Comparisons of Means for Estimating Sea States from an Advancing Large Container Ship

Nielsen, Ulrik Dam; Andersen, Ingrid Marie Vincent; Koning, Jos

Published in:
Proceedings of the PRADS 2013

Publication date:
2013

Citation (APA):
Comparisons of Means for Estimating Sea States from an Advancing Large Container Ship

Ulrik D. Nielsen\(^{1)}\), Ingrid Marie V. Andersen\(^{1)}\) and Jos Koning\(^{2)}\)

\(^{1)}\) Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark
\(^{2)}\) MARIN, 6700 AA Wageningen, The Netherlands

Abstract

The paper deals with sea state estimation from a container carrier (9,400 TEU) en-route. Knowledge of the on-site sea state is fundamental input to any kind of in-service decision support system that evaluates performance of, e.g., accelerations, fuel efficiency, and hull girder strength, related to ship-wave interactions in a seaway. In the paper, sea state estimates are produced by three means: the wave buoy analogy, relying on shipboard response measurements, a wave radar system, and a system providing the instantaneous wave height. The presented results show that for the given data, recorded on five different days of continuous operation, the agreement between the estimating means is reasonable; in terms of both absolute (mean) values and hourly trends of integrated sea state parameters.

Keywords

Sea state estimation; decision support system; wave buoy analogy; wave radar system; instantaneous wave height.

Introduction

Knowledge of the sea state in which a ship is operating is of outmost importance as input parameter for most on-board decision support systems as well as for proper ship performance monitoring with regards to fuel efficiency. The sea state data, i.e. the wave period, the significant wave height and the mean relative wave direction, can be estimated by several means. This includes wave radar systems, analysis of ship response measurements or by forecast and hindcast of meteorological data. For decision support and ship performance systems, accuracy as well as reliability of the sea state estimate is important and hence the method must be robust. Moreover, information about the sea state should be available at actual ship position and at actual time of operation in the seaway. The latter requirement – on-site and on-time availability – of the sea state estimate excludes use of hindcast data when decision support and performance systems are considered.

This paper presents results related to sea state estimates obtained from an in-service (large) container vessel, where the considered means for sea state estimation are based on three different procedures: 1) the wave buoy analogy, 2) a wave radar system, and 3) direct measurement of the instantaneous wave height at ship bow. The first and second authors have worked extensively on the wave buoy analogy during recent years. Fundamentals of the analogy is given in a subsequent section but, directly reflected by the name, the estimation principle follows that of a traditional wave rider buoy, where motion measurements are processed to give the sea state estimate. The other two means for sea state estimation, 2) and 3), are based on commercially available systems, which are, respectively, the WaMoS\(^{\textregistered}\) II wave radar system and The WaveGuide system by Dutch company Radac.

It should be noted that this paper does not contain verification and/or parameter studies related to the wave buoy analogy, in terms of, e.g., numerically generated data. The focus herein is on analysis of full-scale data from a 9,400 TEU container vessel, where the presentation of results from the three estimating means is the central point. In this connection, it is noteworthy that all data have been collected as part of the EU FP7 project TULCS (Tools for Ultra Large Container Ships, project no. 234146).

Sea State Estimation - The Wave Buoy Analogy

Although focus is primarily on results, the following section briefly outlines key points of the wave buoy analogy. Fundamentals of the approach are given but the section is by no means comprehensive and for details the literature should be consulted, e.g. Iseki and Terada (2002), Tannuri et al. (2003), Nielsen (2006, 2008a) and Pascoal et al. (2007, 2008).

If a set of ship responses are assumed stationary and linear with the incident waves, the complex-valued transfer functions, \(\Phi_k(\omega_k, \theta)\) and \(\Phi_j(\omega_j, \theta)\) for the \(k\)th and \(j\)th responses, yield the theoretical relationship between the \(k\)th and \(j\)th components of the cross spectra \(S_{jk}(\omega_k)\) and the directional wave spectrum \(E(\omega_k, \theta)\) through the following integral equation
\[ S_\phi(\omega) = \int_{-\infty}^{\infty} \Phi_x(\omega, \phi) \overline{\Phi_y(\omega, \phi)} E(\omega, \phi) d\phi \]  

(1)

where the bar denotes the complex conjugate, and with \( \phi \) being the heading of the ship (relative to the waves) and \( \omega \) being the encounter frequency. It should be noted that the complex-valued transfer functions are written as functions of only the heading and the encounter frequency, since the implication of changing operational parameters is understood.

