Design of Crashworthy Ship Structures

Rikard Törnqvist
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Preface

This thesis is submitted as a partial fulfilment of the requirements for the Danish Ph.D. degree. The work has been performed in the Section of Maritime Engineering, the Technical University of Denmark, during the period from March 2000 to April 2003. The project was supervised by Associated Professor Bo Cerup Simonsen and Professor Preben Terndrup Pedersen.

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My sincere thanks to my supervisors, Associated Professor Bo Cerup Simonsen and Professor Preben Terndrup Pedersen, for many inspiring discussions. Their help and guidance is highly appreciated.

Thanks to my colleagues in the Section of Maritime Engineering, the technical staff in the MEK department, friends and family for their help, support and kindness. Moreover, thanks to Professor Tony Atkins, the postgraduate students and the technical staff in the Engineering Department, University of Reading, for their kindness, helpfulness and for making my five months’ stay in Reading very interesting and pleasant.

Special thanks to Stephie for her support and understanding during the entire study.

Rikard Törnqvist
Copenhagen, June 2003
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Executive Summary

The main purpose of the project has been to develop a rational procedure for designing new crashworthy side structures for those ship types where it could be expected that a substantial improvement of the crashworthiness and the related safety could be achieved by careful consideration of the structural design. For a tanker vessel or other vessels carrying hazardous cargo, damage is not acceptable if it results in cargo outflow with disastrous consequences to the environment. Likewise, the foundering of a passenger vessel can be disastrous with loss of many human lives.

A major challenge in collision and grounding analysis is the prediction of the onset of fracture and crack propagation in the shell plating. In simulations of accidental loading on ships it is crucial that fracture is determined correctly, as it will influence the global deformation mode and the amount of damage to the hull and thus determine which compartments will be flooded as well as the amount of oil outflow. The most commonly used failure criterion in large-scale FE-simulations has been the equivalent plastic strain. However, it is well known that equivalent plastic strain is not suitable as a fracture criterion when a structure is subjected to biaxial loading. The main aspect of this thesis has therefore been to study the fracture criteria available in the literature and to validate them against various experiments with varying stress and strain states.

Several experiments were conducted where e.g. the hydraulic giant bulger at Reading University was used for investigating fracture initiation and crack propagation in biaxially loaded metal sheets. The capability of various empirical fracture criteria, void growth fracture criteria as well as continuum damage mechanics (CDM) and porosity damage models for predicting fracture initiation was compared with the experimental results. The void growth model by Rice and Tracey was shown to be able to predict fracture initiation and crack propagation for positive triaxial stress states, while the Cockcroft and Latham criterion predicted ductile fracture for low and negative triaxialities. There is also experimental evidence that for triaxial stress states in homogenous compression ($\frac{\sigma_H}{\sigma_{eq}} < -\frac{1}{3}$) fracture will not occur. It is believed that the Cockcroft and Latham criterion is able to model ductile shear fracture correctly, whereas the Rice and Tracey criterion has proved to model ductile fracture due to void growth. Thus, the new proposed combined Rice-Tracey and Cockcroft-Latham (RTCL) criterion is a natural combination that will cover the whole range of triaxialities where the damage algorithm switches between the two triaxiality functions at a triaxiality value of 1/3.
Executive Summary

Only one parameter has to be determined to calibrate the RTCL criterion, which makes the criterion very practical in simulations of engineering problems.

The new RTCL fracture criterion was implemented into an explicit finite element code (LS-DYNA), together with a user-defined material model with a constitutive material algorithm based on $J_2$ flow theory with a radial return algorithm and isotropic hardening. Several large scale experiments available in the literature were simulated to validate the RTCL criterion for complex plate structures subjected to various accidental loadings. The finite element results corresponded well with the experimental results, which proved the usefulness of the RTCL criterion for large plate structures. It was also shown that the element length should be as small as possible to be able to describe the stress and strain localisation as well as to capture various folding patterns. In the large scale experiment simulations an element size of about five times the plate thickness was found to be sufficient to achieve good correlation with the experiments.

Therefore, by use of the RTCL criterion and a sufficiently small element size it was believed that the point of fracture for different structural arrangements could be predicted with reasonable accuracy for various accidental loading scenarios. The energy absorption capability of three crashworthy side structures for a small coaster was investigated, for which the building costs, the operating costs as well the payload were assumed to be almost equal. The simulations revealed that the new structural layout with corrugated panels and the layout with concrete/steel sandwich had less energy absorption capability at inner side shell fracture compared to a conventional ice strengthened structure. This was mainly due to the small distance between the outer and inner hull plating in the two sandwich panels.

