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Analyses of electron runaway in front of the negative streamer channel

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Abstract X-ray and γ-ray emissions, observed in correlation with negative leaders of lightning and long sparks of high-voltage laboratory experiments, are conventionally connected with the bremsstrahlung of high-energy runaway electrons (REs). Here we extend a focusing mechanism, analyzed in our previous paper, which allows the electric field to reach magnitudes, required for a generation of significant RE fluxes and associated bremsstrahlung, when the ionization wave propagates in a narrow, ionized channel created by a previous streamer. Under such conditions we compute the production rate of REs per unit streamer length as a function of the streamer velocity and predict that, once a streamer is formed with the electric field capable of producing REs ahead of the streamer front, the ionization induced by the REs is capable of creating an ionized channel that allows for self-sustained propagation of the RE-emitting ionization wave independent of the initial electron concentration. Thus, the streamer coronas of the leaders are probable sources of REs producing the observed high-energy radiation. To prove these predictions, new simulations are planned, which would show explicitly that the preionization in front of the channel via REs will lead to the ionization wave propagation self-consistent with RE generation.

Plain Language Summary X-ray and γ-ray emissions, observed in correlation with the negative leaders of lightning and long sparks of high-voltage laboratory experiments, are conventionally connected with the bremsstrahlung of high-energy runaway electrons (REs). Here we extend a focusing mechanism, analyzed in our previous paper, which allows the electric field to reach magnitudes, required for a generation of significant RE fluxes and associated bremsstrahlung, when the ionization wave propagates in a narrow, ionized channel created by a previous streamer. Under such conditions we compute the production rate of REs per unit streamer length as a function of the streamer velocity and find that, once a streamer is formed with the electric field capable of producing REs ahead of the streamer front, the ionization induced by the REs creates an ionized channel that allows for self-sustained propagation of the RE-emitting ionization wave independent of the initial electron concentration. Thus, the streamer coronas of the leaders are probable sources of REs producing the observed high-energy radiation.

1. Introduction

This work extends the formulation in Babich et al. (2014a, 2015), describing a mechanism whereby the electric field in the streamer corona of negative leaders of lightning and long laboratory spark is enhanced to a level allowing a production of the X- and γ-ray emissions, as observed in a correlation with natural lightning and in high-voltage laboratory experiments (Dwyer et al., 2004, 2005, 2008, 2012a, 2012b; Nguyen et al., 2008; Rahman et al., 2008; March and Montanya, 2010; Kochkin et al., 2012, 2014, 2015; Ostgaard et al., 2016; Agafonov et al., 2013). The radiation is the bremsstrahlung of high-energy runaway electrons (REs), for which production strong electric fields are required. Direct sensor measurements of electric field inside the leaders or streamers are not possible even in the controlled laboratory discharges due to too small spatial and temporal scales of the domains occupied by the field; therefore, the X- and γ-ray emissions are the only evidence that the electric field intensity reduced to the number density of neutrals $E/N$ during the leader/streamer processes is increased significantly above the reduced intensity of the external field $(E/N)_{ext}$, which is too low for electrons to be energized to the levels allowing the high-energy emissions.

It is known that leaders consist of a hot plasma channel with the streamer corona at the leader tip. Lightning negative leaders and upward positive leaders have been shown experimentally to propagate in a stepwise manner (Rakov and Uman, 2003, sections 5.3.2 and 5.3.3; Dwyer et al., 2005; Bazelyan and Raizer, 2000, section 3.1, and citations therein). Downward positive leaders have been observed to pulse, which suggests that they may also step (Wang and Takagi, 2011; Rakov and Uman, 2003, sections 5.3.2 and 5.3.3), but this has yet to be
explicitly demonstrated or observed. Principles of the stepping process were formulated by Gorin et al. [1976]; its mechanism was studied by observations of rocket-and-wire triggered lightning discharge [Biagi et al., 2010, 2011, 2014].

Because the field is very weak within the highly conductive hot leader channel, the analyses carried out in this paper are limited to further development of the so-called “thermal” [Moss et al., 2006] or “cold” [Dwyer, 2004] runaway in strong electric fields that may occur at heads of streamers constituting the leader corona. The field in the corona of the previous (old) step must be increased to a level that allows the RE generation before the field in this region is reduced by the next step and is reformed at the new tip location.

However, streamers, in initially unionized media, experience strong transversal expansion, which limits the field intensity to very low level [Raizer and Simakov, 1998]. Therefore, Babich et al. [2014a, 2015] proposed that if the leader corona consists of narrow ionized channels, such as formed by streamers, new ionization waves propagating in such channels allow overcoming this limitation. They noted that the limitation of the electric field magnitude due to the transversal streamer expansion can be removed because the secondary ionization waves, propagating in already ionized media, tend to focus. Focusing is a result of ionization wave velocity decrease with the ionization degree decrease [Bazelyan and Raizer, 1998, section 3.1.3]. As the electron concentration in a streamer channel decreases from the inner domain to its periphery, the secondary ionization wave propagates with the highest velocity at the center of the channel. Therefore, the transverse expansion of the ionization wave occurs much slower than the longitudinal wave propagation, and the wave transverse size tends to be limited to a region around the axis of the (old) streamer channel. As a result, the field intensity locally can reach the magnitudes required for the efficient RE generation [Bakhov et al., 2000].