The wave spectrum is advantageously estimated in the wave frequency domain. This means that the speed-of-advance or triple-valued function problem in following sea needs to be considered. This problem, governed by the Doppler Shift, has been properly incorporated by Iseki and Ohtsu (2000), for details see Nielsen (2006).

In terms of matrix notation, Eq. (1) can be written

\[ \mathbf{b} = \mathbf{A} \mathbf{x} \]  

(2)

The vector function \( \mathbf{f}(\mathbf{x}) \) expresses the unknown values of the wave spectrum \( E(\omega, \phi) \), i.e. the spectral components, while the vector \( \mathbf{b} \) contains the elements of \( S_\phi(\omega) \), and the coefficient matrix \( \mathbf{A} \) has elements according to the products of the transfer functions, cf. Eq. (1). In principle, Eq. (2) can be solved for \( \mathbf{x} \) by minimising \( \chi^2(\mathbf{x}) \) with

\[ \chi^2(\mathbf{x}) = \| \mathbf{A} \mathbf{x} - \mathbf{b} \|^2 \]  

(3)

where \( \| \| \) represents the L2 norm. Typically, Eq. (3) is in this context dealt with by parametric or non-parametric – so-called Bayesian – modelling.

**Parametric Modelling**

In the parametric approach, Eq. (3) is solved directly as an optimisation problem, where the directional wave spectrum \( \mathbf{f}(\mathbf{x}) \equiv E(\omega, \phi) \) is introduced as a parameterised spectrum. Herein, \( E(\omega, \phi) \) is taken as a 15-parameter spectrum (Nielsen and Stredulinsky, 2012) that allows for mixed sea conditions, existing in the presence of, e.g., wind waves and swell:

\[ E(\omega, \phi) = \sum_{i=1}^{M} E_i(\omega) \cdot G_i(\omega, \phi) \]

\[ E_i(\omega) = \frac{1}{4} \left( \frac{4 \lambda_i + 1}{\omega_0 \lambda_i} \right)^{\frac{1}{2}} \frac{H_i^2}{\Gamma(\lambda_i) \omega_{1/2}^{\lambda_i}} \]

\[ \times \exp \left[ - \frac{4 \lambda_i + 1}{4} \frac{\left( \frac{\omega}{\omega_0} \right)^{\lambda_i}}{\lambda_i} \right] \]

\[ G_i(\omega, \phi) = A(s) \cos^{\gamma_i} \left( \theta - \theta_{\text{mean},i} \right) \]

\[ A(s) = \frac{s^{2\gamma_i - 1} \Gamma(\gamma_i + 1)}{\Gamma(2\gamma_i + 1)} \]

(4)

In Eq. (4), \( E_i(\omega) \) is a one-dimensional wave spectrum with \( \omega_0, H_i, \) and \( \lambda_i \) being the peak frequency, the significant wave height and the shape parameter, respectively, of the spectrum. \( G_i(\omega, \phi) \) is the directional distribution function, where \( \theta_{\text{mean},i} \) is the mean relative wave direction. \( A(s) \) is a constant to secure normalisation and it is evaluated using the Gamma function on the spreading parameter \( s \). Insertion of Eq. (4) into Eq. (3) implies a nonlinear optimisation problem that can be solved using, e.g., MATLAB® by invoking “fmincon”, which is a built-in function based on sequential quadratic programming.

**Bayesian Modelling**

If the equation system given by Eq. (3) is solved for the actual spectral components of the directional wave spectrum, it implies a highly underdetermined, or otherwise degenerate, equation system. However, through Bayesian modelling (Akaike, 1980), a stable solution is facilitated by the introduction of prior information. Thus, two main assumptions are introduced: 1) the directional wave spectrum is smoothly changing with both frequency and direction, and, 2) the wave spectrum is expected to have negligible values for very low and high frequencies. Details of the Bayesian approach can be found in the literature, e.g. Nielsen (2006, 2008a), but it is worth to mention that, as discussed by Iseki and Ohtsu (2000), the Bayesian approach introduces a stochastic viewpoint where the difference between the left- and the right-hand side of Eq. (2) is taken as a white noise sequence vector \( \mathbf{w} \) with zero mean and variance \( \sigma^2 \). Secondly, to avoid negative spectral estimates, a non-negativity constraint is applied to the wave spectrum by use of a coordinate transformation \( E(\omega, \phi) = \exp(\mathbf{x}) \), e.g. Iseki and Ohtsu (2000).

