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A Higgs at 125.1 GeV and baryon mass spectra derived from a common U(3) Lie group framework

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Baryons are described by a Hamiltonian on an intrinsic U(3) Lie group configuration space with electroweak degrees of freedom originating in specific Bloch wave factors. By opening the Bloch degrees of freedom pairwise via a U(2) Higgs mechanism, the strong and electroweak energy scales become related to yield the Higgs mass 125.085+/-0.017 GeV and the usual gauge boson masses. From the same Hamiltonian we derive both the relative neutron to proton mass ratio and the N and Delta mass spectra. All compare rather well with the experimental values.

We predict neutral flavour baryon singlets to be sought for in negative pions scattering on protons or in photoproduction on neutrons and in invariant mass like $\Sigma_c^+(2455)D^-$ from various decays above the open charm threshold, e.g. at 4499, 4652 and 4723 MeV. The fundamental predictions are based on just one length scale and the fine structure coupling. The interpretation is to consider baryons as entire entities kinematically excited from laboratory space by three impact momentum generators, three rotation generators and three Runge-Lenz generators to internalize as nine degrees of freedom covering colour, spin and flavour.

Quark and gluon fields come about when the intrinsic structure is projected back into laboratory space depending on which exterior derivative one is taking. With such derivatives on the measure-scaled wavefunction, we derived approximate parton distribution functions for the u and d valence quarks of the proton that compare well with established experimental analysis.

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Introduction

In the electroweak sector of the standard model, the recent combination [1] of the Higgs mass determinations among the experimental groups ATLAS and CMS calls for accurate theoretical predictions to match the increased statistics expected when the results from LHC Run 2 at CERN will start to be released.

In the strong sector, the quark flavour model has a *missing resonance problem* since it predicts many more baryon πN-resonances than observed [2, 3, 4]. The quark colour model (QCD) has a confinement problem to construct hadrons analytically from quarks and gluons. We are aware of the successes of QCD in pertubative domains [5, 6] and in lattice gauge theory [7, 8, 9, 10]. Nevertheless we want to stress the two first mentioned issues. The latter, the confinement problem, motivated a radical approach which solves the former as a by-product. We construct the dynamics in a compact, intrinsic space. The spectroscopy can live there and manifest itself in real space as different mass resonances. The compactness means that the potential is periodic in parameter space. This opens for Bloch degrees of freedom. We will have confinement per construction and will see quarks and gluons by projection from the compact configuration space to the laboratory space. In general terms the idea is that a hamiltonian description is the more natural framework for spectroscopy and the lagrangian description more suitable for scattering phenomena, see e.g. [11].

In these proceedings we briefly sketch the correlation between strong and electroweak interactions for baryons leading to (1) and we give a value for $\alpha(m_\mathrm{W})$ from its established value $\alpha(m_\mathrm{Z})$. Figure 1 shows major results which include the result for the Higgs mass [12]

$$m_\mathrm{H}c^2 = \frac{1}{\sqrt{2}}\frac{2\pi}{\alpha}\Lambda = \frac{1}{\sqrt{2}}\frac{2\pi}{\alpha(m_W)}\frac{\pi}{\alpha_e}m_\mathrm{e}c^2 = 125.085 \pm 0.017\ \mathrm{GeV}. \tag{1}$$

This is based on a Higgs mechanism and in excellent agreement with the result $m_\mathrm{H}c^2 = 125.09 \pm 0.24$ GeV from the merged experiments. In (1) Λ is a strong interaction scale. The fine structure coupling α in the first fraction is to be evaluated at bosonic energies whereas the α in the second fraction is to be evaluated at electronic energies, where $\alpha_\mathrm{e} = e^2/(4\pi\varepsilon_0\hbar c) = 1/137.035999074(44)$ [13]. Elsewhere we gave the electroweak energy scale and masses of the gauge bosons. We further derived accurate expressions for the electron to neutron mass ratio, the relative neutron to proton mass shift, the neutral flavour baryon mass spectrum and approximate u and d quark parton distribution functions for the proton [14, 12].