Allowing for the ionization wave focusing, we shall model one part of the leader stepping process, namely, only forward propagation of the ionization wave, which velocity vector $\vec{v}_f$ is directed along the external electric force $-e\vec{E}_{\text{ext}}$. The negative leader is assumed to develop as illustrated in Figure 1 [Babich et al., 2014a, 2015]. According to Bazelyan and Raizer [2000, section 2.7], a scenario of the negative leader development is as follows. At the external boundary of the leader corona, a plasma formation develops, extending along the electric field vector. From both ends of this formation, streamers start in directions to and from the leader head. Further, the plasma formation with streamers transforms in the so-called space leader with negative
and positive coronas. The positive head of the space leader with its corona moves toward the negative head of the main leader channel and, at contact, a united conducting channel is being formed with the potential of the old leader being transferred to the space leader [Moss et al., 2006; Celestin and Pasko, 2011]. The potential jump in the body of the space leader launches new ionization waves along the old streamer channels in the negative streamer corona ahead of the new leader head.

Proceeding from this scenario, Babich et al. [2014a, 2015] modeled the ionization wave propagation in a streamer channel using the diffusion-drift approximation of the fluid equations for free electrons and ions and showed that the intensity of the ionization wave field may reach magnitudes above 240 kV cm⁻¹ [Bakhov et al., 2000] required for efficient generation of REs in the lower atmosphere. They further computed the magnitudes of the ionization wave speed \( v_{\text{fi}} \) space-time distributions of the electron concentration \( n_e(z,\rho, t) \), and self-consistent field intensity \( \vec{E}(z,\rho, t) \) in cylindrical set of coordinates \((\vec{z},\vec{\rho})\) with \( \vec{z} \) and \( \vec{\rho} \) directed accordingly against and orthogonal to the external field vector \( \vec{E}_{\text{ext}} \). On the basis of the obtained results and with the use of a dependence of the RE production rate \( \nu_{\text{run}}(E) \) on the field intensity [Bakhov et al., 2000], the RE production per unit streamer length was determined for different magnitudes of the initial electron concentration in the streamer channel. For an ionization wave propagating a distance \( z_2 - z_1 \) it is

\[
k_{\text{re}} = \frac{N_{\text{re}}(t_{z_2}) - N_{\text{re}}(t_{z_1})}{z_2 - z_1},
\]

where

\[
N_{\text{re}}(t) = \int_0^t \int_{V_{\text{sim}}} \nu_{\text{run}}(E(z,\rho, t')) \cdot n_e(z,\rho, t') \cdot dV \cdot dt'.
\]

Here \( N_{\text{re}}(t_z) \) is the number of REs at time \( t_z \) when the ionization wave front, defined as a position of the field maximum, has reached a point \( z \); \( V_{\text{sim}} \) is the simulation domain volume [Babich et al., 2014a, 2015]. Values of \( v_{\text{fi}} \) and \( k_{\text{re}} \), calculated with \( z_2 = 3.0 \) cm and \( z_2 - z_1 = 0.1 \) cm at the end of simulations in Babich et al. [2014a, 2015], when quasi-state electron concentration and field distributions were achieved, are presented in Table 1, where \( n_e^{\text{ch},a} \) is the initial electron concentration at the channel symmetry axis. It is seen, that \( v_{\text{fi}} \) increases with \( n_e^{\text{ch},a} \) reaching a plateau. The dependence of \( k_{\text{re}} \) on \( n_e^{\text{ch},a} \) is more complicated: initially, \( k_{\text{re}} \) increases with \( n_e^{\text{ch},a} \) reaching a maximum magnitude at \( n_e^{\text{ch},a} = 10^{12} \) m⁻³, and then decreases. This is explained as follows: at small \( n_e^{\text{ch},a} \), the field intensity and, consequently, the velocity \( v_{\text{fi}} \) do not reach large magnitudes due to the streamer head radial expansion, and at large \( n_e^{\text{ch},a} \), the channel conductivity, which limits the field strength, grows faster than at lower \( n_e^{\text{ch},a} \) values.

The function \( \nu_{\text{run}}(E) \), used in equation (2), was computed for a constant field [Bakhov et al., 2000], which is valid if the field intensity does not vary during electron trajectory simulations. However, at large \( v_{\text{fi}} \) magnitudes, this is not the case. For instance, electrons, starting at the point where the field intensity is maximal, lag behind the maximum, if their initial velocity is less than the wave speed \( v_{\text{fi}} \) and enter a region with weaker field, whereas electrons, starting ahead of the field maximum, experience amplifying field because the wave overtakes them. This is not allowed in equation (2). It is necessary to take into account the displacement of electrons during their energizing to the runaway energy range because the field intensity gradient is very large in the vicinity of the field maximum.