In the Bayesian approach, the solution \( \mathbf{x} \) is obtained by maximisation of the product of the likelihood function and the prior distributions. The likelihood function is written as

\[ l(\mathbf{x} | \sigma^2) = \left( \frac{1}{2\pi\sigma^2} \right)^{P/2} \exp \left( -\frac{1}{2\sigma^2} \| \mathbf{A} \mathbf{x} - \mathbf{b} \|^2 \right) \]  

(5)

where \( P \) is the total number of integral equations derived from Eq. (1).

Two prior distributions are considered, and both distributions seek to minimise the sum of the second order difference of the unknown vector \( \mathbf{x} \) in order to smoothen the changes with frequency and direction, respectively, of the wave spectrum (Nielsen, 2006; 2008a and Iseki and Terada, 2002). The prior distributions are therefore defined by the minimisation of the functionals \( \varepsilon_{1\text{mix}} \) and \( \varepsilon_{2\text{mix}} \) (Press et al., 1992)

\[ \sum_{n=1}^{M} \sum_{n=1}^{M} \varepsilon_{1\text{mix}}^2 = \mathbf{x}^T \mathbf{H}_1 \mathbf{x} \]

(6)

\[ \sum_{n=1}^{M} \sum_{n=1}^{M} \varepsilon_{2\text{mix}}^2 = \mathbf{x}^T \mathbf{H}_2 \mathbf{x} \]

(7)

where \( N \) and \( M \) are the number of discrete wave frequencies and discrete headings. If the functionals \( \varepsilon_{1\text{mix}} \) and \( \varepsilon_{2\text{mix}} \) are assumed to be normal distributions with zero mean and variance \( \sigma_{1\text{mix}}^2 \) and \( \sigma_{2\text{mix}}^2 \), respectively, the prior distribution is weighted in terms of so-called hyperparameters \( \mu \) and \( \nu \). Physically, the hyperparameters control the trade-off between the good-fit of the solution to the data and the smoothness, or stability, of
the solution. 

The posterior distribution $p(x|u,v,\sigma^2)$ is proportional to the product of the likelihood function and the prior distribution (Akaike, 1980) and can be written, cf. Nielsen (2008a),

$$p(x|u,v,\sigma^2) = \frac{1}{2\pi\sigma^2} \exp \left( -\frac{1}{2\sigma^2} S(x) \right) \det(u^2H_1 + v^2H_2)^{1/2} \exp \left( -\frac{1}{2\sigma^2} S(x) \right)$$

with

$$S(x) = \|Af(x) - b\|^2 + x^T\left[ u^2H_1 + v^2H_2 \right] x$$

where $c$ in Eq. (8) is a normalising factor, independent of $x$ and the hyperparameters $u$ and $v$.

The optimum values of the hyperparameters are determined by minimising the control criterion $ABIC$, cf. Akaike (1980),

$$ABIC = -2 \ln \int p(x|u,v,\sigma^2)dx$$

With knowledge of the values of the optimum hyperparameters, the best estimate of $x = x^*$ is schematically obtained from (Press et al., 1992),

$$x^* = \left( A^T A + u^2H_1 + v^2H_2 \right)^{-1} A^T b$$

Hence, the spectral components of the wave spectrum are determined. In practice, the solution is achieved through an iterative process and, although many details on algebra and numeric are left out, this finalises the Bayesian approach.