Baryon colour and flavour from a common configuration space

We used a reinterpreted Kogut-Susskind intrinsic [14] Hamiltonian with a lattice-inspired potential (Manton) [18]

$$H\Psi(u) = \frac{\hbar c}{a}\left[-\frac{1}{2}\Delta + \frac{1}{2}\mathrm{Tr}\,\chi^2\right]\Psi(u) = \mathscr{E}\Psi(u) \tag{2}$$

for baryon mass spectra on the Lie group $U(3)$. Next we exploited paired Bloch degrees of freedom [19] to correlate the strong and electroweak energy scales by matching the Higgs field potential to the colour angle potential $\frac{1}{2}\mathrm{Tr}\,\chi^2$ [12], see fig. 2. We use $u = e^{i\chi} = e^{\alpha_k T_k} \in U(3)$ as an intrinsic

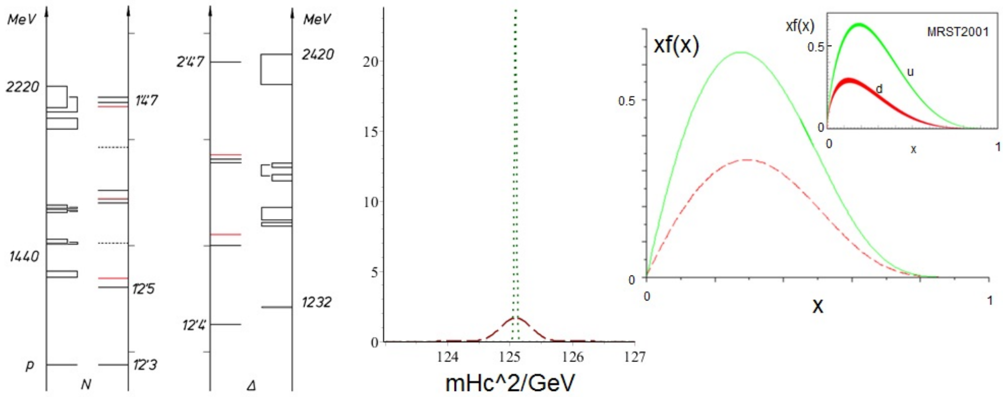

Figure 1: *Left*: Neutral flavour baryon mass spectrum adopted from [12]. The boxes indicate the uncertainty in peak value for all observed four star N and Δ resonances [13]. The lines are approximate predictions [12]. *Middle*: Higgs mass gaussians for comparison of the combined results of the ATLAS and CMS Collaborations [1] (dashed, red) with the present theoretical prediction based on a $U(2)$ Higgs mechanism correlated to a $U(3)$ configuration space for intrinsic baryon dynamics (dotted, green) [12]. The curve widths represent the uncertainties in peak determinations, not resonance widths [15, 16]. *Right*: Approximate parton distribution functions for u and d valence quarks of the proton [14]. For comparison is shown an insert for 10 GeV^2 adapted from the particle data group with other parton distributions erased [17].

configuration variable[1] for baryon colour dynamics with flavour degrees of freedom hidden in the Laplacian, see below. The configuration space $U(3)$ with its nine generators T_k and with nine dynamical variables $\alpha_k \in \mathbb{R}$ is the natural choice for the nine degrees of freedom that can be excited kinematically from laboratory space R^3. The potential $\frac{1}{2}\mathrm{Tr}\,\chi^2$, as well as the Laplacian Δ, can be parametrized by a polar decomposition [21] to yield for integer n

$$\frac{1}{2}\mathrm{Tr}\,\chi^2 = w(\theta_1)+w(\theta_2)+w(\theta_3), \quad \text{with} \quad w(\theta)=\frac{1}{2}(\theta-n\cdot 2\pi)^2, \;\; \theta\in[(2n-1)\pi,(2n+1)\pi] \tag{3}$$

