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The full gyrokinetic electrostatic linearized Landau collision operator is calculated including the equilibrium operator, which represents the effect of collisions between gyrokinetic Maxwellian particles. First, the equilibrium operator describes energy exchange between different plasma species, which is important in multiple ion-species plasmas. Second, the equilibrium operator describes drag and diffusion of the magnetic field aligned component of the vorticity associated with the $E \times B$ drift. Therefore, a correct description of collisional effects in turbulent plasmas requires the equilibrium operator, even for like-particle collisions.

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Gyrokinetic theory plays a central role in analytical and numerical investigations of low-frequency turbulence and the associated anomalous transport in magnetized laboratory plasmas and in nature [1]. The theory eliminates the fast time scale associated with the fast cyclotron gyration from the equations of motion, and reduces the dimensionality of the kinetic equation from six to five. However, gyrokinetic theory was originally developed without taking collisions into account. Collisions strongly influence low-frequency turbulence and are therefore explicitly gyroaveraged when the plasma is nearly in a local equilibrium state [9], the gyrokinetic distribution function leads to the gyrokinetic linearized collision operator describing collisions between particles of species $a$ and $b$. The operator is therefore required in order to describe the influence of collisions on electrostatic turbulence and the associated anomalous transport in magnetized laboratory plasmas. Furthermore, the equilibrium operator is particularly important in multiple ion-species plasmas such as fusion and astrophysical plasmas. The corresponding terms have the same order of magnitude as the test- and field-particle operators. The equilibrium operator is therefore required in order to describe the influence of collisions on electrostatic turbulence and the associated transport correctly. Contrary to conventional wisdom, it is shown that the equilibrium operator must be retained even for like-particle collisions.

The starting point of our derivation is the Landau kinetic equation,

$$\frac{\partial}{\partial t} f_a(z,t) + \dot{z} i \frac{\partial}{\partial \dot{z}} f_a(z,t) = C[f_a; f_b],$$

(1)

where $f_a$ denotes the distribution function of species $a$ having mass $m_a$ and charge $q_a$, and $z = (x, p)$ are noncanonical phase-space coordinates; $x$ denotes particle position and $p = mv$ is the particle momentum. Repeated $i$ and $j$ imply summation throughout this Rapid Communication. The equations of motion $\dot{z} = (\dot{z}, h)$ are here written in terms of the Hamiltonian $h$ and the Poisson bracket,

$$\{f, g\} = \frac{\partial f}{\partial x} \cdot \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \cdot \frac{\partial g}{\partial x} + qB \cdot \frac{\partial f}{\partial p} \times \frac{\partial g}{\partial p}.$$

(2)

The Landau collision operator,

$$C[f_a; f_b] = \frac{\partial}{\partial p_i} \left[ \frac{m_a}{m_b} f_a \frac{\partial H_b}{\partial p_i} - m_a^2 \frac{\partial^2 G_b}{\partial p_i \partial p_j} \frac{\partial f_a}{\partial p_j} \right],$$

(3)

describes drag and diffusion in velocity space due to small-angle Coulomb collisions between species $a$ and $b$. The Landau collision operator in Eq. (3) is here written in terms of

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Rosenbluth potentials [12]

\[
\begin{aligned}
H_b &= -\frac{\Gamma_{ab}}{8\pi} \int d^3z' f_b(z') \delta^3(x-x') \left( \frac{2u^{-1}}{\mu} \right),
\end{aligned}
\]

where \( u = |v - v'| \) is the relative velocity, \( \Gamma_{ab} = (qB_0/e_0)^3 \ln \Lambda, \) and \( \delta^3(x) \) is the Dirac delta function. By employing the properties \( \nabla^2 H_b = H_b \) and \( \nabla^2 H_b = \Gamma_{ab}, f_b \) of the Rosenbluth potentials, the collision operator Eq. (3) can be written in terms [4] of Poisson brackets:

\[
\mathcal{C}[f_a; f_b] = \left[ \{x_i, f_a\}, H_b \right] m_a m_b f_a f_b - m_a^2 \left[ \{x_i, x_j\}, H_b \right] f_a f_b,
\]

which simplifies the transformation of the operator to gyrocenter coordinates.

