



Electrically Small Magnetic Dipole Antennas With Quality Factors Approaching the Chu Lower Bound

Kim, Oleksiy S.; Breinbjerg, Olav; Yaghjian, Arthur D.

Published in:
I E E E Transactions on Antennas and Propagation

Link to article, DOI:
[10.1109/TAP.2010.2046864](https://doi.org/10.1109/TAP.2010.2046864)

Publication date:
2010

Document Version
Early version, also known as pre-print

[Link back to DTU Orbit](#)

Citation (APA):
Kim, O. S., Breinbjerg, O., & Yaghjian, A. D. (2010). Electrically Small Magnetic Dipole Antennas With Quality Factors Approaching the Chu Lower Bound. *I E E E Transactions on Antennas and Propagation*, 58(6), 1898-1906. <https://doi.org/10.1109/TAP.2010.2046864>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Electrically small magnetic dipole antennas with quality factors approaching the Chu lower bound

Oleksiy S. Kim, Olav Breinbjerg, *Member, IEEE*, and Arthur D. Yaghjian, *Life Fellow, IEEE*

Abstract—We investigate the quality factor Q for electrically small current distributions and practical antenna designs radiating the TE_{10} magnetic dipole field. The current distributions and the antenna designs employ electric currents on a spherical surface enclosing a magneto-dielectric material that serves to reduce the internal stored energy. Closed-form expressions for the internal and external stored energies as well as for the quality factor Q are derived. The influence of the sphere radius and the material permeability and permittivity on the quality factor Q is determined and verified numerically. It is found that for a given antenna size and permittivity there is an optimum permeability that ensures the lowest possible Q , and this optimum permeability is inversely proportional to the square of the antenna electrical radius. When the relative permittivity is equal to 1, the optimum permeability yields the quality factor Q that constitutes the lower bound for a magnetic dipole antenna with a magneto-dielectric core. Furthermore, the smaller the antenna the closer its quality factor Q can approach the Chu lower bound. Simulated results for the TE_{10} -mode multiarm spherical helix antenna with a magnetic core reach a Q that is 1.24 times the Chu lower bound for an electrical radius of 0.192.

Index Terms—Electrically small antennas, magnetic dipole, quality factor, Chu limit, spherical modes, surface integral equation

I. INTRODUCTION

ELECTRICALLY small antennas, that is, antennas that are small compared to the free-space wavelength at their frequency of operation, have been a subject of research for many years [1]–[8] and the challenges of these antennas are well known. Nevertheless, in recent years the escalation of miniaturized wireless technology has stimulated new research to develop small antennas with acceptable matching, bandwidth, and radiation efficiency [9]–[12]. In particular, it is of great interest to investigate how closely a small antenna can have its radiation quality factor Q approach the Chu lower bound Q_{Chu} . For an antenna that is resonant (equal stored electric and magnetic energies) and that radiates either an electric-dipole field (TM_{10} spherical mode) or a magnetic-dipole field (TE_{10} mode) at a frequency with free-space wave number k , and with the smallest circumscribing sphere of radius a , this bound can be expressed as [4]

$$Q_{\text{Chu}} = \frac{1}{(ka)^3} + \frac{1}{ka}, \quad ka \ll 1. \quad (1)$$

This work is supported by the Danish Research Council for Technology and Production Sciences within the TopAnt project (<http://www.topant.dtu.dk>).

O. S. Kim and O. Breinbjerg are with the Department of Electrical Engineering, Electromagnetic Systems, Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark (e-mail: {osk,ob}@elektro.dtu.dk)

A. D. Yaghjian is a Research Consultant at 115 Wright Road, Concord MA, 01742 USA.

It is sometimes stated that the Chu lower bound is based on the assumption that the internal volume of the circumscribing sphere does not store any energy — but this is not entirely correct. As pointed out by Chu [2, p. 1170] the internal volume can store the energy that may be needed to make the antenna resonant. For an electrically small TM_{10} electric-dipole antenna, where the external stored electric energy dominates the external stored magnetic energy, the internal volume must store a magnetic energy to compensate the difference of the external stored energies, and vice versa for a TE_{10} magnetic-dipole antenna. In any case, this internal stored energy is taken into account since (1) is based on a total stored energy that is set to twice the maximum of the external stored electric and magnetic energies.

Internal stored energy, beyond that needed to make the antenna resonant, will obviously increase the quality factor above the Chu lower bound. An electrically small, spherical surface current distribution in free space that radiates the TM_{10} electric-dipole field will have a significant internal stored electric energy, while a similar current distribution that radiates the TE_{10} magnetic-dipole field will have a significant internal stored magnetic energy. Investigating a spherical coil antenna radiating the TE_{10} magnetic-dipole field, Wheeler [13] reported a quality factor 3.0 times the first term of the Chu lower bound (1). More recently, Thal [8] showed that the inclusion of the internal stored energy results in a quality factor for electric surface currents radiating the TM_{10} electric-dipole field that is 1.5 times the Chu lower bound, while the quality factor for electric surface currents radiating the TE_{10} magnetic-dipole field is 3.0 times the Chu lower bound. That is,

$$Q_{\text{Thal}}^{\text{TM}_{10}} = 1.5Q_{\text{Chu}}, \quad ka \ll 1 \quad (2a)$$

$$Q_{\text{Thal}}^{\text{TE}_{10}} = 3.0Q_{\text{Chu}}, \quad ka \ll 1. \quad (2b)$$

Thal's derivation [8] is based on his previously derived circuit equivalents for spherical vector waves [14] and is thus in line with the approach of Chu [2]. The authors of this manuscript have followed the more direct approach of Collin and Rothschild [4] to verify (2) by calculating the internal and external stored energies from spatial integrations of the electromagnetic fields [15]. These calculations show that for an electrically small, electric surface current density radiating the TM_{10} electric-dipole field, the internal stored electric energy is 0.5 times the external stored electric energy. Furthermore, for an electrically small, electric surface current density radiating the TE_{10} magnetic-dipole field the internal stored magnetic energy is 2.0 times the external stored magnetic energy.

It is interesting to note that recent electrically small TM_{10} electric-dipole antennas by Best [9], [10] as well as TE_{10}

magnetic-dipole antennas by Best [16] and Kim [17] exhibit quality factors that approach the values in (2) from above, and there is apparently no report in the literature of free-space antennas with quality factors below Thal's lower bounds.

In order to overcome Thal's lower bounds (2) and approach the Chu lower bound (1), an antenna with vanishing internal stored energy is required. For an electrically small, spherical electric surface current density radiating the TM_{10} electric-dipole field, it is easily seen that the internal stored electric energy will increase, not decrease, if the internal volume is filled with a dielectric material. However, for an electrically small, spherical electric surface current density radiating the TE_{10} magnetic-dipole field, the internal stored magnetic energy will decrease, not increase, if the internal volume is filled with a magnetic material; in fact, the internal stored magnetic energy will be inversely proportional to the permeability of the magnetic material. The latter case was pointed out already by Wheeler [13] and confirmed by Thal [8]; specifically, with μ_r denoting the relative permeability the quality factor becomes

$$Q = \left(1 + \frac{2}{\mu_r}\right) Q_{Chu}, \quad ka \ll 1. \quad (3)$$

It is noted, that the expression (3) holds only in the limit of vanishing electrical radius since it is necessary to avoid the cavity resonances that may result from filling a finite-sized spherical surface current density with a high-permeability material. Such resonances, with no external field, will of course make the quality factor approach infinity. Though the expression (3) constitutes a highly interesting limit, it is thus not guaranteed to apply to practical antenna designs of finite size. As will be shown in this manuscript, it does not, and for a given electrical size of the spherical current density, the lowest quality factor is not obtained with the highest permeability, because of resonances.

