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Bisinella, Valentina; Conradsen, Knut; Christensen, Thomas Højlund; Astrup, Thomas Fruergaard

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A GLOBAL APPROACH FOR SPARSE REPRESENTATION OF UNCERTAINTY IN LIFE CYCLE ASSESSMENTS OF WASTE MANAGEMENT SYSTEMS

V. Bisinella,[a], K. Conradsen**, T.H. Christensen*, T.F. Astrup**

* Technical University of Denmark, Department of Environmental Engineering, Miljoevej, Building 113, 2800 Kongens Lyngby, Denmark

** Technical University of Denmark, Department of Applied Mathematics and Computer Science, Richard Petersens Plads, Building 321, 2800 Kongens Lyngby, Denmark

[a] Corresponding author. Tel: +45 4525 1698, E-mail: valenb@env.dtu.dk

Abstract

Purpose Identification of key inputs and their effect on results from Life Cycle Assessment (LCA) models is fundamental. Because parameter importance varies greatly between cases due to the interaction of sensitivity and uncertainty, these features should never be defined *a priori*. However, exhaustive parametrical uncertainty analyses may potentially be complicated and demanding, both with analytical and sampling methods. Therefore, we propose a systematic method for selection of critical parameters based on a simplified analytical formulation that unifies the concepts of sensitivity and uncertainty in a Global Sensitivity Analysis (GSA) framework.

Methods The proposed analytical method based on the calculation of Sensitivity Coefficients (SC) is evaluated against Monte Carlo sampling on traditional uncertainty assessment procedures, both for individual parameters and for full parameter sets. Three full-scale waste management scenarios are modelled with the dedicated waste LCA model EASETECH and a full range of ILCD recommended impact categories. Common uncertainty ranges of 10 % are used for all parameters, which we assume to be normally distributed. The applicability of the concepts of additivity of variances and GSA is tested on results from both uncertainty propagation methods. Then, we examine the differences in discernibility analyses results carried out with varying numbers of sampling points and parameters.

Results and discussion The proposed analytical method complies with the Monte Carlo results for all scenarios and impact categories but offers substantially simpler mathematical formulation and shorter computation times. The coefficients of variation obtained with the analytical method and Monte Carlo differ only by 1 %, indicating that the analytical method provides a reliable representation of uncertainties and allows determination of whether a discernibility analysis is required. The additivity of variances and the GSA approach show that the uncertainty in results is determined by a limited set of important parameters. The results of the discernibility analysis based on these critical parameters vary only by 1 % from discernibility analyses based on the full set but require significantly fewer Monte Carlo runs.

Conclusions The proposed method and GSA framework provide a fast and valuable approximation for uncertainty quantification. Uncertainty can be represented sparsely by contextually identifying important parameters in a systematic manner. The proposed method integrates with existing step-wise approaches for uncertainty analysis by introducing a global importance analysis before uncertainty propagation.

Keywords: LCA, Uncertainty propagation, Analytical methods, Monte Carlo, Global Sensitivity Analysis, Total variance

1 Introduction

Uncertainty analysis is essential for a balanced interpretation and use of Life Cycle Assessment (LCA) in decision-making. In relation to waste, LCA is gaining increased use for quantification of the environmental performance and sustainability of alternative management solutions (Laurent et al. 2014a; Laurent et al. 2014b). LCAs of waste management systems are often relatively complex models, where the results are subject to uncertainty due to combined effects of inherent data variability, unrepresentative datasets, and modelling assumptions (Clavreul et al. 2012). In order to ensure transparency and reliability of modelling of such systems, identification of the most important factors and understanding of the mechanisms by which they influence the results are fundamental.

Several authors have investigated uncertainty in LCA and agree on categorizing the relevant factors as parameter, scenario, and model uncertainties (Heijungs et al. 2005; Lloyd and Ries 2007; Clavreul et al. 2012). As far as parameters are concerned, critical data inputs in waste LCAs are commonly: chemical waste composition, material and energy recovery efficiencies, etc.

However, the relevance of these factors cannot be defined *a priori*. Rather, the importance of a parameter is a global concept, defined by the interaction of the parameter´s sensitivity and uncertainty (Heijungs, 1996). While the sensitivity accounts for the weight of a parameter in the case-specific model configuration, the uncertainty related to a parameter does not depend on the system, but on the parameter´s nature and characteristics. For these reasons, critical factors should be identified with a systematic and rigorous approach that integrates sensitivity and uncertainty on a case-by-case basis, such as life-cycle screening (Heijungs, 1996).

In LCAs of waste management systems, the approach previously suggested in the literature was instead a sequence of contributions, sensitivity and uncertainty analyses (e.g. Clavreul et al., 2012; Heijungs and Kleijn, 2001, Laurent et al., 2014b). For uncertainty analysis, different approaches are available: propagation by analytical or sampling based methods. The analytical approaches are based on the theory of error propagation (Ciroth et al. 2004) and address with differential calculus how input uncertainties propagate into the output uncertainties through the LCA mathematical model (Groen et al. 2014). Multiple analytical expressions based on a wide range of formulations and assumptions are present in the literature (Heijungs et al. 2005; Heijungs 2010; Hong et al. 2010; Clavreul et al. 2012; Imbeault-Tétreault et al. 2013). Alternatively, sampling methods consist of repeatedly calculating the impact scores with inputs randomly sampled from specified probability distributions (Imbeault-Tétreault et al. 2013). Most LCA software tools facilitate uncertainty propagation by means of sampling methods, mostly based on Monte Carlo simulations (Lloyd and Ries 2007).

These two classes of error propagation approaches have been compared by a number of authors with respect to data input needs, formulas, and types of output, in the light of their foundation and implementation in LCA and emphasizing performance-wise differences and similarities. Quantitative comparisons in literature demonstrate that the two methods provide similar results but differ with respect to output type, input data requirements and computing time (Heijungs and Lenzen 2014; Groen et al. 2014). Overall, analytical methods require shorter computing times and facilitate extraction of preliminary uncertainty information when large numbers of input parameters are considered and the input uncertainties are small (Groen et al. 2014). On the other hand, sampling methods produce a sample of results from which several statistics can be computed and provide more types of information (Heijungs and Lenzen 2014). Various drawbacks of these approaches have been emphasized in literature: from analytical methods, the output of a single variance value does not allow visualization of the uncertainty as a probability distribution, nor the performance of discernibility analyses (Heijungs and Lenzen 2014). In particular, the analytical formulation involving Taylor series expansion (Heijungs et al. 2005; Heijungs 2010) was found to be impractical in case of the large and complex scenarios often applied in waste LCAs (Clavreul et al. 2012). Nonetheless, sampling methods involving Monte Carlo techniques are computationally intensive and do not automatically assess the sensitivity and contribution of individual parameters to the total parametrical uncertainty (Hong et al. 2010; Heijungs and Lenzen 2014). Finally, the accuracy of sampling methods is often hampered by the difficulty of assigning the required uncertainty distributions to the often numerous parameters in an LCA model (Hong et al. 2010).

Therefore, the task of identifying critical factors of a waste management LCA on a case-by-case basis becomes highly complicated when all the aspects of these models have to be included for the uncertainty propagation, especially in real-scale case studies and across multiple impact categories. Following the tiered approaches of Heijungs et al. (2005) and Clavreul et al. (2012), sensitivity and uncertainty analyses are most often done separately, where uncertainty propagation is rarely carried out due to its perceived complexity (Laurent et al., 2014b). However, *a priori* and unjustified exclusion of individual parameters does not offer a valid approach to uncertainty propagation. Indeed, the full influence of input parameters with a low sensitivity and a high uncertainty would not be quantified, especially in cases where the uncertainty is propagated only for highly sensitive parameters, e.g. Clavreul et al. (2012), leading to iterative revisions of the model results (Saltelli et al., 2006). A systematic "importance measure" approach that includes analysis of the fundamental connections between sensitivity and uncertainty of individual parameters, commonly known also as Global Sensitivity Analysis (GSA), has been identified as the best practice, *inter alia*, by Saltelli et al. (2006), but has never been applied explicitly by existing literature on waste management LCAs.

GSA aims to ascertain how a specific system depends on the model structure and the information entered into the model. GSA offers a thorough assessment framework that can provide guidance for improving reliability, transparency and credibility of environmental assessments (Kioutsioukis et al. 2004). GSA methods subdivide into variance-based and moment-independent importance measures. The first exploit the law of total variance, or Sobol's functional variance decomposition (Sobol' 2001), and are characterized by identifying the most important variables in a global perspective, thus allowing to focus on improving the quality of critical input data. Similar approaches in LCA literature appear with different names, e.g. "life-cycle screening" (Heijungs et al., 1996), "key issue analysis" (Heijungs et al., 2005), "uncertainty contribution analysis" (Clavreul et al., 2012), and "contribution to variance" (Heijungs and Lenzen, 2014). Moment-independent methods calculate the difference in the uncertainty in output that would be provided by knowing the value of parameters in input, one at a time. However, the separation between unconditional and conditional output densities becomes smaller with a high number of parameters in the model and implementation of this technique might be problematic for models with long computational times (Borgonovo et al., 2011). Variance-based importance measures thus seem to be more suitable for vast models with countless parameters such as waste-LCAs. So far, very little attention has been devoted to the possibility of explaining a large proportion of the variability in environmental impacts with a limited number of parameters (Bala et al. 2010). Only a few studies have proposed a systematic method for identification of the most influential parameters in LCA: Padey et al. (2012) presented a method where a general variance decomposition based on the Sobol' indices was applied to quantify the influence of input parameters on result variability. Meinrenken et al. (2012, 2014) applied parameter screening for a concurrent uncertainty contribution analysis during the data gathering phase. While any cut-off threshold for selection of critical parameters should be based on multiple impact categories rather than a single impact category, existing literature on uncertainty in LCA has focused primarily on climate change impacts (Bourgault et al., 2012). Consistent assessment of uncertainties in the increasingly complex scenarios assessed in state-of-the-art waste LCAs (e.g. a wide range of waste material fractions, treatment, recovery and disposal technologies) requires a more systematic approach to identify critical factors and quantify their contribution to uncertainty.