Response Combinations

Similar to the data processing of classical wave rider buoys, the wave buoy analogy considers a set of three responses simultaneously, since this has shown to give the most accurate and, at the same time, computationally efficient sea state estimate (Nielsen, 2006). Today, most ships are installed with monitoring systems and, typically, numerous sensors provide information about a (large) number of responses. This means that a set of three responses should be selected to achieve the best sea state estimate, and the optimum choice of responses is likely to depend on given operational conditions at the time of estimation. Initial studies towards a dynamic and automatic response selection have been carried out by Andersen and Storhaug (2012). In the referred paper, a conceptual idea is outlined but it is also stated that the idea is not yet ready for practical use. Consequently, the selection of responses is based on a manual choice that should reflect both the filtering issue (a ship acts inherently as a wave filter) and the need for at least one of the considered responses to be asymmetric with respect to wave heading. Further discussions are found in the literature (e.g., Nielsen, 2006, 2008b; Pascoal et al., 2007, 2008; Tannuri et al., 2003), but future work on the wave buoy analogy should focus on a (more) automatic selection process of the optimum response combination.

Vessel and Full-scale Measurements

Main dimensions of the considered vessel are given in Table 1 and a photo of the container vessel is seen in Figure 1. The location of the Radac system can be seen from the photo and the WaMos wave radar was installed on top of the compass deck (portside). Additionally, several sensors for motion and acceleration measurements were mounted at specific (but different) locations. In this paper, consideration is given to sway, heave, roll, and pitch measurements that are given as input to the wave buoy analogy. It is worth to note that stress measurements were also carried out as part of the measurement campaign. Thus, related studies are presented by Andersen et al. (2013a, b), although the focus therein is not on sea state estimation.

Table 1: Main dimensions of ship.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOA</td>
<td>349.0 m</td>
</tr>
<tr>
<td>Beam</td>
<td>42.8 m</td>
</tr>
<tr>
<td>Draught</td>
<td>15.0 m</td>
</tr>
<tr>
<td>DWT</td>
<td>113,000 ton</td>
</tr>
</tbody>
</table>

Fig. 1: Considered vessel and location of instantaneous wave height meters (Radac system).

It should be noted that the analyses of sea state estimates have been made as a post-voyage process, although both the WaMos radar and the Radac systems were part of an integrated system running real-time on the considered vessel. A sketch of the integrated system is seen in Figure 2, where it is noted that overall system design and control was done by MARIN while the wave sensors were installed by SIREHNA.
Based on the four responses considered for the wave buoy analogy, two combinations, or sets, consisting of three responses are formed:

1. \{sway, heave, roll\}
2. \{sway, heave, pitch\}

Although studies have been initiated towards an automatic selection of the optimum set of responses (Andersen and Storhaug, 2012), the two sets, a) and b), have been selected by ‘brute-force manner’ based on experience and previous findings by the first author in his studies of the wave buoy analogy. In particular, the following results for the wave buoy analogy are, conceptually, based on findings by Nielsen and Studulinsky (2012). Thus, in the referred paper, reliable and accurate sea state estimations were made for full-scale data recorded during sea trials (Studulinsky, 2010). In the analysis of data, Nielsen and Studulinsky (2012) found that the best sea state estimates by the wave buoy analogy were obtained when results were based on average values considering different response combinations. This approach is also considered in the present paper. For this reason, the results of the wave buoy analogy – considering both parametric and Bayesian modelling – are based on average values obtained from the combination of sets a) and b). Further, similar studies in this respect are in progress (Nielsen, 2013a; 2013b).

The analysis of data herein is carried out for five days of operation, and for each day 24 hours of continuous data are available. The dates, including approximate geographical positions of the vessel and visual sea state observations, are as follows: 12th August 2011: Gulf of Aden (going West, moderate sea state); 20th August 2011: Mediterranean Sea (going West, mild sea state); 16th September 2011: Gulf of Aden (going East, mild to moderate sea state); 20th September 2011: South of India (going East, mild sea state); and 2nd October 2011: Off Hong Kong (going North West, severe sea state). Table 2 summarises the operational data of the vessel on the considered dates, including the visual observation of the sea state.

