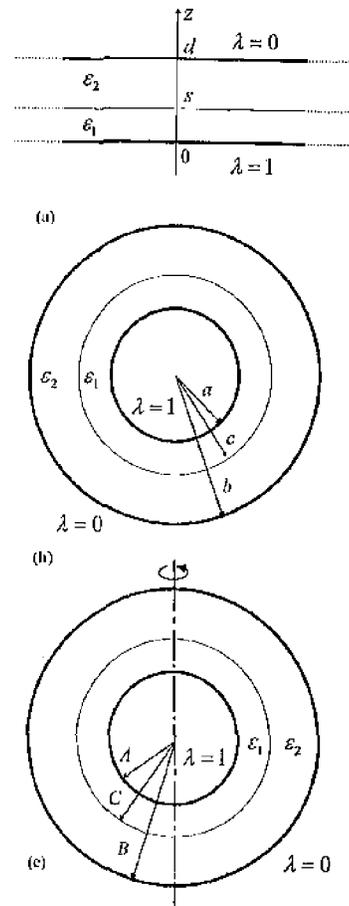
Figure 1. The heterogeneous bulk dielectric systems. (a) planar geometry, (b) cylindrical geometry, (c) spherical geometry.

Three simple geometries which fulfill this requirement are shown in Figure 1; namely a planar, a cylindrical and a spherical geometry. Each supports a two layer dielectric system. Because the derivation of the relevant $\vec{\nabla}\lambda_0$ and $\vec{\nabla}\phi$ functions is straightforward, this is confined to the Appendix. It should be noted that $\lambda = 1$ denotes the detecting electrode.

4 POISSONIAN INDUCED CHARGE

To undertake a comparative assessment of the influence of the dielectric geometry upon PD transients we will assume that the dipole $\vec{\mu}$, associated with the charge which accumulates at the void wall following a PD, is considered a constant in all situations. Furthermore, it will be assumed that the void is more than 10× its greatest linear dimension from any interface, such that the $\vec{\nabla}\lambda$ distribution within the void is effectively uniform; i.e. the existence of an interface does not perturb $\vec{\nabla}\lambda$ in the void. This assumption implies that the concept of h is valid and that (7) may be employed.

To establish the influence of a heterogeneous system, we take as the reference value the induced charge q which occurs in a homogeneous system for the *same* PD ($\vec{\mu}$) at the same void location. Thus, with reference to the induced charge q_n of the heterogeneous system, we have

$$\frac{q_n}{q} = \frac{-\vec{\mu} \cdot h_n \vec{\nabla}\lambda_{0n}}{-\vec{\mu} \cdot h\vec{\nabla}\lambda_0} \tag{12}$$

where $n = 1, 2$ depending in which dielectric medium the void is located. For the homogeneous system λ_0 is independent of the specific dielectric. In the heterogeneous system, if the void is located in a dielectric of the same permittivity as that of the homogeneous reference dielectric, we have equality of h_n and h.

Due to the geometries under consideration, the dipole moment is directed either away from, or towards the coordinate origin. Hence this moment can be expressed as

$$\vec{\mu} = \pm\mu\vec{e}_u \tag{13}$$

where $\vec{e}_u$ is a unit vector tangential with the coordinate u, where u is the relevant coordinate for the geometry in question. Hence upon undertaking the vector operations, (12) is reduced to

$$\frac{q_n}{q} = \frac{d\lambda_{0n}/du}{d\lambda_0/du} \tag{14}$$

Provided the use of h_n in (12) remains valid, (14) indicates that q_n/q is independent of the *void* geometry.

4.1 PLANAR GEOMETRY

Upon substituting the relevant λ_0 gradients (see Appendix) into (14), we obtain for a void in medium 1

$$\frac{q_1}{q} = \frac{\varepsilon_2 d}{\varepsilon_1(d - s) + \varepsilon_2 s} \tag{15}$$

and for a void in medium 2

$$\frac{q_2}{q} = \frac{\varepsilon_1 d}{\varepsilon_1(d - s) + \varepsilon_2 s} \tag{16}$$

These expressions indicate that q_n/q is also independent of the void location in the specific medium, in so far as the h validity requirement is fulfilled. For $\varepsilon_1 = \varepsilon_2$, the ratios in (15) and (16) reduce to unity.

The variations of q_n/q with $\varepsilon_2/\varepsilon_1$ are shown in Figure 2 for several values of s/d. From the Figure it is seen that, in comparison to the homogeneous system, the induced charge of a heterogeneous system is increased when the void is located in the medium of lesser permittivity. Likewise a decrease occurs for void locations in the medium of higher permittivity. When the void is in the lower dielectric (medium 1), the
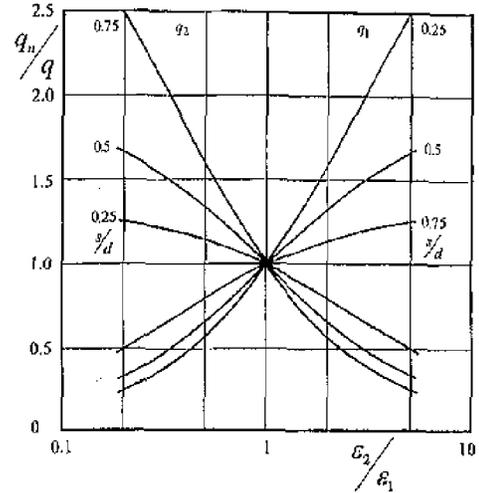