The objective of this paper is to provide a systematic and reproducible method for identification and uncertainty propagation of important parameters in a GSA perspective for application in waste LCA modelling, based on and aiming to simplify and improve existing tiered approaches. Three full-scale waste management alternatives for municipal solid waste in Denmark were modelled with the dedicated waste LCA model EASETECH (Clavreul et al. 2014). A sensitivity analysis was carried out according to the sequential approach described in Clavreul et al. (2012); then, uncertainties were propagated (i) analytically and (ii) by means of Monte Carlo sampling. A discernibility analysis was carried out comparing scenarios modelled based on uncertainty propagation of the full set of parameters and scenarios modelled with uncertainty propagation only of the parameters identified as critical in a GSA perspective. The analysis was performed with normal probability distributions for parameters and for all ILCD recommended impact categories.

2 Methods

2.1 Sensitivity analysis

Sensitivity analysis constitutes a well-established phase in traditional uncertainty quantification approaches, and is the first step in applying GSA to an LCA scenario. Sensitivity analysis identifies how results vary as a consequence of a change in the model input values and is composed by: contribution analysis (results decomposition into processes and substances), perturbation analysis and calculation of sensitivity coefficients. In the perturbation analysis each parameter is increased by a limited numerical amount in a "one-at-a-time" (OAT) manner while keeping all other parameters fixed at their nominal value. For an increment of the value of the input parameter *i*, a new impact score is calculated and compared with the initial score within the same impact category *j*. The following ratios are calculated:

The sensitivity coefficient (SC), which is the ratio between two absolute changes:

$$SC_i^j = \frac{(\Delta \text{ result})^j}{(\Delta \text{ parameter})_i} \approx \frac{\partial z_j}{\partial x_i} \quad (1)$$

The sensitivity ratio (SR), which is the ratio between the two relative changes:

$$SR_i^j = \frac{\left(\frac{\Delta \text{ result}}{\text{initial result}}\right)^j}{\left(\frac{\Delta \text{ parameter}}{\text{initial parameter}}\right)_i} \approx \frac{\partial z_j}{\partial x_i}\frac{x_i}{z_j} \tag{2}$$

With i=1,…,n tested parameters in the model and j=1,…,m impact categories in the characterization method selected for the calculation of the impacts. The second expression complies with Heijungs and Lenzen (2014), where $z=z(x,y)$ is the model result, x_i the input parameter and j the impact category.

2.2 Uncertainty propagation

Parametrical input uncertainties are systematically propagated into output uncertainties with analytical or sampling methods, as previously explained. At this stage, the LCA practitioner chooses whether to represent uncertainty according to the probability or possibility theory. The first assumes that all uncertainties can be represented by single probability distributions, thus referring to stochastic uncertainty related to measured data variability and fluctuations. Here the probability theory is applied as the focus was placed on the uncertainty propagation theory rather than the nature of the data. Laurent et al. (2014b) highlighted how it is a common difficulty to represent the uncertainties of the input data that will be propagated in the model: please refer to the Supporting Information for further instructions on how to provide input uncertainty.

2.2.1 Analytical uncertainty propagation

Analytical uncertainty propagation in LCA has been addressed by a number of authors, with various formulations and assumptions. As explained, *inter alia*, by Ciroth et al. (2004), the analytical approach is based on the theory of error propagation, where the influence of perturbations can be approximated by differential calculus. Using the first order approximation of the Taylor series, the uncertainty associated with the function $z=z(x,y)$, is:

$$V(z) \approx \left(\frac{\partial z}{\partial x}\right)^2 \cdot V(x) + \left(\frac{\partial z}{\partial y}\right)^2 \cdot V(y) + 2\frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y} \cdot COV(x,y) \tag{3}$$

The uncertainty is thus given by the partial first-order derivatives of the function, multiplied by the input uncertainty associated with the parameters, $V(x)$ and $V(y)$. In Eq. (3), the covariance term includes the possibility that the errors of the variables x and y are correlated (Heijungs and Lenzen 2014).

The correlation structure among variables in LCA is rarely investigated (Bojacá and Schrevens 2010), as in most cases the covariance is assumed to be negligible and uncertainties to be independent (Heijungs et al. 2005). In such cases, the covariance can be set to zero and Eq. (3) simplified to:

$$V(z) \approx \left(\frac{\partial z}{\partial x}\right)^2 \cdot V(x) + \left(\frac{\partial z}{\partial y}\right)^2 \cdot V(y) \tag{4}$$

Where *V(x)* and *V(y)* can be associated with any type of underlying distribution. Examples of the values the first order derivative may assume can be found in Heijungs (2010), Hong et al. (2010, 2012) and Imbeault-Tétreault et al. (2013). Eq. (4) can be further approximated according to Heijungs et al. (2005) and the SC method introduced by Clavreul et al. (2012). A small change, Δx of an input parameter x, leads to a change Δz in the result z, as explained in Section 2.1. This leads to the approximation of the first-order derivative with the relative change, and thus SC as in (1).

$$\frac{\partial z_j}{\partial x_i} \approx \frac{\Delta z_j}{\Delta x_i} = SC_x^j \tag{5}$$

The SC method was tested against the Taylor series expansion method by Clavreul et al. (2012). The results varied by less than 0.5 %, confirming that the simpler SC method can be used as a good approximation.

The analytical approach for error propagation presented in this paper is based on the abovementioned findings in literature and the following assumptions:

i) The model is linear. In reality, an LCA model is composed of mixed equations of multiplications and sums of variables, calling into question the validity of the first order approximation. However, as shown by Imbeault-Tétreault et al. (2013), the difference between the results obtained with sampling methods did not justify the use of a more complex analytical method. ii) Independence between model input parameters (univariate distributions). As a first approximation, including covariance for all input parameters was recognized as an unfeasible task also by Huijbregts et al. (2003). iii) An unspecified form of the probability distribution of input parameters. Contrary to Hong et al. (2010, 2012) and Imbeault-Tétreault et al. (2013), Heijungs et al. (2005, 2010 and 2014) highlight that it suffices to specify the first two moments of the distribution (mean and variance). The same approach was followed by Groen et al. (2014).

Schematizing a general LCA with a mathematical relationship as:

$$Y^j = f(X_1, ..., X_n) \tag{6}$$

Where Y is the result score for the impact category j, depending on an n number of input parameters X_i. Then the analytical uncertainty for the individual parameter, i, is given by:

$$V(Y)_i^j \approx \left(SC_i^j\right)^2 \cdot V_{input}(X_i) \tag{7}$$

Where V_{input} is the initial uncertainty associated to the i-th parameter X_i.

When considering all parameters in a scenario, the total parametrical variance corresponds to:

$$V(Y)^j \approx \sum_{i=1}^{n} \left[\left(SC_i^j\right)^2 \cdot V_{input}(X_i)\right] \tag{8}$$

The variance in the result score in a specific impact category, j, will thus be given by the sum of the single parameter uncertainties. These are determined by a fixed initial input uncertainty and the specific SC of the i-th parameter in that impact category.

2.2.2 Uncertainty propagation with Monte Carlo sampling

Selected input parameters are represented by a stochastic variable with a defined probability distribution. The Monte Carlo analysis randomly samples a value within each uncertainty distribution and calculates the LCA impact scores. This is repeated for $k=1,\ldots,N$ number of runs providing k set of results. These LCA result scores can then be evaluated by the associated statistical properties, such as expected value and variance, or by constructing a frequency histogram and computing a probability distribution representing the model results. Independence between model parameters is assumed, as for the analytical uncertainty propagation.

2.3 Discernibility analysis

In an LCA context, discernibility analysis aims at unifying comparative and uncertainty analyses. The comparison between results expressed as probability distributions was addressed by many authors (Huijbregts 1998; Heijungs and Klein 2001; Heijungs et al. 2005; Hong et al. 2010; Clavreul et al. 2012; Imbeault-Tétreault et al. 2013; Heijungs and Lenzen 2014). Hong et al. (2010) underlined how the uncertainty of the difference between scenarios might depend on shared parameters, due to the many processes and characterization factors common between scenarios. According to Clavreul et al. (2012), this can be relevant because some uncertainties may have the same influence on the scenarios and, therefore, no influence on the ranking. The discernibility analysis functions as a *pair-by-pair* evaluation of the difference (Heijungs and Klein 2001; Heijungs et al. 2005; Clavreul et al. 2012) or the ratio (Huijbregts 1998) between scenarios. Results of the discernibility analysis might be easier to communicate by presenting only the percentage of cases where one scenario obtains more favourable results than another, especially if there are more than two scenarios (Heijungs and Kleijn 2001; Heijungs et al. 2005; Clavreul et al. 2012).