Collision statistics shows that the struck ship in a ship-ship collision often has a forward velocity at the point of impact, but in most finite element simulations the struck ship has been at a standstill and fixed in space. A study of different ship-ship collision scenarios, where the effects of material strain-rate sensitivity and surrounding water were included, showed that some of the simplifications in the simplified analytical methods are improper, e.g. neglecting the roll motion of the ships and assuming constant contact force ratio (longitudinal over transverse forces of the struck ship). It is also questionable if the complex interaction of the deformation between the structural members of the two ships in a collision event, e.g. bending of the bulb, can be described by simplified analytical methods. Nevertheless, the simplified methods have shown to give good results for perpendicular ship-ship collisions with the struck ship at a standstill, but their validity for more complex collision scenarios may be questionable.

The hydrodynamic damping effects were insignificant for the studied load cases of two colliding coasters, whereas the effects of material strain rate sensibility and rigid body motions in water were of large importance for the simulation results. The penetration depths at the point of inner side shell rupture varied only slightly for different collision angles and forward speeds when the loading conditions of the two ships were the same. Less energy was, however, dissipated at the point of fracture of the inner side when the struck ship had a forward velocity than when it was at a standstill. For different loading conditions, the penetration
depths as well as the structurally absorbed energy varied with the vertical contact location. If the bulb hit above the double bottom structure much less energy was absorbed by the struck ship than when the bulb hit the double bottom.
Synopsis

Hovedformålet med projektet har været at udvikle en rational metode for design af nye ”crashworthy” sidestrukturer for forskellige skibstyper med en forbedret energi absorptionsevne. For et tankskib eller lignende fartøj, som fragter farligt gods, kan skader på skroget ikke accepteres, idet de fører til forurenring af miljøet med katastrofale følger for omgivelserne.


RTCL kriteriet er blevet implementeret i et explicit FE-program (LS-DYNA), gennem en bruger-defineret materiale model, hvor en konstitutiv materiale algoritme baseret på $J_2$ fly-

Med RTCL kriteriet og tilstrækkeligt små elementer kan det derfor antages at brudinitiering i forskellige strukturelle konfigurationer udsat for ulykkesbelastninger kan bestemmes med tilstrækkelig nøjagtighed. Energi-absorberings-evnen af kapacitet er blevet undersøgt for tre forskellige sidestrukturer i et mindre frigørelstøj, hvor byggeomkostninger, driftsomkostninger samt lasteereve er de samme. De numeriske simuleringer viste at nye strukturelle layouts med korregerede paneler samt beton/stål sandwich paneler havde mindre energi-absorberings-evne sammenholdt med et konventionelt is-forstærket skrog. Det var blandt andet på grund af den korte afstand mellem yder- og inderknækkning, samt at bulben ramte en web i det is-forstærkede skrog.

Statistiske analyser af skibskollisioner har vist, at det påsejledes skib oftest har en hastighed fremad ved kollisionstidspunktet, men i de fleste FE-simuleringer af skibskollisioner er det påsejledes skib stillestående og fastlåst i rummet. I et studie af forskellige kollisions-scenarier, hvor effekterne af ”strain rate” samt det omkringliggende vand var inkluderet, blev det vist, at nogle af forenklingerne i de analytiske metoder ikke er korrekte, f.eks. ikke at inkludere rulle-bevægelser og at antage et konstant kraftforhold mellem langsgående og tværgående kræfter, som virker på det kolliderede skib. Det er også tvivlsomt, om den komplekse interaktion mellem de forskellige strukturelle dele af de to skibe kan beskrives af de analytiske metoder. Disse metoder har derimod vist sig at kunne give gode resultater for vinkelrette kollisioner, når det kolliderede skib er stillestående.