<table>
<thead>
<tr>
<th>( n_e^{\text{ch},a}, ) m⁻³</th>
<th>( t_{z_2}, \text{ns} )</th>
<th>( E_f, \text{kV cm}^{-1} )</th>
<th>( v_{\text{fi}}, \text{m s}^{-1} )</th>
<th>( k_{\text{re}}, \text{m}^{-1} )</th>
<th>( k_{\text{re}}^{\text{MC}}, \text{m}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{10} )</td>
<td>3.86</td>
<td>205</td>
<td>( 1.3 \cdot 10^7 )</td>
<td>2 \cdot 10^4</td>
<td>4.3 \cdot 10^7</td>
</tr>
<tr>
<td>( 10^{11} )</td>
<td>3.80</td>
<td>267</td>
<td>( 2 \cdot 10^7 )</td>
<td>5 \cdot 10^7</td>
<td>2.3 \cdot 10^9</td>
</tr>
<tr>
<td>( 10^{12} )</td>
<td>3.66</td>
<td>296</td>
<td>( 2 \cdot 10^7 )</td>
<td>2 \cdot 10^9</td>
<td>4.6 \cdot 10^8</td>
</tr>
<tr>
<td>( 10^{13} )</td>
<td>3.50</td>
<td>286</td>
<td>( 3 \cdot 10^7 )</td>
<td>9 \cdot 10^8</td>
<td>2.8 \cdot 10^6</td>
</tr>
<tr>
<td>( 10^{14} )</td>
<td>3.31</td>
<td>253</td>
<td>( 3 \cdot 10^7 )</td>
<td>2 \cdot 10^7</td>
<td>2.8 \cdot 10^6</td>
</tr>
<tr>
<td>( 10^{15} )</td>
<td>3.14</td>
<td>204</td>
<td>( 3 \cdot 10^7 )</td>
<td>3 \cdot 10^4</td>
<td>2.8 \cdot 10^6</td>
</tr>
</tbody>
</table>

\*Babich et al. [2014a, 2015].
Below, in section 2, the effect of a moving ionization wave on the production of REs is studied by Monte Carlo simulations. With this goal we use the electron concentration and field intensity spatial distributions [Babich et al., 2014a, 2015] obtained at the end of simulations, which invariable profiles we let to move with the given velocity \( \nu_f \) through a computational mesh. As a result of this motion, in each cell of the mesh, the electron concentration and field intensity vary. At each time step in each cell of the mesh, the RE numbers are computed as a number of electrons capable of overcoming the runaway threshold. In section 3, a scenario of the streamer propagation outside the old ionized channel allowing for the gas preionization by the REs ahead of the wave front is analyzed. Section 4 includes some further comments and conclusions.

2. The Effect of the Ionization Wave Speed on Runaway Electrons Production

We first analyze the most interesting case with the electron concentration \( n_e^{\text{ch},a} = 10^{12} \text{ m}^{-3} \), at which the \( \nu_r \) is at its maximum as calculated by Babich et al. [2014a, 2015] (see Table 1). With this goal, we execute 2-D Monte Carlo simulations and compute the generation rate of REs per unit length along the velocity direction of an ionization wave moving with a constant speed \( \nu_f = 2 \times 10^7 \text{ m s}^{-1} \) corresponding to \( n_e^{\text{ch},a} = 10^{12} \text{ m}^{-3} \) (Table 1). The ionization wave is modeled as the spatial distributions of the electron concentration \( n_e^f(z, \rho) \) and field intensity \( \vec{E}(z, \rho) \), computed at the end of simulations (\( t = 3.6 \text{ ns} \)) in Babich et al. [2014a, 2015].

The electron concentration and field intensity at each moment of time are then \( n_e(z, \rho, t) = n_e^{f}(z - \nu_f t, \rho) \) and \( \vec{E}(z, \rho, t) = \vec{E}(z - \nu_f t, \rho) \). The simulations are carried out with cell sizes \( \Delta z = \Delta \rho = 10^{-5} \text{ m} \) of the computation mesh and time step \( \Delta t = 10^{-14} \text{ s} \).

Following the energy group technique [Babich and Kutsyk, 1995], we divide the electron ensemble in a “reservoir” of low-energy electrons, limited from above to some energy \( \varepsilon_1 \), and “high-energy” electrons with the energies \( \varepsilon > \varepsilon_1 \), the source of which is the “reservoir.” We choose the \( \varepsilon_1 \) magnitude such that the concentration of low-energy electrons to be much higher than that of “high-energy” electrons, such that these are so few that do not affect the transport processes, the field magnitude, and ionization wave configuration. The preionization effect ahead of the ionization wave by electrons moving with a velocity higher than \( \nu_f \) will be discussed in the following section. The “high-energy” electrons produced in the ionization wave are further limited to the energy range \( [\varepsilon_1, \varepsilon_{\text{th}}] \), where \( \varepsilon_{\text{th}} = 4 \text{ keV} \) [Bakhov et al., 2000] is the runaway threshold. Note, limiting from below the energy to the magnitude \( \varepsilon_1 \), above which we carry out simulations, allows us to limit the number of trajectories to be simulated. We do this by using the electron production rate \( v_1(\varepsilon_1, E) \) computed beforehand with the same Monte Carlo code.

