1. Motivation

- Applications: Computer vision, web data analysis, anomaly detection, and data visualization, etc.
- Robust Principal Component Analysis (RPCA): Batch-based, decomposes all data samples (matrix $M$) into low-rank ($L$) and sparse ($S$), e.g., all frames in a video, high computational and memory requirements

$$\min_{L, S} \left[ \|L\|_1 + \lambda \|S\|_1 \right], \text{subject to } M = L + S$$

Challenges
- Online method processing a sequence of signals per time instance from a small set of measurements $y_t = \Phi(x_t + \nu_t)$
- $L_t = L_t + S_t$ into $S_t = [x_1 \cdots x_t]$ and $L_t = [v_1 \cdots v_t]$
- Minimization at time instance $t$

$$\min_{x_t, v_t} \left[ \|L_t - v_t\|_1 + \lambda \|x_t\|_1 \right], \text{subject to } y_t = \Phi(x_t + v_t)$$

where $\lambda > 0$ and $\beta > 0$ are weights across the side information signals, and $W_t$ is a diagonal matrix with weights for each element in the side information signal $u_i$, namely, $W_t = \text{diag}(w_{1}, \ldots, w_{n})$ with $w_{i} > 0$ being the weight for the $i$-th element in the $x_i$ vector.

2. Compressive Online RPCA (CORPCA) With Multiple Prior Information

Problem formulation
- Incorporating multiple prior information: at time instance $t$ we observe $y_t = \Phi(x_t + \nu_t)$ with $y_t \in \mathbb{R}^{m}$ given priors $Z_{t-1}$ and $B_{t-1}$ from $[x_1 \ldots x_{t-1}]$ and $[v_1 \ldots v_{t-1}]$
- Solving the $\ell_1$ minimization problem

$$\min_{x_t, v_t} \left[ \|L_t - v_t\|_1 + \lambda \|x_t\|_1 \right], \text{subject to } y_t = \Phi(x_t + v_t)$$

where $\lambda > 0$ and $\beta > 0$ are weights across the side information signals, and $W_t$ is a diagonal matrix with weights for each element in the side information signal $u_i$, namely, $W_t = \text{diag}(w_{1}, \ldots, w_{n})$ with $w_{i} > 0$ being the weight for the $i$-th element in the $x_i$ vector.

The CORPCA algorithm
- Solving $\ell_1$ minimization via the soft thresholding operator and the single value thresholding operator, at iteration $k + 1$

$$v_{t+1} = \text{arg min} \left[ \|h(v_t) - n_t - \left( (m - 1)^{-\frac{1}{2}} \frac{1}{2} \frac{1}{w_{i}} \sum_{i} W_t |x_i|^{2} \right) \|_{2}^{2} \right]$$

$$x_{i, t+1} = \text{arg min} \left[ \|h(x_{i, t}) - n_{i, t} - \left( (m - 1)^{-\frac{1}{2}} \frac{1}{w_{i}} \sum_{i} W_t |x_i|^{2} \right) \|_{2}^{2} \right]$$

where $h(u) = (1/2) [\Phi(x_i + v_t) - u]$, $y_{i, t} = \sum_{i} W_t |x_i|^{2}$, and $h(v_t)$ is the $\ell_1$ norm of $v_t$.

- Updating weights $\beta$ and $W_j$

- After solving for time instance $t$: Prior updates

$$Z_t = \left( z_t = d_{t-1} + \frac{1}{\beta} \right)$$

$$B_t = \left( \beta \sum_{i=1}^{T} \beta W_i (z_{i} - d_{i}) \right)$$

3. Experimental Results

- Synthetic data
  - Generating low-rank components: $n = 500$, $d = 100$ (training), $q = 100$ (testing), $r = 5$ (rank)
  - $L = UV^T$, where $U \in \mathbb{R}^{500 \times 20}$ and $V \in \mathbb{R}^{20 \times 500}$
  - Generating sparse components with $\|x_0\|_0 = 80$ and $\|x_1 - x_0\|_0 = 100$ obtaining $S = [x_1 \ldots x_5]$
  - Testing on $M = [x_0; \ldots; x_5; \cdots; x_{100}]$
  - Measuring probabilities of successful decomposition, $Pr(\text{success})$, success if $\|x_t - x_{t-1}\|_2 / \|x_t\|_2 \leq 10^{-2}$

- Compressive video foreground-background separation
  - Considering two videos, Bootstrap (60x80 pixels) and Curtain (64x80 pixels) having a static and a dynamic background, respectively
  - Background-foreground video separation with full access to the video data
  - Compressive separating by varying rates on the number of measurements $m$ over the dimension of the data $n$

4. Summary

- Solution for an $\ell_1$ minimization
  - Incorporating efficiently multiple prior information
  - Updating iteratively weights

- The proposed CORPCA algorithm
  - Processing a data vector per time instance using compressive measurements
  - Solving the $\ell_1$ minimization and updating priors for the next instance

- Evaluation of CORPCA on synthetic data and actual video data
  - Outperforming classical compressive sensing (CS) ($\ell_1$ minimization) and CS with single prior information ($\ell_1$ + $\ell_1$ minimization)
  - The superior performance improvement compared to the existing methods

- CORPCA source code, test sequences, and the corresponding outcomes.
- [Available]: https://github.com/huynhlvd/COPRCA