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The Method of Varying Amplitudes for Solving (Non)linear Problems Involving Strong Parametric Excitation

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Abstract

Parametrically excited systems appear in many fields of science and technology, intrinsically or imposed purposefully; e.g. spatially periodic structures represent an important class of such systems [4]. When the parametric excitation can be considered weak, classical asymptotic methods like the method of averaging [2] or multiple scales [6] can be applied. However, with many practically important applications this simplification is inadequate, e.g. with spatially periodic structures it restricts the possibility to affect their effective dynamic properties by a structural parameter modulation of considerable magnitude. Approximate methods based on Floquet theory [4] for analyzing problems involving parametric excitation, e.g. the classical Hill's method of infinite determinants [3,4], can be employed also in cases of strong excitation; however, with Floquet theory being applicable only for linear systems, this is impossible or rather cumbersome for combined parametric and direct excitation, or with nonlinearity.

The present work employs a novel approach, the *Method of Varying Amplitudes* (MVA) [7], for solving linear and nonlinear problems involving combined direct excitation and strong parametric excitation. This approach is inspired by the method of direct separation of motions [1]; it is strongly related to Hill's method of infinite determinants [3,4] and the method of space-harmonics [5], and may be considered a continuation of the classical methods of harmonic balance [3] and averaging [2]. It implies a harmonic series solution with varying amplitudes, but in contrast to averaging methods, the amplitudes are not required to vary slowly. Thus the MVA does not assume the presence of a small parameter in the governing equations, or any restrictions on the sought solution. To illustrate the method several problems are considered:

The *first* problem is vibration suppression in predefined regions of a string subjected to distributed time-periodic loading, by continuous spatial cross-section modulation; this could be relevant for, e.g., oscillations of transmission lines, suspension bridges, and stay cables under rain and wind. Employing the effect of parametric attenuation [3], the problem is reduced to a forced linear Mathieu equation. As a result optimal parameters for the string cross-sectional area modulation are determined for the cases of harmonically, uniformly, and arbitrarily distributed load.

The *second* problem is the determination of eigenproperties for a Bernoulli-Euler beam with periodically and continuously varying spatial properties, as is relevant e.g. for risers and rotor blades. The corresponding governing equation is, in non-dimensional form:

$$\left((1+\chi_I \sin x)\varphi''\right)'' - \delta(1+\chi_A \sin x)\varphi = 0, \quad (1)$$

where $\varphi=\varphi(x)$ represents beam deflection, χ_I and χ_A modulation amplitudes for the beam stiffness and mass per unit length respectively, and δ the squared non-dimensional frequency. As a

result the dispersion relation and eigenproperties are determined, and it is shown that such non-uniform structures are able to sustain long-wave oscillations at comparatively high frequencies.

Thirdly, the effect of weak nonlinearity on the dispersion relation and the frequency band-gaps of a periodic Bernoulli-Euler beam is examined; this is relevant, since applications may demand effects of nonlinearity on structural response to be accounted for. The corresponding governing non-dimensional equation takes the following form:

$$\left((1+\chi_I \sin x)\left(\varphi'' + \mu(\varphi'')^2 \bar{\varphi}''\right)\right)'' - \delta(1+\chi_A \sin x)\varphi = 0, \tag{2}$$

where the parameter μ defines the nonlinearity of the beam stress-strain relation, and the over-bar denotes complex conjugation. As a result a shift of band-gaps to a higher frequency range is revealed, while the width of the band-gaps appears relatively insensitive to (weak) nonlinearity.

Fourthly we analyze the response of perfectly tuned or slightly detuned nonlinear parametric amplifiers; this is of interest e.g. for micro and nanosystems. The model equation considered is:

$$\ddot{x} + \beta\dot{x} + \omega^2(1+p\cos\Omega_p t)x + kx^3 = d\cos(\Omega_d t + \phi), \tag{3}$$

where x represents the amplifier response, d and p the amplitudes of external and parametric excitations with frequencies Ω_p and Ω_d, respectively, ω the natural frequency, and k the nonlinearity coefficient (not necessarily small). For a *detuned* amplifier ($\Omega_p/\Omega_d \neq 2$) the resulting quasi-periodic response is obtained. It appears that large responses may emerge with arbitrarily small external excitation, with the amplifier gain tending to infinity.

Finally, to illustrate that the applicability range of the MVA is not restricted to problems with non-autonomous excitation, self-excited oscillations in autonomous systems are considered for the Van der Pol equation with strong nonlinearity. As a result stationary as well as non-stationary responses are determined, and validated numerically.

The considered examples illustrate that the MVA is an efficient tool for treating problems with strong parametric excitation, combined external and parametric excitation, self-excitation, and nonlinearity. It may be applied as well for strongly nonlinear systems, though with certain restrictions on solution space. [The work is carried out with financial support from the Danish Council for Independent Research and FP7 Marie Curie Actions – COFUND: DFF – 1337-00026.]

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