Fibre waviness and misalignment measurement of unidirectional glass/LPET commingled composites – Effect on mechanical properties

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An experimental investigation was conducted to study the effect of fibre misalignment and waviness on stiffness and strength of unidirectional glass fibre/LPET composites under tensile and compression loading. Analytical models were developed for predicting the fibre misalignment angles and tex factor for three levels of waviness generated during yarn formation. It is shown that the stiffness and strength in both tension and compression loading is reduced as the fibre misalignment angle and tex factor (waviness) increase. The stiffness in both tension and compression are analysed on the basis of the fibre angle dependence calculated for laminate plates with straight fibres (in each ply). The tension strength of the composites is analysed on the basis of the three composite failure modes, tension failure along fibres, shear failure along fibres, and tension failure transverse to fibres. The dominating failure is the shear failure, and an increase of the shear strength is implied from the tension data, and indicates a possible shear strengthening effect of wavy fibres. The compression strength of the composites is analysed on the basis of a model for elastic shear buckling with a further assumption of perfect plasticity in shear beyond the yield stress. The use of the improved shear strength values obtained from the tension data and the fibre angle dependence of the shear modulus, gives calculated compression data which show the same trend in fibre angle dependence as the experimental data, but at a lower level. The difference is 50-100 MPa, such that the experimental data indicates a possible shear strengthening effect for wavy fibres, similar to the observation for the tension data.
1. INTRODUCTION

Many researchers are challenged with trying to find cost-effective solutions for designing large structures by optimizing the materials properties and achieving high end benefits from polymer composites. Composites are made from its constituent materials, such as polymers and synthetic or natural fibres. Processing or mixing of polymer and fibre reinforcements can be done at various levels, which include fibre-fibre, bundle-bundle, and ply-ply level. The process of mixing polymer fibres and fibre reinforcements at filament level by the air texturing method forming a single tow or yarn is called commingling. The resultant yarn is hybrid in nature and normally woven into a fabric form. The other possibility could be winding up the commingling yarns on to a metal frame which can finally form a purely unidirectional pattern. Another advantage with commingled yarns include that during processing the molten polymer inside the fibre preforms can flow shorter distances and allow the material to be moulded with low pressures. The air texturing process (commingling) inevitably introduces some misalignment of the (two or more) fibres. This is of particular importance for the reinforcement fibres and the term fibre waviness is used to describe this situation. Fibre misalignment, i.e. fibres with orientation angles larger than zero relative to the loading direction, normally will reduce mechanical properties such as stiffness and strength. It is therefore of importance to control, measure and evaluate the effect of (normally small) fibre misalignments on mechanical properties. Composites with unidirectional fibre orientation may be particularly sensitive to fibre misalignment under compression loading.

Few researchers studied the role of fibre waviness and misalignment on composite properties. For example, Hsiao and Daniel (1996) investigated the effect of fibre waviness on stiffness and strength reduction of unidirectional composites under compressive loading. Both Young’s modulus and compressive strength decrease as the fibre waviness increases. Material anisotropy is also shown to influence the stiffness and strength reduction with the carbon/epoxy material being much more sensitive to fibre waviness than the glass/epoxy material. The experimental results were in good agreement with the analytical predictions. Similarly, Piggott (1995) reviewed the effect of fibre waviness on the mechanical properties. Apart from strength and stiffness under compression, shear strength and delamination resistance were considered. Due to lack of sufficient experimental data there are no conclusions from the analysis. Yurgartis (1987) studied fibre misalignment and its influence on mechanical properties such as longitudinal compression strength and modulus. The technique presented was able to measure fibre misalignment angle, and can provide a full bivariate distribution such as in-plane and out-of-plane misalignments. Three laminates with a lay-up sequence 0/90, unidirectional, and prepreg carbon fibre/epoxy were considered to study the effect of misalignment on properties. The misalignment angle measurement varies between 0.693 and 1.936 degrees. For measuring the fibre waviness and misalignment experimentally, none of the authors considered fibre level mixing forming a commingled yarn.