Figure 2. Variation of q_n/q with $\varepsilon_2/\varepsilon_1$ for the planar geometry and selected s/d values.

nearer the dielectric interface is to the detecting electrode ($\lambda = 1$), the larger are the relative changes in these induced charges. The opposite behavior occurs when the void is in the upper dielectric (medium 2); i.e. the further the interface is from the detecting electrode the stronger is the effect. As the value of s/d represents the relative thickness of medium 1 these results imply that, when the void is in the dielectric of smaller physical extent, the larger is the relative increase in q_n. Owing to the geometrical symmetry of the planar system, the behavior of q_n is symmetrical with respect to s/d and $(1 - s/d)$.

To simplify comparison with the other geometries the following convention will be adopted; *viz.* the dielectric layer bounding the detecting electrode has permittivity ε_1 and thickness s, while the combined thickness of the two dielectric layers is d.

4.2 CYLINDRICAL GEOMETRY

Substitution of the relevant $\vec{\nabla}\lambda_0$ functions into (14) gives for a void in medium 1

$$\frac{q_1}{q} = \frac{\varepsilon_2 \ln(b/a)}{\varepsilon_1 \ln(b/c) + \varepsilon_2 \ln(c/a)} \tag{17}$$

and for a void in medium 2

$$\frac{q_2}{q} = \frac{\varepsilon_1 \ln(b/a)}{\varepsilon_1 \ln(b/c) + \varepsilon_2 \ln(c/a)} \tag{18}$$

These expressions indicate that q_n/q is also independent of the void location, so long as the previously discussed h requirement is fulfilled. Again, the ratios in (17) and (18) reduce to unity for $\varepsilon_1 = \varepsilon_2$.

When referring to either the cylindrical or spherical geometries, we will use the product Hd as a measure of the geometric non-uniformity. The parameter H is the mean curvature of the inner (detecting) electrode, while d represents the total thickness of the dielectric insulation. Hence for the cylindrical geometry we have

$$H = \frac{1}{2a} \tag{19}$$

and

$$d = b - a \tag{20}$$

As the parameter s represents the thickness of the dielectric bounding the detecting electrode, then for the cylindrical system we have

$$s = c - a \tag{21}$$

For a coaxial electrode system with a radii ratio of c, $Hd = 0.859$, while for the planar electrode system, $Hd = 0$.

The variations of q_n/q with $\varepsilon_2/\varepsilon_1$ for the cylindrical geometry, $Hd = 1$, are illustrated in Figure 3 for the same values of s/d as in the planar case. The pattern of the variations is similar to that shown in Figure 2 for the planar geometry, but now the simple symmetry of the pattern is absent; namely, the increases and reductions in q_1/q are less, while the reverse behavior is exhibited by q_2/q. These results indicate that the *geometric* non-uniformity modifies the influence of the dielectric thickness. We proceed to examine further the influence of the geometric parameter.
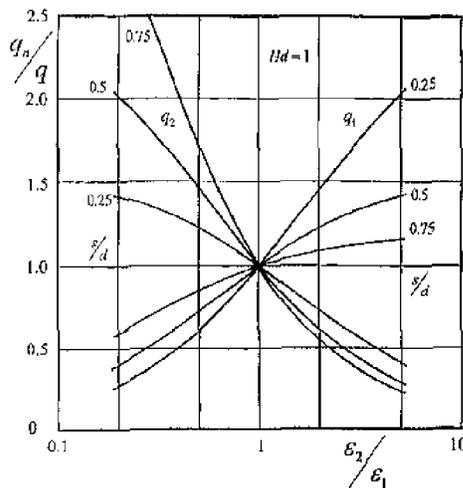

Figure 3. Variation of q_n/q with $\varepsilon_2/\varepsilon_1$ for the cylindrical geometry $(Hd = 1)$ and selected s/d values.

The effect of the non-uniformity of the geometry, *i.e.* the value of Hd, upon the q_n/q behavior is shown in Figure 4. It is clear that increasing the value of Hd leads to a reduction in the increase/decrease of q_1/q about the reference value of unity. This behavior is more pronounced when the void is in the dielectric of lower permittivity, *i.e.* for q_1/q when $\varepsilon_2/\varepsilon_1 > 1$. The opposite trend is exhibited by q_2 in that increasing values of Hd lead to an even larger increases/decreases in the q_2/q behavior about unity.

4.3 SPHERICAL GEOMETRY

With respect to the spherical geometry, the mean curvature of the inner electrode is given by

$$H = \frac{1}{A} \tag{22}$$

while the total insulation thickness is now

$$d = B - A \tag{23}$$

and the thickness of the dielectric bounding the detecting electrode is
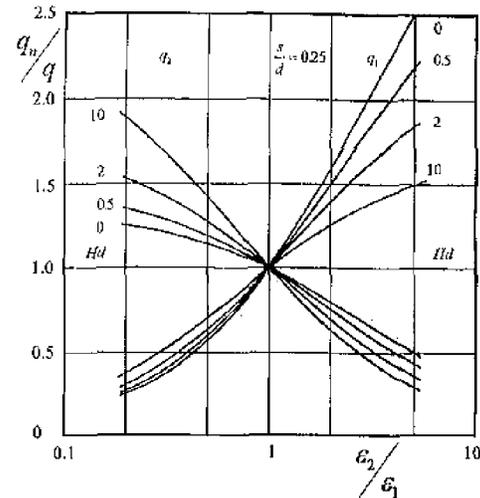
$$s = C - A \tag{24}$$

Figure 4. Variation of q_n/q with $\varepsilon_2/\varepsilon_1$ for the cylindrical geometry and $s/d = 0.25$; Effect of Hd.