The purpose of this work is to investigate the quality factor Q for electrically small current distributions and practical antenna designs radiating the TE_{10} magnetic dipole field; in particular, to determine how close the Q can approach the Chu lower bound. The current distributions are spherical electric surface current densities, and the practical antennas are conducting wires wound on a spherical surface — in both cases enclosing a magnetic or magneto-dielectric spherical core to reduce the internal stored magnetic energy. For the current distribution an analytical solution is derived in terms of spherical vector wave functions that leads to closed-form expressions for the internal and external stored energies as well as the radiated power for arbitrary values of the electrical radius of the sphere and the permeability/permittivity of the core. The influence of these parameters on the quality factor Q can thus be investigated, and the optimal permeability for a given electrical radius can then be determined.

As a practical antenna, the TE_{10} -mode multiarm spherical helix antenna [17] augmented with a spherical magnetic/magneto-dielectric core is investigated. A numerical solution is obtained using the surface integral equation and higher-order method of moments [18].

Some preliminary results of this work were reported previously at the ISAP 2009 conference [19], [20]. This manuscript

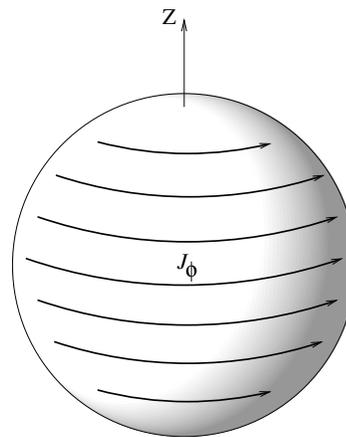


Fig. 1. Ideal magnetic dipole antenna with a magneto-dielectric core.

concentrates on the optimal permeability and the lowest possible Q for a given antenna electrical radius. Furthermore, the influence of the core permittivity on the quality factor Q is investigated both analytically and numerically.

II. SPHERICAL SURFACE CURRENT DENSITY ENCLOSING A MATERIAL CORE

A. Single TE_{10} Mode

The configuration consists of a time-harmonic, spherical, electric surface current density \mathbf{J} of radius a and amplitude J_0 enclosing a magneto-dielectric core with relative permeability μ_r and permittivity ϵ_r , and wave number $k_s = \sqrt{\epsilon_r \mu_r} k$ (Fig. 1). The core material is assumed to be linear, isotropic, homogeneous, lossless and dispersion-less¹. Introducing a spherical $(r\theta\phi)$ coordinate system with its origin at the center of the core, and employing phasor notation with suppressed time factor $\exp(-i\omega t)$, $\omega = kc > 0$ (where c is the speed of light), the surface current density can be expressed as

$$\mathbf{J} = \mathbf{a}_\phi J_0 \sin\theta \quad (4)$$

where \mathbf{a}_ϕ is the azimuthal unit vector. This electric surface current density is thus azimuthally directed with a sinusoidal polar variation, and it is proportional to the electric equivalent surface current density of the Huygens-Love field equivalence principle for a z -directed magnetic dipole located at the coordinate system origin.

1) *Analytical solution:* The radiated field is obtained by expressing the internal field for $r < a$ as a spherical vector wave of the standing wave type, the external field for $r > a$ as a spherical vector wave of the outward propagating type, and then enforcing the appropriate boundary conditions at $r = a$ to determine the complex amplitudes C^- and C^+ of the internal and external fields, respectively. Since the surface current density is azimuthally constant and has a first-order polar variation, this will also be the case for the radiated fields; thus, only the magnetic-dipole spherical vector wave with azimuthal index $m = 0$ and polar index $n = 1$ will be present; that is the TE_{10} mode. Employing the notation of

¹It is assumed that the frequency dispersion of μ_r and ϵ_r is negligible over the relevant bandwidth, and thus, $\mu_r > 0$ and $\epsilon_r \geq 1$ [21].

Hansen [22], the interior and exterior electric and magnetic fields can be expressed as

$$\mathbf{E}^-(\mathbf{r}) = \frac{k_s}{\sqrt{\eta_s}} C^- \mathbf{F}_{101}^{(1)}(\mathbf{r}) \quad (5a)$$

$$\mathbf{H}^-(\mathbf{r}) = -ik_s \sqrt{\eta_s} C^- \mathbf{F}_{201}^{(1)}(\mathbf{r}) \quad (5b)$$

$$\mathbf{E}^+(\mathbf{r}) = \frac{k}{\sqrt{\eta}} C^+ \mathbf{F}_{101}^{(3)}(\mathbf{r}) \quad (5c)$$

$$\mathbf{H}^+(\mathbf{r}) = -ik \sqrt{\eta} C^+ \mathbf{F}_{201}^{(3)}(\mathbf{r}). \quad (5d)$$

In these expressions η and η_s are the intrinsic admittances of free space and the core, respectively. Furthermore, the \mathbf{F} -functions are the power-normalized spherical vector wave functions

$$\mathbf{F}_{101}^{(1)}(\mathbf{r}) = \frac{\sqrt{6}}{4\sqrt{\pi}} \frac{1}{k_s r} \left\{ -\cos k_s r + \frac{\sin k_s r}{k_s r} \right\} \sin \theta \mathbf{a}_\phi \quad (6a)$$

$$\mathbf{F}_{201}^{(1)}(\mathbf{r}) = \frac{\sqrt{6}}{2\sqrt{\pi}} \frac{1}{(k_s r)^2} \left\{ -\cos k_s r + \frac{\sin k_s r}{k_s r} \right\} \cos \theta \mathbf{a}_r - \frac{\sqrt{6}}{4\sqrt{\pi}} \frac{1}{k_s r} \left\{ \sin k_s r + \frac{\cos k_s r}{k_s r} - \frac{\sin k_s r}{(k_s r)^2} \right\} \sin \theta \mathbf{a}_\theta \quad (6b)$$

$$\mathbf{F}_{101}^{(3)}(\mathbf{r}) = -\frac{\sqrt{6}}{4\sqrt{\pi}} \frac{e^{ikr}}{kr} \left\{ 1 + \frac{i}{kr} \right\} \sin \theta \mathbf{a}_\phi \quad (6c)$$

$$\mathbf{F}_{201}^{(3)}(\mathbf{r}) = -\frac{\sqrt{6}}{2\sqrt{\pi}} \frac{e^{ikr}}{(kr)^2} \left\{ 1 + \frac{i}{kr} \right\} \cos \theta \mathbf{a}_r - \frac{\sqrt{6}}{4\sqrt{\pi}} \frac{e^{ikr}}{kr} \left\{ -i + \frac{1}{kr} + \frac{i}{(kr)^2} \right\} \sin \theta \mathbf{a}_\theta \quad (6d)$$

and the C^- and C^+ coefficients are given by

$$C^- = J_0 i a \frac{\sqrt{\eta_s} 4\sqrt{\pi}}{\eta \sqrt{6}} \left(1 + \frac{i}{ka} \right) I \quad (7a)$$

$$C^+ = -J_0 i a \frac{e^{-ika} 4\sqrt{\pi}}{\sqrt{\eta} \sqrt{6}} \left\{ -\cos k_s a + \frac{\sin k_s a}{k_s a} \right\} I \quad (7b)$$

where

$$I = \left\{ \left(-i + \frac{1}{ka} + \frac{i}{(ka)^2} \right) \left(-\cos k_s a + \frac{\sin k_s a}{k_s a} \right) + \frac{\eta_s}{\eta} \left(1 + \frac{i}{ka} \right) \left(\sin k_s a + \frac{\cos k_s a}{k_s a} - \frac{\sin k_s a}{(k_s a)^2} \right) \right\}^{-1}. \quad (8)$$