Therefore the current study considers glass fibres mixed by commingling with low temperature based polyethylene terephthalate (LPET) polymer fibres at filament level. To study the effect of fibre waviness and misalignment on tensile and compression properties three levels of misalignment are introduced into glass fibres. The commingled yarns are used to produce unidirectional composites in the form of plates. The three levels of fibre waviness aim to give three levels of increasing glass fibre misalignment. The fibre volume fraction of the composites is nominally 30 vol% in order to ensure a high quality material, i.e. low porosity.
Fibre misalignment of glass/LPET – effect on mechanical properties

Fig. 1. Schematic representation of sinusoidal fibre waviness. (a) sinus curve and definition of geometric parameters, (b) definition of xy and xz planes.

2. METHODOLOGY

Fibre bundles and yarns are normally characterized by their tex value, i.e. weight in grams per 1000m. For straight fibres in the yarn the tex value can be measured directly by weighing, and it can be calculated from fibre diameter and density. For yarns with “overfeed” it may be difficult to calculate the tex value, while it is fairly simple to measure the tex value by weighing. The “overfeed” can be characterized by a tex factor defined as the actual tex value relative to the tex value for the corresponding straight fibre bundle. The higher tex value tex(n) of the bundle with “overfeed” can be described as a wavy fibre bundle confined to the same reference length as the straight fibre bundle with tex(0). The wavy shape of the fibre bundle is justified by the observations of the yarns in Fig. 2, with three levels of “overfeed” (nominally 1%, 5%, 10%). The wavy shape implies a longer bundle length ‘s’, relative to the length ‘s0’ of the straight fibre bundle, therefore the tex factor is:

\[
\text{Tex factor} = \frac{\text{tex}(n)}{\text{tex}(0)} = \frac{s}{s_0}
\]  

(1)

The transformation of the waviness of the fibre bundle into a fibre misalignment (in the composite) can be done by using a simple (planar) sinus curve to characterize the fibre bundle waviness. The sinus curve is reasonably justified by the observations in Fig. 2. The formal sinus curve is characterized by its wave length \( \lambda \) and amplitude A, as shown in Fig. 1. Furthermore the length of (one period of) the sinus curve is ‘s’. This establishes a relation to the tex factor:

\[
\text{Tex factor} = \frac{\text{tex}(n)}{\text{tex}(0)} = \frac{s}{s_0} = \frac{s}{\lambda}
\]  

(2)

because the wavelength \( \lambda \) corresponds to the (period) length of the straight fibre bundle.

The sinus curve allows calculation of the (local) fibre orientation angle \( \theta \) relative to the length direction (x-axis). The angle \( \theta \) varies between zero and a maximum value, which is determined by the amplitude A relative to the sinus curve wave length \( \lambda \), and thus by the fibre waviness.

The fibre angle \( \theta \) can be described by its distribution \( 0 < \theta < \theta_{\text{max}} \), by the average value \( \theta_{\text{av}} \), and by the maximum value \( \theta_{\text{max}} \).
From the sinus curve the $\theta$ can be found (by simple mathematical derivations), and are

$$\tan \theta_{\text{max}} = \frac{2\pi A}{\lambda}$$  \hfill (3)

$$\tan \theta_{av} = \frac{4A}{\lambda}$$  \hfill (4)

It is noted that

$$\tan \theta_{\text{max}} = \frac{\pi}{2} \tan \theta_{av}$$  \hfill (5)

From the arc length of the sinus curve it is derived that

$$\frac{s}{\lambda} = \sqrt{1 + \frac{256}{\pi^2} \left(\frac{A}{\lambda}\right)^2}$$  \hfill (6)

This establishes a relation between tex factor ($\frac{s}{\lambda}$) and fibre angle:

$$\tan \theta_{\text{max}} = \frac{\pi^2}{8} \sqrt{\left(\frac{s}{\lambda}\right)^2 - 1}$$  \hfill (7)

$$\tan \theta_{av} = \frac{\pi}{4} \sqrt{\left(\frac{s}{\lambda}\right)^2 - 1}$$  \hfill (8)

These equations allow an estimate of fibre angles, based on a planar sinus curve as an approximation for the fibre waviness in a bundle. The estimate can be based directly on the wavy fibre bundle via the tex factor, or based on (microscopical) observations of the sinus curve approximation for the fibre waviness, either on the bundles (Fig. 2) or on the composites (Fig. 3). It should be noted that the potential non-straightness of fibre (typically in unidirectional fibre composites) conveniently can be characterized by “microscopic fibre misalignment” which is small deviations from zero angle in otherwise straight fibres, and “macroscopic fibre waviness” which is numerically larger deviations of angle over larger distances, as described above. The microscopic fibre misalignment could be overlayed on the fibre waviness. The microscopic fibre misalignment can be estimated by methods as described by Kratmann et al. (2009). The “macroscopic fibre waviness” can potentially be treated as described above.