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Symbols

Roman Symbols

\[ A \quad \text{Area} \]
\[ A_0 \quad \text{Initial area} \]
\[ A_{eff} \quad \text{Effective area} \]
\[ A_{jc}, B_{jc}, C_{jc} \quad \text{Material parameters (Johnson-Cook)} \]
\[ A_s \quad \text{Element area} \]
\[ A_w \quad \text{Water plane area} \]
\[ a_0 \quad \text{Material constant (Oyane)} \]
\[ B \quad \text{Specimen width} \]
\[ b \quad \text{Ligament length} \]
\[ b_0, b_1, b_2 \quad \text{Pressure coefficients} \]
\[ C \quad \text{Power law coefficient} \]
\[ C \quad \text{Damping matrix} \]
\[ C(\omega) \quad \text{Damping matrix depending on wave pulsation} \]
\[ C_{cs} \quad \text{Strain rate coefficient (Cowper-Symond)} \]
\[ C_{ijkl} \quad \text{Elastic tangent modulus matrix} \]
\[ C_s \quad \text{Speed of sound} \]
\[ c_N \quad \text{Material parameter (Norris)} \]
\[ c \quad \text{Crack length} \]
\[ c_A \quad \text{Calibration constant (Atkins)} \]
\[ c_{HM} \quad \text{Material parameter (Hancock-MacKenzie)} \]
\[ D \quad \text{Damage} \]
\[ D_1, \ldots, D_5 \quad \text{Calibration constants (Johnson-Cook damage model)} \]
\[ D_{cr} \quad \text{Critical damage value} \]
\[ D_i \quad \text{Damage indicator} \]
\[ d \quad \text{Diameter} \]
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<td>$E$</td>
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<td>$\dot{E}$</td>
<td>Dissipated energy rate</td>
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<td>$L_1, \ldots, L_4$</td>
<td>Element side length</td>
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<tr>
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<td>Definition</td>
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<tr>
<td>$L_s$</td>
<td>Characteristic element length</td>
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<tr>
<td>$l_e$</td>
<td>Element size</td>
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<tr>
<td>$M$</td>
<td>Mass matrix</td>
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<tr>
<td>$M_{RB}$</td>
<td>Rigid-body inertia matrix</td>
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<tr>
<td>$M_A$</td>
<td>Added inertia matrix</td>
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<tr>
<td>$m$</td>
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<tr>
<td>$\bar{m}_{RG}$</td>
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<td>Power law exponent</td>
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<tr>
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<td>Pressure</td>
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<td>$q$</td>
<td>Strain rate exponent (Cower-Symond)</td>
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<td>Material constants (GTN)</td>
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<td>Initial void radius</td>
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<tr>
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<tr>
<td>$R_L$</td>
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<td>Material constant (Lemaitre)</td>
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<td>$s_0$</td>
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<td>$s_{ij}$</td>
<td>Deviatoric stress tensor</td>
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<td>Deviatoric trial stress tensor</td>
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<td>$T$</td>
<td>Temperature</td>
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<tr>
<td>$T^*$</td>
<td>Homologous temperature</td>
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<tr>
<td>$\Delta t$</td>
<td>Time step increment</td>
</tr>
<tr>
<td>$t_0$</td>
<td>Initial thickness</td>
</tr>
<tr>
<td>$U_p$</td>
<td>Plastic deformation energy</td>
</tr>
</tbody>
</table>
\( u, \dot{u}, \ddot{u} \) Displacement and its derivatives

\( V \) Volume

\( \vec{v_G}, \dot{v_G} \) Velocity vector and its derivatives

\( W_0 \) Damage work

\( w \) Width

\( w_0 \) Initial width

\( x_{\text{ref}} \) Reference position

\( Y \) Strain energy density release rate

**Greek symbols**

\( \alpha \) Damage exponent (Bonora)

\( \alpha_c \) Calibration factor (Chaouadi)

\( \alpha_f \) Time scale factor

\( \alpha_{HM} \) Material constant (Hancock-MacKenzie)

\( \alpha_{RT} \) Material constant (Rice-Tracey)