2.4 Law of total variance

GSA variance-based techniques estimate the fractional contribution of each input variable X_i to the variance of Y (Archer et al. 1997). Eq. (4) shows that in the case of independent model parameters, the total uncertainty can be approximated by the sum of the single parametrical uncertainties. The more general case is defined by Sobol's ANOVA-representation (Sobol′ 2001). With a model of the form $Y=f(X_1, X_2, \ldots, X_n)$, the total variance of the model output, $V(Y)$, is decomposed as:

$$V(Y) \approx \sum_{i=1}^{n} V_i + \sum_{i=1}^{n} \sum_{i<z} V_{iz} + \ldots + V_{1,2,\ldots n} \qquad (9)$$

Where i denotes the number of parameters in the model (n), and z the subset of second order interacting parameters. Taking the first order approximation, the so-called "contribution to variance" is the ratio between first order effects (V_i) and the overall variance, also known as Sobol's sensitivity indexes, S_i (Saltelli et al. 2010). The V_i obtained with the first order approximation (Eq. (11)) corresponds to the terms added in Eq. (4), to the single-parameter analytical uncertainty in Eq.(7) and to the contribution to variance (CTV) mentioned in Heijungs and Lenzen (2014).

$$S_i = \frac{V_i}{V(Y)} \qquad (10)$$

Where

$$V_i = V(Y)_i^j \approx \left(\frac{\partial z_i}{\partial x_i}\right)^2 \cdot V(x) \approx \left(SC_i^j\right)^2 \cdot V_{input}(X_i) = CTV(Y^j, x_i) \tag{11}$$

The estimation of S_i indexes allows ranking the input variables according to their importance for the model result (Sobol′ 2001). In the case of an LCA with j impact categories, the single variance contributions can be formulated according to Eq. (7) and the total variance according to Eq. (8). Then, this contribution can be decomposed as:

$$V(Y)^j \approx \sum_{i=1}^{n} V_i^{\,j} \approx \sum_{i=1}^{r} V_i^{\,j} + \sum_{i=r+1}^{n} V_i^{\,j} \tag{12}$$

Where r represents the number of parameters which, summed progressively according to their importance in the model, is required to reach a desired representativeness level of the total parametrical uncertainty in a scenario. Ranking the most important parameters allows prioritization of efforts to improve data quality in a systematic and consistent way. This concept thereby unifies sensitivity and uncertainty related to input parameters into importance in a GSA framework.

3 Case study

3.1 Case study scenarios and impact categories

A hypothetical case study was defined in order to evaluate the combined sensitivity and uncertainty analysis described in the previous sections. The emphasis was placed on the methodological aspects and application of the analysis method rather than on intense data collection; however, the scenarios have been defined to reflect features of full-scale waste management systems in a Danish context. The case study includes three scenarios for management of single family household waste in Denmark in 2013 (Jensen et al. 2013). The scenarios are based mainly on the following technology combinations: S1) Recycling + incineration, S2) Recycling + incineration + anaerobic digestion, and S3) Recycling + landfilling.

The technology processes included in the model were obtained from the EASETECH model database (Clavreul et al. 2014). The study presents results for climate change (GWP), stratospheric ozone depletion (ODP), human toxicity, cancer (HTc) and non-cancer (HTnc) effects, particulate matter (PM), ionizing radiation (IR), photochemical ozone formation (POFP), terrestrial acidification (TA), terrestrial eutrophication (TE), freshwater eutrophication (FE), marine eutrophication (ME), freshwater ecotoxicity (ET), fossil resources depletion (RDfos), metals/minerals depletion (RD). All characterization methods and normalization references are selected among those recommended by European Commission (2010). The case study does not aim to evaluate the latest impact categories, but rather to illustrate the validity of the methodology applied. The time horizon of the study was 100 years. The case study simulates a decision support LCA that involves consequences that result in additionally installed or additionally decommissioned equipment/capacity outside the foreground system of the analysed system. Consequently, the decision context falls within Situation B (meso/macro level decision support for technology scenarios) according to European Commission (2010). The LCI was modelled following a consequential approach and multi-functionality in the model was addressed by substitution. Please refer to the Supporting Information for details on the modelled framework and technologies.

3.2 EASETECH model

All three scenarios were modelled with EASETECH (Clavreul et al. 2014), an LCA model facilitating advanced LCA of waste management systems. The model enables modelling of a reference flow consisting of a mix of material fractions and tracking of substances within the individual material fraction flows, from generation to final release of substances to the environment. The model is particularly well suited for the focus of this paper, since EASETECH allows the use of parameters in all input fields. For each parameter the user can specify one value, a list of values or a probability distribution (normal, uniform, log-normal or triangular). The uncertainty of the obtained LCA results can be propagated with a Monte Carlo simulation tool.

3.3 Case study evaluation approach

The case study was evaluated following the set-up of established tiered approaches involving: (a) sensitivity analysis, (b) uncertainty propagation with Monte Carlo analysis, (c) discernibility analysis. However, the SCs were used to propagate the uncertainty also analytically, and results of the two propagation methods were compared. The concepts of additivity of variances and GSA were applied before the discernibility analysis *(v)* in order to evaluate the insights gained from the parameter screening against the discernibility results of the traditional approach. For illustration and simplicity, the input variance associated to the parameters follows a predefined common uncertainty range of 10 % for all parameters, which are assumed to be normally distributed and with a 95 % confidence interval. Please refer to the Supporting Information for comparisons carried out with other uncertainty ranges and distribution types.

(a) For each scenario and impact category, normalized impact scores were calculated and used for contribution analysis, which allowed identification and parameterization of relevant LCA model inputs. These parameters were then employed for an OAT perturbation analysis. SCs and SRs were calculated with Eq. (1) and (2) for all parameters in the three scenarios for equal parameter variations of +10 %.

(b) The uncertainty propagation was carried out, first for individual parameters and then for the entire set of parameters, for all impact categories and all three scenarios. Then, results obtained analytically (Eq. (7) and (8)) and by means of the Monte Carlo simulations were compared. The Monte Carlo simulation was carried out with increasing number of sampling points (N=1000, 10000, 100000). For each N, the differences from the analytical result were calculated as a percentage. When comparing the variance associated with the entire sets of parameters, the coefficient of variation (CV) was also determined:

$$CV^j = \frac{\sqrt{V(Y)^j}}{Y^j} \tag{13}$$

The CV is specific for each impact category *j* and is expressed as a percentage by dividing the standard deviation associated with the impact category by the respective mean result score. This value provides an indication of how uncertain the average result is.

(v) The compliance between analytical and sampled variance to the concept of additivity of variances was tested by applying Eq. (12) for the individual parameters of each case study scenario and all impact categories. This was

performed based first on the analytical method and then on the sampling populations resulting from the Monte Carlo simulations.

(c) Discernibility analysis based on the pair-wise difference between Monte Carlo results was carried out with varying number of simulation runs (N) and number of parameters included in the simulation (r).

4 Results

The discussion of results focuses mainly on scenario 1 for illustrative purposes, since similar behaviour was observed also for the two other scenarios in all steps of the methodology. Any difference between the three scenarios is specified when needed. Tables and figures regarding scenario 2 and 3 can be found in the Supporting Information.

4.1 LCA results and sensitivity

Figure 1 shows the normalized impact scores for all scenarios and impact categories. Negative values indicate savings, while positive values indicate impacts. The magnitude of the results scores varies between impact categories, depending on scenario and dataset choices. The impact categories with the highest overall PE scores are GWP, HTnc, ME, ET and RDfos. An example of how processes contribute to the net impacts for these impact categories is provided in Figure 2. A detailed contribution analysis describing the remaining impact categories is summarized in the Supporting Information.