Using (53), (54) and (55), the relevant q_n/q expressions are derived from (14). However as the variations of q_n/q display the same behavior as these of the cylindrical geometry, it is not necessary to discuss the spherical geometry in detail.

In brief, it may be stated that, for $Hd \leqslant 1$, the q_n/q variations for the spherical geometry are essentially identical to those exhibited by the cylindrical geometry. For $Hd > 1$, the q_n/q variations begin to differ, in particular at the low s/d values; *i.e.* the increases/reductions in q_1/q for the spherical geometry are smaller than those in the cylindrical case. The reverse behavior is observed with q_2/q.

5 INDUCED CHARGE DUE TO $\delta \vec{P}$ IN HETEROGENEOUS DIELECTRIC SYSTEMS

With respect to the component of the induced charge related to $\delta \vec{P}$, we have upon combining (9) with (8)

$$\frac{q_{pn}}{q_n} = \frac{-\vec{\mu} \cdot (h_n \vec{\nabla} \lambda_{0n} - \vec{\nabla} \phi)}{-\vec{\mu} \cdot h_n \vec{\nabla} \lambda_{0n}} \tag{25}$$

where $n = 1, 2$ depending in which dielectric the void is located. On performing the vector operations, (25) simplifies to give

$$\frac{q_{pn}}{q_n} = 1 - \frac{d\phi/du}{h_n d\lambda_{0n}/du} \tag{26}$$

where u is the relevant coordinate for the geometry under discussion. From (26) it is evident that the polarity of q_{pn}/q_n is dependent upon whether

$$\frac{d\phi/du}{h_n d\lambda_{0n}/du} \lessgtr 1 \tag{27}$$

That is the polarization component may *reverse* polarity. The polarity of the Poissonian induced charge, however, cannot exhibit such behavior; see (14).

Because (26) contains h_n, this implies that q_{pn}/q_n is dependent upon the void geometry. In this study, we will assume the void to be

spherical, in which case we have

$$h = \frac{3\varepsilon_r}{1 + 2\varepsilon_r} \qquad (28)$$

where ε_r is the relative permittivity of the dielectric containing the void.

With reference to q_n/q, its value when associated with the condition $q_{pn}/q_n = 0$ can be deduced by using (26) together with the fact that for a homogeneous system λ_0 and ϕ are synonymous. Thus, for zero polarization component, we obtain

$$\left[\frac{q_n}{q}\right]_0 = \frac{1}{h_n} \qquad (29)$$

where the subscript 0 in (29) refers to this condition. For $\varepsilon_r > 1$, we have $h_n > 1$ and thus (29) implies that $0 < (q_n/q)_0 < 1$.

5.1 PLANAR HETEROGENEOUS DIELECTRIC GEOMETRY

Employing (43), (44) and (45) we obtain for a void in medium 1

$$\frac{q_{p1}}{q_1} = 1 - \frac{\varepsilon_1(d - s) + \varepsilon_2 s}{\varepsilon_2 h_1 d} \qquad (30)$$

and for a void in medium 2

$$\frac{q_{p2}}{q_2} = 1 - \frac{\varepsilon_1(d - s) + \varepsilon_2 s}{\varepsilon_1 h_2 d} \qquad (31)$$

For $\varepsilon_1 = \varepsilon_2$, equality of h_1 and h_2 exists and thus both (30) and (31) reduce to (11).
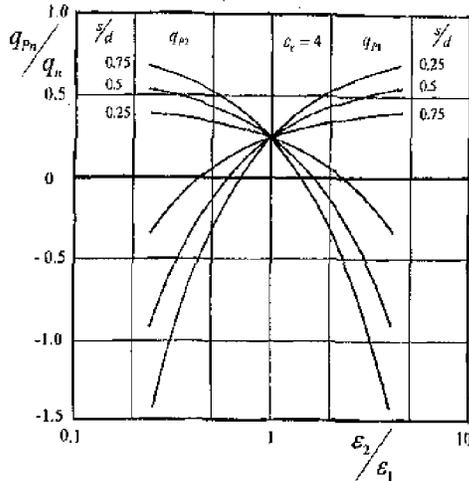

Figure 5. Variation of q_{pn}/q_n with $\varepsilon_2/\varepsilon_1$ for the planar geometry and selected s/d values ($\varepsilon_r = 4$).