The stored energies and the radiated power are determined through direct spatial integration of the radiated fields. In the calculation of the external stored energy the contribution of the propagating field is subtracted from that of the total field to obtain the energy density of the non-propagating field only [4], [6]. With W_E^- , W_H^- , W_E^+ , and W_H^+ denoting the internal electric, internal magnetic, external electric, and

external magnetic stored energy, respectively, it is found that

$$W_E^- = \frac{1}{4\omega} |C^-|^2 \left\{ -\frac{\sin^2 k_s a}{k_s a} + \frac{k_s a}{2} + \frac{\sin 2k_s a}{4} \right\} \quad (9a)$$

$$W_H^- = \frac{1}{4\omega} |C^-|^2 \left\{ -\frac{\sin^2 k_s a}{(k_s a)^3} + \frac{\sin 2k_s a}{(k_s a)^2} - \frac{\cos^2 k_s a}{k_s a} + \frac{k_s a}{2} - \frac{\sin 2k_s a}{4} \right\} \quad (9b)$$

$$W_E^+ = \frac{1}{4\omega} |C^+|^2 \frac{1}{ka} \quad (9c)$$

$$W_H^+ = \frac{1}{4\omega} |C^+|^2 \left\{ \frac{1}{(ka)^3} + \frac{1}{ka} \right\}. \quad (9d)$$

Finally, the radiated power is

$$P_{\text{rad}} = \frac{1}{2} |C^+|^2. \quad (10)$$

It is noted that these expressions hold for arbitrary electrical radius ka of the sphere, and thus allow us to investigate the relationships between ka , relative permeability μ_r and permittivity ϵ_r , and the quality factor Q in the case of finite-sized spheres.

2) *Stored energy*: First, we consider a pure magnetic core, that is, $\epsilon_r = 1$. Figure 2a shows the ratio of the total stored electric energy $W_E = W_E^+ + W_E^-$ to the total stored magnetic energy $W_H = W_H^+ + W_H^-$ vs. free-space electrical radius ka in the range from 0 to 1 for four values of the relative permeability $\mu_r = 1, 2, 8, \text{ and } 100$. For vanishing electrical radius the magnetic energy clearly dominates the electrical energy and the current distribution is thus not resonant. As the electrical radius increases, the electrical energy becomes relatively larger and eventually equals the magnetic energy at the resonances.

The complimentary dependence of the ratio W_E/W_H vs. the relative permeability μ_r for three different values of $ka = 0.1, 0.25, \text{ and } 0.5$ is presented in Fig. 3a. The total dominance of the magnetic energy for $ka = 0.1$ is observed in the entire shown range of μ_r , whereas for $ka = 0.25$ and 0.5 the magnetic energy dominates only away from the resonances.

Figure 2b shows the ratio of the internal stored energy $W^- = W_E^- + W_H^-$ to the total stored energy $W_{\text{Total}} = W_E^- + W_H^- + W_E^+ + W_H^+$ vs. free-space electrical radius ka . In the free-space case, $\mu_r = 1$, the internal energy is seen to constitute $2/3$ of the total energy — and thus it is equal to twice the external energy — for vanishing electrical radius. As the relative permeability μ_r increases to 2, 8, and 100, the energy ratio decreases to 0.5, 0.2, and 0.02, respectively. However, for non-zero electrical radius the dependence of the energy ratio on permeability μ_r is different, and for the internal cavity resonances the total stored energy is actually entirely internal. It is thus seen that the smallest energy ratio is not obtained with the largest permeability for finite values of ka .

The latter observation is also clearly illustrated in Fig. 3b, where the ratio W^-/W_{Total} is plotted (note the logarithmic scale) vs. the relative permeability μ_r in the range from 1 to 500 for three different values of $ka = 0.1, 0.25, \text{ and } 0.5$. The ratio only decreases monotonically for the very small electrical radius $ka = 0.1$, whereas for larger electrical radii

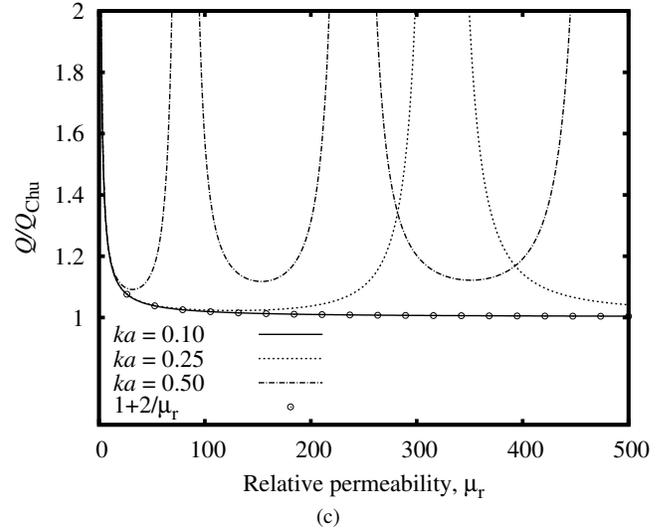
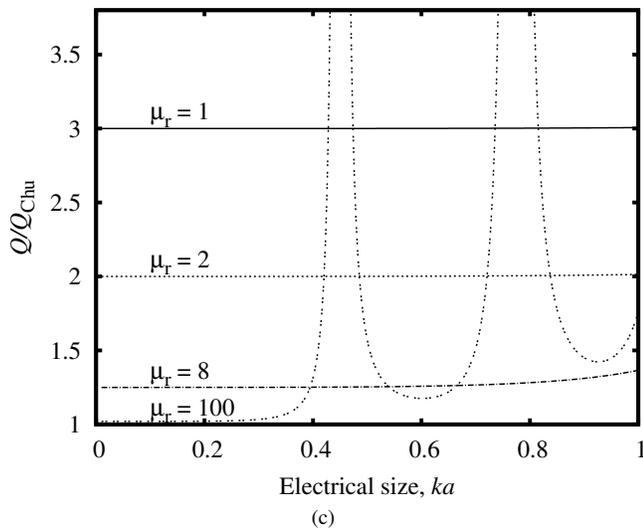
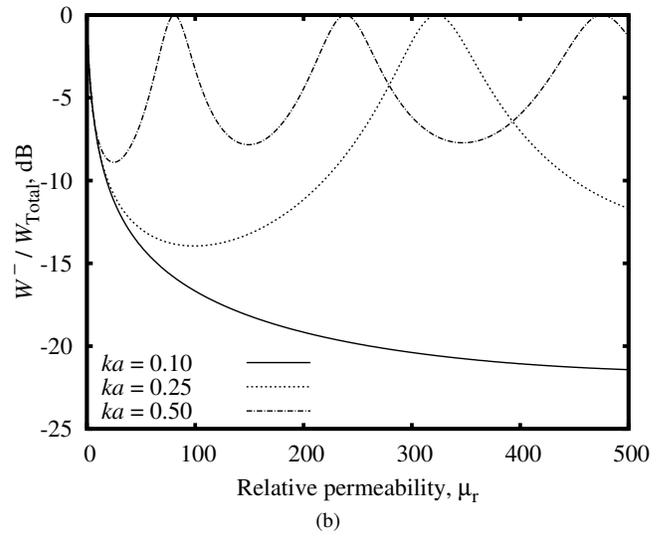
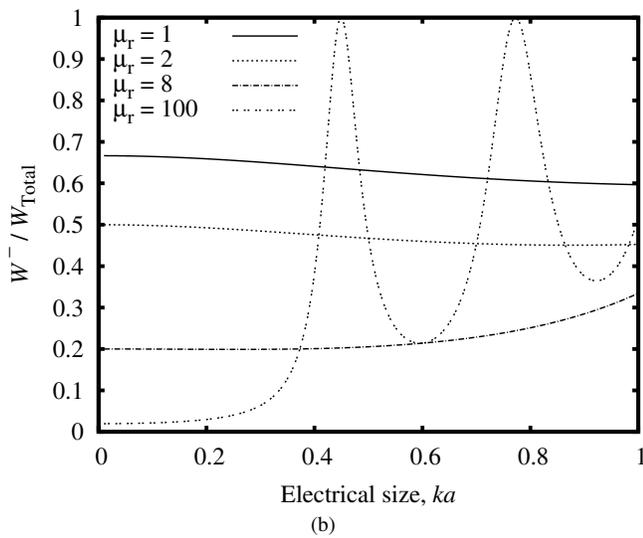
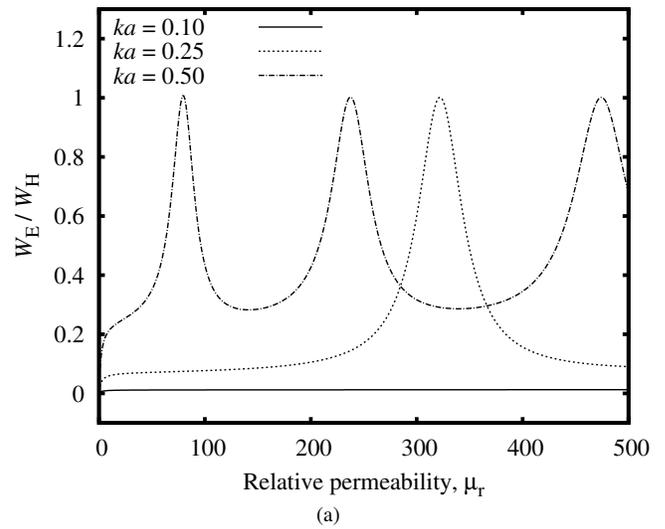
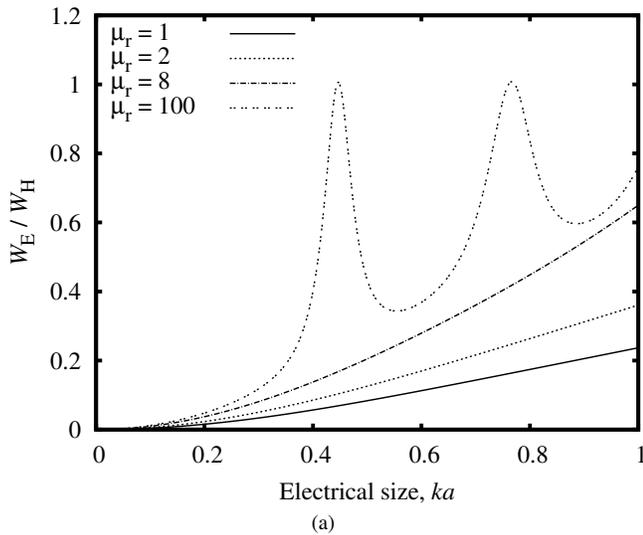


