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Chance-Constrained AC Optimal Power Flow
Integrating HVDC Lines and Controllability

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Abstract—The integration of large-scale renewable generation has major implications on the operation of power systems, two of which we address in this paper. First, system operators have to deal with higher degrees of uncertainty. Second, with abundant potential of renewable generation in remote locations, they need to incorporate the operation of High Voltage Direct Current lines (HVDC). This paper introduces an optimization tool that addresses both challenges by incorporating: the full AC power flow equations and chance constraints to address the uncertainty of renewable infeed, HVDC modeling for point-to-point lines, and optimizing generator and HVDC corrective control policies in reaction to uncertainty. The main contributions are twofold. First, we introduce a HVDC line model and the corresponding HVDC participation factors in a chance-constrained AC-OPF framework. Second, we modify an existing algorithm for solving the chance-constrained AC optimal power flow to allow for optimization of the generation and HVDC participation factors. Using realistic wind forecast data, and a 10 bus system with one HVDC line and two wind farms, we demonstrate the performance of our algorithm and show the benefit of controllability.

Index Terms—AC optimal power flow, chance constraints, HVDC transmission, uncertainty.

I. INTRODUCTION

Power system operators have to deal with higher degrees of uncertainty. Increasing shares of unpredictable renewable generation, and stochastic loads, can lead to additional costs and jeopardize system security if uncertainty is not explicitly considered and addressed. In addition, with abundant renewable potential being available further away from load centers, e.g. off-shore, High-Voltage Direct Current lines (HVDC) become the preferred technology for transmitting large amounts of renewable energy over longer distances. In order to deal with uncertainty, operators carry out both preventive and corrective control actions in their system [1]. HVDC lines and grids can offer corrective control actions in the form of real-time control of active and reactive power flows. The goal of this paper is to propose a chance-constrained AC optimal power flow (AC-OPF) formulation that a) incorporates an HVDC line model and b) allows for an optimization of the generator and HVDC control response to fluctuations in renewable generation.

Existing literature considers uncertainty within the OPF problem using methods such as scenario-based or chance-constrained stochastic programming, or robust optimization methods. Stochastic formulations can include a set of scenarios describing possible realizations of uncertainty, or chance constraints which define a maximum allowable probability of constraint violation. In this paper we focus on the chance-constrained OPF. To deal with the higher complexity arising from the uncertain variables, existing approaches either assume a DC-OPF or use different techniques to achieve a tractable formulation of the chance constrained AC-OPF. Chance-constrained DC-OPF results to a faster and more scalable algorithm, but the DC-OPF is an approximation that neglects losses, reactive power, and voltage constraints. Refs. [2] and [3] formulate a chance constrained DC-OPF assuming a Gaussian distribution of the forecast errors. Alternatively, the works in [4], [5] use a linearization to achieve a tractable formulation of the chance constraints in the AC-OPF and use the full set of AC power flow equations to describe the forecasted system state. These approaches have a higher degree of accuracy as they incorporate the AC power system variables, and they can be shown to be scalable, despite not resulting to a convex optimization problem. For a convex formulation of a chance-constrained AC optimal power flow, the interested reader can refer to [6]–[9]. The work in [10] obtains tractable polynomial approximations to the chance constraints in the AC-OPF using semidefinite programming. Using robust optimization and bi-level programming, the work in [11] achieves a tractable chance-constrained AC-OPF formulation.

Previous work has already dealt with the incorporation of uncertainty and HVDC in a single optimization problem. For example, Refs. [12]–[15] consider stochastic formulations and they do incorporate HVDC lines and HVDC grids. However, they all assume a DC-OPF formulation. The focus of this paper is to avoid most of these simplifications to the extent that it is possible, and instead use the full AC power flow equations as the DC-OPF can lead to substantial errors [16].