The \( \varepsilon_1 \) magnitude is to be chosen to compromise between the accuracy requirements and limitations of a computer capacity limiting the number of simulated trajectories. Therefore, to check whether reasonably chosen \( \varepsilon_1 \) magnitude will affect the results, we carried out test Monte Carlo simulations of \( N_{\text{sim}} = 10^6 \) electron trajectories in the field with the constant intensity \( E = 270 \text{ kV cm}^{-1} \) for two \( \varepsilon_1 \) magnitudes: 100 and 200 eV. A number of physical electrons \( N_{\text{th}}(\varepsilon_1, \varepsilon_{\text{th}}) \), crossing the runaway threshold \( \varepsilon_{\text{th}} = 4 \text{ keV} \) [Bakhov et al., 2000], is connected with an initial number of low-energy electrons \( N_e^{(0)}(\varepsilon_1) \), crossing the border \( \varepsilon_1 \), and relative number of successful simulated trajectories \( N_{\text{th}, \text{sim}}(\varepsilon_1, \varepsilon_{\text{th}})/N_{\text{sim}} \) as follows:

\[
N_{\text{th}}(\varepsilon_1, \varepsilon_{\text{th}}) = N_e^{(0)}(\varepsilon_1) \times \frac{N_{\text{th}, \text{sim}}(\varepsilon_1, \varepsilon_{\text{th}})}{N_{\text{sim}}} \times \frac{N_{\text{th}, \text{sim}}(\varepsilon_1, \varepsilon_{\text{th}})}{N_{\text{sim}}},
\]

where \( v_1 \) is the rate with which the low-energy electrons cross the border \( \varepsilon_1 \).

With the magnitudes of the rate \( v_1(\varepsilon_1 = 200 \text{ eV}) \) and \( v_1(\varepsilon_1 = 100 \text{ eV}) \), available in Bakhov et al. [2000], and computed numbers \( N_{\text{th}, \text{sim}}(\varepsilon_1 = 200 \text{ eV}, \varepsilon_{\text{th}}) \) and \( N_{\text{th}, \text{sim}}(\varepsilon_1 = 100 \text{ eV}, \varepsilon_{\text{th}}) \), a ratio of \( N_{\text{th}}(\varepsilon_1, \varepsilon_{\text{th}}) \) for the above \( \varepsilon_1 \) magnitudes

\[
\frac{N_{\text{th}}(\varepsilon_1 = 200 \text{ eV, } \varepsilon_{\text{th}})}{N_{\text{th}}(\varepsilon_1 = 100 \text{ eV, } \varepsilon_{\text{th}})} = \frac{v_1(\varepsilon_1 = 200 \text{ eV})}{v_1(\varepsilon_1 = 100 \text{ eV})} \times \frac{N_{\text{th}, \text{sim}}(\varepsilon_1 = 200 \text{ eV, } \varepsilon_{\text{th}})}{N_{\text{th}, \text{sim}}(\varepsilon_1 = 100 \text{ eV, } \varepsilon_{\text{th}})}
\]

is close to the unity, meaning that the number \( N_{\text{th}}(\varepsilon_1, \varepsilon_{\text{th}}) \) of runaway electrons rather weakly depends on the choice of \( \varepsilon_1 \) magnitude within a reasonable range. This could be expected because with \( \varepsilon_1 = 100 \text{ eV} \), a number of successful trajectories is lower than with \( \varepsilon_1 = 200 \text{ eV} \), but \( v_1(\varepsilon_1 = 100 \text{ eV}) \) is higher than \( v_1(\varepsilon_1 = 200 \text{ eV}) \).
In the subsequent Monte Carlo simulations with the field profile as in Babich et al. [2014a, 2015], we let $\epsilon_1 = 200$ eV due to larger number of successful trajectories and smaller computation time. Were we to carry out the simulations with $\epsilon_1 = 100$ eV, rather than with $\epsilon_1 = 200$ eV, the difference would only be the additional trajectory section in the range from 100 to 200 eV. The energizing time from 100 to 200 eV is on the order of the difference in the time lag $t_d(\epsilon_1)$ required to cross the border $\epsilon_1$ [Bakhov et al., 2000] $\Delta t_d = t_d(200 \text{ eV}) - t_d(100 \text{ eV}) < 10^{-12} \text{ s}$, during which the field does not significantly vary and the electron displacement is less than the cell size.

The generation rate of the “high-energy” electrons per unit volume is computed by a formula similar to the integrand in equation (2):

$$S_1(z, \rho, t, \epsilon_1) = v_1(E(z, \rho, t), \epsilon_1) n_1(z, \rho, t).$$  

(5)