\( \alpha_l \) Pressure constant

\( \alpha_{11} \) Pressure constant

\( \beta \) Number of values

\( \beta_{\text{fail}} \) Weld failure parameter

\( \delta_{ij} \) Kronecker’s multiplier

\( \varepsilon_0 \) Uniaxial fracture strain, critical damage value

\( \varepsilon_1, \varepsilon_2, \varepsilon_3 \) Principle strain

\( \varepsilon_{br} \) Bridgman strain

\( \varepsilon_{\text{coa}} \) Strain at coalescence

\( \varepsilon_{eq} \) Equivalent strain

\( \varepsilon_f \) Failure strain

\( \varepsilon_{f,\text{uni}} \) Uniaxial failure strain

\( \varepsilon_f(l_e) \) Failure strain

\( \varepsilon_g \) Uniform through thickness strain

\( \varepsilon_{ij} \) Strain tensor

\( \dot{\varepsilon}_{ij} \) Rate of strain tensor

\( \varepsilon_{\text{int}} \) Instability strain

\( \varepsilon_N \) Mean value

\( \varepsilon_n \) Through thickness necking strain

\( \varepsilon_{\text{nom}} \) Nominal strain

\( \varepsilon_{\text{nuc}} \) Strain at nucleation
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\varepsilon_{th}$</td>
<td>Threshold strain</td>
</tr>
<tr>
<td>$\varepsilon_\theta$</td>
<td>Circumferential strain</td>
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<tr>
<td>$\varepsilon_v$</td>
<td>Maximum effective strain</td>
</tr>
<tr>
<td>$\varepsilon_{yp}$</td>
<td>Elastic strain to yield</td>
</tr>
<tr>
<td>$\varepsilon_z$</td>
<td>Axial strain</td>
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<tr>
<td>$\eta_p$</td>
<td>Configuration constant</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Flow potential</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Plastic multiplier</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Potential energy</td>
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<tr>
<td>$\phi$</td>
<td>Flow potential</td>
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<tr>
<td>$\rho$</td>
<td>Density</td>
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<td>Strain ratio</td>
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<td>$\sigma$</td>
<td>Actual stress</td>
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<td>Yield stress</td>
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<td>$\sigma_0^d$</td>
<td>Dynamic flow stress</td>
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<td>$\sigma_1, \sigma_2, \sigma_3$</td>
<td>Principle stress</td>
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<tr>
<td>$\sigma_{avg}^d$</td>
<td>Average dynamic yield stress</td>
</tr>
<tr>
<td>$\sigma_{eq}$</td>
<td>Equivalent stress</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>Fracture stress</td>
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<td>$\sigma_{ij}$</td>
<td>Stress tensor</td>
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<tr>
<td>$\sigma_{ij}^*$</td>
<td>Trial stress tensor</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>Hydrostatic stress</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>Normal stress component</td>
</tr>
<tr>
<td>$\sigma_{nom}$</td>
<td>Nominal stress</td>
</tr>
<tr>
<td>$\tau_t$</td>
<td>Transverse shear stress component</td>
</tr>
<tr>
<td>$\tau_n$</td>
<td>Normal shear stress component</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Wave pulsation</td>
</tr>
<tr>
<td>$\omega, \dot{\omega}$</td>
<td>Rotational velocity vector and its derivatives</td>
</tr>
</tbody>
</table>

**Other symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$\nabla$</td>
<td>Displacement</td>
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Chapter 1

Introduction

1.1 Overview and Background

Today there is a growing public demand to reduce the risk of human lives and oil spillage at sea as well as to minimise the damage caused by ship groundings or collisions. Recently, one of the largest oil pollution catastrophes in the history took place when the single hull tanker the Prestige sank off the coast of Spain, Figure 1.1. Every year about 230 vessels, most short sea vessels, with a registered tonnage of 1.1 million are lost at sea worldwide, claiming over 1000 lives. Moreover, collision and grounding events will continue to occur in spite of continuous efforts to prevent them.

Shipping safety and marine pollution are inextricably linked and the protection of the environment from major disasters as oil spillage is rather complex. The means for protecting the environment are generally divided into two groups: the active and the passive methods. The active methods (e.g. navigation equipment, crew training, traffic control systems) shall prevent the accident from taking place, while the passive methods try to minimise the consequences by e.g. improvements of the crashworthiness of the hull structure or rescue operations.

In a ship-ship collision or a grounding event, the impact energy is mainly absorbed by large structural deformations in the ship structure, and if the construction is allowed to deform without rupture of the side plating, oil spillage or flooding of the cargo holds will not occur. The collision mechanics of the ship structures is, however, a very complex and a highly non-linear phenomenon. It is thus important to establish reliable analysis tools and procedures for evaluating the hull structure response during accidental loading, and in the last decades several methods have been developed for analysing ship structure crashworthiness and the internal mechanics during collision and grounding events.

A classical empirical method for estimating the damage from a collision was proposed by Minorsky (1959). He divided the collision problem into two, the external dynamics and the
Figure 1.1: The single hull tanker the Prestige sinks off the northwestern coast of Spain carrying more than 70,000 tonnes of fuel on 19 November 2002.

internal mechanics. The external dynamics treats the global motions of the ships and the interaction of the surrounding water during the collision event. The internal mechanisms were based on the volume of the damaged steel, which was derived from cases of actual collisions. The Minorsky formula has been widely used in many calculations of the collision resistance of ships, however, the structural arrangements of the striking and the struck ship were not included in the formula. Several attempts have been made on modifying the Minorsky formula to include the effects of the structure layout (Woisin (1979), Pedersen and Zhang (2000)). However, by using empirical methods like these for the design of new crash-worthy ship structures, the analysis could be incorrect as new structures are not represented in the databases from which the formulas are derived. Nevertheless, the formulas are useful for fast estimations of the energy absorption of the ship structure in an accident.

