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# Evaluation of Damping Using Time Domain OMA Techniques

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**Keywords:** Operational Modal Analysis, structural damping, Ibrahim Time Domain, Eigenvalue Realization Algorithm, Polyreference Time Domain, closely spaced modes.

## Abstract

The prevailing Operational Modal Analysis (OMA) techniques provide in most cases reasonably accurate estimates of structural frequencies and mode shapes. In contrast though, they are known to often produce poor structural damping estimates, which is mainly due to inherent random and/or bias errors. In this paper a comparison is made of the effectiveness of three existing OMA techniques in providing accurate damping estimates for varying loadings, levels of noise, number of added measurement channels and structural damping. The evaluated techniques are derived in the time domain and are namely the Ibrahim Time Domain (ITD), Eigenvalue Realization Algorithm (ERA) and the Polyreference Time Domain (PTD). The response of a two degree-of-freedom (2DOF) system is numerically established from specified modal parameters with well separated and closely spaced modes. Two types of response are considered, free response and random response from white noise loading. Finally, the results of the numerical study are presented, in which the error of the structural damping estimates obtained by each OMA technique is shown for a range of damping levels. From this, it is clear that there are notable differences in accuracy between the different techniques.

## Introduction

In 2011, Georgakis and Acampora [1] reported negative aerodynamic damping in the presence of rain for the longest cable on the Øresund Bridge between Denmark and Sweden. The aerodynamic damping was identified from full scale measurements of the bridge using two time domain techniques, the Eigenvalue Realization Algorithm (ERA) [2] and Stochastic Subspace Identification (SSI) [3]. The OMA estimates of damping were identified to be inconsistent and coarse in all cases. The poor damping estimation is thought mainly to be due to inherent random and/or bias errors. The reliability and accuracy of the available techniques in determining damping is unknown. Thus it was felt important to evaluate the robustness and accuracy of the existing OMA techniques for various levels of damping.

The existing parametric OMA techniques are based on impulse responses, free decay responses from structures or pseudo free decays found using random decrement techniques which are estimated as correlation functions [4]. The estimated correlation function from a pure free response is an unbiased estimate and serves as a basis to obtain the lower bound of

the error on the damping estimate. On the contrary the loading on civil engineering structures is rarely controllable and the excitation will therefore lead to a random response. The input correlation function from a random response measurement is finite and estimated, The input for identification is further disturbed by high signal to noise ratios, and it becomes a challenge to determine the physical modes of the system. Moreover for structures with some degree of symmetry, closely spaced modes are often encountered. The proximity of natural frequencies reduces the quality of the estimates [5,6], however it is unknown how much.

Three techniques were chosen for evaluation, namely the Ibrahim Time Domain (ITD), Eigenvalue Realization Algorithm (ERA) and Polyreference Time Domain (PTD). The evaluation of the techniques is based on the performance for configurations of free response and response to white noise loading, separated and closely spaced modes, varying noise levels, and noise modes for varying levels of damping. Numerical results are presented in support of the proposed configurations, giving an overview of the performance of each identification technique for the estimation of each level of structural damping ratio.

## **Theory**

The basic idea of all OMA techniques is to interpret the auto-correlation functions of the structural response as free decays. For the system subjected to white noise loading the correlation function is estimated as a modal decomposition of the correlation function following the derivation by Brincker and Zhang [7].

The basic principle for the time domain techniques is the identification of structural parameters from experimental data placed in the form of an eigenvalue problem. Three time domain techniques were used as implemented in the OMA toolbox[7], where the multiple-output-multiple-input (MIMO) version of the Ibrahim Time Domain technique is derived as introduced by Ibrahim et al. in [8]. This version is based on the use of the block Hankel matrices for the formation of the Topelitz matrices containing information from all free decays, but generalized to multiple input like described in [7]. The average estimate of the system matrix is then formed and used for the eigenvalue problem. The basic principle of the ERA technique similarly uses the estimate of the system matrix to solve an eigenvalue problem. This technique is formulated as in the original version introduced by Pappa et al.[2]. The main difference between the ERA and ITD is that the system matrix in ERA is found through singular value decomposition (SVD) of the block Hankel matrices. The PTD technique is presented by Vold et al [9], where the derived version in this analysis is the authors' interpretation of the PTD technique. This technique uses auto regressive (AR) models. It differs from the approach taken by Vold as the author's version applies correlation functions rather than impulse response functions.