It should be noted that the direct measurement of tex values and thus the tex factor gives an essentially three-dimensional (3D) description of fibre waviness. The observation of the (dry) fibre bundles (Fig. 2) gives a projection of the 3D nature of the bundle onto a plane of observation, and thus the description may be called 2-3D. The composites are normally studied on planar sections of the composites (Fig. 3), and thus the description is clearly 2D, with a well defined plane. This is typically the xy plane or the xz plane, as defined in Fig. 1b. The choice of plane may depend on e.g. the sample cross sectional geometry, where specimens with rectangular cross sectional area may be sensitive to compression loading in particular in the “thin” direction of the specimen, and therefore the fibre waviness on the xz plane will probably be most critical for compressive strength.
3. EXPERIMENTAL MEASUREMENTS

Glass fibres commingled with LPET fibres made into yarns by the air texturing method will have a tendency to become slightly wavy. The wavy states of the yarns depend on the practical setting and other parameters for the air texturing machine. The interaction between the manufacturing parameters and the resulting (compression) properties have been studied in a series of composites made from glass fibres and LPET as matrix. To ensure a low (near zero) porosity the composites are designed to have about 30 vol % of fibres. Deliberately three levels of waviness of fibres are created for the yarns (made by Comfil ApS, DK).

The fibre yarns produced are shown in Fig. 2 and are labeled with nominal overfeed: 1%, 5%, and 10%. The three groups of glass/LPET unidirectional composites with $V_f = 31\%$ were manufactured at Comfil ApS, DK, who also performed the morphological characterization.

3.1 Measurement of waviness. The fibre yarns are characterized by weighing and by microscopy, to obtain tex factors and calculate related fibre orientation angles, both average fibre angles and maximum fibre angles. The results for the microscopy observations and measurements (Figs. 2 and 3) are given in Table 1, where data for sample DDD04 could not be estimated reliably. All data are collected in Table 2, where especially the fibre angles (average and maximum) are listed.

(a) 

(b) 

Fig. 3. Optical images (merged from several smaller images) of polished Glass/LPET composites for quantification of fibre waviness, (a) in-plane (xy), b) through-thickness (xz).
Table 1. Evaluation of fibre waviness parameters of the glass/LPET composites.

<table>
<thead>
<tr>
<th>Plane</th>
<th>Sample</th>
<th>xy</th>
<th>xz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DDD04</td>
<td>DDD05</td>
<td>DDD06</td>
</tr>
<tr>
<td>Nominal tex increase</td>
<td>1%</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>( \lambda ) (mm)</td>
<td>-</td>
<td>16.7</td>
<td>28.0</td>
</tr>
<tr>
<td>( A ) (mm)</td>
<td>-</td>
<td>1.09</td>
<td>2.20</td>
</tr>
<tr>
<td>( \theta_{\max} ) (°)</td>
<td>-</td>
<td>14.7</td>
<td>17.5</td>
</tr>
<tr>
<td>Tex Factor (measured) ( \frac{\sigma}{\lambda} )</td>
<td>-</td>
<td>1.063</td>
<td>1.075</td>
</tr>
<tr>
<td>Tex Factor (calc) ( A &amp; \lambda )</td>
<td>-</td>
<td>1.054</td>
<td>1.077</td>
</tr>
<tr>
<td>( \theta_{\max} ) (°)</td>
<td>-</td>
<td>22.3</td>
<td>26.3</td>
</tr>
</tbody>
</table>

Table 2. Measurement of tex factor and fibre misalignment angles.