In this paper, we propose an iterative chance-constrained AC-OPF for AC grids with HVDC lines, developing further the work described in [4] and elaborated in [17]. Our contributions are a) the integration of a simplified HVDC model and HVDC corrective control policies in the chance-constrained AC-OPF framework, and b) the optimization of both generator and HVDC participation factors within the iterative solution algorithm. The structure of this paper is as follows. Section II states the chance-constrained AC-OPF formulation. In Section III, the HVDC line model and HVDC corrective control policy is explained. Section IV introduces the iterative solution algorithm. Section V evaluates the performance of the proposed approach on a 10 bus test case. Section VI concludes.
II. OPTIMAL POWER FLOW FORMULATION

This section states the chance-constrained AC-OPF and presents a tractable reformulation of the chance constraints, which is based on the work from [4] and [17]. For ease of reference, we follow the notation of [17] wherever possible.

A. Chance-Constrained AC Optimal Power Flow

The chance-constrained AC-OPF aims at determining the least-cost operating point, which reduces the probability of violating the limits of system components to an acceptable level $\epsilon$ for a range of uncertainty realizations. Consequently, the AC-OPF variables, commonly defined in the space of $\mathbf{x} := \{\mathbf{P}, \mathbf{Q}, \mathbf{V}, \theta\}$ variables, are not only subject to one possible set of realizations of the uncertain parameters but also depend on their forecast errors $\omega$. $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{V}$ and $\theta$ denote vectors of nodal active and reactive power injections as well as nodal voltage magnitudes and angles, respectively. We assume wind power forecast errors $\omega$ to be the only source of uncertainty and to follow a multivariate Gaussian distribution with zero mean and known covariance, as the authors in [17] have shown is reasonably accurate, even when $\omega$ is not normally distributed.

The actual wind power realization $\tilde{\mathbf{P}}_W$ is modelled as the sum of its expected value $\mathbf{P}_W$ and the forecast error $\omega$.

$$\tilde{\mathbf{P}}_{W,i} = \mathbf{P}_{W,i} + \omega_i, \quad \forall i \in \mathcal{W},$$

where $\mathcal{W}$ denotes the subset of network nodes with wind generators connected to them. We assume that wind power plants are operated with a constant power factor, which means that their reactive power output follows their active power output, i.e., $\tilde{\mathbf{Q}}_{W,i} = \gamma (\tilde{\mathbf{P}}_{W,i} + \omega_i)$, where the power ratio $\gamma = \sqrt{1 - \cos^2 \phi}$ depends on the power factor $\cos \phi$ and can be a parameter or an optimization variable. The actual realizations of the OPF decision variables are modelled as the sum of their optimal set-points at the forecasted wind infeed $\mathbf{x}$ and their reactions to a change in wind power injection $\Delta \mathbf{x}(\omega)$, i.e., $\tilde{\mathbf{x}}(\omega) = \mathbf{x} + \Delta \mathbf{x}(\omega)$. This gives rise to the following formulation of the chance-constrained AC-OPF:

$$\begin{array}{ll}
\min_{\mathbf{x}} & \mathbf{c}^T \mathbf{P}_G + \mathbf{c}_1^T \mathbf{P}_G + c_0 \\
\text{s.t.} & f_i(\mathbf{x}) = 0 \quad \text{for } i = 1, \ldots, n \\
& \mathbb{P}(g_i(\tilde{\mathbf{x}}(\omega)) \leq 0) \geq 1 - \epsilon \quad \text{for } i = 1, \ldots, m.
\end{array}$$

The chance-constrained AC-OPF (2) – (4) minimizes the total generation cost for the forecasted operating point. The term $\mathbf{P}_G$ denotes the active power dispatch of the generators, and $\mathbf{c}_2$, $\mathbf{c}_1$, $c_0$ denote the quadratic, linear and constant cost factors, respectively. Constraint (3) enforces the $n = 2|\mathcal{N}|$ nodal active and reactive power balance equations for the forecasted operating point where $\mathcal{N}$ represents the set of network nodes. Note that we do not explicitly enforce the power balance for $\omega \neq 0$. Instead, as will be outlined in the following, our formulation ensures satisfaction of the linearized AC equations around the operating point, which in combination with the chosen control policies has been shown to perform well on the non-linear system for reasonable levels of uncertainty [17]. The $m$ inequality constraints in (4) include upper and lower limits on voltage magnitudes, voltage angles, active and reactive power generation as well as active power flows $\mathbf{P}_{\text{line}}$. They are formulated as chance constraints and enforced with a confidence level of $(1 - \epsilon)$. The chance constraints account for the entire range of $\omega$, as they can be analytically reformulated to tractable deterministic constraints, which will be discussed in Section II-A2.