## **Numerical simulations and results**

The numerical simulations were performed for variations of loading, separated and closely spaced modes, noise level and addition of measurement channels. These variations led to 36 tested configurations listed in Table 1. The input was either the correlation functions for the

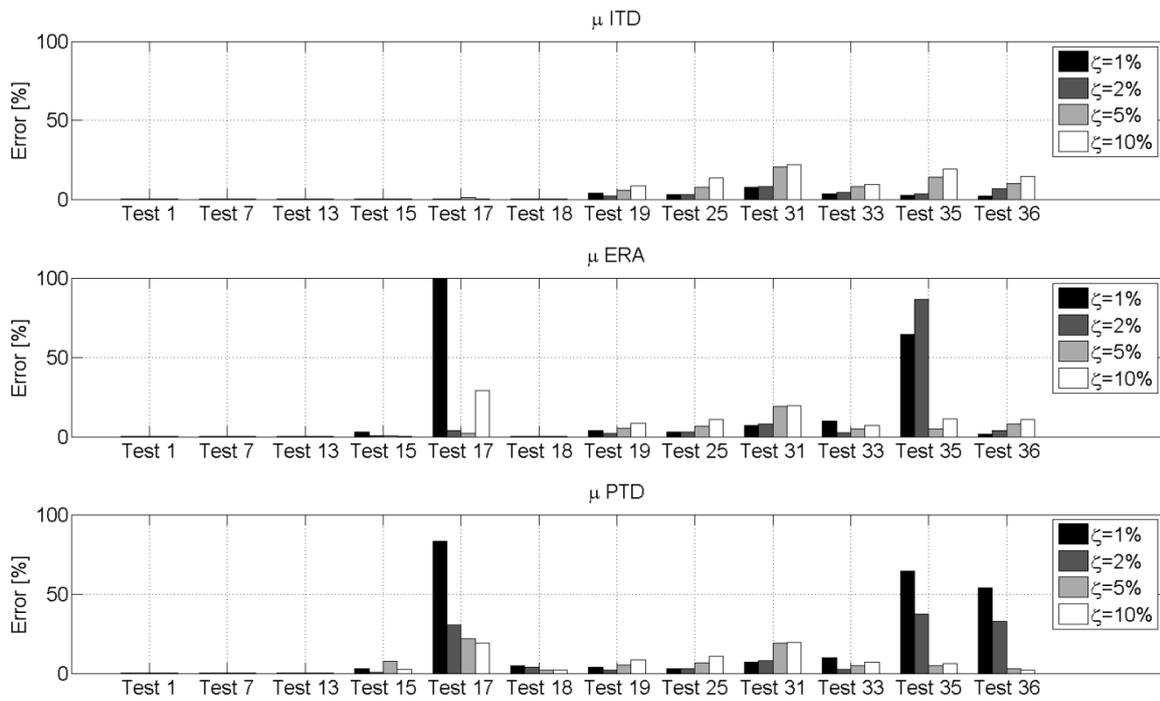
free response of the system or the estimated correlation function of the response to white noise loading.

Identification problems in OMA arise due to limited time series length, as it is generally assumed that the correlation function estimate tend to the exact correlation function when the time series is infinite. Therefore a criterion was set to ensure reasonable estimates of the correlation function, such that the estimated damping includes minimal influence from leakage bias. The time series length was estimated from the maximum correlation time in the response, assuming that the longest correlation time is defined by the lowest natural frequency of interest. Thus the time series length was inversely proportional to the structural damping ratio times the minimum natural frequency of interest, where the damping ratio  $\zeta$  is presented as a percentage of the critical damping. The time step was set to 0.05 sec and was held constant. Increasing the time step, for this system, will limit the noise and vice versa.