Several large-scale experiments of ship collisions and groundings have been made in the past (Nagasawa et al. (1981), Rodd (1996), Woisin (1968)), but as experiments are very time-consuming and in addition expensive large-scale test structures are needed to avoid scale effects. Today large-scale experiments are mainly used for validation of theoretical and numerical methods, e.g. Amdahl and Kavlie (1992).

A technique often used for prediction of the energy absorption during accidents is the simplified analytical methods, where the structure is divided into a small number of basic elements, in which the collapse modes are described in an idealised way, e.g. folding mechanisms, membrane stretching and tearing (see e.g. Zheng and Wierzbicki (1996), Simonsen (1998a), Wierzbicki (1995)) so that they capture the basic characteristics of the structural response. The energy dissipation and the average load are found by summing up the contributions from the active elements. The methods are very useful and give insight into both global and local levels and have been used for e.g. ship-ship collisions, bottom ranking and ship-bridge collisions (see e.g. Simonsen (1998b), Pedersen et al. (1993)). The simplified
1.1 Overview and Background

Analysis methods are well suited for initial estimates as well as Monte Carlo simulations of the absorbed energy in various structural arrangements and loading conditions, Lützen (2001).

The continuous development in computer technology resulting in faster computers has made it possible to simulate ship collisions and groundings by use of finite element methods within a reasonable time span. Large simulations of ship collisions and groundings have been made by e.g. Kitamura (1997), Mizukami et al. (1996) and Lee and Kim (2001). It has been shown that nonlinear finite element simulations are reliable and can account for large deformations, nonlinear material behaviour and contact. However, the prediction and the simulation of ductile crack propagation have not yet been completely understood. The classical structural fracture mechanics is not sufficient to determine structural failure during accidental loading in complex finite element simulations of ship groundings or collisions.

In simulations of accidental loading on ships it is crucial that fracture of the plating is determined correctly, as it will influence the global deformation mode and the amount of damage to the hull and thus determine the magnitude of the energy dissipation. Several large-scale experiments in recent years have given substantial input to the understanding of the structural rupture behaviour. Extensive finite element simulations have shown the need for a general fracture criterion that captures the fracture mechanisms. More reliable calculations can support innovative structural arrangements and lead to increased survivability and lower lifetime costs.

Structural members in a ship structure subjected to lateral loading (e.g. side plating in a collision) will deform and membrane forces will be developed in the plate and a large amount of energy will be dissipated by the plastic straining. The rate of the dissipated energy \( \dot{E} \) at a given deformation rate during the deformation process is given by the volume integral:

\[
\dot{E} = \int_V \sigma_{ij} \dot{\varepsilon}_{ij} dV \tag{1.1}
\]

where \( \sigma_{ij} \) and \( \dot{\varepsilon}_{ij} \) are the stress tensor respectively the rate of the strain tensor. A substantial loss in resistance and energy absorbing capability will take place at the point of fracture and once a crack has been initiated relatively little energy is needed for its propagation. During crushing, the structural members tend to buckle and the deformation is therefore concentrated to plastic hinges. Thus, members subjected to crushing have less energy absorbing capabilities per unit volume compared to membrane deformations, but the members in crushing will continue to absorb a relatively large amount of energy even after the point of fracture. As a result, it is of vital importance that the fracture initiation is correctly predicted in the numerical calculations, as a slightly too late fracture initiation will result in too much dissipated energy leading to smaller penetration depths.
1.2 Objectives and Scope of the Work

The scope of the work is primarily to establish methodologies for crashworthiness prediction and to increase the knowledge of numerical simulations of accidental loading of ships. The effect of several materials and also nonconventional structural arrangements will be analysed with respect to energy absorbing capabilities. For this analysis, the explicit nonlinear finite element procedure will be used, which has proved to be very accurate for crushing of large structures as long as fracture does not occur. Existing methods are to be improved by introduction of general fracture criteria for the base material.