The gyrocenter coordinate transformation is a two-step transformation going from particle coordinates \( z = (x, p) \) through guiding-center coordinates \( Z_{gc} = (X_{gc}, v_{gc}, \mu_{gc}, \theta_{gc}) \) to gyrocenter coordinates \( Z = (X, v_{gc}, \mu, \theta) \). The guiding-center coordinate transformation eliminates the fast cyclotron motion time scale in the equations of motion in the absence of fluctuating electromagnetic fields. For the sake of simplicity, nonuniformity of the magnetic field is neglected throughout. The zeroth-order particle to guiding-center coordinate transformation [13] is defined as \( x = X_{gc} + \rho_0 \) and \( v = \hat{b}(X_{gc})v_{gc} + \hat{v}_l(X_{gc}, \mu_{gc}, \theta_{gc}) \). Here, \( X_{gc} \) is the gyrocenter position, \( \mu_{gc} \) is the guiding-center velocity parallel to \( \hat{b} = B/B_{\parallel} \), which denotes the unit vector parallel to the magnetic field, \( \mu_{gc} \) is the magnetic dipole momentum-like coordinate, and \( \theta_{gc} \) denotes the gyroangle coordinate of the cyclotron motion. The perpendicular component of the velocity is related to the guiding-center coordinates as \( v_{\perp gc} = \sqrt{2\mu_{gc}B/m_{\perp}} = \Omega \rho_0 \times \hat{b} \), where \( \Omega = qB/m \) denotes the gyrofrequency, and \( \hat{b} \) is a co-moving unit vector parallel to the perpendicular velocity vector. The unit vectors form an orthonormal triad \( \hat{b} \times \hat{b} = \hat{v}_l = \partial \phi / \partial \mu = (\hat{e}_1, \hat{e}_2, \hat{e}_3) \) and \( \hat{e}_1 \) and \( \hat{e}_2 \) are local fixed orthonormal basis vectors, which, together with \( \hat{b} \), form a local orthonormal triad. The fixed unit vectors \( \hat{e}_1 \) and \( \hat{e}_2 \) are related to the co-moving unit vectors through the relation \( \hat{b}(X_{gc}, \mu_{gc}, \theta_{gc}) = -\hat{e}_1(X_{gc}, \mu_{gc}, \theta_{gc}) \sin \theta_{gc} - \hat{e}_2(X_{gc}, \mu_{gc}, \theta_{gc}) \cos \theta_{gc} \).

In the second step a fluctuating small-amplitude electric field is introduced. The fluctuating electric field reintroduces the fast time scale associated with the cyclotron motion into the guiding-center equations of motion. The fast time scale can be asymptotically eliminated by adopting the standard nonlinear gyrokinetic ordering, \( \phi \sim \mu \sim \mu \sim \epsilon_3 \ll 1 \) and \( k_\parallel \rho_0 \sim 1 \), where \( \phi \) denotes the electrostatic potential, \( T \) is the temperature, \( \omega \) is the characteristic fluctuation frequency, and \( k_\parallel \) and \( k_\perp \) are the characteristic parallel and perpendicular fluctuation wave numbers, respectively. The resulting guiding-center to gyrocenter coordinate transformation [1] is given as

\[
Z = Z_{gc} + \epsilon_3 S_t Z_{gc} + \mathcal{O}(\epsilon_3^2).
\]

Here, \( S_t = \Omega^{-1} \int_0^\phi d\theta \phi \) is the first-order generating function of the Lie transformation and \( \phi = \phi - \phi \) defines the oscillatory part of the electrostatic potential; \( \langle \phi \rangle = (2\pi)^{-1} \int_0^{2\pi} d\phi \phi \) is the gyroangle average. To first order in \( \epsilon \), the Poisson bracket written in terms of gyrocenter coordinates [14] of two arbitrary functions \( F \) and \( G \) is given as

\[
\{F, G\} = \frac{q}{m} \left( \nabla F \frac{\partial G}{\partial \mu} - \nabla G \frac{\partial F}{\partial \mu} \right) + \frac{\hat{b}}{qB} \cdot \nabla F \times \nabla G.
\]