<table>
<thead>
<tr>
<th>Method</th>
<th>Sample</th>
<th>DDD04</th>
<th>DDD05</th>
<th>DDD06</th>
<th>DDD04</th>
<th>DDD05</th>
<th>DDD06</th>
<th>Tex factor</th>
<th>Average fibre angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fibre weighing 3D</td>
<td>1.011</td>
<td>1.029</td>
<td>1.054</td>
<td>6.7</td>
<td>10.8</td>
<td>14.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibre microscopy 2-D, s/( \lambda )</td>
<td>1.012</td>
<td>1.035</td>
<td>1.081</td>
<td>7.0</td>
<td>11.8</td>
<td>17.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibre microscopy 2-D, A &amp; ( \lambda )</td>
<td>1.007</td>
<td>1.023</td>
<td>1.066</td>
<td>5.3</td>
<td>9.6</td>
<td>16.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>1.010</td>
<td>1.029</td>
<td>1.067</td>
<td>6.3</td>
<td>10.7</td>
<td>16.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composite microscopy 2-D, ( \lambda )</td>
<td>1.063</td>
<td>1.075</td>
<td>-</td>
<td>15.8</td>
<td>17.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composite microscopy 2-D, xy, A &amp; ( \lambda )</td>
<td>1.054</td>
<td>1.077</td>
<td>-</td>
<td>14.7</td>
<td>17.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>1.058</td>
<td>1.076</td>
<td>-</td>
<td>15.3</td>
<td>17.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composite microscopy 2-D, ( \lambda )</td>
<td>1.011</td>
<td>1.021</td>
<td>-</td>
<td>6.7</td>
<td>9.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composite microscopy 2-D, xy, A &amp; ( \lambda )</td>
<td>1.016</td>
<td>1.007</td>
<td>-</td>
<td>8.0</td>
<td>5.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>1.013</td>
<td>1.014</td>
<td>-</td>
<td>7.4</td>
<td>7.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.2 Measurement of tex factor and misalignment angles. The fibre angle values can be estimated directly from the tex factors, i.e. before composite fabrication, or from microscopy measurements (sinus curve), i.e. after fabrication on the final composites themselves. The tex factor determined by weighing of the yarns directly is attractive because the measurement is simple. The various estimates of the tex factor are summarized in Table 2. The signatures of 3-D, 2-3-D and 2-D indicate that the measurements give waviness in 3, 2-3, and 2 dimensions.

The data show for fibres and yarns that the weighing method and the microscopy method are rather similar, with values from weighing being slightly smaller. The tex factors measured on the xy plane are larger than the 3-D tex factor, which again is larger than the tex factor for the xz plane, this is expected from the fabrication procedures of (pressing) flat composite plates. The fibre misalignment angles are calculated from the procedures described above, and the values are listed in Table 2. The fibre angles are used to correlate with the mechanical properties of glass/LPET composite i.e. strength and stiffness (Table 3).
### Table 3. Tensile and compression properties of three glass/LPET composites.

<table>
<thead>
<tr>
<th>Composite Parameters</th>
<th>Units</th>
<th>DDD04</th>
<th>DDD05</th>
<th>DDD06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fibre vol fraction</td>
<td>%</td>
<td>31.1</td>
<td>30.9</td>
<td>31.3</td>
</tr>
<tr>
<td>Matrix vol fraction</td>
<td>%</td>
<td>68.5</td>
<td>68.6</td>
<td>68.2</td>
</tr>
<tr>
<td>Porosity vol fraction</td>
<td>%</td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Fibre based $\theta_{\text{avg}}$ degree</td>
<td></td>
<td>6.3</td>
<td>10.7</td>
<td>16.3</td>
</tr>
<tr>
<td>$\theta_{\text{max}}$ degree</td>
<td></td>
<td>8.1</td>
<td>14.7</td>
<td>24.5</td>
</tr>
<tr>
<td>xy-plane, $\theta_{\text{avg}}$ degree</td>
<td></td>
<td></td>
<td>15.3</td>
<td>17.3</td>
</tr>
<tr>
<td>xy-plane, $\theta_{\text{max}}$ degree</td>
<td></td>
<td></td>
<td>22.3</td>
<td>26.3</td>
</tr>
<tr>
<td>xz-plane, $\theta_{\text{avg}}$ degree</td>
<td></td>
<td></td>
<td>7.4</td>
<td>7.3</td>
</tr>
<tr>
<td>xz-plane, $\theta_{\text{max}}$ degree</td>
<td></td>
<td></td>
<td>12.6</td>
<td>8.3</td>
</tr>
</tbody>
</table>