1) Affine Policies: We model the control policies as affine functions of the uncertainty $\omega$. Conventional generators are assumed to balance fluctuations in active power generation according to their generator participation factors $\alpha$ for each generator $i \in \mathcal{G}$ according to

$$\tilde{\mathbf{P}}_{G,i}(\omega) = \mathbf{P}_{G,i} + \Delta \mathbf{P}_{G,i}(\omega) = \mathbf{P}_{G,i} - \alpha_i \mathbf{1} \omega + \delta^P,$$

where the term $\delta^P$ denotes the contribution to the compensation of the unknown changes in active power losses, $\mathbf{1}$ represents an all-ones row vector of size $|\mathcal{W}|$ and $\mathcal{G}$ denotes the set of nodes with conventional generation. The generator participation factors $\alpha$ are thus defined w.r.t. to the total wind power deviation $\Omega = \sum_{i \in \mathcal{W}} \omega_i$ and can be either pre-determined (e.g., as a result of a reserve procurement) or optimized within the OPF. The condition $\sum_{i \in \mathcal{G}} \alpha_i = 1$ ensures balance of the total power mismatch, i.e., $\sum_{i \in \mathcal{G}} \alpha_i \sum_{i \in \mathcal{W}} \omega_i = \Omega$. Active power losses vary non-linearly with the wind power deviation and are usually compensated by the generator at the reference bus; this results in the loss term $\delta^P$ being equal to zero for generators at PV and PQ buses. All other variables of interest $\Delta \mathbf{x}(\omega) := \{\Delta \mathbf{Q}_G, \Delta \mathbf{V}, \Delta \theta, \Delta \mathbf{P}_{\text{line}}\}$ are modeled similarly,

$$\tilde{x}_i(\omega) = x_i + \Gamma^x_i \omega,$$

where $\Gamma^x_i$ is a $(1 \times |\mathcal{W}|)$ vector defining the response of variable $x_i$ to each wind power deviation. In general, the response is modeled as follows: $\Delta \mathbf{x}(\omega) = \frac{\partial \mathbf{x}}{\partial \omega} \omega = \Gamma^x \omega$, where $\Gamma^x$ represents a matrix of linear sensitivities w.r.t. $\omega$. The term $\Gamma^x$ also includes expressions for the unknown changes in active power losses $\delta^P$ and is derived from the first order Taylor expansion of the AC power flow equations around the forecasted operating point,

$$\begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix} = \mathbf{J}_{\mathbf{x}} \begin{bmatrix} \Delta \theta \\ \Delta \mathbf{V} \end{bmatrix}.$$  

The term $\mathbf{J}$ denotes the Jacobian matrix. The left-hand side of (7) can also be expressed in terms of the wind deviation $\omega$, the power ratio $\gamma$, the generator participation factors $\alpha$ as well as the unknown nonlinear changes in active and reactive power (i.e., $\delta^P$, $\delta^Q$),

$$\begin{bmatrix} \mathbf{I} \\ \text{diag}(\gamma) \end{bmatrix} \omega + \begin{bmatrix} -\alpha \mathbf{H} \\ 0 \end{bmatrix} \omega + \begin{bmatrix} \delta^P \\ \delta^Q \end{bmatrix} = \Psi \omega + \begin{bmatrix} \delta^P \\ \delta^Q \end{bmatrix}.$$  

