



## The Effects of Uncertainty in Speed-Flow Curve Parameters on a Large-Scale Model

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## **The Effects of Uncertainty in Speed-Flow Curve Parameters on a Large-Scale Model: The Danish National Model Case Study**

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**ABSTRACT**

Uncertainty is inherent to transport models and prevents from using a deterministic approach when modelling traffic. Quantifying uncertainty thus becomes an indispensable step to produce more informative and reliable output of transport models. Within traffic assignment models, volume delay functions express the travel time as a function of traffic flows and theoretical capacity of the modelled facility. The US Bureau of Public Roads (BPR) formula is one of the most extensively applied volume delay functions in practice. This study investigated uncertainty in the BPR parameters. Initially, BPR parameters were estimated by analyzing observed traffic data related to the Danish highway network. Then, BPR parameter distributions were generated by using re-sampling Bootstrap technique. Finally, the generated parameter vectors were used to implement sensitivity tests on the four-stage Danish national transport model. The results clearly highlight the importance for modelling purposes of taking into account BPR formula parameter uncertainty, expressed as a distribution of values rather than assumed point values. Indeed, the model output demonstrates a noticeable sensitivity to parameter uncertainty. This is particularly evident for stretches of the network with a high number of competing routes. Model sensitivity was also tested for BPR parameter uncertainty combined with link capacity uncertainty. The resulting increase in model sensitivity demonstrates even further the importance of the implementation of uncertainty analysis as part of a robust transport modelling process.

## INTRODUCTION

By modelling complex systems, transport models are subject to uncertainty that can affect all model components (i.e., context, model structure and methodology, inputs and parameters) to finally propagate to the model output. The main consequence of this inherent uncertainty is that transport models do not provide reliable point estimates of modelled traffic flows and derived measures. Instead, modelled traffic flows are better expressed as a central estimate and an overall range of uncertainty margins articulated in terms of (output) values and likelihood of occurrence (*I*). Uncertainty analysis relates to how uncertainty in each model component propagates to the model output and how to express the model output as a distribution, so reflecting the overall uncertainty present in the model.

The assignment algorithms of large-scale transport models often use static volume delay functions to express travel time as a function of traffic flow and theoretical capacity of the modelled facility. However, travel time is not just a function of flow and it is in fact affected by a number of different factors, such as downstream bottlenecks and resulting spillback or less than ideal weather conditions, causing drivers to drive slower. Consequently, a problem arises whenever traffic data output of static models are used to feed cost benefit analysis. In these cases, in order to produce valuable information, a necessary step is to address uncertainty in the volume delay functions by quantifying the sensitivity of the model output to the variability of the volume delay functions components.

Volume delay functions can be divided in three main groups (2): hyperbolic, polynomial and exponential. The US Bureau of Public Roads (BPR) formula, belonging to the polynomial group and proposed in its original version in 1964 (3), is one of the most extensively applied volume delay functions in practice. The BPR formula, given free flow travel time, observed flow and link capacity uses parameters to represent different relationships between travel time and (modelled) flow-to-capacity ratios. Usually, the values for the parameters are pre-defined, based on assumptions and practice. However, as for any other model components, the BPR formula parameters have inherent uncertainty that originates from both the ignorance of the modeler of the true value of the parameters (epistemic uncertainty) and the stochastic behavior of the (true) parameters itself (ontological uncertainty), which potentially vary by driver behavior, time of the day, weather conditions and link characteristics.

An approach widely used in the transportation literature to quantify model uncertainty is to run model sensitivity tests by using distributions of input and parameters, and output of stochastic sampling procedures. For this purpose, re-sampling techniques such as Bootstrap (4) have been used to generate model parameter distributions. Re-sampling approaches have a clear advantage compared to other sampling procedures. In fact, they do not require modelers' knowledge or assumptions about the shape of the parameter distributions, which becomes instead the output of the re-sampling methodology itself. Bootstrap has been implemented in many studies on transport uncertainty by Brundell-Freij (5), Hugosson (6), De Jong et al. (7), Matas et al. (8) and Petrik et al. (9). Bootstrap defines the parameter distributions by recalibrating the model parameters for a number of model samples, which are generated from the original sample by re-sampling with replacement.