Fig. 2. Energy and quality factor as functions of ka ($\epsilon_r = 1$): (a) Ratio of electric and magnetic stored energies; (b) Ratio of internal and total stored energies; (c) Quality factor (normalized by Chu lower bound).

Fig. 3. Energy and quality factor as functions of μ_r ($\epsilon_r = 1$): (a) Ratio of electric and magnetic stored energies; (b) Ratio of internal and total stored energy; (c) Quality factor (normalized by Chu lower bound).

the initial decrease reaches a minimum whereafter the ratio actually increases.

3) *Quality factor*: The radiation quality factor Q is defined as 2π times the ratio of the stored electric and magnetic energy

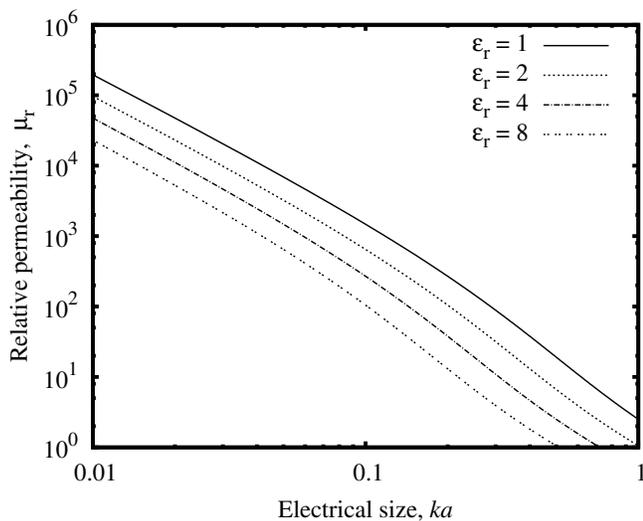


Fig. 4. (a) Values of μ_r , for which the relative error between the exact Q and expression (3) becomes larger than 1%, as a function of ka with ϵ_r as a parameter.

to the radiated power per period. However, in order to establish a resonant system (equal electric and magnetic stored energy) the stored energy is set to twice the maximum of the electric and magnetic energies. It is understood that the lesser of the two has been increased to equal the larger by use of, e.g., a lumped-component tuning circuit. For the TE_{10} magnetic-dipole antenna, the magnetic energy dominates for small electrical radii but the electric energy may dominate at larger radii. Thus, the quality factor Q is determined from (9) and (10) as

$$Q = 2\omega \frac{\max(W_H^- + W_H^+, W_E^- + W_E^+)}{P_{\text{rad}}}. \quad (11)$$

Figure 2c shows the quality factor Q , normalized by the Chu lower bound Q_{Chu} , vs. free-space electrical radius ka of the magnetic sphere for different relative permeabilities μ_r . For $\mu_r = 1$ (non-magnetic core) the normalized Q is equal to 3 for vanishing ka and it remains fairly constant for $ka < 1$ (this agrees with Thal [8, Table I]). For $\mu_r = 2, 8,$ and 100 the normalized Q equals 2, 1.25, and 1.02, respectively, for small ka in agreement with (3). This is also demonstrated in Fig. 3c, where the ratio Q/Q_{Chu} is plotted as a function of μ_r for different values of the free-space electrical radius ka . In the shown range of the relative permeability, the curve for the smallest sphere with $ka = 0.1$ very closely follows the expression (3).

For larger values of ka the resonance behavior of the fields in the core manifests itself in the fact that the curves start diverging from (3). In the following this divergence is investigated for a general magneto-dielectric core, that is, the relative permittivity ϵ_r is allowed to be greater than 1. Figure 4 shows the values of permeability μ_r , for which the relative error between the exact Q (11) and (3) becomes larger than 1%, as a function of ka with the permittivity ϵ_r as a parameter. A general observation is that the smaller ka and the lower ϵ_r the larger is the range of μ_r in which the expression (3) holds.