where θ_j are dynamic torodial angles from the three eigenvalues $e^{i\theta_j}$ of the configuration variable u and the trace folds out in periodic parametric potentials, see fig. 2. The Laplacian is [21, 23]

$$\Delta=\sum_{j=1}^{3}\frac{1}{J^2}\frac{\partial}{\partial\theta_j}J^2\frac{\partial}{\partial\theta_j}-\sum_{i<j,k\neq i,j}^{3}\frac{K_k^2+M_k^2}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)}, \quad \text{where} \quad J=\prod_{i<j}^{3}2\sin\left(\frac{1}{2}(\theta_i-\theta_j)\right). \tag{4}$$

The scale $\Lambda=\hbar c/a$ is given via a space projection $x_i = a\theta_i$ where the length scale a is matched to the classical electron radius r_e by $\pi a = r_e = e^2/(4\pi\varepsilon_0 m_e c^2)$ [14]. The toroidal generators $T_j = -i\frac{\partial}{\partial\theta_j}$ correspond to parametric momenta with standard commutators

$$p_j=-i\hbar\frac{1}{a}\frac{\partial}{\partial\theta_j}=\frac{\hbar}{a}T_j, \quad [p_j, a\theta_i]=-i\hbar\delta_{ij}. \tag{5}$$

To solve the intrinsic Schrödinger equation (2) for stationary baryonic states we factorized the wavefunction [14] into a toroidal part τ and an off-torus part Υ to match the polar decomposition

[1] For a general discussion of group elements as elementary physical variables see ref. [20].

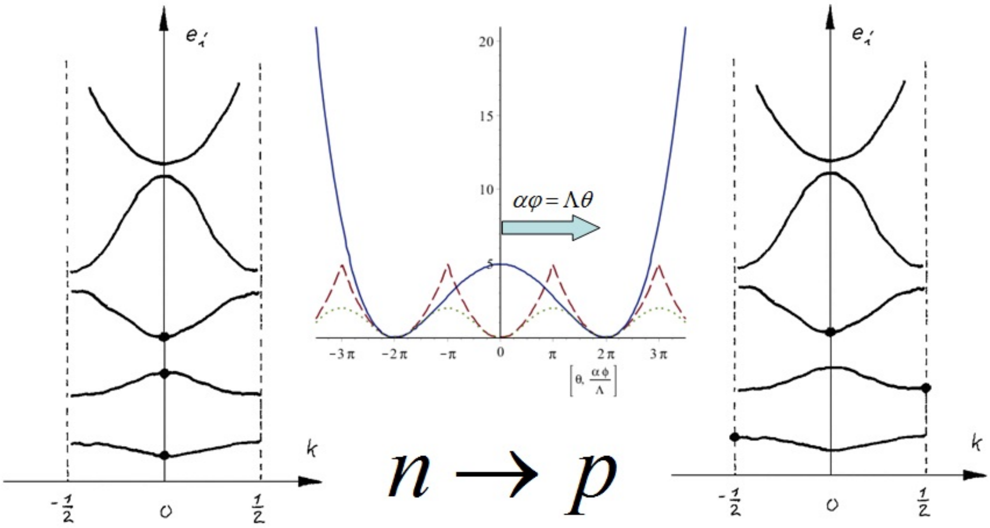

Figure 2: *Left and right*: Reduced zone schemes [19] for Bloch wave numbers for the neutron state (left) and the proton state (right) [12]. *Middle*: Higgs potential (solid, blue) matching the Manton-inspired potential [18] (dashed, red) and the Wilson-inspired potential [22] (dotted, green). The Manton and Wilson inspired potentials yield the same value for the Higgs mass and the electroweak energy scale whereas only the Manton inspired potential gives a satisfactory reproduction of the baryon spectrum [12].

in (4)

$$\Psi(u) = \tau(\theta_1, \theta_2, \theta_3)\Upsilon(\alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9), \tag{6}$$

where the six α_ks trace out the off-toroidal degrees of freedom. The parton distribution functions [14] in fig. 1 were generated by space projections of the toroidal part $R = J\tau$ of the measure-scaled wavefunction $\Phi = J\Psi$ via the exterior derivative [24] $dR = \psi_j d\theta_j$. The differential form coefficients ψ_j are interpreted as quark field colour components.