Fracture in a metal structure has often been simulated by use of an equivalent plastic strain. However, it is well known that a failure criterion based only on the equivalent strain is generally not valid in biaxially loaded plates. Ductile fracture by application of void growth models has been treated in the literature (see Atkins (1997), Gurson (1977), Rice and Tracey (1969), Hancock and Mackenzie (1976), McClintock et al. (1966), Cockcroft and Latham (1968) and Norris (1978)). Due to the large structural models and the fact that the simulation must be solved in a realistic computational time, the finite elements in a ship structure have to be quite large. This is contrary to the fracture phenomenon, which is a very local problem. Therefore, to obtain reliable calculation results, simple failure criteria for simulating ductile fracture will be derived and implemented in the finite element code - LS-DYNA - and experiments are to be performed to verify the developed mathematical model.

In ship structures a large amount of welds exists and the crashworthiness of the ship depends highly on the weld strength and quality. When welds fail during a collision, each structural part may deform independently, causing a much greater damage. Thus, it is crucial in a collision simulation that the welds are modelled correctly. Fracture initiation and propagation in welds will be investigated.

The rigid body motions of the colliding ships are often neglected in finite element analyses. Nevertheless, the effect of the external energy may be very important to the damage extension, see Pedersen and Zhang (1998). A time domain analysis of these motions and an implemented program for treating the external energy balance during the ship-ship collision simulation will be used to demonstrate the effects of the external dynamics.

The thesis is composed as follows:

In Chapter 2 a number of fracture criteria and damage models are presented. Several of the presented criteria and models have been implemented into the finite element program. A new proposed fracture criterion (RTCL) is presented, which covers the whole range of triaxialities.

In Chapter 3 a comprehensive study of the fracture criteria and damage models in various stress and strain states is presented. For validation of the criteria and damage models
1.2 Objectives and Scope of the Work

Various experiments are conducted, e.g. the novel test setup with the giant hydraulic bulger is used. The experimental and numerical simulations are described in detail and the results are compared.

Chapter 4 is devoted to validation of the RTCL fracture criterion by simulations of several different large-scale experiments of various collision or grounding scenarios. Several experiments of ship collisions and groundings have been carried out since the early 1960’s, and some of these have been selected for the validation of the two finite element fracture criteria for typical ship structures under various loadings.

In Chapter 5 failure of welds is briefly discussed and various modelling techniques are presented.

Chapter 6 presents various improved crashworthy ship structures and a few finite element simulations of these.

Chapter 7 gives examples of a number of modelling techniques for simulations of ship collisions. Eight different collision scenarios of two colliding coasters are simulated and the structural responses are compared. An explosion simulation in the cargo hold is also presented.

Chapter 8 contains the conclusions and the recommendations for future work.
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Chapter 2

Ductile Fracture in Metals

2.1 Introduction

The most commonly used failure criterion in large scale FE-simulations has been the equivalent plastic strain. It is, however, well known that equivalent plastic strain is not suitable as a fracture criterion when a structure is subjected to biaxial stress states and even in simple uniaxial tensile tests the plastic strain is inappropriate. Thus, a major challenge in collision and grounding analysis is the prediction of the onset of fracture and crack propagation in the shell plating. In simulations of accidental loading on ships it is crucial that the fracture is determined correctly, as it will influence the global deformation mode and the amount of damage to the hull and thus determine which compartments will be flooded. The governing damage processes in materials are highly influenced by the stress triaxiality, which should somehow be accounted for in the constitutive material model or in the damage criterion.

Fracture has long been of high interest to researchers, as it is of vital relevance in many engineering fields. The classical linear elastic fracture mechanics (LEFM) theory assumes that a single parameter, such as the stress intensity factor $K_I$, describes the conditions at the crack tip. Therefore, the value of $K_I$ at fracture can be considered as a material constant. LEFM is only valid as long as the nonlinear material deformation is confined to a small region at the crack tip. The elastic-plastic fracture mechanics parameters, such as the crack tip opening displacement (CTOD) and the $J$-contour integral, describe the crack tip conditions for elastic-plastic materials, however, for excessive plasticity or large crack growth, the methods become size- and geometry-dependent. In two-parameter fracture mechanics, such as the $J$-$Q$ theory, an additional parameter characterises the crack tip conditions, but the methodology is more descriptive than predictive and is therefore not used in practice. In the last decades much work has also been devoted to the prediction of fatigue failure, where the plastic field around the crack tip is relatively small. Ductile fracture is, however, characterised by large plastic flow before fracture initiation and around the crack tip and is today still an important topic for researchers.
Chapter 2. Ductile Fracture in Metals