From Fig. 2c and 3c we can also conclude that the Chu lower bound can be approached with modest values of μ_r ;

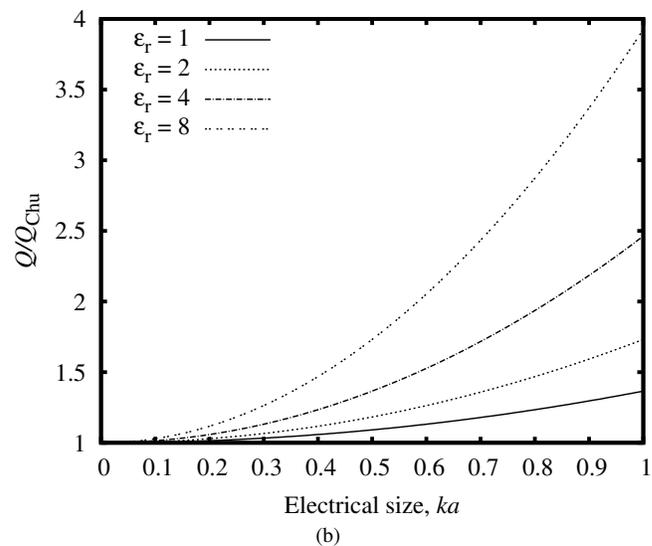
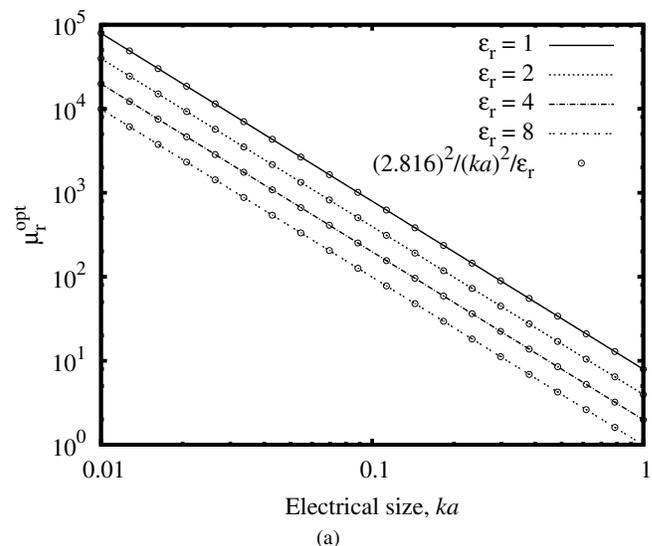


Fig. 5. (a) Optimal permeability μ_r^{opt} , yielding the lowest Q for a given ka and ϵ_r ; (b) The corresponding lowest achievable ratio Q/Q_{Chu} — the lower bound for a magnetic dipole antenna with a magneto-dielectric core.

e.g., within 25% for $\mu_r = 8$ and $ka < 0.5$. Furthermore, the smallest Q is not necessarily obtained with the largest μ_r . For a given ka and permittivity ϵ_r there is an optimum permeability μ_r^{opt} for which the ratio Q/Q_{Chu} is minimal. These values μ_r^{opt} have been determined numerically and are plotted in Fig. 5a as a function of ka with the core permittivity ϵ_r as a parameter. The curves in Fig. 5a very accurately follow the expression

$$\mu_r^{\text{opt}} = \frac{1}{\epsilon_r} \frac{(2.816)^2}{(ka)^2} \quad (12a)$$

$$\text{or} \quad \sqrt{\epsilon_r \mu_r^{\text{opt}}} ka = k_s^{\mu} a = 2.816. \quad (12b)$$

The minimum achievable ratios Q/Q_{Chu} corresponding to μ_r^{opt} (12a) are plotted in Fig. 5b, from which it is seen that in order to reach the Chu lower bound a magnetic dipole antenna should have a vanishing electrical size ka . For finite values of ka , the curve corresponding to $\epsilon_r = 1$ shows the lower bound for a magnetic dipole antenna with a magneto-dielectric core.

B. Higher-Order Modes

In the investigation of Section II-A above, it is assumed that there is only a single TE_{10} mode excited in the system. The situation changes in the presence of higher-order spherical modes that might be excited in a practical antenna when the current distribution deviates from (4). If these modes are not suppressed, they increase the stored energy inside the magnetic core and, in the worst case of a resonance, make the Q diverge. In particular, the TM modes with the polar index $n = 1$ are problematic, since in a magnetic sphere they generate the lowest frequency resonance. Whereas the first TE_{10} -mode resonance occurs at $k_s a = 4.493$, the first resonance for the TM_{1m} modes is at $k_s a = 2.744$. Consequently, the presence of any of the TM_{1m} modes makes the optimum $k_s^{\mu} a = 2.816$ found in Section II-A a poor choice. Furthermore, the range of usable core permeabilities μ_r is now limited by the TM_{1m} -mode resonance, i.e., it is reduced to almost half of the range for the single TE_{10} -mode case. The optimum core permeability μ_r^{opt} and the minimum achievable Q for a given ka depend in each particular case on the power ratios between TM_{1m} and TE_{10} modes as well as on the presence of other spherical modes.

III. MULTIARM SPHERICAL HELIX (MSH) ANTENNA

In this section we extend the theory presented in the previous Section II by numerical results obtained for a practical TE_{10} -mode antenna configuration with a magneto-dielectric core. We employ the magnetic dipole MSH antenna first reported in [17]. This antenna consists of multiple wire arms twisted into two symmetric hemispherical helices and excited by a curved dipole placed at the equator of the antenna (Fig. 6). As shown in [17], where an air-core version of the MSH antenna is thoroughly investigated, the resonance frequency of this self-resonant electrically small antenna is determined by the length of the wire arms, whereas the input resistance at the resonance is nearly independently controlled by the length of the excitation dipole. By placing a material core inside the antenna we change its resonance frequency and thus the electrical size. To avoid this change we adjust the length of the arms for each value of permeability/permittivity so that the resonance frequency is kept constant at $f_0 = 300$ MHz.

For our investigation we have chosen a 4-arm antenna configuration with the wire radius of $r_w = 0.5$ mm. The length of the excitation dipole is quantified in angular units; the half-length is $\alpha = 143^\circ$. The antenna is assumed to be lossless and fed by a delta-gap voltage generator in the middle of the excitation dipole. To ensure a well-behaved numerical solution the radius of the core a is always set 1 mm less than the helix radius r_0 .

The Q is calculated from the antenna input impedance $Z(\omega)$ using the expression from [7]

$$Q \approx \frac{\omega_0}{2R_0} |Z'(\omega_0)| = \frac{\omega_0}{2R_0} \sqrt{[R'(\omega_0)]^2 + [X'(\omega_0)]^2} \quad (13)$$

where $R(\omega)$ and $X(\omega)$ are the input resistance and reactance, respectively, and $R_0 = R(\omega_0)$ with ω_0 being the resonance frequency.

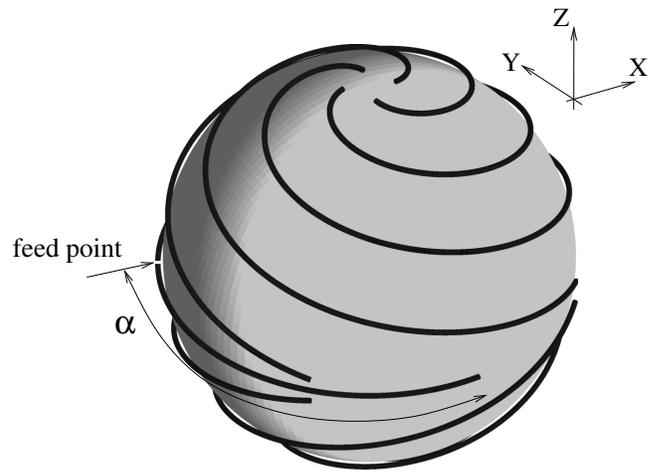


Fig. 6. The TE_{10} -mode MSH antenna with a spherical core.