The variation of q_{pn}/q_n is shown in Figure 5 for $\varepsilon_r = 4$ together with the pre-selected s/d values. To ensure that the q_{pn}/q_n calculations were undertaken for realistic $\varepsilon_2/\varepsilon_1$ values, the range of this parameter was restricted to

$$\frac{1}{\varepsilon_r} \leqslant \frac{\varepsilon_2}{\varepsilon_1} \leqslant \frac{10}{\varepsilon_r} \qquad (32)$$

From Figure 5 it is observed that the behavior of q_{pn}/q_n is similar to that shown in Figure 2 for q_n/q; i.e. q_{pn}/q_n increases when the void is located in the dielectric with the lesser permittivity. In marked

contrast, the reduction which occurs when the void is in the medium of higher permittivity can be of such a degree that the polarity of q_{pn} is *reversed*. The influence of s/d with respect to the magnitude of q_{pn}/q_n is similar to that observed with q_n/q; see Figure 2.
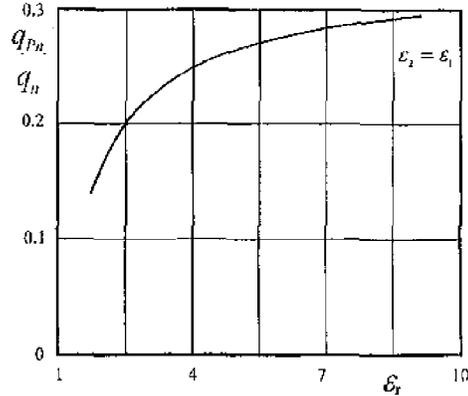

Figure 6. Variation of q_{pn}/q_n with ε_r for $\varepsilon_2/\varepsilon_1 = 1$.

For $\varepsilon_r = 6$, the variation of q_{pn}/q_n with $\varepsilon_2/\varepsilon_1$ is similar to that shown in Figure 5 for $\varepsilon_r = 4$. The magnitude of q_{pn}/q_n is however increased. This increase is proportional to the increased value of q_{pn}/q_n for $\varepsilon_1 = \varepsilon_2$, i.e. to the crossover value observed in Figure 5. For this condition, the variation of q_{pn}/q_n with ε_r is illustrated in Figure 6. With the heterogeneous system and $\varepsilon_r = 2$, a similar variation to that shown in Figure 5 is found to be exhibited by q_{pn}/q_n. However, as can be understood from the reduced cross-over value for $\varepsilon_r = 2$ shown in Figure 6, there is a corresponding reduction in the q_{pn}/q_n values with respect to those for $\varepsilon_r = 4$.

5.1.1 POLARITY REVERSAL OF POLARIZATION COMPONENT

With respect to the polarity reversal of q_{pn}/q_n, the value of $\varepsilon_2/\varepsilon_1$, corresponding to $q_{p1}/q_1 = 0$ is obtained from (30)

$$\left[\frac{\varepsilon_2}{\varepsilon_1}\right]_c = \frac{d - s}{dh_1 - s} \qquad (33)$$

while from (31) the appropriate $\varepsilon_1/\varepsilon_2$ value for $q_{p2}/q_2 = 0$ is

$$\left[\frac{\varepsilon_1}{\varepsilon_2}\right]_c = \frac{s}{d(h_2 - 1) + s} \qquad (34)$$

The subscript c in (33) and (34) refers to the fact that the permittivity ratio has a critical value. As the polarity reversal occurs when the void is in the dielectric of higher permittivity, this implies that the values of the ratios in (35) and (36) are < 1, and that the permittivity in the denominator represents the medium containing the void.

The variations of the above permittivity ratios with s/d for different ε_r values are shown in Figure 7. For increasing values of s/d, it is clear that $(\varepsilon_1/\varepsilon_2)_c$ increases, while $(\varepsilon_2/\varepsilon_1)_c$ decreases correspondingly. These results indicate that, for the dielectric containing the void, the smaller the physical dimensions are of this dielectric the more $(\varepsilon_2/\varepsilon_1)_c$ and $(\varepsilon_1/\varepsilon_2)_c$ tend to an upper limiting value. From (33) it is evident that as $s/d \to 0$

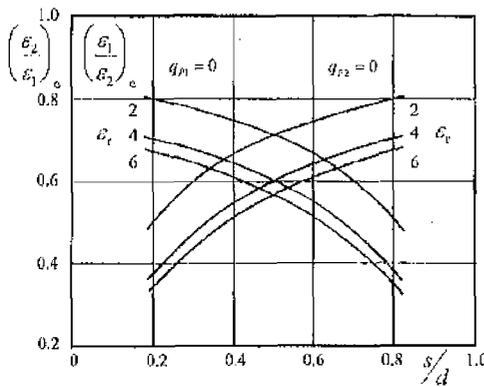$$\left[\frac{\varepsilon_2}{\varepsilon_1}\right]_c \to \frac{1}{h_1} \qquad (35)$$

Figure 7. Variation of 'zero' permittivity ratios with s/d for selected ε_r values.

while from (34) we find that as $s/d \to 1$

$$\left[\frac{\varepsilon_1}{\varepsilon_2}\right]_c \to \frac{1}{h_2} \tag{36}$$

As the permittivity ratios for practical situations lie in the range 0.4 to 0.7, the possibility for polarity reversal of q_{pn}/q_n is real. Planar systems are frequently employed in laboratory studies.