At the best of our knowledge, no attempt has been made so far to estimate uncertainty in the BPR formula parameters from the analysis of observed data and to analyze its effect on traffic assignment results of large-scale models. For this purpose, observations of the Danish highway network were obtained from the Hastrid dataset that is owned by the Danish Road Directorate. Non-linear regression analyses were implemented to allow the calibration of the values of the BPR formula parameters simultaneously. Afterwards, parameters were

repeatedly calibrated on 10,000 Bootstrap samples to generate parameter distributions. Finally, selected percentiles of the distributions were used to run sensitivity tests on the Danish national transport model (LandsTrafikModellen, LTM). In addition, a scenario investigating LTM sensitivity to BPR parameter uncertainty combined with link capacity uncertainty was tested. The link capacity uncertainty was quantified by creating vectors of capacity values through the implementation of Monte Carlo simulation.

The next section provides a description of the methodology applied to estimate the BPR parameter distributions, including a description of the datasets used for the research and the Bootstrap sampling technique. After a brief description of the LTM, the following section illustrates and discusses the results from the sensitivity tests run. The conclusions from this research are presented in the last section of this paper.

## METHODOLOGY

### Time-Flow Relationship: the BPR Formula

In traffic assignment models a common way to describe the relationship between travel time and traffic flows is the BPR formula (3):

$$TT_r = FFT_r * \left\{ 1 + \alpha * \left[ \frac{Flow_r + \gamma (Flow'_r)}{Capacity_r} \right]^\beta \right\} \quad (1)$$

where  $TT_r$  is the congested travel time on link  $r$ ,  $FFT_r$  is the free flow time on link  $r$ ,  $Flow_r$  is the traffic volume on link  $r$ ,  $Capacity_r$  is the capacity of link  $r$ ,  $Flow'_r$  refers to the traffic volume on the opposite direction of link  $r$  (relevant only in case of non-separated lanes), and  $\alpha$ ,  $\beta$  and  $\gamma$  are volume-delay parameters. Specifically,  $\alpha$  represents the ratio between free flow speed and speed at capacity,  $\beta$  determines how steeply the curve bends once the capacity is reached, and  $\gamma$  captures the effect of speed reduction due to opposite traffic in roads with non-separated lanes.

The BPR formula can be modified to express the relationship between speed (instead of congested time) and flow-to-capacity ratio, as illustrated by Nielsen and Jørgensen (10) and Fagnant and Kockelman (11):

$$S_r = \frac{FFS_r}{1 + \alpha * \left[ \frac{Flow_r + \gamma (Flow'_r)}{Capacity_r} \right]^\beta} \quad (2)$$

where  $S_r$  is the observed average speed on link  $r$  and  $FFS_r$  is the velocity in free flow conditions on link  $r$ . The use of either the time-flow or the transformed speed-flow formulas is generally data-driven, namely is dependent on the availability of data concerning either travel times or travel speeds. For example, the current study considers observations from a dataset of travel speeds, and hence uses the transformed speed-flow formula for the calibration of the BPR parameters. It is important to stress that the transformed formula implies an approximation. In fact, the speed is measured by local detectors, so it does not reflect precisely the link travel time, but rather is expression of the overall link conditions. On the top of our knowledge, no attempt has been done so far to quantify this discrepancy.

In general, criticisms have been moved to the BPR formula. As pointed out by Downing et al. (12), depending on the choice of the parameter values the BPR formula may result insensitive to volume changes until demand exceeds capacity, when the predicted

speed drops abruptly. Nevertheless, other studies proved that with an appropriate choice of parameter values specific for road type, the BPR formula offers comparable or even better goodness of fit to observed data than other volume delay functions (13).

Another drawback is that the BPR formula results correct to model travel time only when the traffic flow is below capacity. In fact, when traffic flow reaches capacity (in figure 1 the point corresponding to flow at capacity  $FC$  and the related speed at capacity  $SC$ ), the curve representing the BPR formula takes the shape of the dotted curve on the right of  $FC$ . Instead, the observed traffic behavior is tendentially close to the pattern described by the bold line. To overcome this issue, it was suggested expressing the flow-capacity ratio in terms of density-density at maximum flow ratio (13). With this approach in fact, the speed-flow observations assume an s-shape that is possible to model.