In the coordinate representation [25] we have [14] e. g.

$$K_1 = a\theta_2 p_3 - a\theta_3 p_2 = \hbar\lambda_7 \quad \text{and} \quad M_1/\hbar = \theta_2\theta_3 + \frac{a^2}{\hbar^2}p_2 p_3 = \lambda_6. \tag{7}$$

The λ_ks are Gell-Mann generators. The K_ks and the M_ks commute as

$$[M_k, M_l] = [K_k, K_l] = -i\hbar\varepsilon_{klm}K_m. \tag{8}$$

The sign for the K-commutators signifies body fixed coordinates for intrinsic spin as in nuclear physics [26]. From the Casimir operator $C_1 = \lambda_k\lambda_k$ one gets [14, 12] by repeated use of (5) the Okubo structure [27, 28]

$$K(K+1) + M^2 = a' + b'y + c'\left[\frac{1}{4}y^2 - i(i+1)\right] \tag{9}$$

for the nominator in the second term in (4) which contains the baryon flavour structure. Here a', b' and c' are constants and y and i are hypercharge and isopin respectively.

Strong electroweak correlation for a Higgs field

We match the intrinsic colour phase angle θ and the Higgs field by correlating the energies in a balancing trailing Ansatz $\Lambda\theta = \alpha\phi$ and fit the potential V_{H} of the Higgs field ϕ with the troughs $n = \pm 1$ neighbouring to the generic one at $n = 0$. This gave us the weak energy scale $v = \sqrt{2}\phi_0$ [12] from a 2π shift of θ, see fig. 2, with the fit

$$V_{\mathrm{H}}(\phi) = \delta^2 - \frac{1}{2}\mu^2(\phi^\dagger\phi) + \frac{1}{4}\lambda^2(\phi^\dagger\phi)^2; \quad \delta^2 = \frac{1}{8}\phi_0^2, \ \ \mu^2 = \frac{1}{2}, \ \ \lambda^2 = \frac{1}{2}\frac{1}{\phi_0^2}, \ \ \phi_0 = \frac{2\pi}{\alpha}\Lambda. \tag{10}$$

From this follows $m_{\mathrm{H}}c^2 = \frac{1}{\sqrt{2}}\phi_0$ giving (1). The present v differs from the standard model value v_{SM} by a factor $\sqrt{V_{\mathrm{ud}}}$ involving the u-d element of the Cabibbo-Kobayashi-Maskawa quark flavour mixing matrix [29], thus $v_{\mathrm{SM}} = v\sqrt{V_{\mathrm{ud}}} = 246.85$ GeV [12] .

Two component Higgs to open U(2) Bloch degrees of freedom

For the 2π-shift in Fig. 2 to occur, spontaneous period doublings are needed in the toroidal part $R = J\tau$ of the measure-scaled wavefunction. Since the labelling of θ_j is arbitrary, τ is symmetric in these, and since J is antisymmetric, R can be expanded on Slater determinants

$$b_{pqr} = \varepsilon_{ijk} b_p(\theta_i) b_q(\theta_j) b_r(\theta_k), \tag{11}$$

where p,q,r are natural number labels for orthogonal solutions to the one-dimensional Schödinger equation

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial\theta^2} + w(\theta)\right] b_p(\theta) = e_p b_p(\theta). \tag{12}$$