With the gyrokinetic Poisson bracket (7) and the gyrocenter coordinate transformation (6) at hand, the gyroaveraged kinetic equation (1), written in terms of gyrocenter coordinates, becomes

\[
\frac{d}{dt} \langle F_a \rangle + \frac{d}{dt} \langle \hat{Z}_i F_a \rangle = \sum_{a,b} \langle \mathcal{C}[F_a; (F_b)] \rangle.
\]

where \( \hat{Z}_i \) is the gyrokinetic distribution function. This conventional wisdom is most probably e.g., Refs. [16–20], that the equilibrium part vanishes in like-order in \( \epsilon \), when evaluated using two Maxwellians \( \delta F_a, \delta F_b \) is assumed small compared to the equilibrium distribution function \( \delta F_a/F_{Ma} \) and \( F_{Ma} = N(2\pi T/m_{\perp})^{-3/2} \exp(-m_{\parallel}^2/2T_{\parallel}) \), where \( N \) and \( T \) are the equilibrium gyrocenter density and temperature, respectively. The amplitude of \( \delta F_a \) is assumed small compared to the equilibrium distribution function \( \delta F_a/F_{Ma} \sim \epsilon_3 \). In the following calculations, no distinction will be made between the gyrokinetic expansion parameter and the degree of magnetization \( \epsilon \sim \epsilon_3 \ll 1 \).

The \( \delta F_a \) gyrokinetic equation is formally of order \( \epsilon_2 \). In the calculations of the gyroaveraged collision operator, terms up to second order must therefore be retained. The test-particle \( \delta [F_{Ma}, \delta F_a] \) and the field-particle \( \delta F_{Ma} \) operators are of order \( \epsilon^2 \), whereas the equilibrium collision operator \( \delta F_{Ma} \delta F_a \) is formally of order \( \epsilon \), and hence first- and second-order contributions must be retained.

When the gyrokinetic linearized Landau collision operator enters the widely applied \( \delta F_a \) gyrokinetic equation (1), which governs the time evolution of the gyroangle-independent perturbed distribution function \( \delta F_a \). The equilibrium distribution function is taken as the lowest-order stationary solution to the zeroth-order gyrokinetic Landau collision operator. This conventional wisdom is most probably e.g., Refs. [16–20], that the equilibrium part vanishes in like-order in \( \epsilon \), when evaluated using two Maxwellians \( \delta F_a, \delta F_b \) and \( \delta F_{Ma} = N(2\pi T/m_{\perp})^{-3/2} \exp(-m_{\parallel}^2/2T_{\parallel}) \) with the same temperature \( T_{\parallel} \), \( n_0 \) is the equilibrium particle density. However, the Landau collision operator does not vanish [3] when operating on two gyrokinetic Maxwellians \( F_{Ma} \). Using the gyrocenter coordinate
transformation, given in Eq. (6), the gyrokinetic Maxwellian can be written as $F_M(Z) = f_M(z)[1 - q(\phi(z))/T] + O(\epsilon^2)$, showing that $F_M$ and $f_M$ are not identical, and hence $C[F_M, F_M]$ contains a second-order contribution which must be retained.

We note that $F_{Ma}(Z)(1 - q(\phi)(Z)/T) = f_{Ma}(z)[1 - q\phi(x)/T] + O(\epsilon^2)$, and hence the like-particle gyrokinetic equilibrium collision operator can be written as

$$C[F_{Ma}, F_{Ma}] = C \left[ F_{Ma} \frac{q(\phi)}{T} , F_{Ma} \right] + C \left[ F_{Ma}, F_{Ma} \frac{q(\phi)}{T} \right],$$

which can be evaluated using the gyrokinetic test-particle operator Eq. (15) and the field-particle operator Eq. (20) operator given later, since the arguments $F_{Ma} \frac{q(\phi)}{T}$ are gyroangle independent.

For unlike-particle collisions we must explicitly calculate the gyroaverage of the gyrokinetic equilibrium collision operator $C[F_{Ma}, F_{Mb}]$ retaining all second-order terms. The second-order contributions originate from the velocity derivatives and the Rosenbluth potentials when expressed in gyrocenter coordinates. The velocity derivatives $\frac{\partial}{\partial q} = m(x)$ give rise to second-order contributions because the particle position $x = X + \rho + \epsilon \rho_1$ includes the first-order gyrocenter displacement $\rho_1 = -\{S_1, X + \rho\}$. When expressed in gyrocenter coordinates the relative velocity in the Rosenbluth potentials Eq. (4) becomes

$$u = [1 - \{S_1, Z_i\} \rho_{Z_i} - \{S_1', Z_i'\} \rho_{Z_i'}] u_0,$$

where $S_1'$ and $\{\cdot, \cdot\}$ denote the first-order generating function and the Poisson bracket expressed in primed coordinates, respectively; $v_0 = \tilde{b} v_i + v_{\perp}(Z)$ denotes the zeroth-order velocity and $u_0 = [v_0 - v_0]$. Inserting the transformed relative velocity (10) into the Rosenbluth potentials, we get