*[Insert figure 1 about here]*

Despite the criticism, in static assignment models the BPR formula is commonly used and accepted for practical reasons. Among others, with the BPR formula the speed-flow relationship curve is “continuous even beyond capacity and differentiable”, as argued by Nielsen and Jørgensen (10).

### **Hastrid Dataset and Parameter Calibration**

This study intended to calibrate the BPR formula parameters, and hence used information regarding the Danish highway network that was contained in the dataset Hastrid, owned by the Danish Road Directorate. The Hastrid dataset contains observations for vehicle flow and average speed by time intervals of 15 minutes. The data used in the present analysis were collected in September 2009 from 3 count stations located in north east part of Zealand. Two count stations were located on the highway M11, called “Holbækmotorvejen”, connecting Holbæk, in the north-west part of Zealand, with the south-west suburbs of Copenhagen. The third count station was instead located on the highway M16, called “Hillerødmotorvejen”, connecting Hillerød, in the north part of Zealand, with the northern suburbs of Copenhagen. Table 1 summarizes the main characteristics of the three sections where the count stations were located while figure 2 shows their geographical location on the highway network.

*[Insert table 1 about here]*

*[Insert figure 2 about here]*

In order to perform the parameter calibration, the 15 minute data were transformed into hourly data by summing the 15 minute vehicle flow observations and averaging the corresponding observed speeds. The flow-to-capacity ratio was calculated as density-density at a maximum flow ratio (13). The density of maximum flow was defined at 28 passenger cars per kilometer per lane, corresponding to the value suggested by the Highway Capacity Manual (14) of 45 passenger cars per mile per lane. Finally, the free flow speed was calculated for each section as corresponding to the average observed speed at density-density at a maximum flow ratio lower than 0.5.

However, this approach may result in curves with a long tail on the right hand side (15). This would imply the acceptance of relatively high speeds in situations over capacity, thus leading to an overestimation of the network accessibility. Thus, the density-density at the maximum flow ratio approach was partially modified to better model severe congested conditions. Accordingly, for the calibration we used the value  $X$ , calculated as:

$$\begin{aligned}
 X &= D/D_{\max} \text{ if } D/D_{\max} < 1 \\
 X &= 1 + 0.2 * (D/D_{\max}) \text{ if } D/D_{\max} \geq 1
 \end{aligned}
 \tag{3}$$

where  $D/D_{\max}$  is the density-density at the maximum flow ratio. As can be seen, for severe congested conditions, i.e.  $D/D_{\max} \geq 1$ , the density-density at the maximum flow ratio values were reduced to avoid unreasonably high congested values.

The upper part of figure 3 graphically shows the observed average speed plotted against  $X$ . Overall, the observed speed-flow relationship on the three links shows a trend consistent with what theoretically expected. As can be noticed, the majority of the observations cluster around the free flow speed of approximately 110km/h for low levels of congestion (corresponding to  $X < 1$ ). Only a few observations unexpectedly register free flow speed also in congested conditions (corresponding to  $X > 1$ ), probably due to count mistakes. Besides, there is a cluster of observations corresponding to speeds around 75km/h for low levels of congestion. These observations are probably related to trucks in the inner lane, which have speed limits of 80km/h (10).

*[Insert figure 3 about here]*

The parameter calibration, implemented using the statistical software SAS, resulted in  $\alpha = 0.33$  and  $\beta = 4.04$ . With respect to the Danish road network, Hansen (15) defined a range of values between 0.5 and 2 for  $\alpha$  and between 1.4 and 11 for  $\beta$ . Thus, for validation purposes, vehicle speeds resulting from the BPR formula and the calibrated values of  $\alpha$  and  $\beta$  were calculated and compared with observed average speeds through both regression analysis and visual inspection. Results from the regression analysis were satisfactory ( $R^2 = 0.9764$ ) as well as the ones from the visual inspection of the pattern of the speed estimated from the BPR formula, depicted in the bottom part of figure 3.