To compute the $v_1$ dependence on the field intensity, we use the approach that Bakhov et al. [2000] used for computing the runaway rate $v_{\text{run}}(E)$ and the same set of elementary cross sections, taking into account elastic scattering, excitation of electronic shells, and impact ionization. As the rate $S_1(z, \rho, t, \epsilon_1)$ in formula (5) depends on the local electric field and electron concentration, it is necessary that the locality principle be satisfied; i.e., the kinetics of low-energy electrons be governed by the local field. In the field with the intensity $E > 200$ kV cm$^{-1}$ during a time lag $t_d \approx 2 \cdot 10^{-12}$ s required to cross the border $\epsilon_1 = 200$ eV (electron velocity $v_\epsilon = 0.8 \cdot 10^7 \text{ m s}^{-1}$) [Bakhov et al., 2000], low-energy electron moves for a distance $eE t_d^2/2m_e$ of $3.5 \cdot 10^{-6}$ m, which is less than the cell sizes $\Delta z = \Delta \rho = 10^{-5}$ m of the computational mesh. Since the electron velocity $v_\epsilon$ is of the order of the wave speed $v_f$ and the vector $\vec{v}_f$ is directed along the external electric force $-eE_{\text{ext}}$, the electron concentration and field intensity vary insignificantly with time. The chosen time step $\Delta t = 10^{-14}$ s is significantly less than $t_d \approx 2 \cdot 10^{-12}$ s, which, in its turn, is significantly less than the characteristic time of the field variation $\tau_E \approx L_E/\nu_f > 5 \cdot 10^{-11}$ s, where $L_E$ is the width of the field wave profile. Here we use $\lambda_E \equiv (d \ln E/dz)^{-1} \approx 5 \cdot 10^{-4}$ m for the strongest part of the field profile from [Babich et al., 2014a; Babich et al., 2015], where $E \geq 240$ kV cm$^{-1}$ required for the electron runaway. Naturally, the total profile width and, consequently, $\tau_E$, are larger. Thus, the locality principle is satisfied.

Because we use an ionization wave propagating without attenuation or dispersion, i.e., with invariable (in the reference frame connected with the wave) electron concentration and field profiles and magnitudes, all disk layers with a thickness equal to the step $\Delta z = 10^{-5}$ of the computational mesh operate as sources of electrons with the energy $\epsilon_1$ during the wave passage through each of them. They produce the same number of “high-energy” electrons, although with some time delay according to the equation $S_1(z_1, \rho, t) = S_1(z_2, \rho, t - (z_2 - z_1)/v_f)$, where $z_1$ and $z_2$ are the coordinates of the disks. In view of our goal to compute the RE numbers per unit length, it is sufficient to execute the simulations of electrons with the initial energy $\epsilon_1$ generated only in a single disk layer of thickness $\Delta z$. The simulations do not account for the preionization due to REs.

We place the disk layer in the vicinity of the coordinate $z_0$ far ahead of the coordinate $z_{\text{max}}$ of the field maximum $[z_0 - z_{\text{max}} = 0.5-1 \text{ cm}$ at the start of the computation; Figure 2]. In each cell of the mesh with a volume $\Delta V = 2\pi\rho\Delta\rho\Delta z$ in each time interval $[t, t + \Delta t_{\text{gen}}]$, an ensemble, containing $\Delta N_1 = S_1(z, \rho, t) \cdot 2\pi\rho\Delta\rho\Delta z \cdot \Delta t_{\text{gen}}$ electrons with the energy $\epsilon_1$, is generated. We use $\Delta t_{\text{gen}} = 10^{-13}$ s and simulate the further ensemble behavior with a time step of $\Delta t = 10^{-14}$ s by a trajectory of one test electron with initial energy $\epsilon_1$ and weighted with $\Delta N_1$. The technique of trajectory simulations follows Bakhov et al. [2000]. The simulations are carried out until the test electron achieves the threshold $E_{\text{th}}$, or decelerates down to the range $\lesssim 1$ eV. In the first case the energy $E_{\text{th}}$ is prescribed to all $\Delta N_1$ electrons; this is statistically reasonable owing to small size and large number of the cells, small $\Delta t_{\text{gen}}$, and, accordingly, large number of simulated trajectories (hundreds of thousands of trajectories are simulated simultaneously). After each time step $\Delta t_{\text{gen}}$, an additional ensemble of new “high-energy” electrons in each cell of the layer is generated, which trajectories are simulated as described above. The sought RE numbers are computed, summing the weights of all test electrons achieving the threshold $E_{\text{th}}$: $N_{\text{re}} = \sum_i \Delta N_1(E_{\text{th}})$. Sensitivity trial computations with twofold decrease of $\Delta t_{\text{gen}}$ (twofold decrease in $\Delta N_1$) give essentially the same RE number. Note, unlike the fluid calculations by Babich et al. [2014a, 2015], the Monte Carlo simulations account for the spatial and temporal changes in the field conditioned to the wave motion.
Unlike equation (1), the RE number per unit length born by the ionization wave moving with a set constant velocity \(v_f\) is calculated as follows:

\[
k_{re}^{MC}(z_0, v_f) = \frac{n_{re}}{\Delta z} = \frac{1}{\Delta z} \int_0^{t_{max}} dN_{re}(z_0, v_f, t), \quad (6)
\]

where \(dN_{re}(z_0, t)\) is the number of high-energy electrons created in the layer \([z_0, z_0 + \Delta z]\) and reached the runaway threshold \(\epsilon_{th}\) within the time range \([t, t + dt]\) and \(t_{max}\) is the time required to simulate the ionization wave passage through the layer \([z_0, z_0 + \Delta z]\). Using equation (6), instead of (1) allows reducing a number of simulated electron trajectories and increasing the calculation accuracy.