A. Magnetic Core

First, we consider two antenna configurations of radii $r_0 = 40$ mm (electrical radius $k(r_0 + r_w) = 0.254$) and $r_0 = 30$ mm ($k(r_0 + r_w) = 0.192$), respectively, with a pure magnetic core ($\epsilon_r = 1$). The radii of the core are $a = 39$ mm and $a = 29$ mm for the first and second configuration, respectively. The quality factor Q normalized to the Chu lower bound Q_{Chu} is plotted in Fig. 7a as a function of the core permeability μ_r . It is observed that the ratio Q/Q_{Chu} steeply decreases from its initial free-space value — $Q/Q_{\text{Chu}} = 3.40$ for $r_0 = 40$ mm and $Q/Q_{\text{Chu}} = 3.32$ for $r_0 = 30$ mm — to a level below 1.5 as the permeability reaches only $\mu_r = 10$. The rapid drop of the stored magnetic energy inside the core requires a corresponding decline of the TM_{20} mode (Fig. 8a), since this mode acts as a distributed reactive matching circuit providing the stored electric energy needed to make the antenna self-resonant. The strength of the TM_{20} mode is proportional to the number of turns, and the latter reduces as well (Fig. 7b). The input resistance R_0 also initially reduces to about 50-60 ohms and then remains relatively stable until the first resonance is reached (Fig. 7c). This resonance occurs at roughly $\mu_r \approx 130$ for $r_0 = 40$ mm and at $\mu_r \approx 240$ for $r_0 = 30$ mm, which in both cases correspond to $k_s a \approx 2.8$, i.e., to the first TM_{11} -mode internal resonance in a magnetic spherical core ($k_s a = 2.744$). Thus, the TM_{11} mode present in the antenna radiation spectrum (Fig. 8b) gives rise to a spike in the Q/Q_{Chu} curve.

Having passed the first resonance, the Q for the configuration with $r_0 = 40$ mm monotonically grows with μ_r and reaches the TE_{10} -mode internal core resonance at $\mu_r \approx 330$, or $k_s a \approx 4.5$. The minimum value of the Q is found below the TM_{11} resonance at $\mu_r \approx 45$ for $r_0 = 40$ mm and at $\mu_r \approx 60$ for $r_0 = 30$ mm. The corresponding ratios Q/Q_{Chu} are 1.28 and 1.24. This result is consistent with the observations in Section II, i.e., the lowest achievable ratio Q/Q_{Chu} decreases with the antenna electrical size.

B. Magneto-Dielectric Core

For the core permittivity larger than unity ($\epsilon_r > 1$), the core resonances, which are functions of $k_s a = \sqrt{\epsilon_r \mu_r} ka$, shift

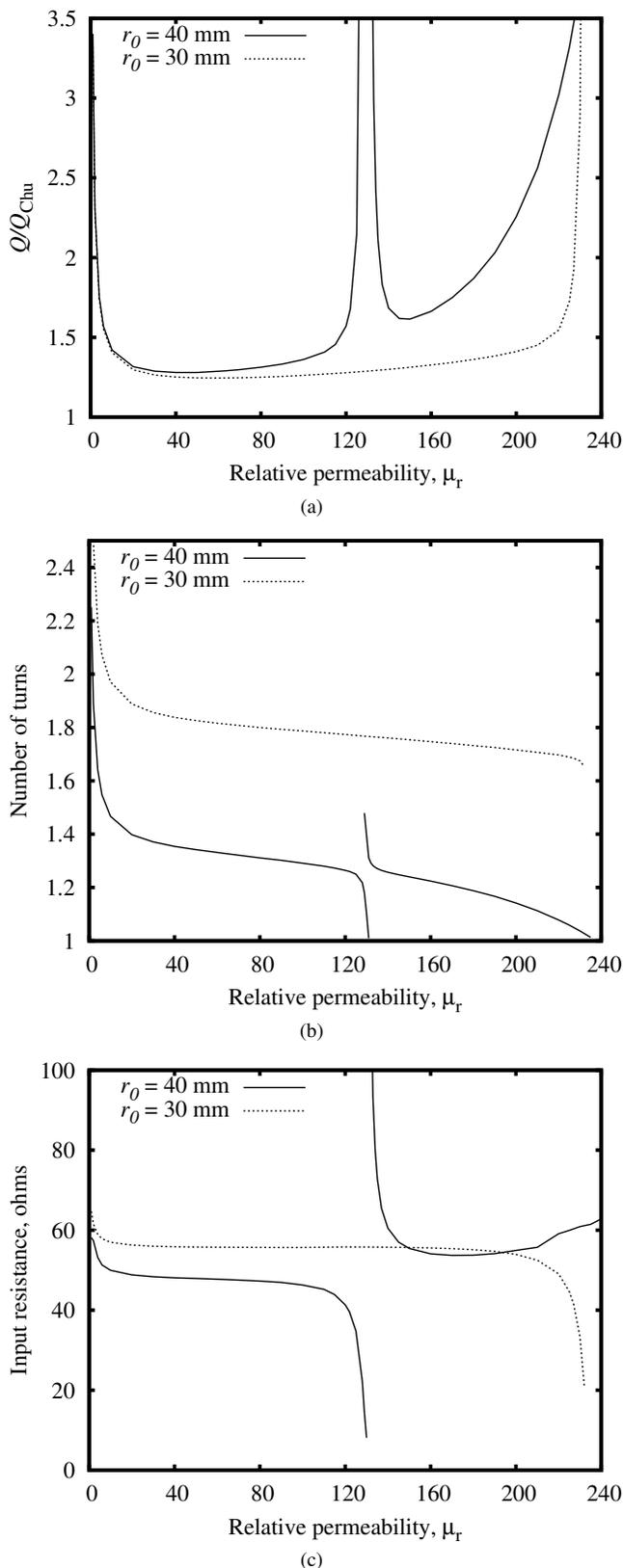


Fig. 7. Numerical results for the TE₁₀-mode MSH antenna with a magnetic core ($\epsilon_r = 1$). (a) Quality factor (normalized by Chu lower bound); (b) Number of turns; (c) Input resistance R_0 .

towards the lower values of μ_r (Fig. 9a); and the minimum values of the ratio Q/Q_{Chu} increase. Again, the result is