### **Quantification of Uncertainty in the BPR Formula Parameters**

In order to produce BPR parameter distributions, the re-sampling technique Bootstrap (4) was used. The Bootstrap method investigates the accuracy of an estimator  $\theta$  based on the assumption of considering the original sample, originating  $\theta$ , as the population. Bootstrap consists in a three step procedure. Firstly, from the original sample of  $n$  observations a number of samples are generated through (re)sampling with replacement. All Bootstrap samples contain  $n$  observations as the original sample. The replacement approach guarantees that each observation in the original sample has a constant probability  $1/n$  to be drawn; as a consequence the Bootstrap samples have a high probability of differing from each other. Secondly, the estimator  $\theta$  is calculated for each Bootstrap samples. Thirdly, the new  $\theta$  values obtained are analyzed to infer the accuracy of the estimator by using some uncertainty measures such as variance or standard deviation.

One restriction to the use of Bootstrap is that it can be only implemented for variables which are the output of calibration processes and only when the sample is available. Thus, it cannot be applied to variables observed, assumed or imported. Besides, it is important to notice that the Bootstrap method has two downsides. Firstly, there is no rule defining the correct number of Bootstrap samples to generate, although the number should be large and, in theory, tendentially infinite. Secondly, the results are constrained by the quality of the original sample, given that the Bootstrap samples do not increase the amount of information there contained.

Using as original sample the one used for the parameter calibration, 9999 Bootstrap samples were created and the calibration process was repeatedly implemented for each of

them. The resulting parameter statistics are summarized in Table 2. Also the coefficients of variation (CV) are reported and henceforward used as a measure of uncertainty. Table 2 also shows selected percentiles of the distribution. The sensitivity tests on the LTM were run based on these values rather than for all 10,000 parameter values (9,999 from the Bootstrap samples plus one of the original calibration) because of the extremely long run times of the LTM model. Finally, figure 4 graphically shows the resulting distributions for  $\alpha$  and  $\beta$ .

*[Insert table 2 about here]*

*[Insert figure 4 about here]*

### **Link Capacity Uncertainty**

Despite this study focuses on BPR parameter uncertainty, also the other variables of the BPR formula, namely  $FFT_r$  (or  $FFS_r$ ),  $Flow_r$  and  $Capacity_r$ , potentially have inherent uncertainty. A comprehensive analysis of model uncertainty should include also the assessment of model sensitivity to the uncertainty of these variables. However, with respect to LTM,  $FFT_r$  is based on legal speed limits and  $Flow_r$  depends upon trip generation processes, thus only uncertainty inherent to link capacity has been investigated.

As previously highlighted, Bootstrap can only be applied to calibrated variables. Thus, Monte Carlo simulation has been implemented in order to quantify link capacity uncertainty. Triangular distributions were used in order to avoid illogical sampling results, such as negative or too high capacity values. The limits of the triangular distributions were defined as  $\pm 25\%$  of the capacity link value provided in the LTM network description. The resulting vector values were used in combination with BPR parameter values resulting from the Bootstrap procedure to run sensitivity tests on the LTM model. In this way it was possible to analyze the combined effect of the two uncertainty sources (i.e., BPR parameters and link capacity) on the model. As for the Bootstrap vectors, only selected percentiles from the Monte Carlo simulations were used to run the sensitivity tests.

## **CASE STUDY**

### **The LTM**

The LTM is meant to establish a unified reference model for transport policy analysis and project evaluation in Denmark (16). The model relies on two main data sources: the Danish travel survey, namely Transportvane Undersøgelsen (TU), and the Danish national register. TU is a national survey on-going from 1992 that contains travel information from around 1000 individuals per month, while the national register provides socioeconomic information for the entire Danish population. The model zone system is based on four different aggregation levels going from the more disaggregated up to the more aggregated: level 3 (regional level, 3670 zones), level 2 (national level, 907 zones), level 1 (strategic level, 176 zones) and level 0 (municipality level, 98 zones).