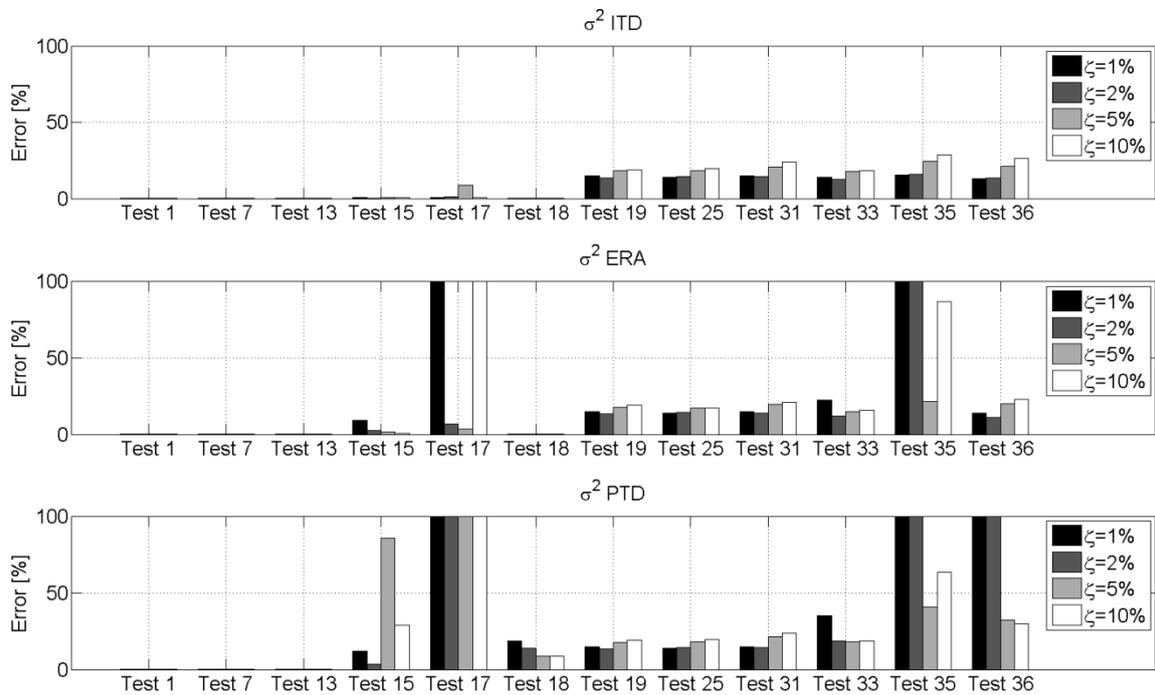
The time window of the correlation function was adjusted by removing the first five data points to avoid the influence of measurement noise and the amplitude of the correlation function determined the length of the time window. The correlation function was truncated at the point where the amplitude was below 20% of the maximum amplitude thus excluding the noise tail. The time windows were of equal length at each DOF for each random correlation function.

**Table 1: Tests denotes the variation of simulation configurations.  $R_y(\tau)$  denotes the correlation function based on the free response of the system.  $\hat{R}_y(\tau)$  denotes the estimated correlation function from white noise loading .**

Correlation function	Frequency	Noise level	Additional channels	Test
$R_y(\tau)$ (computed from the free response of the system)	$f_1 = 1.00\text{Hz}$ $f_2 = 3.00\text{Hz}$	0‰	0	<b>1</b>
			2	2
		1‰	0	3
			2	4
		2‰	0	5
			2	6
	$f_1 = 1.00\text{Hz}$ $f_2 = 1.20\text{Hz}$	0‰	0	<b>7</b>
			2	8
		1‰	0	9
			2	10
		2‰	0	11
			2	12
	$f_1 = 1.00\text{Hz}$ $f_2 = 1.02\text{Hz}$	0‰	0	<b>13</b>
			2	14
		1‰	0	<b>15</b>
			2	16
		2‰	0	<b>17</b>
			2	<b>18</b>
$\hat{R}_y(\tau)$ (estimated from white noise loading of the system)	$f_1 = 1.00\text{Hz}$ $f_2 = 3.00\text{Hz}$	0%	0	<b>19</b>
			2	20
		1‰	0	21
			2	22
		2‰	0	23
			2	24
	$f_1 = 1.00\text{Hz}$ $f_2 = 1.20\text{Hz}$	0‰	0	<b>25</b>
			2	26
		1‰	0	27
			2	28
		2‰	0	29
			2	30
	$f_1 = 1.00\text{Hz}$ $f_2 = 1.02\text{Hz}$	0‰	0	<b>31</b>
			2	32
		1‰	0	<b>33</b>
			2	34
		2‰	0	<b>35</b>
			2	<b>36</b>