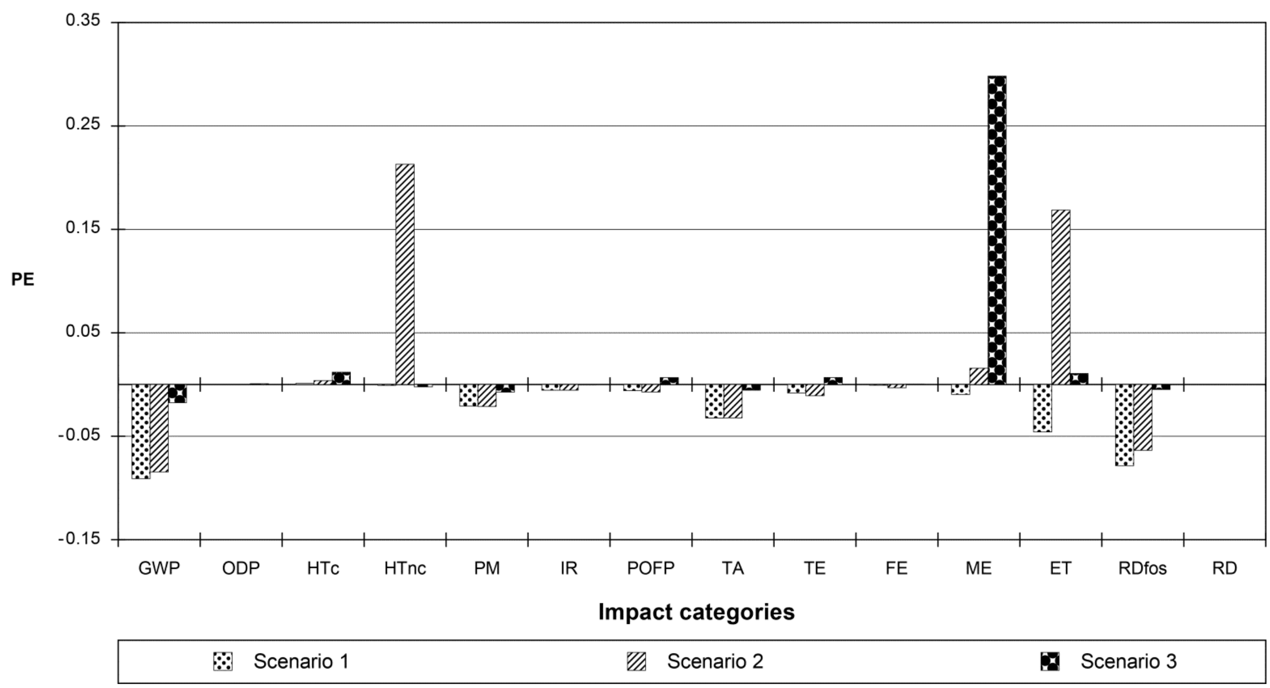

Fig. 1 Normalized impact scores for the three waste management scenarios. The impact categories are: climate change (GWP), stratospheric ozone depletion (ODP), human toxicity, cancer effects (HTc), human toxicity, non-cancer effects (HTnc), particulate matter (PM), ionizing radiation (IR), photochemical ozone formation (POFP), terrestrial acidification (TA), terrestrial eutrophication (TE), freshwater eutrophication (FE), marine eutrophication (ME), freshwater ecotoxicity (ET), fossils depletion (RDfos), metals/minerals depletion (RD).

The contribution analysis allowed selection of a total of eighty parameters for each scenario, including aspects such as waste characteristics (relevant for input specific emissions), process specific features of technologies, fuel consumption, distances driven, recycling and recovery rates. The amount of the most abundant waste fractions was also parameterized, maintaining the same reference flow. A complete list of the parameters selected based on the contribution analysis is available in the Supporting Information.

Table 1 provides the results of the perturbation analysis as SR and SC for the selected sensitive parameters and impact categories for scenario 1. The parameters are shown in Table 1 in hierarchical order according to their sensitivity, which can be positive or negative depending on the sign of the result score. The SCs cannot be directly compared, having different units. The SRs have the same unit, but caution needs to be paid for comparisons across scenarios and impact categories. Scenario 1 presents overall low SRs with values above 2 (corresponding to a variation in results of over 20 % with a 10 % variation in parameters) for the HTnc, IR, ME and FE impact categories. These impact categories also show the highest SRs for scenario 2. Scenario 3 presents somewhat higher SRs compared to S1 and S2, with the highest SR of 11 in the ET impact category (data not shown). The differences in SR values are due to the fact that the delta between results generated in the OAT is divided by the original result score, as shown in Eq. (2). Therefore, impact categories with small-magnitude scores are likely to have higher SR values with the same OAT delta between results. For this reason, the choice of the most sensitive parameters should not be based on comparisons between SRs of different impact categories, and the effect of parameter variations should be carefully evaluated within the individual impact categories and scenarios.

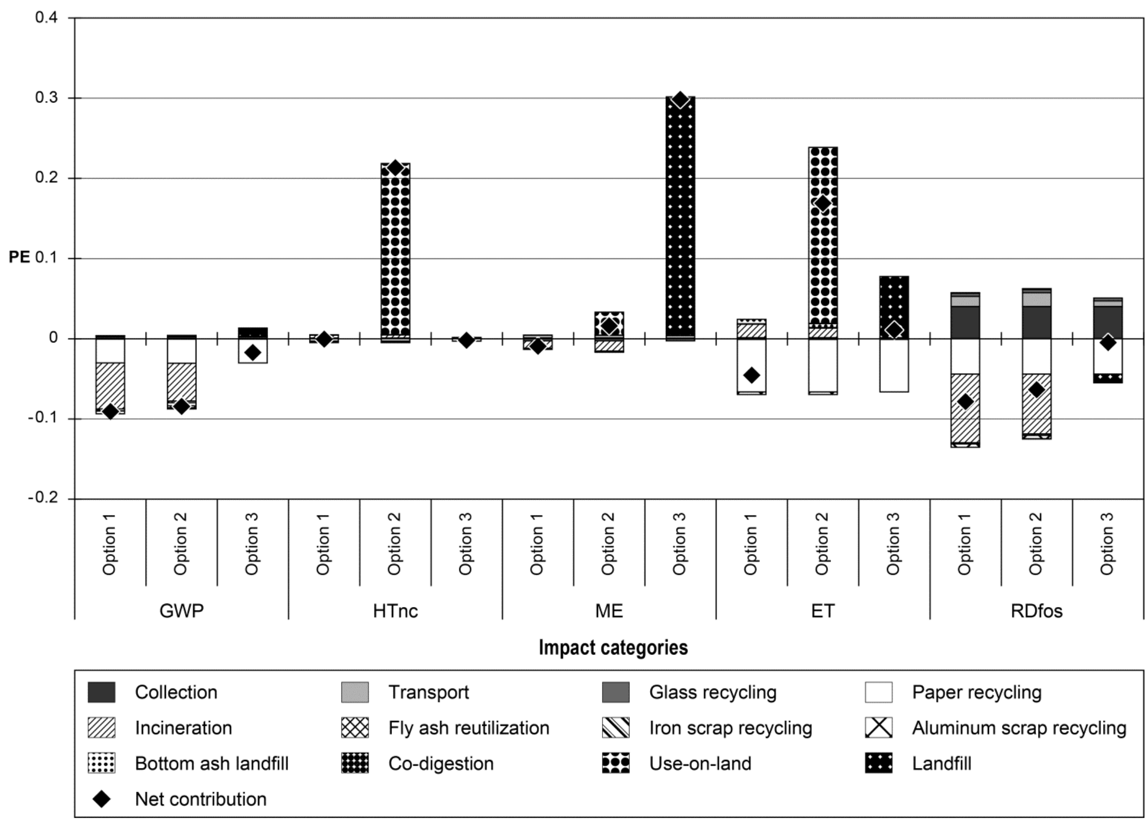

Fig. 2 Contribution analysis of the normalized results for selected impact categories.

Table 1. Sensitivity ratio (SR), sensitivity coefficient (SC), analytical uncertainty and sampled uncertainty for selected highly sensitive parameters in scenario 1 and selected impact categories. The sampled uncertainty results from increasing number of Monte Carlo sampling points (N). Variances for "Amount, vegetable waste" could not be obtained through Monte Carlo sampling as the random sampling would result in different reference flows for all the simulated results, thereby not allowing comparability.

Parameter name	SR	SC	Variance						
			Analytical	Monte Carlo					
				$N=10^3$		$N=10^4$		$N=10^5$	
[unit]	[-]	[PE/unit]	**[PE^2]**	[PE^2]	Difference from analytical	[PE^2]	Difference from analytical	[PE^2]	Difference from analytical
Global warming potential (GWP)									
Electricity recovery	5.9E-01	-2.4E-01	**7.1E-06**	7.4E-06	*5%*	7.2E-06	*2%*	7.1E-06	*0%*
Water content, vegetable waste	-5.1E-01	6.0E-02	**5.4E-06**	5.0E-06	*-8%*	5.2E-06	*-4%*	5.1E-06	*-5%*
Paper recycling	4.0E-01	-4.4E-02	**3.4E-06**	3.2E-06	*-4%*	3.5E-06	*3%*	3.4E-06	*0%*
Heat recovery	3.3E-01	-4.1E-02	**2.3E-06**	2.3E-06	*2%*	2.2E-06	*0%*	2.3E-06	*0%*
Segregated paper	3.2E-01	-5.0E-04	**2.1E-06**	2.0E-06	*-4%*	2.1E-06	*1%*	2.1E-06	*0%*
Amount, vegetable waste	-2.8E-01	9.6E-05	**1.6E-06**	-	-	-	-	-	-
Water content, animal food waste	-1.5E-01	2.4E-02	**4.6E-07**	4.6E-07	*-1%*	4.4E-07	*-4%*	4.4E-07	*-4%*
Particulate matter (PM)									
Electricity recovery	6.0E-01	-5.7E-02	**4.0E-07**	4.2E-07	*6%*	3.9E-07	*-1%*	3.9E-07	*-1%*
Water content, vegetable waste	-5.0E-01	1.4E-02	**2.7E-07**	1.9E-07	*-32%*	1.9E-07	*-31%*	1.9E-07	*-31%*
Paper recycling	3.8E-01	-9.5E-03	**1.6E-07**	1.6E-07	*-3%*	1.6E-07	*2%*	1.6E-07	*1%*
Segregated paper	3.0E-01	-1.1E-04	**9.7E-08**	9.5E-08	*-2%*	9.8E-08	*1%*	9.7E-08	*0%*
Amount, vegetable waste	-2.7E-01	2.1E-05	**8.0E-08**	-	-	-	-	-	-
NOx incineration	-2.2E-01	5.5E+00	**5.5E-08**	5.6E-08	*2%*	5.5E-08	*0%*	5.5E-08	*-1%*
Heat recovery	1.6E-01	-4.7E-03	**2.9E-08**	2.9E-08	*0%*	2.8E-08	*-3%*	2.8E-08	*-2%*
Water content, animal food waste	-1.5E-01	5.4E-03	**2.4E-08**	1.7E-08	*-29%*	1.6E-08	*-32%*	1.7E-08	*-31%*
Marine eutrophication (ME)									
NOx incineration	-3.1E+00	3.5E+01	**2.2E-06**	2.2E-06	*3%*	2.1E-06	*-1%*	2.2E-06	*0%*
Heat recovery	2.7E+00	-3.6E-02	**1.7E-06**	1.8E-06	*1%*	1.7E-06	*0%*	1.7E-06	*1%*
Water content, vegetable waste	-2.6E+00	3.3E-02	**1.6E-06**	1.1E-06	*-31%*	1.2E-06	*-27%*	1.1E-06	*-28%*
Electricity recovery	1.3E+00	-5.8E-02	**4.1E-07**	4.4E-07	*7%*	4.2E-07	*2%*	4.1E-07	*0%*
Amount, vegetable waste	-8.8E-01	3.2E-05	**1.8E-07**	-	-	-	-	-	-
Water content, animal food waste	-7.9E-01	1.3E-02	**1.4E-07**	1.0E-07	*-27%*	1.0E-07	*-29%*	1.0E-07	*-28%*
Heating value, vegetable waste	5.9E-01	-3.1E-04	**8.0E-08**	8.3E-08	*3%*	8.0E-08	*0%*	8.0E-08	*0%*
Heating value, plastic waste	5.7E-01	-1.5E-04	**7.4E-08**	7.7E-08	*4%*	7.4E-08	*0%*	7.3E-08	*0%*