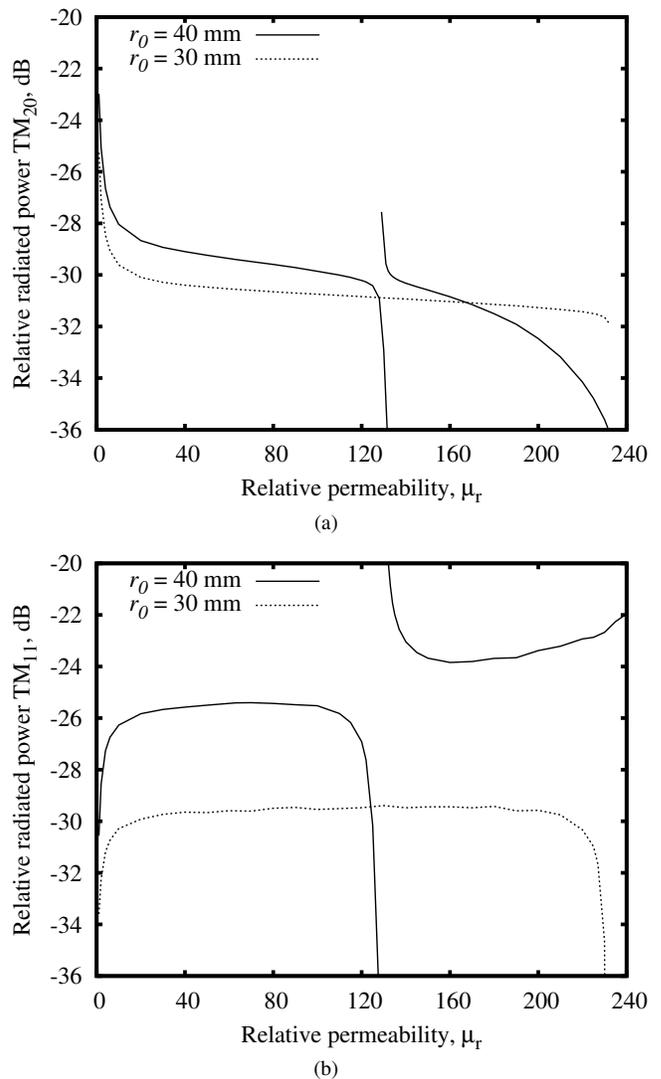


Fig. 8. Numerical results for the TE₁₀-mode MSH antenna with a magnetic core ($\epsilon_r = 1$). Radiated power of the (a) TM₂₀ and (b) TM₁₁ modes normalized to the total radiated power.

consistent with the theoretical prediction illustrated in Fig. 5.

The higher ϵ_r the more electric energy is stored in the core, and, as a consequence, less number of turns is required to make the antenna self-resonant (Fig. 9b). The input resistance R_0 also decreases (Fig. 9c), which, if necessary, can be easily compensated by truncating the excitation dipole [17].

IV. CONCLUSION

An analytical solution has been derived for the TE₁₀-mode magnetic dipole field radiated by a spherical, electric surface current density enclosing a magneto-dielectric core. Through direct spatial integration of the radiated fields, this solution led to closed-form expressions for the internal and external electric and magnetic stored energies, as well as the radiated power, and thus the radiation quality factor Q for arbitrary values of the electrical radius. It is shown that the internal stored energy reduces with increasing permeability μ_r of the core and, for vanishing free-space electrical radius, $ka \ll 1$, the radiation quality factor Q approaches the Chu lower bound

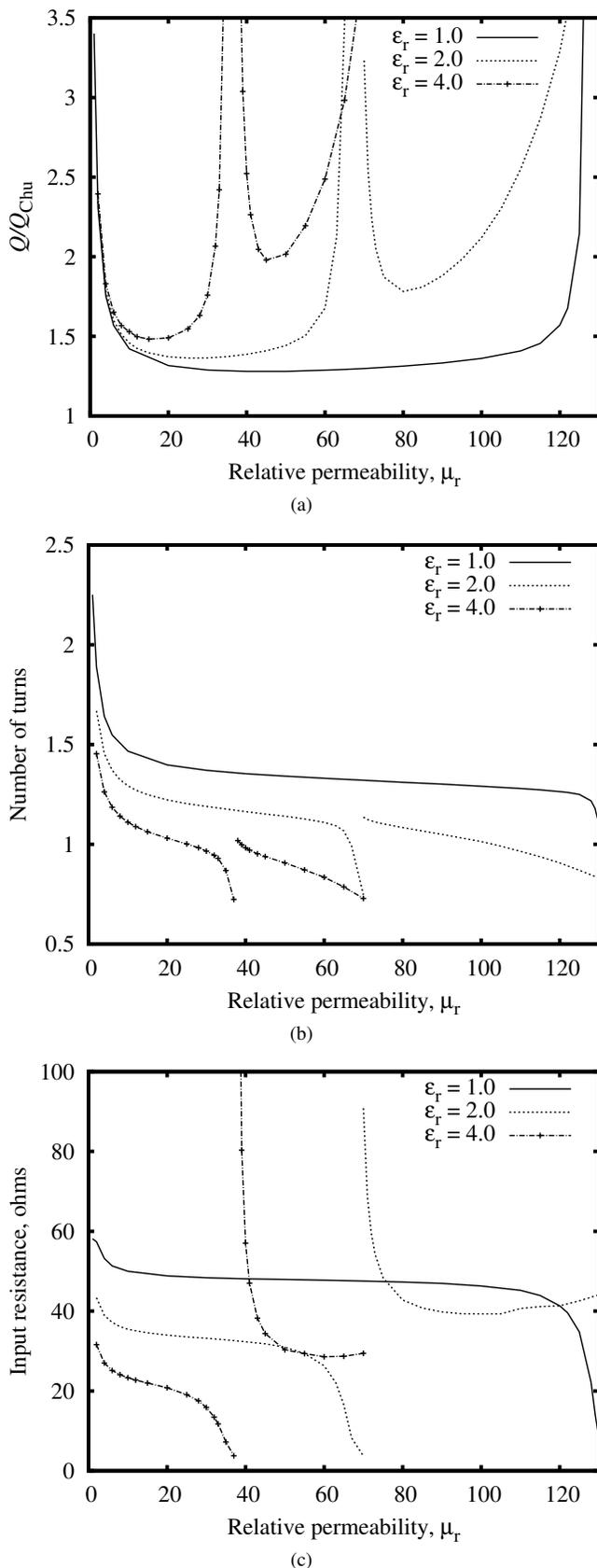


Fig. 9. Numerical results for the TE₁₀-mode MSH antenna with a magneto-dielectric core. (a) Quality factor (normalized by Chu lower bound); (b) Number of turns; (c) Input resistance R_0 .

Q_{Chu} arbitrarily closely as $\mu_r \rightarrow \infty$, in agreement with the prediction of Wheeler [13]. For finite-sized spheres the internal resonances of the magneto-dielectric core limit the minimum value of the quality factor Q . However, it was seen that the Chu lower bound can be approached with even modest values of the permeability μ_r , e.g., within 25% for $\mu_r = 8$ and $ka < 0.5$. On basis of the analytical solution, for a given antenna size and permittivity ϵ_r the minimum value of the quality factor Q was determined numerically and it was found to occur for the relative permeability μ_r^{opt} that satisfies the relation (12a). By substituting μ_r^{opt} corresponding to $\epsilon_r = 1$ into (7)-(11) the lower bound for a magnetic dipole antenna with a magneto-dielectric core is established.

The analytical results are consistent with numerical simulations conducted for the TE₁₀-mode multiarm spherical helix antenna with a magneto-dielectric core. The radiation quality factor $Q = 1.24Q_{\text{Chu}}$ is obtained for the antenna with free-space electrical radius $ka = 0.192$.

Generally, the smaller the antenna the closer its radiation Q can approach the Chu lower bound. In addition, a magnetic core with $\epsilon_r = 1$ and the absence of the parasitic TM_{1m} modes are prerequisites for the lowest possible Q of a magnetic dipole antenna.