These Slater determinants require paired period doublings in order to cancel the denominator in the second term of the Laplacian (4). Thus the Higgs field to absorb the phase changes must be a two-component complex structure in which case we can expand e.g. on [12]

$$g_{pqr} - g_{qpr} = e^{ir\theta_3} e^{i(p+q)\frac{\theta_1+\theta_2}{2}} 2i\sin\left((p-q)\frac{\theta_1-\theta_2}{2}\right); \quad g_{pqr} = e^{ip\theta_1} e^{iq\theta_2} e^{ir\theta_3} \tag{13}$$

even for half odd-integer p,q. For integer p,q,r eq. (2) can be solved with exact integrals by a Rayleigh-Ritz method to give, e.g. the dimensionless ground state $\mathrm{E_n} = \mathscr{E}_{\mathrm{n}}/\Lambda$ which yields

$$\frac{m_{\mathrm{e}}}{m_{\mathrm{n}}} = \frac{\alpha(m_{\mathrm{n}})}{\pi}\frac{1}{\mathrm{E_n}} = 1/1838.\cdots \tag{14}$$

In particular the base

$$b_{pqr} = \varepsilon_{ijk}\cos(p\theta_i)\cos(q\theta_j)\cos(r\theta_k), \tag{15}$$

gives rise to scarce singlets [12] of which especially the ones at 4228 MeV and 4499 MeV around the open charm threshold at 4324 MeV are intriguing in the light of the recent LHCb charged pentaquark observations in that area [30]. For half odd-integer p,q we have not found a proper selection rule for the base (13) that would make the Rayleigh-Ritz method viable. The spectrum in fig. 1 is an approximation based on (12) but now allowing for 4π periodicity. This leads to a diminished ground state $\mathrm{E_p} = e_1' + e_2' + e_3$ that gives the relative neutron to proton mass shift $(\mathrm{E_n} - \mathrm{E_p})/\mathrm{E_p} = 0.13847$ % which compares rather well with the value $(m_{\mathrm{n}} - m_{\mathrm{p}})/m_{\mathrm{p}} = 0.137842$ % calculated from the masses that are known experimentally with eight significant digits [13].

Effective fine structure coupling at W boson energies and the resulting Higgs mass

We estimate the effective fine structure coupling at W boson energies from its known value at Z boson energies based on available analytical expressions. In general at energy scale μ we have the total correction $\Delta\alpha$ defined by

$$\alpha(\mu) = \frac{\alpha_e}{1-\Delta\alpha}. \tag{16}$$

The best known α near m_W is the modified minimal subtraction value at m_Z [31]

$$\alpha^{-1}_{\overline{MS}}(m_Z) = 127.940 \pm 0.014. \tag{17}$$

We estimate $\alpha^{-1}_{\overline{MS}}(m_W)$ based on $\alpha^{-1}_{\overline{MS}}(m_Z)$ by translating between renormalization schemes. From the Particle Data Group [31] we have with known constants a_0, a_1 and a_2

$$\Delta\alpha_{\overline{MS}}(m_Z) - \Delta\alpha(m_Z) = \frac{\alpha_e}{\pi}\left[a_0 - \frac{7}{4}\ln\left(\frac{m_Z^2}{m_W^2}\right) + \frac{\alpha_s(m_Z)}{\pi}a_1 + \frac{\alpha_s^2(m_Z)}{\pi^2}a_2\right]. \tag{18}$$

For the effect of scale changes in the strong coupling constant α_s we use Schwartz [32]. For the radiative corrections we use [33]

$$\alpha^{-1}_{\mathrm{rad}}(m_Z) - \alpha^{-1}_{\mathrm{rad}}(m_W) = -\frac{1}{3\pi}\ln\left(\frac{m_Z^2}{m_W^2}\right) \Big/ \left(1 - \frac{3\pi/4}{\alpha_e^{-1} + 3\pi/4}\right) = -0.02725... \tag{19}$$