$$\left( H_{0a} F_{Mb} \right) = \left( H_{0a} \right) - \frac{q_{ad} \tilde{b}}{m_a v_0} \left( H_{0a} \right) - \left( h^{\phi}_{a} \right);$$

where $H_{0a}$ and $G_{0a}$ are the Rosenbluth potentials evaluated with the zeroth-order relative velocity $[v_0 - v_0]$, $G_{0b} = dG_{0b}/dv$, and

$$\left( h^{\phi}_{a} \right) = -\frac{\Gamma_{ab}}{8\pi \int d\theta' Z^M \delta(x' - x)} \times \left\{ S_1' \left[ \frac{2|v_0 - v_0|^{-1}}{|v_0 - v_0|} \right] \right\},$$

are the field-particle-like parts that originate from the coordinate transformation of $\phi$ entering the relative velocity (10). The two first terms in $H_{0a} [F_{Ma}]$ and in $G_{0a} [F_{Ma}]$, Eq. (11), are time independent because they are evaluated with $F_{Ma}$.

Therefore, the calculations of these terms resemble those of the test-particle operator $C[\delta F_a, F_{Ma}]$. It is therefore possible to pull the time dependence outside of the primed coordinate integrals. We note that the zeroth-order

$$\langle C^T_M \rangle = \int d^3k e^{ik\cdot x} \left\{ \left[ 1 - \frac{m_a}{m_b} \right] v_i F_{Ma} \frac{m_a v_0^2}{T_a} + \frac{q_{ab} \tilde{b} \rho_{10} k^2 \phi_0}{2T_a} \left[ J_0(k_{\perp} \rho_0) + J_2(k_{\perp} \rho_0) \right] \right\} + \frac{\Gamma_{ab}}{8\pi \int d\theta' Z^M \delta(x' - x)} \times \left\{ S_1' \left[ \frac{2|v_0 - v_0|^{-1}}{|v_0 - v_0|} \right] \right\},$$

where $v_0 = v_0^2 + 2\mu_B/m_a$, and $v_s = \frac{(m_a v_0)^{-1}}{dH_{0b}/dv_0}$, $v_D = -2(m_a^2 v_0^3)^{-1} dG_{0b}/dv_0$, and $v_{||} = -2(m_a^2 v_0^3)^{-1} d^2G_{0b}/dv_0^2$ denote the standard slowing-down, deflection, and parallel velocity diffusion frequencies [21], respectively. For completeness we also give the gyroaverage of the test-particle collision operator [3]

$$\langle C[\delta F_a, F_{Mb}] \rangle = v_i \left[ \frac{m_a}{m_b} - 1 \right] \left\{ 2\mu_B \frac{\partial}{\partial \mu} + v_i \frac{\partial}{\partial v_i} \right\} \delta F_a + \frac{\Gamma_{ab}}{8\pi \int d\theta' Z^M \delta(x' - x)} \times \left\{ S_1' \left[ \frac{2|v_0 - v_0|^{-1}}{|v_0 - v_0|} \right] \right\},$$

where $v_0 = v_0^2 + 2\mu_B/m_a$, and $v_s = \frac{(m_a v_0)^{-1}}{dH_{0b}/dv_0}$, $v_D = -2(m_a^2 v_0^3)^{-1} dG_{0b}/dv_0$, and $v_{||} = -2(m_a^2 v_0^3)^{-1} d^2G_{0b}/dv_0^2$ denote the standard slowing-down, deflection, and parallel velocity diffusion frequencies [21], respectively. For completeness we also give the gyroaverage of the test-particle-like contribution $\langle C_{fg} \rangle$ of the equilibrium operator. At first sight, these terms have the intractable property of having a time-dependent function $S_1'(Z, t)$ inside the primed coordinate integral. However, the time dependence of $S_1'$ is due to the electric potential $\phi$, which is a function of particle position $x$ only. It is therefore possible to pull the time dependence outside of the primed coordinate integrals. We note that the zeroth-order