The traditional and inexpensive way of evaluating dynamic fracture toughness is to use the Charpy test, but both the specimen size as well as the impact velocity are limited. The transition from high energy absorbing, ductile void growth fracture to low energy, brittle, cleavage fracture in bcc\textsuperscript{1} metals is believed to be governed by the local stress state. Both temperature and the loading rate affect the stress state at the crack tip together with the geometry of the specimen. Thick specimens lead to a plane strain state at the crack tip, which can result in brittle fracture, while thin specimens lead to a plane stress state ahead of the crack tip. Mirza et al. (1996) investigated the effect of the triaxiality and the strain rate on the fracture characteristics of ductile materials. They found that transition between ductile and brittle fracture, as well as the strain at failure, is strongly dependent on the stress triaxiality and the loading rate. Other studies (e.g. Pussegoda et al. (1996), Jones (1989)) have also shown the effects of high strain rates on the material characteristics and the fracture toughness, and in Section 3.5 the strain rate effects will be further discussed.

In the following plastic instability together with different fracture criteria and damage models available in the literature, which include the effect of the stress triaxiality, will be briefly described. Several of the criteria and damage models have been implemented into the FE-program LS-DYNA to study their usefulness in different applications.

2.2 Plastic Instability

The onset of fracture is often preceded by extensive plastic flow when a typical steel structure is subjected to accidental loading. The strain distribution in the structure can be very complex and in some regions highly non-uniform. Plates with localisation and high strain gradients are critical with respect to fracture and it has long been of high interest to engineers to understand the failure mechanisms.

When a plate is subjected to uniform large deformations, the strain initially spreads along the length of the specimen and at a certain point the forming of a neck is initiated. Keeler (1965) and Goodwin (1968) introduced the forming limit diagram (FLD), Figure 2.1, which gives the major and minor in-plane strain pairs at necking and was developed for sheet metal forming. The FLD’s are constructed from experimental data obtained by using sheets that are biaxially loaded at different strain ratios until the occurrence of necking. Furthermore, there are two types of necks in sheets, the diffuse neck and the localised plain strain neck, of which the diffuse neck is similar to necking in axisymmetric tension in a round bar. The diffuse neck corresponds to a very gradual and mild thickness reduction, where the extent of the necking is several times longer than the plate thickness. Diffuse necking is therefore hard to detect in plate forming operations and as the sheets can often be subjected to additional deformation without failure, the diffuse necking is not considered as critical, whereas the localised necking actually determines the formability of the sheet metal. An extension of the forming limit diagram is the fracture forming limit diagram (FFLD), which includes the

\textsuperscript{1}Body centred cubic
2.2 Plastic Instability

major and minor in-plane strain pair at fracture. Consideré (1885) predicted the initiation of diffuse necking in uniaxial tensile tests. His criterion corresponded to a maximum load \( P_{\text{max}} \), where the point of necking is characterised by

\[
\partial P = 0 = \partial (\sigma A) = A \partial \sigma_1 + \sigma_1 \partial A
\]  

(2.1)

By assuming pure plastic deformation and constant volume, the criterion can be rewritten as

\[
\frac{\partial \sigma_1}{\partial \varepsilon_1} = \sigma_1
\]  

(2.2)

where \( \varepsilon \) is the natural strain and the index 1 indicates the major principal direction (loading direction). For a strain hardening law described by a power law according to

\[
\sigma_{\text{eq}} = C \varepsilon_{\text{eq}}^n
\]  

(2.3)

where \( C \) and \( n \) are material constants, the strain at necking becomes

\[
\varepsilon_1 = \varepsilon_{\text{eq}} = n
\]  

(2.4)

The strain limit for diffuse necking in biaxially loaded isotropic sheets can be found from a similar approach, derived by Swift (1952), when the major principal strain reaches the value

\[
\varepsilon_1 = \frac{2n(1 + \rho_R + \rho_R^2)}{(1 + \rho_R)(2\rho_R^2 - \rho_R + 2)}
\]  

(2.5)

during proportional loading at a constant strain ratio of \( \rho_R = \frac{\varepsilon_2}{\varepsilon_1} \).