Simulations with \(n_e^{ch} = 10^{12} \text{ m}^{-3} \text{ and } v_f = 2 \cdot 10^7 \text{ m s}^{-1}\) give the RE number per unit length \(k_{re}^{MC} = 2.3 \cdot 10^9 \text{ m}^{-1}\), which is comparable to \(k_{re} = 2 \cdot 10^9 \text{ m}^{-1} \text{ [Bakhov et al., 2014a; Babich et al., 2015]} \) (Table 1). Hence, within the model and simulation accuracy, it is not necessary in this case to account for the field variations during the electron energizing. The wave field does not undergo significant variations during time \(t_{max}\) which is orders of the magnitude less than the simulation time in \(Babich et al. [2014a, 2015]\).

To determine the validity range in \(v_f\) of the rate \(k_{re}\) estimated in \(Babich et al. [2014a, 2015]\), we carry out simulations with different \(v_f\) magnitudes using the same distributions of the electron concentration \(n_e(z - v_f t, p)\) and field intensity \(\vec{E}_{out}(z - v_f t, \rho)\) as earlier, again keeping the \(v_f\) magnitude constant. The obtained \(k_{re}^{MC}\) as a function of \(v_f\) is displayed in Figure 3 along with the following analytical evaluation:

\[
k_{re}(z_0) = \frac{1}{\Delta z} \int_0^{t_{max}} dN_{re}(z_0, t), \quad (7)
\]

where

\[
dN_{re}(z_0, t) = \rho_{sim} \int_0^{t_{max}} \rho_{run}(E(z_0, \rho, t)) n_e(z_0, \rho, t) \cdot 2 \pi \rho d\rho. \quad (8)
\]

Here the radial size of the computation area used, \(\rho_{sim} = 6 \text{ mm}\), is the same as the in \(Babich et al. [2014a, 2015]\) and \(\rho_{run}(E)\) is taken from \(Bakhov et al. [2000]\).

It is seen that for \(v_f < 2 \cdot 10^7 \text{ m s}^{-1}\), the estimations and results of Monte Carlo simulations are rather close, whereas at \(v_f > 3 \cdot 10^7 \text{ m s}^{-1}\) equation (7) with integrand (8) strongly overestimates the RE numbers. The reason is that equation (8) is valid, provided the field varies weakly during the delay time \(t_d \approx 10^{-11} \text{ s}\) required for the production of the first RE with the energy \(\epsilon_{th} = 4 \text{ keV} \text{ [Bakhov et al., 2000]}, i.e., } \epsilon_f = \epsilon_{th}/v_f > t_d, \text{ where } \epsilon_f \text{ and } \lambda_f \text{ are the wave temporal and spatial scales. For } \epsilon_f = (d \ln E/dz)^{-1} \approx 5 \cdot 10^{-4} \text{ m in the region near the}
Here we use the same distributions from Babich et al. [2014a, 2015], and our simulations are therefore not fully self-consistent. This, however, does not affect the results because the changes in the distributions due to the wave motion in the time $t_{\text{max}}$ considered are small. For instance, $t_{\text{max}} \approx z_{\text{max}}/v_f \approx 5 \cdot 10^{-3} \text{ m/s}$, which is in the nanosecond range.

The values of $k_{\text{MC}}^{\text{RE}}$, computed for different $n_{e,\text{ch},a}^{\text{max}}$ magnitudes, are shown in the last column of Table 1. They vary from $3 \cdot 10^8$ to $2 \cdot 10^9 \text{ m}^{-1}$. A comparison with $k_e$ magnitudes computed in Babich et al. [2014a, 2015] and presented in Table 1 gives the same validity range of the $v_f$ estimations.

From the simulation results we find that the electrons with the energy $e_1$, starting when the field intensity in the disk layer $\Delta z$ is $E \geq 240 \text{ kV cm}^{-1}$, contribute most to the RE generation. Owing to this, we are able to compute the RE numbers by simulating trajectories of “high-energy” electrons starting not only in the space domain limited to one $\Delta z$-thick layer of the computational mesh but also trajectories starting in all cells of the mesh, in which at the moment of the trajectory start, the field strength exceeds $240 \text{ kV cm}^{-1}$. In such a case we simulate the moving volumetric source of electrons with the energy $e_1$ giving start to new trajectories (Figure 2), unlike to the case with immobile source in the thin $\Delta z$ layer. This “volumetric” $k_{\text{MC}}^{\text{RE}}$, computed from the simulations and using equation (1), is shown in Figure 3 as a function of $v_f$. The difference from the $k_{\text{MC}}^{\text{RE}}$ computed with a single thin disk layer ($\Delta z = 10^{-3} \text{ m}$) and considering the full range of fields in that layer, does not exceed 10%, although much fewer trajectories, $\sim 20,000$, were simulated at each moment of time.