**Figure 1: Bias error presented as percentage deviation of the mean of the estimated damping from the true value of the damping of the three time domain OMA techniques for varying structural damping ratios and simulations denoted test 1, 7, 13, 15, 17, 18, 19, 25, 31, 33, 35 and 36. Top: Ibrahim Time Domain (ITD). Middle: Eigenvalue Realization Algorithm (ERA). Bottom: Polyreference Time Domain (PTD).**



**Figure 2: Random error presented as the standard deviation relative to the true value in percent for three time domain OMA techniques for varying structural damping ratios and simulations denoted test 1, 7, 13, 15, 17, 18, 19, 25, 31, 33, 35 and 36. Top: Ibrahim Time Domain (ITD). Middle: Eigenvalue Realization Algorithm (ERA). Bottom: Polyreference Time Domain (PTD).**

The two mode shapes of the system were random and geometrically orthogonal, and the closeness of the modes was adjusted by the natural frequencies of the modes, where both modes had equal damping. Noise was simulated as white noise with a level of either 1‰ or 2‰ of the maximum value of the correlation function. For each level of noise the damping estimate was evaluated with the addition of two measurement channels.

Using the three time domain techniques, each identification was repeated 100 times. The results are depicted for 12 chosen configurations for the evaluation of the damping estimation. These are the test numbers in bold in Table 1. The mean  $\mu$  and the standard deviation  $\sigma^2$  absolute percentage error of the damping estimation are illustrated in Figure 1-2 for each identification technique and structural damping ratios of 1%, 2%, 5% and 10%.

## Discussion

The OMA techniques identify the lower bound of the error for free response input, and the error level of the damping estimate arising from the estimation of the correlation function from white noise loading. The lower bound of the error on the damping estimate is of an order of magnitude  $10^{-7}$ , as the noiseless correlation function from free response is an ideal input. Once the modes are closely spaced the error on the damping estimate increases. This is predominantly the case for ERA and PTD, whereas the damping estimated by the ITD is less sensitive to the modes. It is also clear that the ITD technique is less sensitive to signal noise, as a high noise to signal ratio results in a damping estimate with larger mean error and standard deviation from the identification with ERA and PTD. Real measurements always include noise, and therefore the fit of the correlation function includes the noise, which is nonphysical information disturbing the identification of the physical system, resulting in larger mean errors in the damping estimation. For higher levels of damping the fit is worse, as the nonphysical information from the noise is becoming more dominant whilst the correlation function decays faster when the damping is higher. Including additional measurement channels improves the identification and reduces the mean and standard deviation of the error on the damping estimate.

When the system is subjected to white noise loading, the mean error of the is identical for the ERA and PTD techniques, with 19% mean error with closely spaced modes and a structural damping ratio of 10%. The ITD is however less robust than the ERA and PTD when the correlation function is an estimate with no signal noise, as the mean error reaches 21% for a system with closely spaced modes and structural damping ratio of 10% .

Further it is important to note how the addition of a signal noise level of 1‰ improves the mean and standard deviation error for a system subjected to random loading. For higher noise levels, the ITD technique is again a more robust identification technique, for all damping levels. On the contrary ERA and TPR are significantly sensitive to noise for low damping ratios when the modes are close, reaching a mean error of 86%.

## Summary

The damping estimates using the ERA and PTD techniques show considerable sensitivity to closeness of modes and noise, whereas the ITD has proven to be more robust for the current

configurations. For structures with large structural damping ratios it is clear that the identification will be poorer especially in the presence of closely spaced modes. This effect can be minimized to some extent by including additional noise modes.

It is important to note that in this paper the damping estimate of a 2DOF linear system with normal modes and proportional damping has been evaluated. Due to the non-proportional nature of damping and the possible presence of non-linearities, the modal damping ratio identification should be examined in future for complex modes.

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