4.2 Uncertainty propagation

4.2.1 Single parameters

Table 1 compares by percentage difference single-parameter variances obtained analytically and by Monte Carlo sampling. The values shown refer to the same selected parameters and impact categories discussed in section 4.1. Results for all impact categories are provided in the Supporting Information.

Analytical uncertainty values were calculated with Eq. (7). The results follow the same hierarchical order of the SRs due to the common uncertainty range chosen for the analysis. The Monte Carlo results were obtained running the uncertainty analysis selecting one parameter at a time. The two methods differ in terms of time required for the calculation: for the analytical method, results were obtained in a few seconds using a simple spreadsheet, while time ranged from seconds to tens of minutes for the Monte Carlo simulations (depending on the parameter and impact category). The variances obtained with Monte Carlo show similar values to those obtained analytically. For most parameters, the average difference from the analytical value was reduced from around 10 % for N=1000 to close to 0 % with N=100000. When the number of samples is higher, the difference between results reduces considerably due to the reduced "randomness" brought by larger number of samples within the distribution. Additionally, the reduction of the percentage difference appears not to be related to the SR nor SC of the parameter.

For a few parameters and impact categories, differences of the order of 30 % between the analytical and the sampled values were observed (increasing N only marginally decreased the differences). These specific parameters are related to the moisture content of the waste (S1-S3) and to the landfill characteristics (S3). In both cases, differences are due to a high interdependency between parameters that is specific to waste LCA studies. In waste modelling, the moisture content is analytically characterized as a complementary of the contents of total solids and affects the chemical composition of the waste modelled in the scenarios. Thus, varying the water content of LCAs based on waste sampling data can cause variations of the chemical composition. Likewise, in case of parameters describing landfill characteristics, e.g. leachate production and emissions to the environment may be interdependent. Hence, the calculated SCs reflect variations that are a consequence of correlations, while the input variance required to obtain the analytical uncertainty refers to one parameter only. Potentially, also the Monte Carlo results can be misleading. The simulation independently propagates the uncertainties of individual parameters, while correlated results would be visible only by implementing multivariate distributions (Bojacá and Schrevens 2010). This behaviour does not always occur with all water content-related parameters, but only for those impact categories that are affected by variations of the waste composition, e.g. the toxicity categories.

The analytical method was considerably faster to implement than Monte Carlo sampling. The values obtained for the single parameters suggest that the analytical method can provide a good approximation of the variance of most parameters in the LCA model. The results of the Monte Carlo sampling better fit the analytical scores with higher number of sampling points; this, however, requires longer simulation times and manual selection of the parameters for the simulations. Moreover, the analytical calculation would respond quickly to a change of distribution type and uncertainty range of the parameters. With different uncertainty ranges, the hierarchy of analytical uncertainties would also change, representing the *importance* of the parameters within the scenarios. Uncertainty associated to parameters with correlations can only be approximated by both methods. The analytical method further allows evaluation of potential contribution to variances caused by changes in the waste composition; this is currently not possible with Monte Carlo simulations as the random sampling may result in changes to the reference flow and thereby the functional unit of the scenarios. However, for most of the tested impact categories the analytical method demonstrated that the waste composition was highly relevant for the impact results.

4.2.2 All parameters

Table 2 provides a selection of the uncertainty analysis results for all parameters in scenario 1 and all impact categories. When all parameters are included in the uncertainty propagation, the resulting variance value is a single score representing the total parametrical variance for the scenario. The results thus comply with the concept of additivity of variances explained in the methodology section. The analytical variance was calculated with Eq. (8) for each impact category and results are shown in the upper part of Table 2. These values were compared to the normalized result scores by calculation of the CV with Eq. (13). The highest variation around the mean was observed for ME at 27 %. The lowest CVs were around 5 %.

For each impact category and Monte Carlo run, the lower part of Table 2 presents the difference between sampled variance and the analytical variance, and the associated CV. The mean values obtained from the Monte Carlo results show negligible differences with respect to the LCIA result scores (<0.5 %, data not shown). The difference between sampled an analytical variance was in average around 6 % for N=1000, about half of that observed in Table 1 for single parameters. Comparisons in literature between other analytical methods and Monte Carlo simulations showed similar outcomes. The same average difference within about 5 % for a full scenario was observed for Hong et al. (2010) and Heijungs and Lenzen (2014), while lower differences were found by Imbeault-Tétreault et al. (2013) and Groen et al. (2014). For increasing N, the differences with the analytical variances were reduced for most impact categories, except when the parameters were related to the moisture content of the waste. However, the Monte Carlo simulations indicated differences in variances of maximum 14 %, suggesting that the effect of interdependent parameters is "absorbed" when considering entire scenarios. The CVs based on Monte Carlo simulations showed marginal differences, within 1 %, with the CVs calculated from the analytical variance values.

In the case of uncertainty analysis for full parameter sets, the difference in the speed performance between the two uncertainty propagation methods was enormous, as also illustrated by Heijungs and Lenzen (2014). The analytical method only required summing of the values calculated with Eq. (7) in a spreadsheet, while the Monte Carlo simulations required manual selection of parameters and calculation times ranging from tens of minutes to hours, depending on the impact category.

Once again, the analytical variance provides results very similar to the ones of the Monte Carlo. The simulation results showed reduced differences from the analytical values with increasing number of sampling points. These observations are in accordance with the concept of additivity of variances, since the analytical values sufficiently well represent the total parametrical uncertainties of a scenario. The analytical uncertainty method can provide a fast approximation of the total scenario parametrical variance, useful for immediate determination of standard deviations for each impact result score.

Table 2. Variance obtained by analytical and sampling methods for waste management scenario 1. The Monte Carlo results were obtained for various numbers of sampling points (N).