5.2 CYLINDRICAL HETEROGENEOUS DIELECTRIC GEOMETRY

Using (48), (49) and (50) we obtain for a void in medium 1

$$\frac{q_{p1}}{q_1} = 1 - \frac{\varepsilon_1 \ln(b/c) + \varepsilon_2 \ln(c/a)}{\varepsilon_2 h_1 \ln(b/a)} \tag{37}$$

and for the void in medium 2

$$\frac{q_{p2}}{q_2} = 1 - \frac{\varepsilon_1 \ln(b/c) + \varepsilon_2 \ln(c/a)}{\varepsilon_1 h_2 \ln(b/a)} \tag{38}$$

As before, for $\varepsilon_1 = \varepsilon_2$, we obtain equality of h_1 and h_2 and hence (37) and (38) reduce to (11).
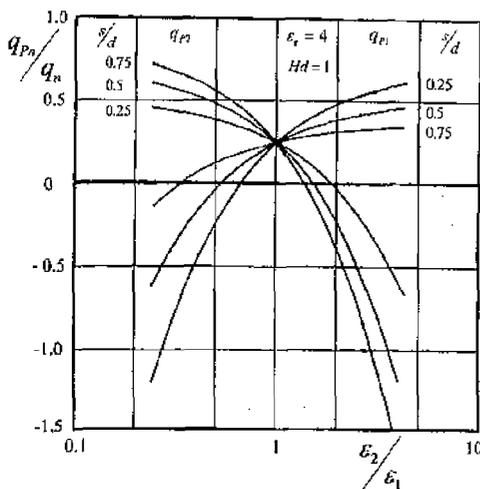

Figure 8. Variation of q_{pn}/q_n with $\varepsilon_2/\varepsilon_1$ for the cylindrical geometry ($Hd = 1$) and selected s/d values ($\varepsilon_r = 4$).

The variations of q_{pn}/q_n with $\varepsilon_2/\varepsilon_1$ for the cylindrical geometry, $Hd = 1$, are shown in Figure 8. It is clear that these variations are similar to those of the planar geometry; cf. Figure 8 with Figure 5. The primary difference is the absence of symmetry in the case of the cylindrical geometry. This results from the increases/reductions in q_{p1}/q_1 being less, while those for q_{p2}/q_2 are larger.
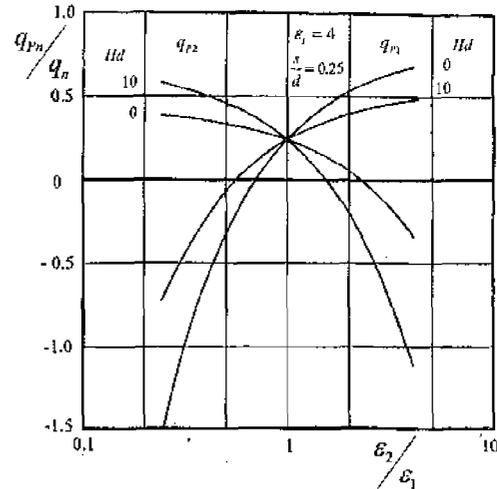

Figure 9. Variation of q_{pn}/q_n with $\varepsilon_2/\varepsilon_1$ for the cylindrical geometry and $s/d = 0.25$: Effect of Hd ($\varepsilon_r = 4$).

The influence of system geometry is fully illustrated in Figure 9 with the variation of q_{pn}/q_n for $Hd = 0$ and $Hd = 10$. In comparison with Figure 8 it is evident that, for the parameter ranges considered, q_{pn}/q_n is more sensitive to s/d than to Hd.

The effect of ε_r upon q_{pn}/q_n for the coaxial geometry is similar to that as discussed previously for the planar geometry, with the addition of the asymmetry shown in Figure 8.

5.2.1 POLARITY REVERSAL OF POLARIZATION COMPONENT

For the cylindrical geometry, the value of $\varepsilon_2/\varepsilon_1$ corresponding to $q_{p1}/q_1 = 0$ is given by

$$\left[\frac{\varepsilon_2}{\varepsilon_1}\right]_c = \frac{\ln(b/c)}{h_1 \ln(b/a) - \ln(c/a)} \tag{39}$$

while the appropriate $\varepsilon_1/\varepsilon_2$ value for $q_{p2}/q_2 = 0$ is

$$\left[\frac{\varepsilon_1}{\varepsilon_2}\right]_c = \frac{\ln(c/a)}{h_2 \ln(b/a) - \ln(b/c)} \tag{40}$$

The variation of these permittivity ratios with Hd for the selected values of s/d are illustrated in Figure 10 for $\varepsilon_r = 4$. Essentially the same behavior is exhibited for $\varepsilon_r = 2$ and $\varepsilon_r = 6$. For Hd values of practical interest for solid insulation ($Hd < 2$), the permittivity ratios generally lie in the range 0.4 to 0.7. Such values imply that the polarity reversal of q_{pn} is quite possible in practical systems.