The terms $\mathbf{I}$, $\mathbf{H}$ and $\mathbf{0}$ denote $(|\mathcal{N}| \times |\mathcal{W}|)$ identity, all-ones and zero matrices, respectively. The matrix of Generation Distribution Factors (GDF) $\Psi$ depends linearly on $\alpha$ and $\gamma$ [18]. In accordance with common practices in power system operations, some variables are assumed not to change under
different wind power realizations, such as the voltage magnitude at PV and reference buses, the voltage angle at the reference bus and the reactive power injection at PQ buses. We summarize the nonzero changes of unknown active and reactive power injections in \( \delta := [\delta_{P,i}^c, \Delta Q_{r,f}, \Delta Q_{P,V}]^T \). Analogously, \( \Delta \hat{x} \) denotes the nonzero changes in voltage magnitudes and angles, i.e., \( \Delta \hat{x} := [\Delta \theta_{P,V}^T, \Delta \theta_{P,Q}^T, \Delta V_{P,Q}^T]^T \). Replacing the left-hand side in (7) with (8) and rearranging the resulting system of equations according to the groups of zero and nonzero elements, i.e.,

\[
\begin{bmatrix}
\delta \\
\hat{x}
\end{bmatrix} = \begin{bmatrix}
J_{mod}^I \\
J_{mod}^II
\end{bmatrix} \begin{bmatrix}
\Psi_{mod}^I \\
\Psi_{mod}^II
\end{bmatrix} \begin{bmatrix}
0 \\
\Delta \hat{x}
\end{bmatrix} - \begin{bmatrix}
\Psi_{mod}^I \\
\Psi_{mod}^II
\end{bmatrix} \omega,
\]

allows us to derive expressions (10) and (11) for the change in variables as a function of the uncertainty \( \omega \).

\[
\Delta \hat{x} = (J_{mod}^{IV})^{-1} \Psi_{mod}^{IV} \omega = \Gamma^x \omega \\
\delta = (J_{mod}^{II})(J_{mod}^{IV})^{-1} \Psi_{mod}^{II} - \Psi_{mod}^{IV} \omega = \Gamma^\delta \omega
\]

(9)

(10)

(11)

\( J_{mod} \) and \( \Psi_{mod} \) denote the modified Jacobian and GDF matrices, where the rows and columns have been rearranged according to \( \delta \) and \( \Delta \hat{x} \). Thus, the linear sensitivities \( \Gamma^x \) depend on the GDF matrix \( \Psi \), which is a linear function of the generator participation factors \( \alpha \) and the power ratio \( \gamma \).

2) Reformulating the Chance Constraints: Given the linear dependency of the OPF variables on \( \omega \) in the region around the operating point and the assumption of a multivariate normal distribution for \( \omega \), we are able to reformulate the chance constraints (4) to tractable deterministic constraints. The linear chance constraint \( \mathbb{P}(x_i + \Gamma^x_i(\Psi)\omega \leq x_i^{max}) \geq 1 - \epsilon \) is reformulated to

\[
x_i \leq x_i^{max} - \Phi^{-1}(1 - \epsilon) \sqrt{\Gamma^x_i \Sigma(\Gamma^x_i)^T},
\]

where \( \Phi^{-1} \) denotes the inverse cumulative distribution function of the Gaussian distribution. It can be observed that the original constraint \( x_i \leq \frac{1}{\sigma_i^2} \) is tightened by an uncertainty margin \( \lambda^x_i := \Phi^{-1}(1 - \epsilon) \sqrt{\Gamma^x_i \Sigma(\Gamma^x_i)^T} \), which secures the system against variations in wind infeed [4]. Given the dependency of \( \Gamma^x \) on \( \Psi \), optimizing over the generation response \( \alpha \) explicitly represents its impact on the uncertainty margins of the remaining variables within the optimization.

III. HVDC Line Modeling

In this section, we present a model to include HVDC lines in the chance-constrained AC-OPF and we introduce HVDC participation factors to allow for corrective control. We assume that the HVDC lines are modeled as presented in Fig. 1 with individual active and reactive power injections \( P_{HVDC}, Q_{HVDC} \) at the two AC buses the HVDC line is connected to. The set \( (i,j) \in \mathcal{N}_{HVDC} \) denotes the corresponding AC buses. We approximate the active and reactive power capability of the converter as a rectangular box with the following constraints:

\[
P_{HVDC}^{min} \leq P_{HVDC} \leq P_{HVDC}^{max} \\
Q_{HVDC}^{min} \leq Q_{HVDC} \leq Q_{HVDC}^{max}
\]