		GWP	**ODP**	**HTc**	**HTnc**	**PM**	**IR**	**POFP**	**TA**	**TE**	**FE**	**ME**	**ET**	**RDfos**	**RD**
Analytical method															
Analytical variance	**[PE^2]**	**2.25E-05**	**1.01E-11**	**2.41E-09**	**1.29E-07**	**1.13E-06**	**6.70E-08**	**1.32E-06**	**3.45E-06**	**5.26E-06**	**1.91E-09**	**6.55E-06**	**2.23E-05**	**4.18E-05**	**1.34E-12**
Coefficient of variation	[%]	-5.2%	-4.8%	4.1%	-48.7%	-5.1%	-4.8%	-19.6%	-5.7%	-27.9%	-6.9%	-26.7%	-10.3%	-8.2%	-6.1%
Monte Carlo simulation															
Sampled variance [PE^2]	$N=10^3$	**2.38E-05**	**1.00E-11**	**2.53E-09**	**1.34E-07**	**1.08E-06**	**6.60E-08**	**1.15E-06**	**2.97E-06**	**4.59E-06**	**1.87E-09**	**5.72E-06**	**2.27E-05**	**4.07E-05**	**1.32E-12**
	$N=10^4$	**2.22E-05**	**1.00E-11**	**2.31E-09**	**1.21E-07**	**1.04E-06**	**6.66E-08**	**1.19E-06**	**2.99E-06**	**4.94E-06**	**1.95E-09**	**5.86E-06**	**2.25E-05**	**3.73E-05**	**1.36E-12**
	$N=10^5$	**2.22E-05**	**1.01E-11**	**2.34E-09**	**1.21E-07**	**1.03E-06**	**6.67E-08**	**1.21E-06**	**3.02E-06**	**4.77E-06**	**1.91E-09**	**6.04E-06**	**2.22E-05**	**3.90E-05**	**1.33E-12**
Difference of sampled variance from the analytical variance [%]	$N=10^3$	*5.7%*	*1.2%*	*5.0%*	*4.0%*	*4.6%*	*1.6%*	*13.2%*	*14.0%*	*12.7%*	*1.7%*	*12.6%*	*1.8%*	*2.7%*	*1.4%*
	$N=10^4$	*1.2%*	*1.0%*	*4.2%*	*6.0%*	*8.3%*	*0.6%*	*10.4%*	*13.5%*	*6.0%*	*2.3%*	*10.4%*	*0.9%*	*10.8%*	*1.7%*
	$N=10^5$	*1.4%*	*0.1%*	*2.9%*	*5.5%*	*8.9%*	*0.5%*	*8.2%*	*12.7%*	*9.2%*	*0.0%*	*7.7%*	*0.3%*	*6.7%*	*0.3%*
Coefficient of variation [%]	$N=10^3$	-5.4%	-4.8%	4.2%	-50.0%	-5.0%	-4.7%	-18.2%	-5.3%	-26.0%	-6.8%	-24.9%	-10.4%	-8.2%	-6.1%
	$N=10^4$	-5.2%	-4.8%	4.0%	-47.1%	-4.9%	-4.8%	-18.5%	-5.3%	-27.1%	-7.0%	-25.3%	-10.4%	-7.8%	-6.1%
	$N=10^5$	-5.2%	-4.9%	4.0%	-47.5%	-4.9%	-4.8%	-18.7%	-5.4%	-26.6%	-6.9%	-25.7%	-10.3%	-8.0%	-6.1%

The highest uncertainties around the mean result scores can thus be instantaneously quantified, e.g., with the CV (Eq. (13)) and by means of error bars. This is of great value in comparative LCAs, since the analytical method allows fast identification of the impact categories presenting potentially overlapping results and requiring discernibility analysis.

4.3 Global sensitivity analysis perspective

The variance associated with the single parameters was rank ordered (*r*) and a partial variance was calculated progressively for increasing *r* with Eq. (12). Figure 3 illustrates the behaviour of impact category results for the three scenarios. The *y* axis shows the percentage of the total analytical variance reached with the number of parameters included in the propagation (*r*) at the corresponding point of the *x* axis. It is evident that all impact categories have a similar behaviour with respect to the scenario uncertainty; most of the uncertainty could be represented by very few parameters. A high level of representation was reached within the ten parameters shown in the graph. In this case, six parameters represent a good compromise between the number of parameters selected for the representation and the average represented variance, which is about 90 % for all impact categories and waste management scenarios. Some impact categories show a clearly steeper behaviour, for example ODP for scenario 1 and 2. Table 3 summarises the highest-ranking parameters identified with the GSA framework and shows the associated percentage of represented analytical variance at the corresponding number of parameters included in the uncertainty propagation. White cells indicate the set of parameters that are required to reach 90% of represented uncertainty. Progressive determination of variances was also carried out with Monte Carlo simulations, confirming how the precision of the simulations increased with increasing number of samplings, ultimately reaching the analytical calculations. The Monte Carlo results fit well the analytical behaviour, and the uncertainty reaches an asymptotic behaviour within the limited number of parameters considered (see Supporting Information for details).

The analytically calculated uncertainty and a GSA perspective can be used to identify the parameters that are actually needed to appropriately represent the uncertainty in each impact category. The number of parameters depends on the scenario and the impact category and should be determined on a case-by-case basis. The focus on multiple impact categories in this study has demonstrated how the number of important parameters is limited as parameters are "shared" between impact categories. For scenario 1, considering the six highest ranking parameters in all impact categories corresponds to a total of ten parameters out of the initial eighty (in bold in Table 3). A significant simplification of the uncertainty representation.

Uncertainty "concentrated" in a few parameters highlights the vulnerability of the decisional process based on LCA results when, for example, a single external process carries the majority of the uncertainty associated with an impact category. A similar observation was highlighted also by Hong et al. (2010) indicating that some parameters could contribute with large shares (>75 %) to total the uncertainty in some impact categories. Figure 3 suggests that the uncertainty is controlled to a large extent by the 6 – 7 most important parameters within each impact category, while the remaining parameters contribute to a lower extent in reaching the asymptotical total variance. This confirms that when looking at the entire set of parameters, the error brought by the interdependent parameters is reduced when the parameters do not have a high scenario importance, while the error remains significant when the parameters are the one of the "important" parameters in an impact category.

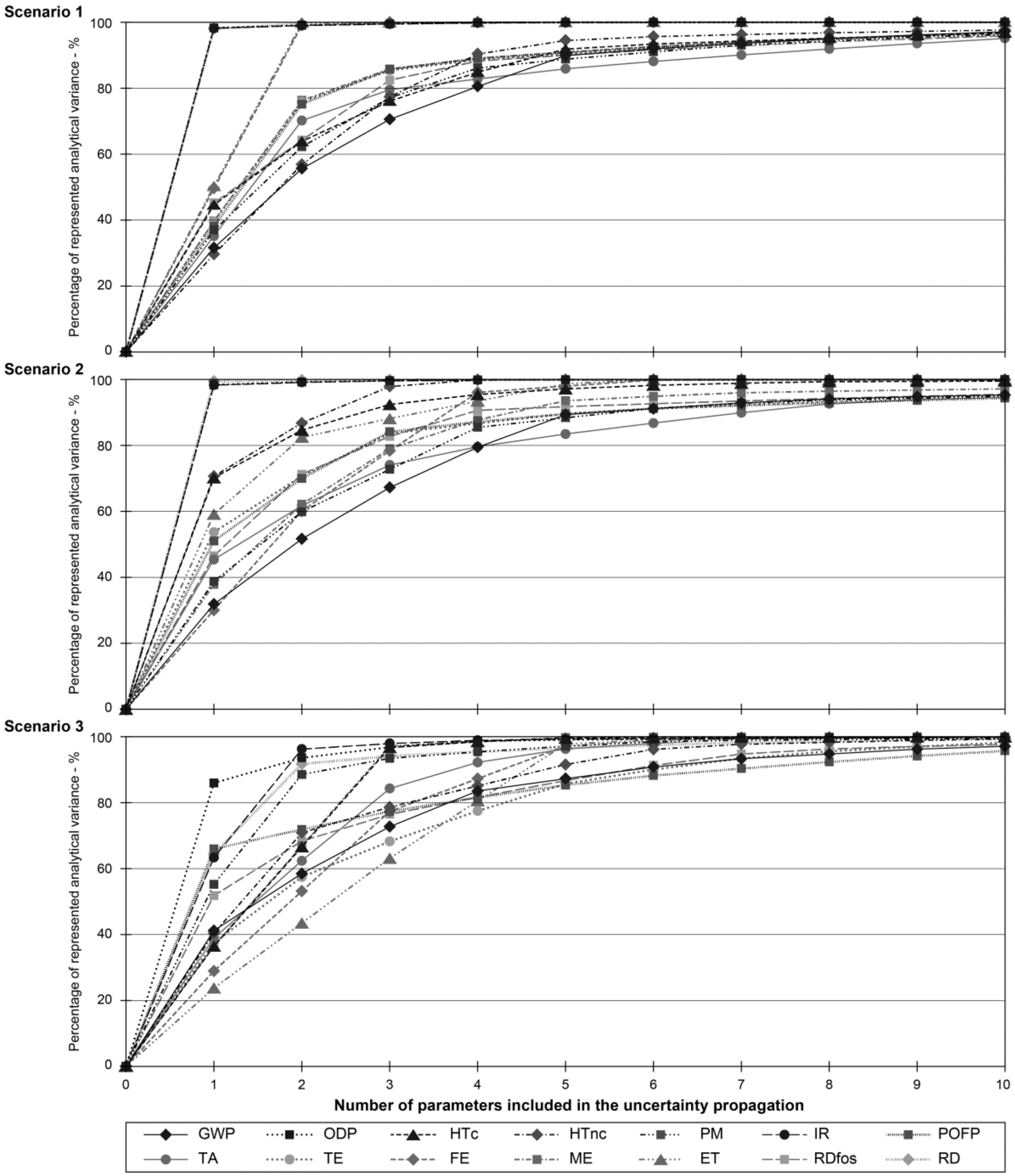

Fig. 3 Percentage of the total analytical variance reached with a variable number of parameters included in the propagation for the three waste management scenarios. The lines represent the impact categories.

Table 3. Ranking of parameters identified with the GSA framework and associated percentage of represented analytical variance for scenario 1 at the corresponding number of parameters included in the uncertainty propagation. White cells indicate the set of parameters that are required to reach 90% of represented uncertainty; parameters in bold are the subset of ten parameters governing most of the uncertainty in the scenario.