Figure 5 graphically describes the model framework, which is based on four stages for the passenger demand model and three stages for the freight demand model. At the initial stage, the model assumptions exogenous to the model are defined, specifically population, employment, and the road and transit networks. In the second stage, the model consists of two parallel segments, the passenger demand model and the freight demand model. Both these models feed the assignment model that defines the route choice equilibrium. The equilibrium solution provides in turn feedback to the demand models.

*[Insert figure 5 about here]*

As can be noticed, the passenger demand model is divided into two sequential models: the strategic model, which defines strategic choices, and the passenger model, which delineates transport related choices. The models are linked in a random utility framework. At the upper level, the strategic model defines the prerequisites for the passenger model. The passenger model then provides information to the assignment model which in turn sends feedback, in terms of accessibility measures, to both the strategic and the passenger models.

This study focuses on the passenger road assignment model. The model is tour-based and the model structure can be divided into two main sub-models modeling the primary tour activity of the day and the intermediate stop activities (conditional on the primary activity). A limitation is imposed so that a tour can consist of a maximum of four trips (i.e., home-stop; stop-main destination; main destination-stop; stop-home) and only two tours are allowed per individual per day.

More in detail, the passenger road assignment model is a link-based model solved by the Method of Successive Averages (MSA) to reach Stochastic User Equilibrium (SUE). The chosen route to travel by mode  $k$  between origin zone  $i$  and destination zone  $j$  is the one that minimizes the cost of travelling calculated at the link level as:

$$C_{ijk} = \omega_l L_{ijk} + \omega_{ff} FFT_{ijk} + \omega_{tc} TC_{ijk} + \omega_c c_c + \varepsilon_{ijk}$$

where  $C_{ijk}$  is the cost of travelling by mode  $k$  from zone  $i$  to zone  $j$  using link  $r$ ,  $L_{ijk}$  is the length of the link  $r$  by mode  $k$  from zone  $i$  to zone  $j$ ,  $FFT_{ijk}$  is the free flow travel time,  $TC_{ijk}$  is the extra travel time due to congestion,  $c_c$  represent monetary cost of travelling (varying according to mode and purpose),  $\varepsilon_{ijk}$  is the vector of residuals, and the  $\omega$ 's are the parameters associated to the respective variable. The relationship between travel time and traffic flows is based on the BPR formula.

## Results and Discussion

The results from the sensitivity test runs on the LTM traffic assignment are summarized in tables 3 and 4. The upper part of the tables (Scenario 1) shows results for model sensitivity to BPR parameter uncertainty. The bottom part (Scenario 2) illustrates instead results for model sensitivity to BPR parameter uncertainty and link capacity uncertainty combined.

Table 3 shows the links average CV referring to vehicle-kilometer (Veh-Km) and average speed (AvgSpeed) for both the entire network and the highway links only. As can be seen, the mean CV values for both Veh-Km and AvgSpeed are low, reflecting low model sensitivity to the BPR parameters uncertainty. However, it is worth to remind that uncertainty was quantified only for parameters  $\alpha$  and  $\beta$  referring to highways links, which amount approximately to the 5% of the network. Besides, the parameter uncertainty resulting from the Bootstrap approach was high neither for  $\alpha$  (CV 0.09) nor for  $\beta$  (CV 0.054). As expected, the combined effect of BPR parameters uncertainty and links capacity uncertainty (scenario 2) increases the model uncertainty for both the overall network and the highways links.

The mean Veh-Km CV for highway links is lower than that for all links, despite the uncertainty was represented only in highway links. This comes as no surprise. In fact, for highway links the traffic demand can be assumed less elastic to changes in travel time (defined by the BPR formula) as compared to journeys using urban or local network. This assumption is primarily due to the lower number of competitive routes which characterizes journeys on highway facilities. Nevertheless, due to the differences in capacity, a small percentage variation in demand of traffic for highway links may easily result in a high variation for the links of the competitive routes that absorb the diverted traffic. This explains why the CV values for highway links result lower than for the overall network. With respect

to AvgSpeed, the model appears to be insensitive. The reason can be probably traced in lower congestion levels which characterize the overall network.