5.3 SPHERICAL HETEROGENEOUS DIELECTRIC GEOMETRY

By employing (53), (54) and (55), we obtain from (26) the relevant q_{pn}/q_n expressions for this geometry. From these expressions it is
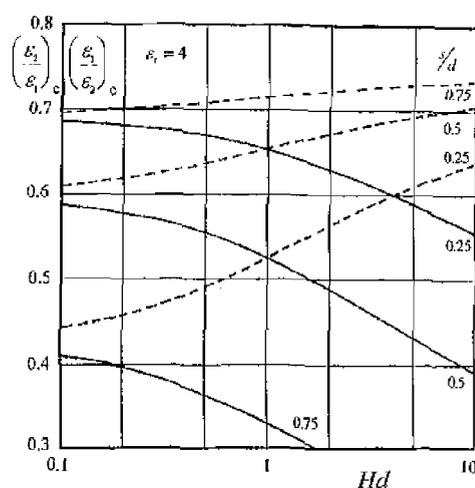
Figure 10. Variation of 'zero' permittivity ratios with Hd for the cylindrical geometry and selected s/d values ($\varepsilon_r = 4$). —: $(\varepsilon_2/\varepsilon_1)_c$ for $q_{p1} = 0$. ---: $(\varepsilon_1/\varepsilon_2)_c$ for $q_{p2} = 0$.

readily demonstrated that the variations of q_{pn}/q_n exhibit the same characteristics as were discussed previously with respect to the corresponding q_n/q; i.e. the spherical q_{pn}/q_n behavior is similar to that of the cylindrical geometry. Consequently, it is unnecessary to discuss in detail the q_{pn}/q_n results for the spherical geometry.

6 DISCUSSION

6.1 GENERAL REMARKS

The Poissonian induced charge q has been shown mathematically to consist of two components [2]. One component q_μ is related directly to the space charge created in the void by the PD. The other component q_p is associated with the change in the polarization of the solid dielectric arising from the action of the electric field produced by the void space charge.

The ability to decompose mathematically the Poissonian induced charge into these two components relates to the use of the λ and ϕ functions to evaluate induced charge. The former accounts for the dielectric polarization implicitly, while the latter function does so explicitly [2].

The combined use of these functions allows quantitative relations for q_p to be deduced, viz. (6) or (9). Each of these expressions contains three independent vectors, and thus for heterogeneous bulk dielectric systems the behavior of q_p is complex. Consequently, to obtain insight into this behavior, the present study has been restricted to situations in which the three vectors are parallel. For heterogeneous bulk dielectric systems such a requirement can be fulfilled by simple geometries, e.g. planar, cylindrical and spherical electrode systems. The planar geometry ($Hd = 0$) enables the basic features of a heterogeneous system to be deduced, but is limited in that the effects of system non-uniformity cannot be addressed. However, this aspect can be fully considered through cylindrical and spherical systems, as such geometries can be used to generate a wide variation in non-uniformity; e.g. $0 < Hd < 10$.

6.2 INFLUENCE OF HETEROGENEOUS BULK DIELECTRIC SYSTEMS UPON INDUCED CHARGE

By comparing the Poissonian induced charge associated with a PD in a void located within a heterogeneous system with that of a corresponding homogeneous system, it has been possible to illustrate the influence of the former. As the same void geometry is considered in both situations, the comparison is made independent of the void geometry.

Under the assumption of a constant PD (i.e. constant dipole moment), it is demonstrated that the magnitude of the Poissonian induced charge can vary significantly, e.g. ±50%. For the present two dielectric system, this behavior is seen to be dependent upon the ratio of the permittivities and within which dielectric the void is located.

The induced charge magnitude is further influenced by the degree of non-uniformity of the system geometry; i.e. symmetrical induced charge variations ($Hd = 0$), vs. asymmetrical variations ($Hd > 0$). The larger the value of Hd is, the higher is the degree of asymmetry. For the cylindrical and spherical geometries essentially identical induced charge variations are produced for $Hd < 1$.

The asymmetry in the induced charge variation for non-uniform geometries ($Hd > 0$) could be interpreted as indicating that, depending on void location, there was a preferential detecting electrode, i.e. the outer instead of the inner electrode. With respect to void PD, the induced charge is proportional to $\vec{\nabla}\lambda_0$ not λ_0; see (8). For the present 2 electrode geometries $\vec{\nabla}\lambda_0$ is independent of which electrode is denoted the detecting electrode, and thus no preferential electrode exists.

The magnitude of the induced charge is also influenced by the physical dimensions of the dielectric containing the void; viz. the smaller the dielectric is in extent, the larger is the change in magnitude. This aspect is clearly illustrated by the symmetry in the planar results.

6.3 INFLUENCE OF CHANGES IN BULK DIELECTRIC POLARIZATION UPON INDUCED CHARGE

The ratio of the polarization component to the Poissonian induced charge was employed to illustrate the manner in which a heterogeneous dielectric system affected the induced charge associated with changes in the polarization of the bulk dielectric. The polarization component increased when the void was located in a dielectric of lesser permittivity, while a decrease occurred when located in a dielectric of higher permittivity. This reduction was observed to be so marked that the polarity of the polarization component was reversed.

For the $q_{pn}/q_n = 0$ condition, the permittivity ratios for practical situations lie in the range 0.4 to 0.7. Hence the chance of this condition arising in practice is high, particularly if, for $Hd > 0$, the inner dielectric is selected to have the higher permittivity. This is common practice to reduce operating stress levels.

The system geometry and the dielectric dimensions also influence the q_{pn}/q_n ratio. These variations reflect those encountered for q_n/q for the heterogeneous/homogeneous systems.

In our analytical approach, it should be noted that the λ and ϕ functions are employed to evaluate directly the charge induced on the detecting electrode. This charge is an *integral* property of the field produced by the PD. Hence the evaluation procedure cannot provide detailed information on the point properties of the field created by the PD. Thus although the polarity reversal behavior of the polarization component is revealed, an in-depth understanding of the phenomenon will necessitate a full field solution. Such a solution is outside the scope of the present study.