### Tensile Properties

<table>
<thead>
<tr>
<th>E-modulus (GPa)</th>
<th>Strength (MPa)</th>
<th>Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.8 ± 0.5</td>
<td>619 ± 26</td>
<td>0.0262</td>
</tr>
<tr>
<td>22.1 ± 0.3</td>
<td>450 ± 19</td>
<td>0.0228</td>
</tr>
<tr>
<td>20.4 ± 0.5</td>
<td>367 ± 19</td>
<td>0.0208</td>
</tr>
</tbody>
</table>

### Compression Properties

<table>
<thead>
<tr>
<th>E-modulus (GPa)</th>
<th>Strength (MPa)</th>
<th>Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.1 ± 0.4</td>
<td>-530 ± 43</td>
<td>-0.0214</td>
</tr>
<tr>
<td>22.6 ± 0.3</td>
<td>-424 ± 36</td>
<td>-0.0197</td>
</tr>
<tr>
<td>20.6 ± 0.8</td>
<td>-349 ± 15</td>
<td>-0.0178</td>
</tr>
</tbody>
</table>

### MECHANICAL CHARACTERIZATION

In order to quantify the effect of fibre waviness and misalignment on strength and stiffness reduction of (nominally unidirectional) composites, both tensile and compression tests were conducted. Both tensile and compression tests were conducted as per standards ISO 527 and ISO 14126.

Tension tests are performed in a standard Instron machine, and mechanical extensometers were mounted back-to-back on the specimen surface within the gauge section to measure strain data. The stress-strain curves indicate the performances of three laminate series. All test samples showed specimen failure as expected within the gauge region.

Compression tests are conducted with a test fixture designed and developed by Risø DTU. This fixture is basically a Mechanical Combined Loading (MCL) fixture which can transmit both shear loading and end loading. The end loading is accomplished by inserting metal shims in the gap between the ends of the specimen and the base of the fixture. The test fixture is accepted in composite community to test the structural materials with a minimum specimen buckling and more accuracy in test results (see Bech et al. 2011). Strain gauges were mounted to measure specimen deformation during compression loading.

The experimental test results from tension and compression loading for the three series of glass/LPET laminates is summarized in the following Table 3.

### RESULTS AND DISCUSSIONS

The results are presented in Figs. 4, 5, 6, and 7 as the mechanical properties versus the fibre misalignment angle as measured above for the three composite systems. The stiffness in tension, the stiffness in compression, and the strength in tension and the strength in compression all show a decrease in values with increasing fibre angle. The various estimates for the fibre angle are shown, to indicate the range for the average angle, the maximum angle, as well as the
estimates from the xy-plane and the xz-plane. There does not seem to be any special measure which could be preferred for a rational analysis of the data. The average values will be used in the further analysis, in order to simplify the models for the stiffness and strength as a function of fibre angle.

![Graph](image)

**Fig. 4.** Tension stiffness as a function of fibre angle; the various estimates for fibre angles are shown. The curves are based on the laminate plate calculations.

5.1 Stiffness in tension and in compression. There exists no model for stiffness as a function of fibre angle for wavy fibres. Therefore the traditional laminate plate calculation for ply-based laminates will be used as a first attempt to describe the stiffness as a function of fibre angle. The laminate calculation is performed both for off-axis fibre angle (+θ), and for angle-ply fibre configuration (±θ). The theoretical curves for ply-based laminate stiffness are calculated with fibre stiffness: $E_f = 72$ GPa and matrix stiffness $E_m = 3.1$ GPa, and with composite fibre volume fraction $V_f = 0.31$. In Fig. 4 and Fig. 5, the brown curve represents the off-axis laminate (+θ), and the yellow curve represents the angle-ply laminate (±θ). It is assumed that for stiffness at small strains, the stiffness will be the same in tension and in compression, and therefore the same calculated curves are used for tension data in Fig. 4 and for compression data in Fig. 5. The figures show a general agreement between data, both for tension and for compression, with the laminate based calculation for angle ply fibre configuration. It does not seem unreasonable that the wavy fibres give a similar “blocking” of the deformation as does the angle-ply configuration for laminate plates.
Fibre misalignment of glass/LPET – effect on mechanical properties

Fig. 5. Compression stiffness as a function of fibre angle; the various estimates for fibre angles are shown. The curves are based on the laminate plate calculations.