REFERENCES

- [1] H. A. Wheeler, "Fundamental limitations of small antennas," *Proc. IRE*, vol. 35, no. 12, pp. 1479–1484, 1947.
- [2] L. J. Chu, "Physical limitations of omni-directional antennas," *J. Appl. Phys.*, vol. 19, no. 12, pp. 1163–1175, 1948.
- [3] R. F. Harrington, *J. Research National Bureau of Standards-D, Radio Propag.*, vol. 64D, no. 1, pp. 1–12, 1960.
- [4] R. Collin and S. Rothschild, "Evaluation of antenna Q," *IEEE Trans. Antennas Propag.*, vol. 12, no. 1, pp. 23–27, Jan. 1964.
- [5] R. L. Fante, "Quality factor of general ideal antennas," *IEEE Trans. Antennas Propag.*, vol. 17, no. 2, pp. 151–155, Mar. 1969.
- [6] J. S. McLean, "A re-examination of the fundamental limits on the radiation Q of electrically small antennas," *IEEE Trans. Antennas Propag.*, vol. 44, no. 5, pp. 672–676, May 1996.
- [7] A. D. Yaghjian and S. R. Best, "Impedance, bandwidth, and Q of antennas," *IEEE Trans. Antennas Propag.*, vol. 53, no. 4, pp. 1298–1324, Apr. 2005.
- [8] H. L. Thal, "New radiation Q limits for spherical wire antennas," *IEEE Trans. Antennas Propag.*, vol. 54, no. 10, pp. 2757–2763, Oct. 2006.
- [9] S. R. Best, "The radiation properties of electrically small folded spherical helix antennas," *IEEE Trans. Antennas Propag.*, vol. 52, no. 4, pp. 953–960, Apr. 2004.
- [10] —, "Low Q electrically small linear and elliptical polarized spherical dipole antennas," *IEEE Trans. Antennas Propag.*, vol. 53, no. 3, pp. 1047–1053, Mar. 2005.
- [11] H. R. Stuart, S. R. Best, and A. D. Yaghjian, "Limitations in relating quality factor to bandwidth in a double resonance small antenna," *IEEE Antennas Wireless Propagat. Lett.*, vol. 6, no. 11, pp. 460–463, 2007.
- [12] A. Erentok and R. W. Ziolkowski, "Metamaterial-inspired efficient electrically small antennas," *IEEE Trans. Antennas Propag.*, vol. 56, no. 3, pp. 691–707, Mar. 2008.
- [13] H. A. Wheeler, "The spherical coil as an inductor, shield, or antenna," *Proc. IRE*, vol. 46, no. 9, pp. 1595–1602, 1958.
- [14] H. L. Thal, "Exact circuit analysis of spherical waves," *IEEE Trans. Antennas Propag.*, vol. 26, no. 2, pp. 282–287, Mar. 1978.
- [15] O. Breinbjerg, "Stored energy and quality factor for spherical dipole antennas in free space," Technical University of Denmark, Tech. Rep., 2009.
- [16] S. R. Best, "A low Q electrically small magnetic (TE mode) dipole," *IEEE Antennas Wireless Propagat. Lett.*, vol. 8, pp. 572–575, 2009.
- [17] O. S. Kim, "Low-Q electrically small spherical magnetic dipole antennas," *IEEE Trans. Antennas Propag.*, in press.

- [18] E. Jørgensen, O. S. Kim, P. Meincke, and O. Breinbjerg, "Higher order hierarchical legendre basis functions in integral equation formulations applied to complex electromagnetic problems," in *Proc. IEEE Antennas and Propagation Soc. Int. Symp.*, vol. 3A, Washington DC, Jul. 2005, pp. 64–67.
- [19] O. Breinbjerg and O. S. Kim, "Minimum Q electrically small spherical magnetic dipole antenna – theory," in *Proc. Int. Symp. Ant. Propagat. (ISAP 2009)*, Bangkok, Thailand, October 20–29 2009.
- [20] O. S. Kim and O. Breinbjerg, "Minimum Q electrically small spherical magnetic dipole antenna – practice," in *Proc. Int. Symp. Ant. Propagat. (ISAP 2009)*, Bangkok, Thailand, October 20–29 2009.
- [21] A. D. Yaghjian, "Power-energy & dispersion relations for diamagnetic media," in *Proc. IEEE Antennas Propag. Soc. Int. Symp.*, Charleston, SC, June 1–5 2009.
- [22] J. E. Hansen, *Spherical Near-Field Antenna Measurements*. London, U.K.: Peter Peregrinus, 1988.



Oleksiy S. Kim received the M.S. and Ph.D. degrees from the National Technical University of Ukraine, Kiev, in 1996 and 2000, respectively, both in electrical engineering. In 2000, he joined the Antenna and Electromagnetics Group at the Technical University of Denmark (DTU). He is currently an associate professor with the Department of Electrical Engineering, ElectroScience Section, DTU.

His research interests include computational electromagnetics, metamaterials, electrically small antennas, photonic bandgap and plasmonic structures.



Olav Breinbjerg (M'87) was born in Silkeborg, Denmark on July 16, 1961. He received the M.Sc. and Ph.D. degrees in electrical engineering from the Technical University of Denmark (DTU) in 1987 and 1992, respectively. Since 1991 he has been on the faculty of the Department of Electrical Engineering (formerly Ørsted-DTU, Department of Electromagnetic Systems, and Electromagnetics Institute) where he is now Full Professor and Head of the Electromagnetic Systems Group including the DTU-ESA Spherical Near-Field Antenna Test Facility. Olav

Breinbjerg was a Visiting Scientist at Rome Laboratory, Hanscom Air Force Base, Massachusetts, USA in the fall of 1988 and a Fulbright Research Scholar at the University of Texas at Austin, Texas, USA in the spring of 1995. Olav Breinbjerg's research is generally in applied electromagnetics — and particularly in antennas, antenna measurements, computational techniques and scattering — for applications in wireless communication and sensing technologies. At present, his interests focus on meta-materials, antenna miniaturization, and spherical near-field antenna measurements. He is the author or co-author of more than 40 journal papers, 100 conference papers, and 70 technical reports, and he has been, or is, the main supervisor of 10 Ph.D.projects. Olav Breinbjerg has taught several B.Sc. and M.Sc. courses in the area of applied electromagnetic field theory on topics such as fundamental electromagnetics, analytical and computational electromagnetics, antennas, and antenna measurements at DTU, where he has also supervised more than 70 special courses and 30 M.Sc. final projects. Furthermore, he has given short courses at other European universities. He is currently the coordinating teacher at DTU for the 3rd semester course 31400 Electromagnetics, and the 7–9th semester courses 31428 Advanced Electromagnetics, 31430 Antennas, and 31435 Antenna Measurements in Radio Anechoic Chambers. Olav Breinbjerg received a US Fulbright Research Award in 1995. Also, he received the 2001 AEG Elektron Foundation's Award in recognition of his research in applied electromagnetics. Furthermore, he received the 2003 DTU Student Union's Teacher of the Year Award for his course on electromagnetics.



Arthur D. Yaghjian (S'68–M'69–SM'84–F'93–LF'09) received the B.S., M.S., and Ph.D. degrees in electrical engineering from Brown University, Providence, RI in 1964, 1966, and 1969.

During the spring semester of 1967, he taught mathematics at Tougaloo College, MS. After receiving the Ph.D. degree he taught mathematics and physics for a year at Hampton University, VA, and in 1971 he joined the research staff of the Electromagnetics Division of the National Institute of Standards and Technology (NIST), Boulder, CO.

He transferred in 1983 to the Electromagnetics Directorate of the Air Force Research Laboratory (AFRL), Hanscom AFB, MA, where he was employed as a Research Scientist until 1996. In 1989, he took an eight-month leave of absence to accept a visiting professorship in the Electromagnetics Institute of the Technical University of Denmark. He presently works as an independent consultant in electromagnetics. His research in electromagnetics has led to the determination of electromagnetic fields in materials and "metamaterials," the development of exact, numerical, and high-frequency methods for predicting and measuring the near and far fields of antennas and scatterers, the design of electrically small supergain arrays, and the reformulation of the classical equations of motion of charged particles.

Dr. Yaghjian is a Life Fellow of the IEEE, has served as an Associate Editor for the IEEE and URSI, and is a member of Sigma Xi. He has received best paper awards from the IEEE, NIST, and AFRL.