	Number of parameters included in the uncertainty propagation Parameter (*percentage of represented analytical variance*)									
Impact category	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
GWP	**Electricity recovery** *32%*	**Water content, vegetable waste** *56%*	**Paper recycling** *71%*	**Heat recovery** *81%*	**Segregated paper** *90%*	Water content, animal food waste *92%*	Heating value, vegetable waste *94%*	Heating value, plastic waste *95%*	Fossil carbon content, plastic waste *96%*	Heating value, animal food waste *97%*
ODP	**Aluminium recycling** *98%*	**Paper recycling** *99%*	**Segregated paper** *99%*	**Electricity recovery** *100%*	**Water content, vegetable waste** *100%*	Water content, animal food waste *100%*	Heating value, vegetable waste *100%*	Heating value, plastic waste *100%*	Electricity consumption, paper recycling *100%*	Heating value, animal food waste *100%*
HTc	**Electricity recovery** *45%*	**Paper recycling** *64%*	**Segregated paper** *76%*	**Water content, vegetable waste** *85%*	**Aluminium recycling** *92%*	**Gravel recycling** *93%*	Heating value, vegetable waste *94%*	Heating value, plastic waste *95%*	**Glass recycling** *96%*	Steel recycling *97%*
HTnc	**Electricity recovery** *30%*	**Paper recycling** *57%*	**Segregated paper** *77%*	**Water content, vegetable waste** *90%*	**Aluminium recycling** *94%*	**Water content, animal food waste** *96%*	Heating value, vegetable waste *96%*	Heating value, plastic waste *97%*	Steel recycling *97%*	**Glass recycling** *98%*
PM	**Electricity recovery** *37%*	**Water content, vegetable waste** *62%*	**Paper recycling** *77%*	**Segregated paper** *86%*	**NOx incineration** *89%*	**Heat recovery** *91%*	**Water content, animal food waste** *93%*	**Aluminium recycling** *94%*	Heating value, plastic waste *96%*	Heating value, vegetable waste *96%*
IR	**Aluminium recycling** *98%*	**Paper recycling** *99%*	**Segregated paper** *99%*	**Electricity recovery** *100%*	**Water content, vegetable waste** *100%*	**Gravel recycling** *100%*	**Water content, animal food waste** *100%*	Heating value, vegetable waste *100%*	Heating value, plastic waste *100%*	Heating value, animal food waste *100%*
POFP	**NOx incineration** *38%*	**Heat recovery** *75%*	**Water content, vegetable waste** *86%*	**Electricity recovery** *89%*	**Water content, animal food waste** *91%*	Heating value, vegetable waste *93%*	Heating value, plastic waste *94%*	**Glass recycling** *95%*	Heating value, animal food waste *96%*	NOx paper recycling *96%*
TA	**Heat recovery** *35%*	**Water content, vegetable waste** *70%*	**Electricity recovery** *80%*	**NOx incineration** *83%*	**Glass recycling** *86%*	**Water content, animal food waste** *88%*	**Aluminium recycling** *90%*	**Segregated glass** *92%*	**Paper recycling** *94%*	Heating value, vegetable waste *95%*
TE	**NOx incineration** *39%*	**Heat recovery** *76%*	**Water content, vegetable waste** *85%*	**Electricity recovery** *89%*	**Water content, animal food waste** *90%*	Heating value, vegetable waste *92%*	Heating value, plastic waste *94%*	**Glass recycling** *95%*	Heating value, animal food waste *96%*	NOx paper recycling *96%*
FE	**Glass recycling** *50%*	Segregated glass *99%*	**Paper recycling** *100%*	**Segregated paper** *100%*	**Gravel recycling** *100%*	**Electricity recovery** *100%*	**Aluminium recycling** *100%*	Fuel consumption residual waste collection *100%*	**Water content, vegetable waste** *100%*	Distance, residual waste transportation *100%*
ME	**NOx incineration** *40%*	**Heat recovery** *76%*	**Water content, vegetable waste** *86%*	**Electricity recovery** *89%*	**Water content, animal food waste** *91%*	Heating value, vegetable waste *93%*	Heating value, plastic waste *94%*	Glass recycling *95%*	Heating value, animal food waste *96%*	NOx paper recycling *96%*
ET	**Paper recycling** *50%*	**Segregated paper** *100%*	Aluminium recycling *100%*	**Electricity recovery** *100%*	**Glass recycling** *100%*	Segregated glass *100%*	Zinc emission, iron recycling *100%*	Water content, plastic waste *100%*	**Water content, vegetable waste** *100%*	Fuel consumption residual waste collection *100%*
RDfos	**Electricity recovery** *45%*	**Water content, vegetable waste** *64%*	**Paper recycling** *82%*	**Segregated paper** *88%*	Fuel consumption residual waste collection *90%*	**Water content, animal food waste** *92%*	Distance, residual waste transportation *93%*	Fuel consumption, residual waste transportation *94%*	Heating value, vegetable waste *95%*	Heating value, plastic waste *96%*
RD	**Gravel recycling** *98%*	**Water content, vegetable waste** *100%*	**Water content, animal food waste** *100%*	Water content, yard waste *100%*	Water content, diapers waste *100%*	Water content, paper waste *100%*	Water content, advertisements waste *100%*	Water content, plastic waste *100%*	Water content, dirty paper waste *100%*	Water content, newsprints waste *100%*

4.4 Discernibility analysis

Table 4 reports the results of the discernibility analysis performed on scenario 1 versus scenario 2. Comparisons with scenario 3 and between scenario 2 and 3 were not relevant since variations around mean result values were not overlapping; this information is provided by the standard deviations calculated from the CVs of Table 2. Discernibility analysis requires a result population obtained by the Monte Carlo sampling. Here, impact scores are obtained based on modelling with 6 and 80 parameters included in the simulation as well as different numbers of sampling points in the Monte Carlo.

The six parameters identified in Section 4.3 were sufficient to carry out the discernibility analysis and provided similar results as when all 80 parameters were included. Moreover, the results differ only by 1 % between the various Monte Carlo simulations with different numbers of sampling points. Performing discernibility analysis only on the parameters identified as important with an analytical uncertainty propagation and a global sensitivity perspective is much faster and efficient. Only a small number of simulations is required to understand in how many cases scenario 1 is preferable over scenario 2, thereby shortening the computational time that is typically required and solving the time-consuming nature of the discernibility analysis addressed by Heijungs and Kleijn (2001).

Regarding shared parameters and processes between scenarios, the analytical method suggests that it is very unlikely that parameters might have the same influence in two scenarios, since this *importance* is a combination between given parametrical uncertainty, which might be the same, and sensitivity, which will likely be different. Therefore, discernibility analysis can estimate only in how many cases one scenario is preferable over another, or the uncertainty of the difference between scenarios, but not totally eliminate the influence of shared parameters.

5 Discussion

5.1 Combining SCs with analytical uncertainty propagation

The results confirmed previous findings in literature regarding comparison between analytical and sampling methods. The analytical method provides good approximation of the sampled results while being substantially simpler, as also observed by Hong et al. (2010). The differences are mainly related to simulation speed and the type of results provided (Heijungs and Lenzen 2014; Groen et al. 2014). For either propagation methods, the contributions and sensitivity analysis steps are fundamental, as selection of parameters for the uncertainty analysis is traditionally subjective. Application of the dedicated waste LCA model EASETECH was essential in this context, since the model allows tracking of impact contributions from substances and material flows, in addition to internal and external processes. The parameterization feature allowed easy switching from OAT to uncertainty propagation with the Monte Carlo.

For all scenarios, the SC (Eq. (5)) was found to provide valuable representations of the derivative in the error propagation formula (Eq. (4)), with differences from the results of Monte Carlo sampling within the error ranges of other analytical propagation methods. Moreover, by utilizing the SC, the proposed analytical propagation method therefore compels the practitioner to carry out a thorough sensitivity analysis. The mathematical calculations required for the uncertainty analysis are considerably simplified, thereby increasing transparency of the analysis: LCA practitioners may easily connect the single parameters to their uncertainty, and evaluate the uncertainty results as a consequence of the

Table 4. Discernibility analysis results for selected impact categories in the comparison between scenario 1 and scenario 2. Comparisons are carried out for different number of parameters (i) included in the Monte Carlo simulation and sampling points (N).

	GWP	**ODP**	**PM**	**IR**	**POFP**	**TA**	**TE**	**RDfos**	**RD**
i=6									
$N=10^3$	85%	42%	41%	42%	15%	50%	16%	97%	91%
$N=10^4$	86%	41%	42%	42%	14%	53%	16%	97%	91%
$N=10^5$	85%	41%	42%	42%	14%	53%	16%	97%	91%
i=80									
$N=10^3$	84%	41%	43%	42%	15%	50%	17%	96%	92%
$N=10^4$	86%	41%	42%	42%	15%	51%	17%	97%	90%
$N=10^5$	85%	42%	43%	42%	15%	52%	17%	96%	91%

The choice of this analytical method allows propagating uncertainties up to the normalization level, because the SCs were calculated from the normalized result scores. In the same fashion, uncertainties can be propagated for the characterized impacts when SC is calculated from the characterized result scores. While expressing an uncertainty for the normalization factor in the SC formula could be implemented easily, including the characterization factors in the SC formula (Eq. (5)) would not be possible in the same simplified fashion. For this purpose, the matrix LCA formulation suggested by Heijungs (2010) would be more appropriate.