Expressing the lower and upper active and reactive HVDC converter limits \( P_{HVDC}^{min}, P_{HVDC}^{max}, Q_{HVDC}^{min}, Q_{HVDC}^{max} \) as a function of the nominal converter rated power \( S_{nom}^{HVDC} \) and assuming that \( P_{HVDC}^{min} = -P_{HVDC}^{max} \) yields:

\[
-m_{p,c} S_{nom}^{HVDC,c} \leq P_{HVDC,c} \leq m_{p,c} S_{nom}^{HVDC,c} \forall c \in \mathcal{C} \\
m_{q,c} S_{nom}^{HVDC,c} \leq Q_{HVDC,c} \leq m_{q,c} S_{nom}^{HVDC,c} \forall c \in \mathcal{C}
\]

The set \( \mathcal{C} \) denotes the HVDC converters. The resulting feasible operating region is visualized in Fig. 2. For a more detailed modeling of the active and reactive power capability of HVDC converter the interested reader is referred to [20]. In order to link the active power injections between the two AC buses that the HVDC line is connected to, an active power balance constraint has to be included. To model the DC system losses \( P_{loss} \), we use a constant loss term \( \epsilon \) defined as a share of the nominal apparent power rating for buses \( (i,j) \in \mathcal{N}_{HVDC} \):

\[
P_{HVDC,i} + P_{HVDC,j} + P_{loss} = 0 \quad \text{with} \quad P_{loss} = 2\epsilon S_{nom}^{HVDC}
\]

(13)

(14)

(15)

(16)

This term gives an estimate of the HVDC converter losses. Note that we neglect the DC line losses, and that the reactive power injections at both AC buses \( (i,j) \in \mathcal{N}_{HVDC} \) can be chosen independently from each other within the HVDC converter limits. To allow for corrective control, we assign a participation factor \( \beta_c \) for each HVDC converter similarly to the case of generators. As the HVDC line itself cannot generate power, the participation factor is positive at one end of the HVDC line and negative at the other end, i.e. \( \beta_i = -\beta_j \) for buses \( (i,j) \in \mathcal{N}_{HVDC} \). This controllability can be used to e.g. reroute power to reduce congestion in case of forecast errors. The GDF matrix \( \Psi \) is modified as follows:

\[
\Psi = \begin{bmatrix}
I - (\alpha + \beta)H \\
\text{diag}(\gamma)
\end{bmatrix}
\]

(17)

(18)

The HVDC participation factors \( \beta \) are nonzero only for the converter connected AC buses and its sign depends on which end of the HVDC line the AC bus is connected to. Similar to the engineering constraints of the AC grid, the converter
limits need to be considered as chance constraints in order to ensure secure operation with sufficient probability throughout the uncertainty range, e.g.,

\[
\begin{align*}
&P(-m_{p,c}^{\text{nom}} \leq P_{\text{HVDC},c} + \beta_c, \omega) \geq 1 - \epsilon \quad \forall c \in C, \quad (19) \\
&P(m_{p,c}^{\text{nom}} \geq P_{\text{HVDC},c} + \beta_c, \omega) \geq 1 - \epsilon \quad \forall c \in C. \quad (20)
\end{align*}
\]

These can be reformulated for each converter \(c \in C\):

\[
\begin{align*}
-m_{p,c}^{\text{nom}} &+ \Phi^{-1}(1 - \epsilon)\sqrt{\beta_c} \Sigma \beta_c^T \leq P_{\text{HVDC},c} \quad (21) \\
m_{p,c}^{\text{nom}} &- \Phi^{-1}(1 - \epsilon)\sqrt{\beta_c} \Sigma \beta_c^T \geq P_{\text{HVDC},c}. \quad (22)
\end{align*}
\]

Note that the uncertainty margins \(\lambda^{\text{HVDC}}\) introduced in (21) and (22) depend linearly on the HVDC participation factor \(\beta\). The degree of controllability is determined by \(\alpha\) and \(\beta\), both of which can be either pre-determined or optimized within the chance-constrained AC-OPF.