Fig. 6. Tension strength as a function of fibre angle; the various estimates for fibre angles are shown.
5.2 Strength in tension. A simple model exists for the off-axis strength of unidirectional composites, where maximum stresses for three failure modes are implied (Stowell and Liu 1961). Tension failure of the composite along fibre direction is given by

$$\sigma_{11} = \frac{\sigma_{11}(0)}{(\cos \theta)^2}$$  \hspace{1cm} (9)

Shear failure of the composite parallel to the fibres is given by

$$\sigma_{12} = \frac{\tau_{12}(0)}{(\sin \theta \cos \theta)}$$  \hspace{1cm} (10)

Tension failure of the composite transverse to fibre direction is given by

$$\sigma_{22} = \frac{\sigma_{22}(0)}{(\sin \theta)^2}$$  \hspace{1cm} (11)

Since the actual fibre angles are in the range 0° to about 30°, the focus will be on this fibre angle range, i.e. the first two equations and in particular the equation for shear failure. For the tension failure the constant $\sigma_{11}(0)$ is the strength of the unidirectional composite; the value is estimated from the simple law of mixtures, and is calculated to be 1120 MPa. For the shear failure the constant $\tau_{12}(0)$ is estimated from the lower bound equation for shear strength of the composite, and is calculated to be 36 MPa. The curves calculated with these two constants are shown in Fig. 8 as the blue curve (tension failure) and the green curve (shear failure). It is clear that the experimental data show higher strength values than the green curve. The shear failure equation has been used to estimate (by back calculation) the effective value of $\tau_{12}(0)$ for the individual
Fibre misalignment of glass/LPET – effect on mechanical properties

data points. As an attempt to quantify this fibre angle dependence, the $\tau_{12}(0)$ values are plotted as a function of $\sin \theta$, including the theoretical lower bound of 36 MPa at $\theta = 0$. The data form approximately a linear relationship which can be written

$$\tau_{12}(0) = \tau_{12}(0)(1 + 7 \sin \theta)$$

(12)

With this value in the shear failure equation, the curve for off-axis strength is shown in Fig 8 as the red curve. As expected this empirical equation gives an improved description of the experimental off-axis tension strength for composites with wavy fibres. This observation indicates that the wavy fibres give a shear strengthening effect to the composite, so that the shear properties are expected to be above the simple estimate for shear strength of composites.

5.3 Strength in compression. A model for strength in compression has been developed, based on the observation of highly localized kinking of small fibre bundles. This mechanism was analyzed by elastic shear buckling by Rosen (1965) and further developed by Budiansky (1983) with the assumption of perfect plasticity in shear beyond the yield stress. The equation for this mechanism is given by

$$\sigma_{11}(B) = \frac{G_{12}}{1 + \theta / \gamma}$$

(13)

where $G_{12}$ is the composite shear modulus, $\theta$ is fibre misalignment angle, and $\gamma$ is the shear yield strain of the composite. The challenge is to establish reliable values for the shear modulus and in particular the shear yield strain.
The shear modulus for the composite is estimated from the laminate plate calculation for increasing fibre angle, both for off-axis fibre angle (+θ), and for angle-ply fibre configuration (±θ). The shear modulus values for the three (average) fibre angles (6.3°, 10.6°, 16.3°) are listed in Table 4. Since shear deformation is not part of a distinction between tension and compression loading, it is assumed that the shear properties estimated from the tension strength in section 5.2, are also usable in the context of compression loading. The maximum shear strength values \( \tau_{12}(0) \) estimated from the tension data are also listed in Table 4. The estimate of the shear yield strain is based on the schematic shear stress – shear strain curve shown in Fig 9. The maximum shear strength values are combined with the shear modulus (+θ) and the shear modulus (±θ), respectively, to establish the curve(s) in Fig 9. The shear yield strains estimated are listed in Table 4, together with the shear strain based on perfect plasticity (Budiansky 1983).