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relative velocity $u_0$ is periodic in the relative gyroangle $\theta' - \theta$. Therefore, the relative velocity $u_0$ can be decomposed [7] into a Fourier series $u_0 = \sum_{n=-\infty}^{\infty} u_{0n} e^{i(n\theta'-\theta)}$. Using the Fourier series decomposition and the Bessel integral representation $J_n(x) = \frac{1}{2\pi} \int_0^{2\pi} d\theta \ e^{i n \theta} \cos(n \theta)$, the $\theta'$ integrals in the field-particle-like Rosenbluth potentials $g^\phi$ and $h^\phi$, Eq. (12), can be evaluated,

$$
\left(g_{nk}'\right) = \sum_{n=-\infty}^{\infty} \int d^3 k \ e^{i k \cdot x - i n \theta} i q_b \phi_b \left(h_{nk}' \ g_{nk}'\right),
$$

where

$$
\left(h_{nk}' \ g_{nk}'\right) = -\frac{\Gamma_{ab}}{8\pi} \int d\mu' d\nu' 2\pi F_{M_H} \left[\delta\rho_0 - J_0(k_{\perp} \rho_0)J_n(-k_{\perp} \rho_0) \frac{\partial}{\partial \mu'} - \sum_{l=1}^{n} \frac{n}{2l} k_{\perp} \frac{\partial \rho_0}{\partial \mu'} \left(J_{n+1}(-k_{\perp} \rho_0) - J_{n-1}(-k_{\perp} \rho_0)\right) \right] \left(2u_n^{-1} \ u_n\right).
$$

and $\delta_{nm}$ denotes the Kronecker delta. Note that the integrals in Eq. (17) are time independent, and can be precomputed and reused for all time steps in numerical simulations.

The $\theta'$ integrals in the field-particle operator Rosenbluth potentials are treated in a similar way:

$$
\left(H_b[\delta F_b]\right) = \sum_{n=-\infty}^{\infty} \int d^3 k \ e^{i k \cdot x - i n \theta} \left(h_{nk} \ g_{nk}'\right),
$$

where

$$
\left(h_{nk} \ g_{nk}'\right) = -\frac{\Gamma_{ab}}{8\pi} \int d\mu' d\nu' 2\pi F_{a} \left[\delta\rho_0 - J_0(k_{\perp} \rho_0)J_n(-k_{\perp} \rho_0) \frac{\partial}{\partial \mu'} - \sum_{l=1}^{n} \frac{n}{2l} k_{\perp} \frac{\partial \rho_0}{\partial \mu'} \left(J_{n+1}(-k_{\perp} \rho_0) - J_{n-1}(-k_{\perp} \rho_0)\right) \right] \left(2u_n^{-1} \ u_n\right).
$$

and $\delta F_b$ is the Fourier transform of $\delta F_b$. The integrals in the field-particle operator are time dependent. But since the gyroangle integral has been calculated, finite Larmor radius effects can be precomputed numerically, hence reducing the dimensionality of the integral from three to two.

The gyroaveraged field-particle operator $\langle C[F_{Ma}, \delta F_b]\rangle$ and the gyroaveraged field-particle-like part of the equilibrium collision operator $\langle C_M \rangle$ are combined and are given as

$$
\langle C_M \rangle + \langle C[F_{Ma}, \delta F_b]\rangle = \int d^3 k \ e^{i k \cdot x} \left\{ \frac{m_a}{m_b} F_{Ma} \langle \delta F_b \rangle \right\} \left(1 - \frac{q_b m_b}{q_a m_a}\right) + \sum_{n=-\infty}^{\infty} F_{Ma} J_{-n}(k_{\perp} \rho_0) \left[\left(1 - \frac{m_a}{m_b}\right) \frac{2\mu}{\partial \mu} + \frac{\partial v_{\perp}}{\partial \mu} \right] \left(\frac{n^2 T_a}{2 \mu B} + \frac{2 B \mu B - T_a}{2 \mu} \frac{\partial}{\partial \mu} \right) + \frac{\mu B(2 \mu B - T_a)}{B^2} \frac{\partial^2}{\partial \mu^2} + \frac{2 \mu B v_{\perp} T_a}{B} \frac{\partial^2}{\partial \mu v_{\perp}} + \frac{m_a v_{\perp} T_a - T_a}{2 m_a} \frac{\partial}{\partial v_{\perp}} \right\} \left(2 \langle g_{nk}' \right) - q_b \phi_b h_{nk} \left(\frac{n^2 T_a}{2 \mu B} + \frac{2 B \mu B - T_a}{2 \mu} \frac{\partial}{\partial \mu} \right) \left(2 \langle g_{nk}' \right) - q_b \phi_b g_{nk} \left(\frac{n^2 T_a}{2 \mu B} + \frac{2 B \mu B - T_a}{2 \mu} \frac{\partial}{\partial \mu} \right)
$$