### IV. Iterative Chance-constrained AC-OPF Optimizing Generator and HVDC Control Policies

The reformulated chance-constrained AC-OPF (23) – (27) considering HVDC lines extends the variable set \(x\) to include the active and reactive power set-points of the HVDC converters (\(P_{\text{HVDC}}, Q_{\text{HVDC}}\)) and possibly \(\alpha\) and \(\beta\) if the corrective control actions provided by conventional generators and HVDC lines are optimized within the same framework.

\[
\begin{align*}
\min_{x} & \quad c_2^T P_G^2 + c_1^T P_G + c_0 \\
\text{s.t.} & \quad f^{\text{ac}}(x) = 0 \\
& \quad f^{\text{dc}}(P_{\text{HVDC}}) = 0 \\
& \quad x \leq x^{\max} - \lambda^{\alpha}(\alpha, \beta) \\
& \quad x \geq x^{\min} + \lambda^{\alpha}(\alpha, \beta) \quad \text{(27)}
\end{align*}
\]

It introduces for each HVDC line an additional power balance equation (25) according to (17) considering the losses in the DC system. All inequality constraints are tightened with their corresponding uncertainty margins \(\lambda^{\alpha}(\alpha, \beta) = [\lambda^{P_G}, \lambda^{Q_\text{G}}, \lambda^V, \lambda^{P_{\text{HVDC}}, \lambda^{Q_{\text{HVDC}}}}(\alpha, \beta)]\). The uncertainty margins do not only depend on the generator and HVDC participation factors but also on the Jacobian matrix of the AC power flow equations as can be observed in (10) and (11). Including the Jacobian terms as optimization variables would introduce even more non-linearities in the AC-OPF and thus, substantially increase the complexity of the problem. To this end, the authors in [17] have introduced a computationally efficient iterative solution algorithm, which decouples the uncertainty assessment (i.e., the derivation of the uncertainty margins) from the optimization.

To maintain computational efficiency, we extend the iterative framework of [17] and evaluate the Jacobian at each iteration for the current operating point. In [17], the uncertainty margins were constants and were computed in an outer iteration. In the current paper, the sensitivity factors are constants, while \(\alpha\) and \(\beta\) are kept as optimization variables, which allows us to optimize these at the expense of adding non-linear (but convex) second order cone (SOC) terms. We define the steps in Algorithm 1, where subscript \(\text{opt}\) denotes the optimal solution of an OPF. The algorithm converges as the change in uncertainty margins between two consecutive iterations falls below a defined tolerance value \(\rho\).

**Algorithm 1 Iterative Chance-Constrained AC-OPF Optimizing Generator and HVDC Corrective Control Policies**

1: Set iteration count: \(k \leftarrow 0\)
2: initialize \(\lambda^{x,0} = 0\)
3: while \(\|\lambda^{x,k} - \lambda^{x,k-1}\|_{\infty} > \rho\) do
4: if \(k = 0\) then
5: solve (23) – (27) for \(x \backslash \{\alpha, \beta\}\) and obtain \(x^{\text{opt}}_{k}\) 
6: evaluate Jacobian at \(x^{k}_{\text{opt}}\).
7: else
8: include \(\lambda^{x,k}(\alpha^k, \beta^k)\) according to (26) and (27) 
9: solve (23) – (27) to obtain \(x^{\text{opt}}_{k}\) 
10: evaluate Jacobian, \(\Gamma^{k}_{\text{opt}}\) and \(\lambda^{k}_{\text{opt}}\) at \(x^{\text{opt}}_{k}, \alpha^{k}_{\text{opt}}\) and \(\beta^{k}_{\text{opt}}\)
11: end if
12: derive expressions for \(\Gamma^{x,k+1}\) and \(\lambda^{x,k+1}\) as functions of optimization variables \(\alpha^{k+1}\) and \(\beta^{k+1}\)
13: \(k \leftarrow k + 1\)
14: end while.

### V. Test Case and Simulations

We specify the simulation setup for the 10 bus test case. In the first part, we show the benefit of optimizing the generator participation factors for the proposed iterative chance-constrained AC-OPF. In the second part, we include an HVDC line to relieve congestion in the AC system and investigate optimizing both the generator and HVDC control policies. The optimization problems are implemented with YALMIP [21] in MATLAB and are solved with IPOPT [22].