The shear modulus values and the shear strain values are used in eq.(13) to establish the (four) curves in Fig. 10. The general trend for the model curves follows the experimental data, but at lower (numerical) levels. The effect of shear modulus values for off-axis and for angle-ply fibre configurations, respectively, is rather small, while the effect of definition of shear yield strain has a larger effect. In all cases there is a positive additional effect of the fibres being wavy. The
tension data indicate (Fig 4 and 5) that the shear modulus (± 0) for angle-ply configuration is most realistic. For these curves in Fig 10, the numerical differences between experiments and models are about 120 MPa for yield strain $\gamma_y$, and about 40 MPa for shear yield strain $\gamma_B$ based on perfect plasticity.

To illustrate the complete curve for compression strength, the $\theta$ dependence of the shear modulus $G_{12}(\pm \theta)$ is estimated from the analytical equation for the shear modulus for off-axis fibre configuration. A good approximation (less than 5% error) in the range of 0° to 25°, is:

$$G_{12}(\pm \theta) = G_{12}(0)/(\cos \theta)^4$$  

(14)

The value of $G_{12}(0)$ is taken as the calculated value (2.07 GPa) from the laminate plate calculation, and this value is higher than the lower bound calculation (giving 1.58 GPa). The $\theta$ dependence of the shear strength is given by eq. (12). With these expressions the plasticity based yield strain can be calculated and the (complete) curve for compression strength can be calculated as a function of fibre angle, and is plotted in Fig. 11. The general trend of curve and data points is approximately the same, with a numerical difference of about 50 MPa. This may be a yet un-identified effect of the waviness of the fibres.

A calculation of the compression strength with the use of the shear modulus $G_{12}(\pm 0)$ for angle-ply fibre configuration gives a very similar curve, shifted upwards by about 10 MPa, see Fig. 11. Thus the effect on compression strength is rather small for the fibre configuration with respect to “blocking” of deformation.

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![Fig. 10. Compression strength as a function of fibre angle; only fibre angle average values are shown. The four groups of (three) data points, connected by lines, represent the two definitions of shear yield strain, and the two estimates of shear modulus (off-axis and angle-ply fibre configurations).](image-url)
6. CONCLUSIONS

An experimental investigation was conducted to study the effect of fibre misalignment and waviness on stiffness and strength of unidirectional glass fibre/LPET composites under tensile and compression loading. Analytical models were developed for predicting the fibre misalignment angles and tex factor for three levels of waviness generated during yarn formation. It is shown that the stiffness and strength in both tension and compression loading is reduced as the fibre misalignment angle and tex factor (waviness) increase. The stiffness in both tension and compression are analysed on the basis of the fibre angle dependence calculated for laminate plates with straight fibres (in each ply). The experimental data agree reasonably well with the angle-ply fibre configuration, indicating that the wavy fibres (partly) “block” the shear deformation in a manner similar way to the +θ plies and the −θ plies in laminates with straight fibres. The tension strength of the composites is analysed on the basis of the three separate failure modes, tension failure along fibres, shear failure along fibres, and tension failure transverse to fibres, each mechanism being active in separate fibre angle ranges. The dominating failure is the shear failure of the composite, and an increase of the shear strength is implied from the tension data, and indicates a possible shear strengthening effect of wavy fibres. The compression strength of the composites is analysed on the basis of a model for elastic shear buckling with a further assumption of perfect plasticity in shear beyond the yield stress. The use of the improved shear strength values obtained from the tension data and the fibre angle dependence of the shear modulus, gives calculated data which show the same trend in fibre angle dependence as the experimental data, but at a lower level. The calculation of the complete curve of compression strength versus fibre angle gives a difference of about 50 MPa, such that the experimental data indicates a possible shear strengthening effect for wavy fibres, similar to the observation for the tension data.
Fibre misalignment of glass/LPET – effect on mechanical properties

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