For lepton and hadron loop corrections we use expressions from Jegerlehner [12, 34]. We update the quark masses to $m_{u,d,s,c,b,t} = 0.023, 0.048, 0.095, 1.275, 4.18, 173.21$ GeV [35] as a generalization of Jegerlehner's $\Delta\alpha^{(5)}_{\mathrm{had}}$ and use $\alpha_s = 0.120$ [31]. With the lepton masses $m_{e,\mu,\tau} = 0.510998928, 105.6583715, 1776.82$ MeV [36] and the updated quark masses we can find

$$\Delta\alpha(m_W) - \Delta\alpha(m_Z) = \left[\Delta\alpha_{\mathrm{lep}}(m_W) - \Delta\alpha_{\mathrm{lep}}(m_Z)\right] + \left[\Delta\alpha_{\mathrm{had}}(m_W) - \Delta\alpha_{\mathrm{had}}(m_Z)\right] + \left[\Delta\alpha_{\mathrm{rad}}(m_W) - \Delta\alpha_{\mathrm{rad}}(m_Z)\right] \tag{20}$$

to yield -0.001300728595 (saving redundant digits for later round off). Then we have

$$\Delta\alpha(m_W) = \Delta\alpha(m_Z) + [\Delta\alpha(m_W) - \Delta\alpha(m_Z)] = 0.05790868410. \tag{21}$$

From (21) we can return to the modified minimal subtraction scheme by use of (18) for m_W to get

$$\Delta\alpha_{\overline{MS}}(m_W) = \Delta\alpha(m_W) + [\Delta\alpha_{\overline{MS}}(m_W) - \Delta\alpha(m_W)] = 0.06610069472 \tag{22}$$

which yields $\alpha_{\overline{MS}}(m_W) = 1/127.9778244$ in (16) to be used for the Higgs mass in (1). To get an estimate on the uncertainties in the Higgs mass result (1) we have done calculations with and without the top quark included ($n_f = 6$, respectively $n_f = 5$), compared with the Jegerlehner quark mass parameters and done calculations with two different values of the strong coupling constant at the Z-scale, namely $\alpha_s(m_Z) = 0.120$ and $\alpha_s(m_Z) = 0.1184$ respectively. The uncertainty on $\alpha_{\overline{MS}}(m_Z)$ is stated in (17) and represents the largest contribution. Thus the calculated Higgs mass can be formulated as $m_H c^2 = 125.085 \pm 0.014(\alpha_Z) \pm 0.0066(n_f) \pm 0.0063(\alpha_s) \pm 0.0003(m_{\mathrm{quark}})$ GeV. In fig. 1 we compare this with the recent combined result $m_H c^2 = 125.09 \pm 0.21(\mathrm{stat.}) \pm 0.11(\mathrm{scale}) \pm 0.02(\mathrm{other}) \pm 0.01(\mathrm{theory})$ GeV from the CMS and ATLAS collaborations [1].

PoS(EPS-HEP2015)097

Conclusion

The agreement is excellent between the calculated Higgs mass $m_{\mathrm{H}}c^2 = 125.085 \pm 0.017$ GeV - based on fine structure constants evaluated at electronic and W-bosonic energies - and the combined experimental result for Higgs production in proton-proton collisions with decays either $H \to \gamma\gamma$ or $H \to ZZ \to 4l$ from the groups ATLAS and CMS at CERN, $m_{\mathrm{H}}c^2 = 125.09 \pm 0.24$ GeV. Our estimate of $\alpha_{\overline{\mathrm{MS}}}(m_{\mathrm{W}})$ is to be confirmed by thorough renormalization group evolution. The above agreement on the Higgs mass raises the question of whether - as expected in the present work - all experimental pp-channels for Higgs particle resonances should give the same result. Since new planned experiments at the LHC Run 2 are expected to give accurate values of the Higgs mass according to higher statistics, it is crucial (for us) to see if our prediction continues to coincide with the experimental value, i.e. to see whether the type of production channel and particle decay is decisive for the experimental Higgs mass value.

In the baryon sector, we predict neutral charge, neutral flavour singlets in invariant mass spectra of $\Sigma_c^+(2455)D^-$ above the open charm threshold, e.g. at 4499, 4652 and 4723 MeV. Such singlets are particular for the present model. The singlets should also be visible in negative pions scattering on protons.

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PoS(EPS-HEP2015)097