3. The Effect on the Streamer Propagation of the Air Preionization by Runaway Electrons

We now show that even with the $k_e$ values far away from the maximal set magnitudes, the additional ionization by RE beam ahead of the wave can exceed the initial electron concentration $n_{e,\text{ch},a}^{\text{max}}$. Figure 4 shows radial distribution of the REs concentration

$$f_e(\rho) = n_e(\rho, e_1)/\int_0^{\rho_{\text{max}}} n_e(\rho, e_1) 2\pi\rho d\rho$$

and the initial concentration distribution of low-energy electrons in the old streamer channel

$$f_e(\rho) = n_e^{\text{ch}}(\rho)/\int_0^{\rho_{\text{max}}} n_e^{\text{ch}}(\rho) 2\pi\rho d\rho,$$

strongest part of the wave field, where $E \geq 240 \text{ kV cm}^{-1}$ [Babich et al., 2014a, 2015], we obtain an inequality $v_f < \lambda_f/E \lambda_{\text{max}} < 5 \cdot 10^{-3} \text{ m/s}$, which is consistent with our Monte Carlo results.

The rate $k_{\text{MC}}^{\text{RE}}$ depends strongly on the wave velocity decreasing by orders of the magnitude with only a threefold increase in $v_f$. The reason is that the velocities of electrons in the range $[e_1, e_{\text{th}}]$ are within the range $[0.75, 3.75] \cdot 10^7 \text{ m/s}$; hence, for $v_f \geq 3.75 \cdot 10^7 \text{ m/s}$, electrons lag behind the high-field region such that the runaway probability strongly decreases.

Note, in the fluid simulations the field and electron concentration distributions change with time; in particular, the field profile widens and amplitude increases [Babich et al., 2014a, 2015].

Figure 3. A dependence of the RE number per unit streamer length on the ionization wave velocity at $n_{e,\text{ch},a}^{\text{max}} = 10^{12} \text{ m}^{-3}$. Estimations using equation (8) (dotted line), simulations using a source of electrons with the energy $e_1$ localized in the layer with thickness $\Delta z$ (solid line), and simulations with the RE volumetric generation (markers).
and consider the REs, born, for instance, along the length $l = 10^{-3}$ m passed by the wave front. Then, the RE contribution to the channel ionization can be estimated as

$$\Delta n_e^e = \left( n_{re} \cdot n_e \right) / \pi \rho_{old}^2,$$

where the number of low-energy electrons per unit length $n_e$, produced by an electron with the threshold energy $E_{th} = 4$ keV, can be estimated as the ratio of the specific energy losses $L_{e}$ to the ionization energy cost in air $\Delta \epsilon$. Using for $L_{e}$, Bethe’s formula for electron energy losses per unit length [Bethe and Ashkin, 1953] at atmospheric number density of neutrals and $\Delta \epsilon = 34$ eV [cf. Dwyer and Babich, 2011, and citations therein], we obtain $n_{e,1} \approx 10^{5}$ m$^{-3}$.

Even with the overestimated RE beam radial size $\rho_{old} \approx 5 \cdot 10^{-4}$ m and underestimated RE number $k_{re} \cdot l = 10^{4}$, computed using rather low $k_{re} = 10^{7}$ m$^{-1}$ (Figure 3), we get $\Delta n_e^e \approx 10^{15}$ m$^{-3}$, which exceeds the maximal $n_e^{ch.a.j}$ magnitude in Table 1. Hence, the ionization wave propagation is determined by the air ionization by the REs ahead of the wave front instead of being governed by the old channel plasma parameters. Thus, the preionization anisotropy behind the wave front, required according to Babich et al. [2014a, 2015] to focus the wave in order to reach the field intensity magnitudes of $\geq 240 \text{ kV cm}^{-1}$ that allow efficient RE generation, can be provided with a RE beam itself once the beam has been created. This suggests that the process is self-sustained.

The old streamer channel provides the anisotropic preionization only at the start of a new streamer before appearing even rather small number of REs but sufficient for producing the preionization with $\Delta n_e^e > n_e^{ch.a.j}$. Further, the streamer can propagate independently and not necessarily along the old channel. Naturally, direct measurements of local fields as strong as $240 \text{ kV cm}^{-1}$ are absent. However, as pointed out in the introduction, the observations of bremsstrahlung X- and $\gamma$-rays, correlated with electromagnetic pulses of leader steps [e.g., Dwyer et al., 2004, 2005, 2012b], suggest that high-energy REs are generated, which, in turn, is evidence that the electric field intensity in the stepping process is increased to at least $240 \text{ kV cm}^{-1}$ in a region close to the leader tip.

REs, even in small numbers, say 10, create per unit length a rather large number of seed electrons ($\sim 10 \cdot N_{e,1} \approx 10^{6}$ m$^{-1}$). It is almost the same as the number of seed electrons per unit length of the old streamer channel $n_e^{ch.a.j} / \pi \rho_{old}^2 = 10^{12}$ m$^{-3}$, which is sufficient for the ionization wave focusing. Hence, the streamer, once formed, may propagate in a casual direction as a result of the field fluctuations, as small RE numbers ($\sim 10$) are generated. As a result, new channels may be formed, in which the current grows and charge is accumulated, strengthening the local field and thus promoting the further channel growth. In this process, the new channel can be narrower than that of the old streamer.