6.4 PRACTICAL ASPECTS

To fully exploit the present theoretical study, the location of the PD was restricted only with distance from an interface: see Section 4. This freedom of location does not exist in practical situations, as a minimum electric field strength is necessary to create the conditions for PD development. Such conditions are invariably fulfilled only in the vicinity of the conductor with greatest curvature. For the cylindrical and spherical geometries, this implies that with the PD void located in the inner dielectric (medium 1) the results of the theoretical study relate directly to situations readily encountered in practice.

It has been shown previously for a homogeneous dielectric system that a quantitative evaluation of a PD transient requires a knowledge of the void location, geometry and dimensions, void gas pressure and composition and the void orientation with reference to the applied field [4, 5]. For heterogeneous dielectric systems, the present study indicates that the situation becomes even more complex.

In the practical world, it is highly unlikely that all the necessary information required to undertake a quantitative evaluation of PD measurements would be available. Consequently such measurements can only be handled in terms of pattern recognition. However to undertake meaningful comparisons with previous data, such that trends become identifiable, the measurements should be restricted to similar equipment produced by the same manufacturer. In this way the many parameters mentioned above would remain essentially unchanged.

Should however a manufacturer change the insulation materials employed in the production process, then this could have a significant effect on the magnitude of the recorded PD transients. For example, if the permittivity ratio were altered, then, for the same PD, the recorded transient would show either an increase or a decrease. This in turn could lead to an incorrect assessment of the status of the insulation, particularly if the criterion for assessment were based on a fixed pC level of activity. Consequently any change in the insulation materials should be accompanied by the acquisition of a new reference set of PD data.

7 CONCLUSIONS

WE have examined the Poissonian induced charge in terms of two components; one due directly to the void space charge, and one due to changes in the dielectric polarization brought about by the former.

For a heterogeneous bulk dielectric system, it is demonstrated that changes in the dielectric polarization can significantly influence the magnitude of the Poissonian induced charge; e.g. a pronounced *reversal*

in the polarity of the polarization component is possible. This behavior is not observed in a homogeneous dielectric system. For a two dielectric system, the scale of this influence is dependent primarily upon the ratio of the dielectric permittivities and within which dielectric the void is located. The physical dimensions of the different dielectrics and the geometry of the dielectric system exert a similar, although lesser effect upon the magnitude of the induced charge.

On the basis of these observations, we conclude that a correct quantitative interpretation of PD transients in terms of void discharge phenomena cannot be achieved for practical, heterogeneous bulk dielectric systems. Moreover, fixed pC level assessment should be treated with extreme caution.

8 APPENDIX

8.1 PLANAR ELECTRODE GEOMETRY

Consider a two layer dielectric bounded by planar electrodes. If in rectangular coordinates, the electrodes are represented by $z = 0$ and $z = d$, then the dielectric interface is taken as $z = s$, with $s < d$. The permittivity of the upper dielectric is ε_2 for which $s \leqslant z \leqslant d$, while that of the lower is ε_1, for which $0 \leqslant z \leqslant s$.

If the lower electrode is used as the detecting electrode, then the boundary conditions for the λ_0 function are $\lambda_0 = 1$ for $z = 0$ and $\lambda_0 = 0$ for $z = d$. Hence the λ_0 functions of the two media are given by

$$\lambda_{01} = \frac{\varepsilon_1(d - s) + \varepsilon_2(s - z)}{\varepsilon_1(d - s) + \varepsilon_2 s} \tag{41}$$

for $0 \leqslant z \leqslant s$, and

$$\lambda_{02} = \frac{\varepsilon_1(d - z)}{\varepsilon_1(d - s) + \varepsilon_2 s} \tag{42}$$

for $s \leqslant z \leqslant d$, where the λ_0 subscripts, 1 and 2, refer to the lower and upper regions, respectively. On differentiating with respect to z, we obtain the relevant expressions for the associated λ_0 gradients

$$\vec{\nabla}\lambda_{01} = \frac{-\varepsilon_2 \vec{e}_z}{\varepsilon_1(d - s) + \varepsilon_2 s} \tag{43}$$

and

$$\vec{\nabla}\lambda_{02} = \frac{-\varepsilon_1 \vec{e}_z}{\varepsilon_1(d - s) + \varepsilon_2 s} \tag{44}$$

where $\vec{e}_z$ is a unit vector in the positive z direction. For a homogeneous medium, $\lambda_0 = \phi$ and thus for a planar system we have

$$\vec{\nabla}\lambda_0 = \vec{\nabla}\phi = -\frac{\vec{e}_z}{d} \tag{45}$$

Both (43) and (44) reduce to this expression for $\varepsilon_1 = \varepsilon_2$.

8.2 CYLINDRICAL ELECTRODE GEOMETRY

We consider a coaxial cylindrical geometry with a two layer dielectric. If, in cylindrical coordinates, the inner electrode is represented by the surface $r = a$, while $r = b$ is the inner surface of the outer electrode, then $r = c$ represents the dielectric interface with $a < c < b$. The permittivity of the inner dielectric is ε_1 for which $a \leqslant r \leqslant c$, while that of the outer is ε_2, for which $c \leqslant r \leqslant b$.