*[Insert table 3 about here]*

Table 4 shows the total network travel time, divided into free travel time and congested. As can be seen, the corresponding CV for both free and congested times are very low. This is consistent and reflects the low variability resulting from the analysis of the AvgSpeed. However, links capacity uncertainty has a high impact on congested time uncertainty, which increases from 0.01 to 0.2.

*[Insert table 4 about here]*

Despite overall the model showed low sensitivity to BPR parameter variation, the demand of traffic for some links revealed instead high elasticity, resulting in a maximum mean Veh-Km CV of 0.931 and 1.360 for Scenario 1 and Scenario 2, respectively. Thus, in order to analyze differences within the network, the data set was divided in three groups including links with Veh-Km CV lower than 0.1 (Group 1), between 0.1 and 0.5 (Group 2) and higher than 0.5 (Group 3). Statistics referring to the three groups are shown in table 5.

*[Insert table 5 about here]*

As can be noticed, the majority of the links shows a modest or null sensitivity, consistently with the results for the overall model. Only a few links, included in the third group, show instead very high sensitivity, but because of their low number at least part of them are considered outliers. More interesting for modelling purposes are instead the links included in the second group. Most of them (around 200 in both scenarios) should be no cause for concern, given that they represent international Danish traffic and the relatively high variability is probably due to the low number of observations in absolute values. However, the remaining ones, for a total of 107 (scenario 1) and 241 (scenario 2) links, mainly refer to short, mid-distance road types (“hovedvej” and “trafikvej”) potentially hosting commuting traffic. As a consequence, the assessment of projects planned to be implemented in the areas of the network where they are located can be highly affected by their inherent uncertainty. In fact, in case of changes in the network due, for example, to structural changes or transport policy, the high sensitivity they demonstrated may cause the traffic to divert from the originally modelled routes. In areas characterized by a dense network, and hence many competitive routes, these changes can easily cause a shock wave throughout the surrounding network.

## CONCLUSIONS

This paper describes the results of a study carried out to test the LTM sensitivity to BPR parameters ( $\alpha$  and  $\beta$ ) uncertainty. BPR parameter uncertainty was quantified using Bootstrap re-sampling approach. The speed and flow data used to calibrate the BPR parameters and, successively, to implement the Bootstrap analysis, refer to three highway links part of the Danish road network. Also model sensitivity to link capacity uncertainty, combined with BPR parameter uncertainty, was tested. The model output analyzed were (i) vehicle-kilometer and average speed at the link level and (ii) travel resistance at network level.

The results confirm the importance of uncertainty analysis as a decision tool for transportation projects. In fact, although the LTM as a whole proved to be quite inelastic to the variability in the BPR formula parameters, some links showed high elasticity. Any assessment of projects potentially affecting traffic flow on those links should then take into consideration this elasticity and integrate uncertainty analysis in the decision process.

More in detail, the results clearly highlight the importance for modelling purposes of taking into account BPR formula parameter uncertainty, expressed as a distribution of values, rather than assumed point values. The increasing amount of traffic data available nowadays, due to the diffusion and improvements of technology, allow in fact to estimate specific traffic delay formula parameters for different facilities and projects. This is an opportunity that should not be missed in order to produce more reliable modelled traffic results. Besides, when combined with uncertainty analysis, it may produce the necessary information required to increase the quality of the decision process and to develop robust or adaptive plans.

Limitations and avenues for further research should be acknowledge to this study. Firstly, a possible limitation relates to the limited amount of count stations providing the traffic data the analysis is based upon. Further research could use a higher number of count stations, with a wider geographical distribution, in order to calibrate parameter values more representative for the overall network. Nonetheless, the results clearly underline the importance of taking into account parameter uncertainty and their essence would likely not change but rather improve from additional data. Secondly, further analysis including urban and rural facilities parameters uncertainty would provide a more comprehensive picture on the topic, including the possibility of developing a class reference approach for uncertainty analyses of such kind. Lastly, due to the characteristics of the LTM and the scope of the study, the analysis presented in this paper did not quantify the effects on the model output deriving from uncertainty in the BPR formula variables free flow speed and link flows. Further research could investigate this issues, depending on the model tested and the objectives of the analysis.