In summary, the gyroaveraged gyrokinetic linearized Landau collision operator valid for arbitrary temperatures and densities, including the previously neglected equilibrium operator $\langle C[F_{Ma}, F_{Mb}]\rangle$, becomes

$$
\langle C[F_a, F_b]\rangle = \langle C[\delta F_a, F_{Mb}]\rangle + \langle C[F_{Ma}, \delta F_b]\rangle + \langle C_M \rangle + \langle C_F \rangle,
$$

where the gyroaveraged test-particle operator $\langle C[\delta F_{ab}, F_{Mb}]\rangle$ and the test-particle-like part $\langle C_M \rangle$ of $\langle C[F_{Ma}, F_{Mb}]\rangle$ are given in Eqs. (15) and (14), respectively. The field-particle operator $\langle C[F_{Ma}, \delta F_b]\rangle$ and the field-particle-like part $\langle C_F \rangle$ of the equilibrium operator are defined in Eq. (20). Finite Larmor radius effects are accounted for in all parts, and can be precomputed and reused in subsequent time steps when implemented in numerical simulations.

The first-order terms in $\langle C_F \rangle$ are responsible for collisional energy exchange [10]. The energy exchange rate between species $a$ and $b$ is proportional [11] to $(T_b - T_a) v_{ab} m_a / (m_a + m_b)$, and is therefore mainly important in multiple ion-species plasmas. The magnitude of the electron-tritium energy exchange rate is smaller than the deuteron-tritium rate by a factor $\sqrt{m_e / m_T}$. According to the gyrokinetic ordering the second-order terms in $\langle C_M \rangle$ and $\langle C_F \rangle$, all of which depend on the electric potential $\phi$, are of the same order as $\langle C[\delta F_a, F_{Mb}]\rangle$ and $\langle C[F_{Ma}, \delta F_b]\rangle$. The second-order terms in $\langle C_M \rangle$ and $\langle C_F \rangle$ can be described as “polarization-density” test- and field-particles colliding with Maxwellian particles. The “polarization-density” test- and field-particle collisions appear in the gyrokinetic collision operator because collisions take place between particles and not gyrocenters. The polarization-density particles represent deviations described by $\rho_1$ of particle positions from the unperturbed particle position due to electric fluctuations. This interpretation is analogous to the polarization density [1] entering the gyrokinetic Gauss’s law, which describes how the charge density given a gyrocenter density function is altered due to electric fluctuations. To
lowest order in the long wavelength limit (LWL) $k_\perp^3 \rho_0^3 \sim \epsilon$ [which corresponds to $n = 0$ and $l = 1$ in Eq. (17)], these terms are proportional to the magnetic field aligned component of the $E \times B$ vorticity $\hat{b} \cdot \nabla \times \mathbf{u}_E$, and hence describe the effect of collisions on the particle motion associated with the electric field. The test- and field-particle operators describe drag and diffusion in gyrocenter velocity ($v_\parallel, \mu$) space, and gyrocenter spatial diffusion perpendicular to the magnetic field. But the test- and field-particle operators also describe drag and diffusion of $E \times B$ and diamagnetic vorticity, which can be illustrated by taking moments of the gyrokinetic distribution function $F$. The zeroth gyrofluid [22] moment $N_0 = \int d\mu d\mathbf{v}_\parallel d\theta B \delta \mathcal{F}_a \delta(X - r)$ in a quasineutral simple plasma can, in the LWL, be expressed in terms of standard particle fluid moments as $N_i \simeq n - \nabla_\perp^2 \left( \frac{\rho_i n}{2} \right) - \frac{\rho_i}{\Omega_i} \nabla_\perp^2 \phi_B$, e.g., spatial diffusion of $\delta F_a$ in $\langle \mathcal{C}[\delta F_a, F_{ML}] \rangle$, among other effects, describes viscous damping of $E \times B$ vorticity. A correct description of collisional effects on, e.g., momentum and energy associated with $E \times B$ drift motion, therefore requires the full gyrokinetic collision operator, including the equilibrium part which was previously neglected. The equilibrium operator must also be retained for like-particle collisions. However, for like-particle collisions the equilibrium operator (9) can be evaluated using the test-particle and field-particle operators.

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