#### A. Simulation Setup

The considered 10 bus system is shown in Fig. 3. The grid parameters are provided in [23]. In this system configuration, the flow of power is from the upper left to the main load units at buses 7 to 10 and the transmission line from bus 2 to bus 10 is congested. Two wind farms are located at buses 10 and 4 with a maximum power of 1.0 GW and 2.5 GW, respectively. To compute the covariance matrix \(\Sigma\) of the forecast errors, we use realistic day-ahead wind forecast.
scenarios from [24]. The forecasts are based on wind power measurements in the Western Denmark area from 15 different control zones collected by the Danish transmission system operator Energinet. We select control zone 7 and 9 at time step 4 to correspond to the wind farms at bus 2 and 10, respectively. In order to construct the covariance matrix we draw 100 random samples from this data. The forecasted wind infed is computed as the mean of these 100 samples.

To evaluate the performance of the proposed approaches we use two metrics. First, we compute the cost of uncertainty which is the increase in generation cost by including chance constraints, i.e. considering uncertainty explicitly. Second, we perform an in- and out-of-sample analysis to compute the empirical constraint violation probability. For in-sample analysis we draw 10'000 samples from a Gaussian distribution and for out-of-sample analysis we use 10'000 samples from the realistic forecast data. For the Monte Carlo Analysis we assume a minimum violation limit of 0.1% to exclude numerical errors. The following assumptions are made regarding the system constraints and specifications. Upper and lower voltage limits of 1.1 p.u. and 0.9 p.u. are assumed. The maximum allowable constraint violation limit is set to $\epsilon = 5\%$. As we consider the active branch flow limit we set the maximum active branch limit to 80% of the apparent branch flow limit. The wind farm power factor $\gamma$ is set to 1.

### B. Optimization of Generator Participation Factors

In this section, for a 10 bus test case, we show the benefit in terms of generation cost of optimizing the generator participation factors $\alpha$ instead of assigning uniform participation factors. The fixed participation factors are $\alpha = [0.2 0.2 0.2 0.2 0.2]$, i.e. each generator equally compensates the deviation in wind power. We compare the performance of an AC-OPF without considering uncertainty, the iterative chance-constrained AC-OPF (CC-AC-OPF) with fixed generator participation factors and the latter (CC-AC-OPF) with optimizing the generator participation factors. For the 10 bus test case, the cost of uncertainty evaluates to 2.03% for fixed participation factors. This can be reduced to 0.79% by optimizing the participation factors.

The results for the Monte Carlo Analysis for in- and out-of-sample testing are shown in Table I. Both in the in- and out-of-sample analyses the AC-OPF without considering uncertainty leads to large empirical violation probabilities for the active generator limits and the active branch flow limits as the response of generators to the wind power deviations is not considered. Voltage violations are observed as well. In case we use the proposed iterative chance-constrained AC-OPF with fixed and optimized generator participation factors we reduce the empirical violation probability both in- and out-of-sample very close to the desired 5%. The remaining minor mismatch can be either attributed to a wrong estimation of the mean and covariance in the out-of-sample analysis or to the approximation we make by using the first-order Taylor expansion to linearize the system behaviour around the forecasted operating point. Note that the forecast errors drawn from the realistic forecast data are not Gaussian distributed and the observed violations out-of-sample can therefore be larger. However, we observe that they are still close to the desired 5% indicating good performance of the proposed algorithm.

If we optimize the generator participation factors, we obtain $\alpha_{\text{opt}} = [0.0 0.30 0.57 0.0 0.13]$. In Fig. 4 we compare the generation dispatch and the uncertainty margins for the three formulations. We can observe that by optimizing the participation factors the generator response is shifted to the generators G2, G3 and G5 with mainly generator G3 compensating the wind power mismatch. The cheap generators G1 and G4 operate at their maximum power output for the forecasted system operating state.