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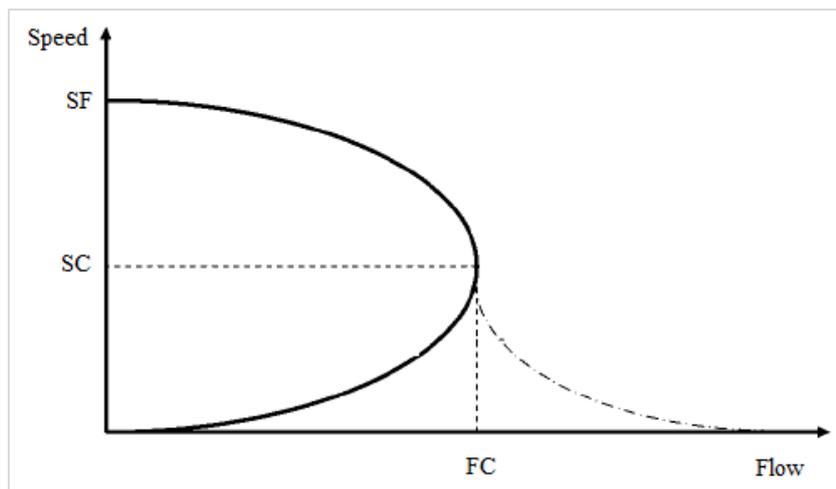
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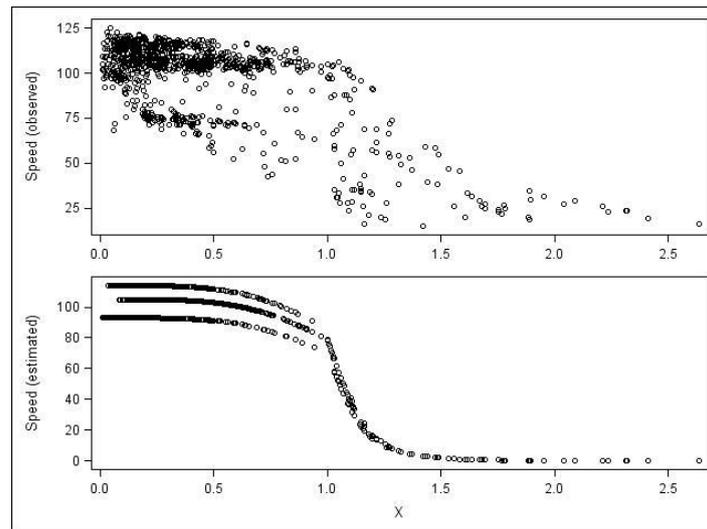
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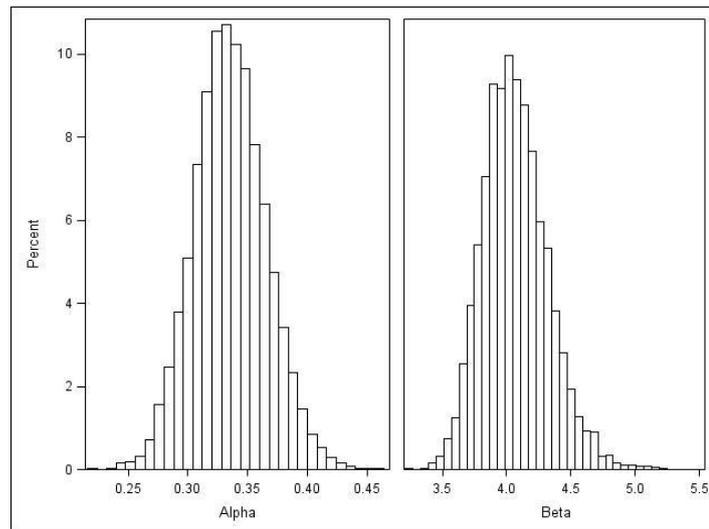
**FIGURE 5** Assumed relationship between speed and traffic flow.



