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# Fundamental limitations to gain enhancement in slow-light photonic structures

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**Abstract:** We present a non-perturbative analysis of light-matter interaction in active photonic crystal waveguides in the slow-light regime. Inclusion of gain is shown to modify the underlying dispersion law, thereby degrading the slow-light enhancement.

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## 1. Introduction

Light propagation in periodic structures can be significantly slowed down in the spectral vicinity of band edges. In a scattering picture, the light is undergoing multiple back and forth reflections with a longer effective path length in the periodic medium, thus effectively making a wavepacket advancing more slowly. Consequently, the available time for light-matter interactions is prolonged and a variety of nonlinear and linear processes may be enhanced [1, 2]. In particular, the slow-light enhancement of linear processes scale with the group index itself, while nonlinear processes scale with  $n_g^n$  ( $n$  is the order of nonlinear process). From a device prospective, enhancement implies that size and power requirements can be reduced proportionally to the  $n_g$ . Considering gain, there is a common expectation that if a material with net gain  $g_0$  is incorporated in a periodic medium, such as Bragg stacks, photonic crystals (PhC) or metamaterials, the gain will effectively be enhanced to  $g_{\text{eff}} \sim n_g^0 g_0$ , where  $n_g^0$  is the group index associated with the underlying dispersion relation  $\omega_0(k)$  of the passive structure.

We analyze the modification of the dispersion due to gain, and show that a large gain will eventually jeopardize the desired slow-light dispersion supported by the periodic system, thus suppressing the slow-light induced light-matter interaction enhancement. On the other hand, a small amount of material gain is shown to be beneficial. Importantly, devices employing quantum-dot gain material may thus display a superior performance.

## 2. Gain and group index

For PhC waveguides, the firm light confinement and strong structural dispersion make them attractive candidates for both compact photonic devices as well as for fundamental explorations of light-matter interactions [2]. In this work, we model gain by including a small imaginary part  $\epsilon''$  to the base material of the PhC. For a specified real-valued frequency  $\omega$  we find the associated complex-valued  $k$  from a matrix eigenvalue problem. Physically, the group index and the photonic density of states (PDOS) thus remain unchanged if changing from loss to a corresponding gain, while there of course is a change from a net loss to a net gain when inspecting the changes in  $k''$ . This shows, that in general gain will introduce many of the same challenges for the slow light, which are known to be a serious issue in the context of absorption.

As an example, Figure 1. shows the effective gain  $g_{\text{eff}} = 2k''$  (right-hand axis) versus  $g_0$  evaluated at  $\omega^*$  (where the propagation is initially slowest) for a slow-light W1 PhC waveguide made in a semiconductor material with gain. Recalling the introductory discussion we anticipate an enhancement proportional to  $n_g$  for low gain. Clearly,  $g_{\text{eff}}$  starts out with a big slope in the low-gain limit, i.e. gain is greatly enhanced. However, at the PDOS singularity associated with the band edge  $n_g(g_0) \propto g_0^{-1/2}$  [3], and consequently

$$g_{\text{eff}}(g_0) \propto n_g(g_0)g_0 \propto g_0^{1/2}. \quad (1)$$

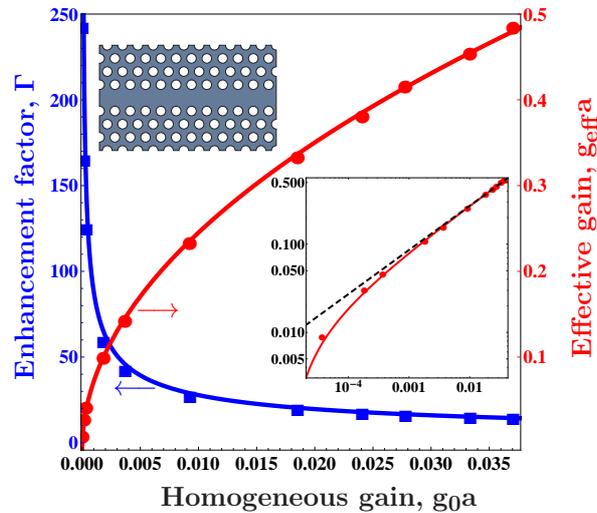


Fig. 1. Example of slow-light enhanced gain in W1 PhC waveguide with embedded semiconductor layer displaying gain. The red data points (right-hand axis) show slow-light enhanced gain  $g_{\text{eff}}$  versus homogeneous gain  $g_0$ , evaluated at  $\omega^*$  where the group index is initially maximal. The red solid line shows a fit to the anticipated square-root dependence, Eq. (1), while the inset (log-log scale) exhibits minor quantitative deviations from a strict square-root dependence (dashed black line) due to a slight detuning from the band-edge singularity. The blue data points (left-hand axis) shows the corresponding gain enhancement  $\Gamma$ .

Indeed, this effect is supported by the full numerical data (red points) and the indicated square-root dependence (red line). The slow-light enhancement factor  $\Gamma = g_{\text{eff}}/g_0$  (blue line, left-hand axis) is correspondingly large for low  $g_0$ . In the example,  $\omega^*$  is slightly detuned from the singularity and a more detailed analysis yields  $n_g \propto (\text{const.} + g_0)^{-1/2}$  [4]. Consequently, a deviation from the square root dependence for small  $g_0$  takes place (see inset). In this analysis we have implicitly assumed that the passive structure itself is ideal and with a diverging group index. Of course, disorder and imperfections will inevitably be present no matter the effort invested in the fabrication of the PhC. Ensemble averaging over disorder configurations will have the same overall effect on the PDOS as gain or absorption will have; singularities become smeared and the group index assumes a finite value. Clearly, such broadening can not be compensated by the addition of gain and the achievable effective gain may turn out lower than the estimate given above.

### 3. Conclusions

Adding gain to a periodically structured photonic material changes the dispersion properties and the slow-light enhanced gain in a complex manner. By both analytical examples and a numerical study we have illustrated how a large material gain degrades the slow-light properties supported by the corresponding passive structure, thereby eventually limiting the effective gain enhancement.

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