**Table 2: Operational parameters on the specific dates.**

<table>
<thead>
<tr>
<th>Dates</th>
<th>Draft [m]</th>
<th>Speed [knots]</th>
<th>Sea state (visual obs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12th Aug.</td>
<td>14.2</td>
<td>21.0-23.5</td>
<td>Moderate</td>
</tr>
<tr>
<td>20th Aug.</td>
<td>14.2</td>
<td>24.0-25.0</td>
<td>Mild</td>
</tr>
<tr>
<td>16th Sep.</td>
<td>14.0</td>
<td>17.0-18.0</td>
<td>Mild/moderate</td>
</tr>
<tr>
<td>20th Sep.</td>
<td>14.0</td>
<td>11.5-13.5</td>
<td>Mild</td>
</tr>
<tr>
<td>2nd Oct.</td>
<td>15.0</td>
<td>9.5-14.0</td>
<td>Severe</td>
</tr>
</tbody>
</table>

**Results and Discussions**

Comparisons of the different means for sea state estimations are made for four (integrated) sea state parameters: Significant wave height $H_s$, zero-upcrossing period $T_z$, peak period $T_p$, and relative mean wave heading $\chi$. The relative heading is defined so that $\chi = 180$ deg. is head sea and $\chi = 0$ deg. is following sea; values in between are either positive, indicating waves approaching on starboard, or negative, corresponding to waves approaching on portside.

Results of the sea state estimations are presented in Figures 3, 4, 5, 6, and 7. It is noted that the five figures are included in separate subsections, representing the outcome on the individual dates listed in Table 2. Each figure is composed of four plots corresponding to the four sea state parameters ($H_s$, $T_z$, $T_p$, $\chi$) considered. In the plots, the x-axis spans the 24-hours period on the given day, and sets of the sea state parameters have been estimated for each hour. The input to the wave buoy analogy, in terms of time series data, is taken as the middle 20-minutes period within each hour, and the same period is considered for extracting (mean value) estimates of the WaMos and the Radac systems. It is noteworthy that the Radac system, i.e. the instantaneous wave height meter, does not facilitate information about (relative) wave heading. Moreover, the instantaneous wave height is measured in the ‘encounter domain’. Thus, in order to compare the characteristic wave periods ($T_z$ and $T_p$) estimated by Radac with the other estimating procedures, it is necessary to transform into the ‘true domain’. This transformation is not possible without the combined knowledge of the speed and the wave heading of the vessel. In general, information about the wave heading is not available if the sea state, including all relevant parameters, has not been estimated by some other means. In this study, the wave heading is known as a result of WaMos and/or the wave buoy analogy. Herein, the heading of WaMos is used to transform the encountered characteristic periods of the Radac system into the true domain.

Another general remark is related to the filtering effect of a ship considering associated wave induced responses. Inherently, this phenomenon can have an influence on the results by the wave buoy analogy, since the phenomenon basically means that wave energy may not be properly recognised at relatively short wave periods (Nielsen, 2008b). Similarly, the estimates of wave period(s) by WaMos can be compromised due to limitations in antenna revolution speed and image resolution. In theory, the Radac system is the only system capable to recognise the entire range of wave periods but, on the other hand, sprays from wave impacts and local wave-hull disturbances may affect estimates negatively. Obviously, the relative (or absolute) wave measure as obtained by the Radac system can itself be used as input to the wave buoy analogy; see, e.g. Nielsen (2008b).

In the following subsections, the main findings and comments related to the sea state estimates for the specific dates are addressed in bullet lists immediately following the corresponding figures.
Main findings and comments:

- In agreement with visual observations, all estimating means find significant wave heights, $H_s$, representing a moderate sea state, reducing to mild at the end of the day.
- Both approaches of the wave buoy analogy estimate less energy, reflected by $H_s$, than the other two means for most of the considered period. Specifically, little difference is seen between the parametric and the Bayesian approach.
- In general, estimates of the zero-upcrossing period, $T_z$, agree well. The picture is not completely the same with the peak period, $T_p$, and, somewhat peculiarly, the Radac system is consistently on the low side with $T_z$ but on the high side with $T_p$.
- The agreement between estimates of the relative mean wave heading, $\chi$, is good; the three estimating means indicating bow-quartering waves approaching on portside.
Main findings and comments:

- The significant wave height increases during the day, and the trend is reflected by all estimating means. The actual values of $H_s$ represent a mild to moderate sea state which is in agreement with the visual observations, and generally the estimates of the four means deviate only little from each other.

- Relatively small variations in the estimated wave periods ($T_z$ and $T_p$) are observed between WaMos and both approaches of the wave buoy analogy. Results of the Radac system agree, on the other hand, poorly and show rather unrealistic values in some cases.

- The relative mean wave heading is ranging between bow-quartering (approaching on starboard) and head sea, which is found for all three means, although the Bayesian method has two off-values.