The assumption of linearity of model equations is considered reasonable. The EASETECH model is based on a layered computational structure (Clavreul et al. 2014) and the inventory, the characterization and normalization layers are based on linear equations. Non-linearity only rarely occurs in the material flow layer, and the relatively small differences between sampling and analytical methods' results do not warrant a more complicated method.

The assumption of independent uncertainties is not valid in cases involving interdependencies between parameters. The results of the analytical uncertainty analysis diverge from those of the Monte Carlo. However, both methods are based on the hypothesis of independent variables, and both can only provide an approximated value for the uncertainty when parameters are correlated. The practitioner could address this by changing the modelling of processes or technologies involving interdependent parameters (thereby essentially "de-coupling" the parameters) or further investigating the correlation. In the latter case, correlation would enlarge the Monte Carlo sampling space, by rotating the main axes of the distribution away from the coordinate axes and thus elongating the probability distribution (Bojacá and Schrevens 2010). Huijbregts et al. (2003) suggested identifying the parameters that contribute most to the output uncertainty before carrying out a correlation analysis. However, this would separate the correlation analysis from the global perspective, where the correlation plays a fundamental role in the definition of a parameter's importance (Eq. (9)).

The results of Table 2 and Figure 3 comply with the concepts of additivity of variances and of the total parametrical scenario variance. Thereby, the GSA perspective could be applied allowing a unification of sensitivity and uncertainty concepts. The results of the global importance analysis can be used to reduce the number of parameter uncertainties needed in the uncertainty propagation. Traditionally, the number of parameters has been reduced only based on the sensitivity analysis, thereby excluding potential importance of very uncertain parameters. The focus on multiple impact categories confirmed that the threshold for selection of important parameters should be based on multiple impact categories, since this allows representation of a large proportion of variability in impact results based on a limited number of parameters.

5.2 Applicability to other cases and validity of the results shown

As anticipated in 2.2.1, the presented analytical method is not limited to a specific probability distribution type. Any differences between distribution types are contained in the expression of the input variance assigned to the model parameters (Eq. (7), V_{input}). Although the percent differences presented in the results refer to the specific assumptions of 10 % uncertainty and normal distributions, compliance between the analytical and Monte Carlo sampling methods and to the concepts of additivity of variances was observed for higher uncertainty ranges and other distribution types, including cases with extremely skewed distributions and mixed uncertainty ranges and distribution choices (details in the Supporting Information). When the uncertainty range is common between all parameters, their resulting hierarchy and contribution to the output variance is unchanged, allowing the selection of the same important parameters presented in the results in the article (Table 3). When uncertainty ranges and distribution types are mixed, the compliance to the GSA concepts is still verified. The hierarchies and contribution to variances change, but a selection of a smaller set of fifteen parameters was found sufficient to carry out a discernibility analysis across fourteen impact categories.

The freedom of choice between distribution types is especially useful in case of waste LCAs where negative values often should be excluded and log-normal distributions would provide better representation of uncertainties, e.g. consumption of materials and emissions (Clavreul et al. 2012). As described in existing literature, possibility theory would be more suited to represent characteristics of waste management systems typically characterized by qualitative data. In such cases, information from fuzzy sets could be converted to, e.g., uniform probability distributions, and the SC method could still provide an approximated result of the uncertainty.

5.3 Revised step-wise approach for quantitative uncertainty assessment

Based on the results, a modification to the existing step-wise approach for quantitative uncertainty assessment of waste LCAs (Clavreul et al., 2012) is suggested. The revised step-wise approach, compared to the traditional and the tested approach, is presented in Figure 4. The well-established first steps of the traditional approach are still essential. A contribution analysis (Step 0) is fundamental to correctly parameterize and include the most contributing features of the system for the sensitivity analysis (Step 1). Calculation of SRs provides a quick ranking of parameters according to sensitivity, which should be analysed contextually within all impact categories and case study scenarios. High ranking parameters might foster further data collection, especially in preparation for the uncertainty analysis. These conventional steps constitute also the basis for applying the GSA framework. The SCs, obtained with a sufficiently small percentage

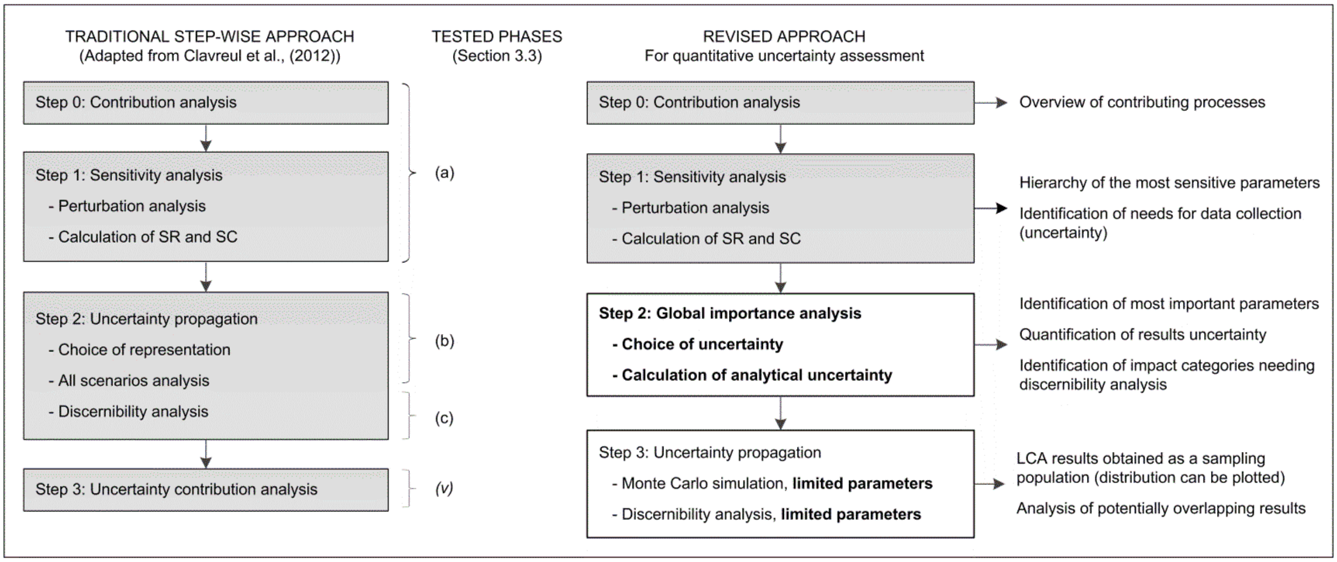

Fig. 4 Revised sequential approach for quantitative global importance and uncertainty analysis compared to a traditional step-wise approach and the tested phases.

variation in the perturbation calculation, provide a "slope factor" to which the input uncertainties associated with each parameter can be multiplied in the global importance analysis (Step 2). The sensitivity and uncertainty terms can thus be combined and the parameters systematically ranked according to their importance. As highlighted in Meinrenken et al. (2012) this could be carried out concurrently to the data gathering phase. Additionally, this step provides quantification of the variability (e.g. CVs, Table 2) and reliability (e.g. ODP uncertainty depending on just one parameter, Figure 3) of the result scores, with negligible difference to Monte Carlo simulations. In comparative LCAs, the LCA practitioner could identify at this stage which impact categories would require a Monte Carlo simulation for the discernibility analysis. Hierarchically ranking the parameters according to importance allows systematic selection of how many parameters (*r*) are required to reach a sufficient percentage of representativeness of the uncertainties in each impact category. Then, the uncertainty propagation (Step 3) can be carried out just for these few parameters and with a limited number of Monte Carlo runs, for the discernibility analysis or representation of the LCA results by probability distribution functions. Within the impact categories for which a discernibility analysis is necessary, it is still possible to perform a combined sensitivity analysis as suggested by Clavreul et al. (2012).

6 Conclusions

A traditional step-wise approach for quantification of uncertainty in LCA was applied in a comparative study including three full-scale waste management systems modelled with the EASETECH LCA model. Uncertainties were propagated by an approximated analytical approach as well as with Monte Carlo simulation. Uncertainty propagation both for single parameters and full parameter sets (eighty) over fourteen ILCD recommended impact categories was included. The analytical method was examined in a Global Sensitivity Analysis (GSA) framework and critically important parameters were selected for completion of discernibility analysis for the specific impact categories and scenarios where results were potentially overlapping. The proposed analytical method for uncertainty propagation provided a transparent and simplified mathematical formulation as far as the normalized result level. The analytical method was evaluated against alternative Monte Carlo sampling and provided quantitatively similar results, but with considerably smaller computational efforts. The law of total variance and application of the GSA perspective played a pivotal role for simplification of uncertainty quantification by the proposed method. It was demonstrated that only few parameters are needed to represent most of the uncertainty in a scenario. The selection of these critical parameters should be carried out contextually to the system modelled and considering multiple impact categories. Consequently, research and efforts to minimise uncertainties can be focused on the important parameters, while other parameters can be fixed within an appropriate range without compromising the LCA results. This was further confirmed by discernibility analysis, which provided the same results based only on the few critical parameters identified through the global importance analysis. A new step-wise approach for uncertainty quantification was proposed to improve reliability, transparency and credibility of LCA practices. The waste management system modelled functioned as a real-scale case, suggesting that the presented approach constitutes a systematic method for quantification of the full importance of parameter sensitivity and uncertainty, applicable to any LCA study.

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