To prove the idea of the self-consistent streamer development in the RE mode, simulations with self-consistent description of the RE kinetics and the ionization wave propagation are planned. Such
simulations can be executed using the group approach developed by Babich and Kutsyk [1995], according to which the electron assembly is divided into three energy groups: low-energy electrons, intermediate-energy electrons, and REs in the energy range ε > ε_{th}. According to this approach, the low-energy electrons are described by the kinetic equation; the probability for electrons “flowing out” off the low-energy range and into the RE range is calculated with ignoring elastic collisions; the RE kinetics is described by the fluid equation in the diffusion-drift approximation.

Now an improved group approach is available. For describing the kinetics of low-energy (ε < ε_1) electrons in the electric field, it is sufficient to use the numerical code using the diffusion-drift approximation for the fluid equations, similar to that developed by Babich et al. [2014a, 2015]. To simulate the electron kinetics in the intermediate energy range [ε_1, ε_{th}], it is reasonable to use the approach described above. The RE kinetics in the energy range ε > ε_{th} can be simulated by means of a simplified Monte Carlo code [Lehtinen et al., 1999; Babich et al., 1998, 2001a, 2001b], in which a model of continuous energy losses and Rutherford’s equation with shielding for the angular scattering are used or by means of the group equations for the electron distribution function moments [Babich and Kudryavtseva, 2007; Babich and Bochkov, 2011].

4. Conclusions

In the framework of the problem of high-energy REs and bremsstrahlung generation by leaders of the long spark discharges, including those of the natural lightning, we have analyzed the evaluations of the RE numbers per unit streamer length carried out by Babich et al. [2014a, 2015] on the basis of results of numerical simulations of the streamer development using diffusion-drift approach for the fluid equations. By means of Monte Carlo simulations, we show that the evaluations in Babich et al. [2014a, 2015] are rather accurate at the ionization wave velocities in the range υ_f < 2 · 10^7 m s^{-1}, but at higher velocities, results of and evaluations manifold differ. The computed RE numbers per unit streamer length vary in the range from about 3 · 10^6 to 2 · 10^9 m^{-1}, depending on the υ_f magnitude.

In this paper we consider discharges in the open atmosphere in rather weak external electric fields with the intensity many-fold below the maximum of the electron drag force F_{max} ≈ 270 kV cm^{-1} [Nguyen et al., 2008; Rahman et al., 2008; March and Montanya, 2010; Kochkin et al., 2012, 2014, 2015; Ostgaard et al., 2016; Agafonov et al., 2013], when the total charge of REs only constitutes insignificant portion of the electricity accumulated in the source: thundercloud (lightning) or capacitor bank (laboratory long spark), unlike the discharges in superstrong external fields with the intensity close to or even higher than F_{max} [e.g., Babich et al., 1990, 2014b; Babich, 2003, chapter 5, 2005; Tarasenko and Yakovlenko, 2004; Yalandin et al., 2011, 2012, and citations therein]. The ionization waves in discharges developing in superstrong fields propagate with speeds close to that of light in free space, and it is recognized that such discharges develop self-consistently with a production of large numbers of high-energy REs, which preionize a gas ahead of the ionization wave front [Babich et al., 1990; Babich, 2003, section 4.2, 2015; Yalandin et al., 2011, 2012].

In this paper, we show that even at rather low ionization wave speed, υ_f < 2 · 10^7 m s^{-1}, the concentration of low-energy electrons produced by small numbers of REs in front of the streamer can significantly exceed the initial concentration of low-energy seed electrons in the old streamer channel. Hence, an adequate mechanism of the streamer propagation in lightning and long laboratory spark discharges also is self-consistent with the RE generation. The above analyses were executed using the electron concentration and field intensity profiles from Babich et al. [2014a, 2015] computed at atmospheric air density. To support a propagation of lightning leader/streamer channels, the process of the RE generation should work in the span of altitudes typical for the lightning discharges, i.e., from ground level to tens of kilometers. All results obtained at the ground level can be generalized to streamers at other altitudes (air densities) of interest using similarity laws, according to which the streamer spatial dimensions (such as a length and tip width) and electric field scale with the air density N as ~1/N and ~N, respectively [Pasko et al., 1998; Liu and Pasko, 2004; Moss et al., 2006]. As a result, numbers of REs and maximum energy they gain in the streamer tip remain the same for similar streamers at different air densities (assuming constant gas temperature). Therefore, the effect of REs on the propagation of lightning leader/streamer channels is almost the same at different altitudes.
We considered the electron runaway in strong electric fields of the heads of streamers constituting the leader corona. However, electron energizing depends on the $E/N$, not simply on $E$. Therefore, there is a chance that electrons can be energized in the hot leader channel. Really, due to the heated gas expansion, the air density in the channel can be reduced to a level, at which the $E/N$ is increased up to a magnitude sufficiently high for producing high-energy REs.

The simulations of the runaway process carried out in this paper are not self-consistent because the moving distributions of electron concentration and the field strength do not vary self-consistently with the RE generation. But the Monte Carlo code developed in this work can be used in the streamer model with self-consistent generation of high-energy REs. This methodology will be applied in our continued work.

References


Tarasenko, V. F., and S. I. Yakovenko (2004), The electron runaway mechanism in dense gases and the production of high-power subnanosecond electron beams, Phys.-Usp. 47, 887–971.