Main findings and comments:

- As seen from the plot of $H_s$, estimates of the energy contained in the sea state are rather low. The observation applies to all four means and is in line with visual observations. The result of the Radac system is, however, slightly off compared to the three other estimating means, being consistently to the higher side.

- A fair agreement is seen between the estimated wave period(s), although the Radac results are standing out the most, similarly to estimations at the other dates.

- Small variations are observed between the parametric and the Bayesian approach when the relative mean wave heading is considered but both methods estimate waves approaching on starboard as bow-quartering to head sea. Estimates by WaMos indicate, on the other hand, a beam sea during the entire day.
Main findings and comments:

- All the four estimating means show the same trend: The sea state increases during the first half of the day and reaches a relatively severe condition from around noon. Some variations are seen in the values of $H_s$ with results of the wave buoy analogy being a lower bound (6-10 m) whereas Radac results are an upper bound (10-13 m). Note the difference in scale on the y-axis compared to the plots for the other dates.

- Considering the zero-upcrossing period, the WaMos estimates yield an upper bound whereas the Radac estimates yield a lower bound; an observation almost applicable to the whole day. The same observation cannot be made with respect to the peak period, where the Radac system finds the highest values consistently.

- Reasonable agreement is seen for the relative mean wave heading (with a few exceptions) considering estimates by the parametric approach and by WaMos. However, results of the Bayesian method are in most cases deviating significantly.

Wave Spectra by the Wave Buoy Analogy

The wave buoy analogy provides the complete (frequency-directional) distribution of energy as its solution. In addition to the integrated sea state parameters, examples of wave spectra can therefore be studied. Below, a few results are shown for both the Bayesian approach and the parametric approach. It is noteworthy that, in the individual case, the wave spectrum is, as mentioned previously, based on one 20-minutes period of time series data considering set(s) of responses.

Figure 8 shows an example of the wave spectrum obtained on 12th August at 17:30. In the specific 1-D plot, it is observed that the parametric method produces a spectrum with slightly less energy than the Bayesian method. Despite a small difference between the energy in the spectra it is interesting to note how close the two spectra are shape-wise; keeping in mind that the Bayesian method solves for the individual spectral components in the complete directional wave spectrum. In the lower plots of Figure 8, the directional wave spectrum is seen as polar diagrams mapped as contour plots. In the plots, the absolute vessel heading is 0 degrees and, thus, the plots are used to infer that the relative mean wave heading is about -110 deg. Obviously(!), both observations – less energy in the parametric spectrum and that of the relative mean wave heading – can be seen from the upper and lower plots, respectively, in Figure 3.

Another example of wave spectra produced by the wave buoy analogy can be seen in Figure 9. In the figure, the wave spectrum on 2nd October at 21:30 is illustrated. Based on the integrated frequency spectrum (upper plot) it is evident that, in this case, the parametric and the Bayesian approaches yield wave spectra which are slightly different shape-wise. The total amount of energy in the individual spectra is almost the same, but the Bayesian method estimates a spectrum with a sharper
peak compared to the parametric method. In terms of practical applications, it is likely that this difference, by itself, will have little influence. However, a more fundamental problem is visible from the polar plots at the bottom of Figure 9. It is seen that both methods estimate two local peaks but the Bayesian method has the two peaks separated at two very distinct headings which is not the case for the parametric method. The reason for the discrepancy between the two methods is not known but, because of the agreement between the relative mean wave heading of WaMos and the parametric method, it is likely that the result of the Bayesian method is inconsistent. It is noteworthy that this type of problem has been observed in only a few cases.

In the future, it would be of interest to develop a procedure – used with the wave buoy analogy – to automatically select an optimum set of responses.

Acknowledgement

The authors sincerely thank their colleague Prof. Jørgen Juncher Jensen for many useful discussions. The full scale data used has been obtained through the EU project TULCS (Tools for Ultra Large Container Ships), project no. 234146. Fruitful discussions with the project partners are acknowledged and, in particular, thanks are given to Dr. Quentin Derbanne, Bureau Veritas, for providing transfer functions and valuable comments in this respect.

References


---

**Fig. 9: Typical wave spectra on 2nd Oct. around noon by wave buoy analogy: frequency spectrum (top) and directional spectrum (bottom).**