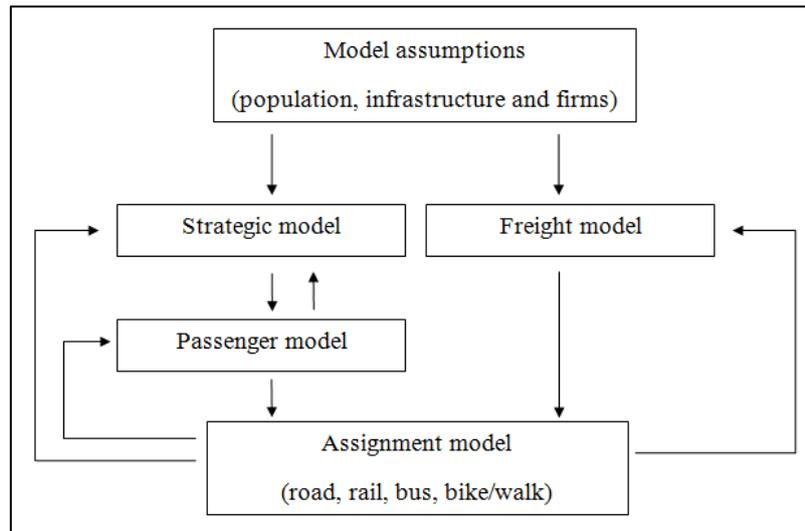
**FIGURE 6** Sections location on the Danish (Zealand) highway network



**FIGURE 7** Speed plotted against the density-density ratio.



**FIGURE 8 Alpha and Beta distributions.**



**FIGURE 5** The Lands Trafik Modellen (LTM) framework.

**TABLE 6 Characteristics of the Hastrid Dataset**

Highway	Section	Section Length	Capacity	Lanes	Observations
Holbæk (M11)	Taastrup - Fløng	1.460 km	4200	3	1,141
Holbæk (M11)	Ringstedvej - Roskilde	0.953 km	3400	2	1,582
Hillerød (M16)	Farum - Skovbrynet	3.701 km	4200	2	1,229

NOTE: 1mi=1.61km.

**TABLE 7 Bootstrap Parameters Statistics and Distribution Percentiles**

Parameter statistics											
Parameter	Estimate	StDev	Min	Max	CV						
Alpha	0.335	0.030	0.216	0.462	0.090						
Beta	4.070	0.254	3.238	5.373	0.062						
Distribution percentiles											
Parameter	P1	P10	P20	P30	P40	P50	P60	P70	P80	P90	P99
Alpha	0,27	0,30	0,31	0,32	0,33	0,33	0,34	0,35	0,36	0,37	0,41
Beta	3,55	3,76	3,86	3,93	3,99	4,04	4,12	4,18	4,27	4,40	4,77

**TABLE 8 Veh-Km and AvgSpeed CV Statistics**

		All links		Highway links	
		Veh-Km	AvgSpeed	Veh-Km	AvgSpeed
Scenario 1	Min	0	0	0	0
	Max	0.931	0.055	0.052	0.055
	Mean	0.011	0.000	0.003	0.001
	StDev	0.026	0.001	0.003	0.003
Scenario 2	Min	0	0	0	0
	Max	1.360	0.070	0.111	0.070
	Mean	0.015	0.001	0.012	0.007
	StDev	0.029	0.003	0.010	0.009

**TABLE 9 Network Travel Time (Hours)**

		Mean	St Dev	CV
Scenario 1	Free time	17,727,618	18,012	0.001
	Cong time	935,988	9,738	0.010
Scenario 2	Free time	17,461,650	30,483	0.001
	Cong time	961,328	192,646	0.200

**TABLE 10 Veh-Km CV by Groups**

	Group 1	Group 2	Group 3
Observations	33,385	307	25
Min	0	0.100	0.501
Max	0.099	0.494	0.931
Mean	0.009	0.189	0.573
StDev	0.010	0.089	0.110
Observations	33265	442	10
Min	0	0.100	0.507
Max	0.099	0.481	1.360
Mean	0.013	0.178	0.859
StDev	0.013	0.088	0.392