If the complete inner electrode is used to detect the PD transient, then the boundary conditions for the λ_0 function are $\lambda_0 = 1$ for $r = a$ and $\lambda_0 = 0$ for $r = b$. Hence the λ_0 functions of the two media are given by

$$\lambda_{01} = \frac{\varepsilon_1 \ln(b/c) + \varepsilon_2 \ln(c/r)}{\varepsilon_1 \ln(b/c) + \varepsilon_2 \ln(c/a)} \tag{46}$$

for $a \leqslant r \leqslant c$, and

$$\lambda_{02} = \frac{\varepsilon_1 \ln(b/r)}{\varepsilon_1 \ln(b/c) + \varepsilon_2 \ln(c/a)} \tag{47}$$

for $c \leqslant r \leqslant b$. The λ_0 subscripts, 1 and 2, refer now to the inner and outer regions, respectively.

Upon differentiating with respect to r, we obtain the relevant expressions for the associated λ_0 gradients

$$\vec{\nabla}\lambda_{01} = \frac{-(\varepsilon_2/r)\vec{e}_r}{\varepsilon_1 \ln(b/c) + \varepsilon_2 \ln(c/a)} \tag{48}$$

and

$$\vec{\nabla}\lambda_{02} = \frac{-(\varepsilon_1/r)\vec{e}_r}{\varepsilon_1 \ln(b/c) + \varepsilon_2 \ln(c/a)} \tag{49}$$

where $\vec{e}_r$ is a unit vector perpendicular to the axis of the coaxial system and directed away from the inner electrode.

For a homogeneous medium, $\lambda_0 = \phi$ and thus for a coaxial system we have

$$\vec{\nabla}\lambda_0 = \vec{\nabla}\phi = \frac{-\vec{e}_r}{r \ln(b/a)} \tag{50}$$

Both (48) and (49) reduce to this expression for $\varepsilon_1 = \varepsilon_2$.

8.3 SPHERICAL ELECTRODE GEOMETRY

Consider a concentric spherical geometry with a two layer dielectric. If, in spherical coordinates, the inner electrode is represented by the surface $R = A$, while $R = B$ is the inner surface of the outer electrode, then $R = C$ represents the dielectric interface with $A < C < B$. The permittivity of the inner dielectric is ε_1 for which $A \leqslant R \leqslant C$, while that of the outer is ε_2, for which $C \leqslant R \leqslant B$.

If the complete inner electrode is used to detect the PD transient, then the boundary conditions for the λ_0 function are $\lambda_0 = 1$ for $R = A$ and $\lambda_0 = 0$ for $R = B$. Hence the λ_0 functions of the two media are given by

$$\lambda_{01} = \frac{\varepsilon_1(1 - C/B) + \varepsilon_2(C/R - 1)}{\varepsilon_1(1 - C/B) + \varepsilon_2(C/A - 1)} \tag{51}$$

for $A \leqslant R \leqslant C$, and

$$\lambda_{02} = \frac{\varepsilon_1(C/R - C/B)}{\varepsilon_1(1 - C/B) + \varepsilon_2(C/A - 1)} \tag{52}$$

for $C \leqslant R \leqslant B$. The λ_0 subscripts, 1 and 2, refer to the inner and outer regions.

Differentiating with respect to R leads to the relevant expressions for the corresponding λ_0 gradients

$$\vec{\nabla}\lambda_{01} = -\frac{1}{R}\frac{\varepsilon_2(C/R)\vec{e}_R}{\varepsilon_1(1 - C/B) + \varepsilon_2(C/A - 1)} \tag{53}$$

and

$$\vec{\nabla}\lambda_{02} = -\frac{1}{R}\frac{\varepsilon_1(C/R)\vec{e}_R}{\varepsilon_1(1 - C/B) + \varepsilon_2(C/A - 1)} \tag{54}$$

where $\vec{e}_R$ is a unit vector tangential with the radial coordinate and directed away from the inner electrode.

For a homogeneous medium, $\lambda_0 = \phi$ and thus for a spherical system we have

$$\vec{\nabla}\lambda_0 = \vec{\nabla}\phi = -\frac{1}{R}\frac{(B/R)\vec{e}_R}{(B/A) - 1} \tag{55}$$

For $\varepsilon_1 = \varepsilon_2$, both (53) and (54) reduce to this expression.

ACKNOWLEDGMENT

REFERENCES

[1] A. Pedersen, G. C. Crichton and I. W. McAllister, "The Theory and Measurement of Partial Discharge Transients", IEEE Trans. Elect. Insul., Vol. 26, pp. 487–497, 1991.

[2] A. Pedersen, G. C. Crichton and I. W. McAllister, "The Functional Relation Between Partial Discharges and Induced Charge", IEEE Trans. Diel. & Elect. Insul., Vol. 2, pp. 535-543, 1995.

[3] A. Pedersen, G. C. Crichton and I. W. McAllister, "Partial Discharge Detection: Theoretical and Practical Aspects", IEE Proc. - Science, Measurement and Technology, Vol. 142, pp. 29–36, 1995.

[4] G. C. Crichton, P. W. Karlsson and A. Pedersen, "Partial Discharges in Ellipsoidal and Spheroidal Voids", IEEE Trans. Elect. Insul., Vol. 24, pp. 335–342, 1989.

[5] I. W. McAllister, "Partial Discharges in Spheroidal Voids: Void Orientation", IEEE Trans. Diel. Elect. Insul., Vol. 4, pp. 456-461, 1997.