### C. Including HVDC Line and HVDC Control Policies

We replace the AC line between buses 2 and 10 with an HVDC line of $S_C^{\text{nom}} = 4\text{ GVA}$, and investigate the relief of congestion and decrease of the cost of uncertainty. We assume the converters are of the multi-modular converter (MMC) technology and that the total losses per converter station are approximately $c = 1\%$ per HVDC converter according to [25], and for the active and reactive power capability of the converter the limits are chosen as $m_p = 0.8$, $m_{q_{\min}} = 0.4$, $m_{q_{\max}} = 0.5$ [19]. The generation cost for the AC-OPF

---

**TABLE I**

**EMPIRICAL CONSTRAINT VIOLATION PROBABILITY FOR 10 BUS TEST CASE WITHOUT HVDC LINE**

<table>
<thead>
<tr>
<th>Constraint limits on</th>
<th>P</th>
<th>Q</th>
<th>V</th>
<th>$P_{\text{line}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-sample analysis with 10'000 samples (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AC-OPF (w/o uncertainty)</td>
<td>49.0</td>
<td>0.0</td>
<td>6.7</td>
<td>49.7</td>
</tr>
<tr>
<td>CC-AC-OPF (fixed $\alpha$)</td>
<td>5.3</td>
<td>0.0</td>
<td>2.8</td>
<td>5.3</td>
</tr>
<tr>
<td>CC-AC-OPF (opt. $\alpha$)</td>
<td>4.9</td>
<td>0.0</td>
<td>2.9</td>
<td>4.9</td>
</tr>
<tr>
<td>Out-of-sample analysis with 10'000 samples (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AC-OPF (w/o uncertainty)</td>
<td>43.2</td>
<td>0.0</td>
<td>4.6</td>
<td>49.2</td>
</tr>
<tr>
<td>CC-AC-OPF (fixed $\alpha$)</td>
<td>5.8</td>
<td>0.0</td>
<td>3.4</td>
<td>6.1</td>
</tr>
<tr>
<td>CC-AC-OPF (opt. $\alpha$)</td>
<td>5.8</td>
<td>0.0</td>
<td>3.4</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Fig. 4. A comparison of (a) normalized generation dispatch and (b) uncertainty margins for active power for AC-OPF without considering uncertainty and the chance-constrained AC-OPF with fixed and optimized generator participation factors. Note that lower active limits of all generators is zero.
without considering uncertainty is decreased by 4.3% due to upgrading the AC to the HVDC line and thereby reducing the congestion level of the system. In case we assume fixed generator participation factors $\alpha = [0.2 0.2 0.2 0.2 0.2]$ and HVDC participation factor $\beta = 0$, the cost of uncertainty amounts to 2.2%. By optimizing both the generator and HVDC participation factors, the cost of uncertainty can be reduced to 0.0%, i.e. the available HVDC and generator controls are sufficient to absorb the uncertainty associated with the two wind farms without any cost increase.

In Table II, the empirical constraint violation probability for an AC-OPF without considering uncertainty, an iterative CC-AC-OPF with fixed $\alpha$ and $\beta$ and an iterative CC-AC-OPF with optimized $\alpha$ and $\beta$ is shown. We observe again that without considering uncertainty, large violations of the generator active, voltage, and active branch flow limits occur. Both the CC-AC-OPF with fixed and optimized $\alpha$ and $\beta$ achieve a satisfactory performance in- and out-of-sample. For the considered test case, the optimized generator participation factors evaluate to $\alpha = [0.0 0.0 1.0 0.0 0.0]$ and the optimized HVDC participation factor $\beta$ evaluates to 0.1032.

VI. CONCLUSIONS

The iterative chance-constrained AC-OPF formulation outlined in [4], [17] was extended to account for a) HVDC lines and HVDC corrective control policies, and b) the optimization of both generator and HVDC participation factors within the iterative solution algorithm. Using realistic forecast data, for a 10 bus test case with one HVDC line and two wind farms, we showed a reduction in generation cost by optimizing both the generator and HVDC control response to wind power fluctuations. While not considering uncertainty in the AC-OPF leads to substantial constraint violations, our proposed approach achieves an excellent performance as shown in both in- and out-of-sample analyses. Future work includes a) detailed HVDC converter modeling, b) interconnected AC and HVDC grids, and c) data-driven approaches [26].

REFERENCES


