Compressive Online Robust Principal Component Analysis with Multiple Prior Information

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1. Motivation

- Applications: Computer vision, web data analysis, anomaly detection, and data visualization, etc.

- Robust Principal Component Analysis (RPCA): Batch-based, decomposes all data samples (matrix $M$) into low-rank ($L$) and sparse ($S$), e.g., all frames in a video, high computational and memory requirements.

$$\min_{L, S} \|L\|_2 + \lambda \|S\|_1 \text{ subject to } M = L + S$$

Challenges:

- Online method processing a sequence of signals per time instance from a small set of measurements $y_t = \Phi(x_t + v_t)$

$$M_t = L + S_t \text{ into } S_t = [x_1, x_2 \ldots, x_t] \text{ and } L_t = [v_1, v_2 \ldots, v_t]$$

- Minimization at time instance $t$

$$\min_{L_t, S_t} \|L_t\|_2 + \lambda \|S_t\|_1 \text{ subject to } y_t = \Phi(x_t + v_t)$$

where $\lambda > 0$ and $\beta > 0$ are weights across the side information signals, and $W_0$ is a diagonal matrix with weights for each element in the side information signal $x_t$, namely, $W_0 = \|w_0\|_0 \geq \|w_0\|_1$, with $w_0 > 0$ being the weight for the $i$-th element in the $x_i$ vector.

2. Compressive Online RPCA (CORPCA) With Multiple Prior Information

Problem formulation:

- Incorporating multiple prior information: at time instance $t$ we observe $y_t = \Phi(x_t + v_t)$ with $y_t \in \mathbb{R}^n$ given priors $Z_{t-1}$ and $B_{t-1}$ from $\{x_1, \ldots, x_{t-1}\}$ and $\{v_1, \ldots, v_{t-1}\}$

$$\min_{L_t, S_t} \|L_t\|_2 + \lambda \|S_t\|_1 \text{ subject to } M_t = L_t + S_t$$

Solving the $\ell_1$ minimization problem:

$$\min_{x_t, v_t} \|H^{-1}(x_t, v_t; Z_{t-1}, B_{t-1})\|_2^2 + \lambda \|W^2(x_t - Z_{t-1})\|_1 + \beta \|B_{t-1}, v_t\|_1, \text{ subject to } y_t = \Phi(x_t + v_t)$$

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The CORPCA algorithm:

- Solving $\ell_1$ minimization via the soft thresholding operator and the single value thresholding operator, at iteration $k = 1$

$$\begin{align*}
\hat{x}_t^{(1)} &= \arg \min_{x_t} \|h(y_t) - (\|y_t\|_2^2 + \lambda \|W(x_t - y_t)\|_1)\|_2^2 \\
\hat{v}_t^{(1)} &= \arg \min_{v_t} \|h(y_t) - (\|y_t\|_2^2 + \lambda \|W(x_t - y_t)\|_1)\|_2^2
\end{align*}$$

where $f(x_t)$ is a $(1/2) \|\Phi(x_t + v_t) - y_t\|_2^2$ function, $y_t = \sum_{i=1}^{n} y_i^{(1)}$, and $h(x_t) = \|B_{t-1}, v_t\|_1$

- Updating weights $\beta$ and $W_{0j}$

- After solving for time instance $t$: Prior updates

$$Z_t := \{z_1, \ldots, z_{t-1}\}$$

$$B_t = U_t(:, 1 : d) \Gamma_{t}^{1/2} S_t(1 : d, 1 : d) V_t(1 : d, 1 : d)^{T}$$

3. Experimental Results

Compressive video foreground-background separation

- Considering two videos, Bootstrap (60x80 pixels) and Curtain (64x80 pixels) having a static and a dynamic background, respectively.

- Background-foreground video separation with full access to the video data.

- Compressive separating by varying rates on the number of measurements $m$ over the dimension of the data $n$

4. Summary

Solution for an $\ell_1$ minimization

- Incorporating efficiently multiple prior information

- Updating iteratively weights

The proposed CORPCA algorithm

- Processing a data vector per time instance using compressive measurements

- Solving the $\ell_1$ minimization and updating priors for the next instance

Evaluation of CORPCA on synthetic data and actual video data

- Outperforming classical compressive sensing (CS) ($\ell_1$ minimization) and CS with single prior information ($\ell_1$ minimization)

- The superior performance improvement compared to the existing methods

CORPCA source code, test sequences, and the corresponding outcomes.

[Available]: https://github.com/huytanhoc/corppca