Hill (1952) found that for the drawing region \( (\varepsilon_1 > 0, \varepsilon_2 < 0) \) the limit strain pair at the initiation of a localised neck for an isotropic power law material is found from

\[
\varepsilon_1 + \varepsilon_2 = n \quad \text{for} \quad \varepsilon_2 < 0
\]  

(2.6)

While the plasticity theory predicts the onset of diffuse necking and localised necking in the drawing region, the localised necking cannot be determined by conventional plasticity theory in the biaxial tensile loading range \( (\varepsilon_1 > 0, \varepsilon_2 > 0) \). The forming limits can only be predicted by assuming initial defects in the sheet, Marciniak and Kuczunski (1967), or by making use of a perturbation analysis for studying the rate of growth of instabilities, Dudzinski and Molinari (1991). Instead an empirical formula is often used for the stretching range, Parmer and Mellor (1978)

\[
\varepsilon_1 - (1/2)\varepsilon_2 = n \quad \text{for} \quad \varepsilon_2 > 0
\]  

(2.7)

In the following ductile fracture will be discussed together with different criteria and damage models for predicting the onset of ductile fracture.
Chapter 2. Ductile Fracture in Metals

2.3 Ductile Fracture Models

The first to be interested in fracture for bodies subjected to large permanent deformations were the metal formers, who wished to avoid cracking in e.g. plate forming processes. They discovered that ductile fracture was not only dependent on plastic deformation, but also on the state of stress. Several simple criteria were developed for predicting the onset of fracture, but it was found that if a criterion worked well for a particular metal forming process, the same criterion failed to predict fracture for processes where the stress and strain state was completely different. Most accepted criteria depended on both the stress and strain state and on the history of stresses and strains in the fracture locus. The empirical criteria predicted that fracture would occur when a function including the stress and strain state, $f_{emp}(\sigma, \varepsilon)$, achieved a critical value, $D_{cr}$, which was characteristic of the material, according to

$$D_{cr} = \int f_{emp}(\sigma, \varepsilon) d\varepsilon$$  \hspace{1cm} (2.8)

Bridgman (1952) showed that the plastic deformation at the point of fracture is highly dependent on the triaxial state of stress. This was shown by conducting tension tests with circularly notched specimens, where the results revealed that the hydrostatic pressure influenced the growth of voids in the material. He found that the highest hydrostatic stress, $\sigma_H$, occurs in the centre of the necked specimens and is calculated from:

$$\left(\frac{\sigma_H}{\sigma_{eq}}\right)_{max} = \frac{1}{3} + \ln\left(\frac{d}{4R_n} + 1\right)$$  \hspace{1cm} (2.9)

where $R_n$ is the notch radius and $d$ is the diameter of the minimum cross-section. The ratio $\frac{\sigma_H}{\sigma_{eq}}$ is called the triaxiality where the hydrostatic stress and the equivalent stress are defined.
2.3 Ductile Fracture Models

as

\[ \sigma_H = \frac{\sigma_{ii}}{3} \]  

(2.10)

respectively

\[ \sigma_{eq} = \left( \frac{3}{2} s_{ij} s_{ij} \right)^{1/2} \]  

(2.11)

The deviatoric stress tensor is defined as \( s_{ij} = \sigma_{ij} - \delta_{ij} \sigma_H \) where \( \delta_{ij} \) is Kronecker’s multiplier. The effective fracture strain \( \varepsilon_{eq}^f \) is calculated from the initial value of the diameter, \( d_0 \), and the diameter after failure, \( d_f \):

\[ \varepsilon_{eq}^f = 2 \ln \frac{d_0}{d_f} \]  

(2.12)

Theoretical studies on the micromechanics, void growth and void coalescence for simple stress and strain states were later conducted, with the pioneering work of McClintock (1968) and Rice and Tracey (1969). McClintock modelled the microstructure in 2-D by an array of hollow circular cylinders in a matrix subjected to plastic deformation. The cylinders grew in volume and changed their shape depending on the stress state and when the cylinders touched each other fracture occurred. Rice and Tracey later developed a 3-D model, simulating the growth of a single spherical void in a matrix subjected to a remote stress field. Most criteria derived from the void growth models were of the type:

\[ D_{cr} = \int_{\varepsilon_{eq}^{th}}^{\varepsilon_{eq}^f} f \left( \frac{\sigma_H}{\sigma_{eq}} \right) d\varepsilon_{eq} \]  

(2.13)

which were a function of the triaxiality and the equivalent plastic strain. The damage evolution begins when the threshold strain \( \varepsilon_{eq}^{th} \) is reached, and when a critical value \( D_{cr} \) is attained fracture has occurred.

Physically, ductile failure occurs in an elasto-plastic material mainly by nucleation of voids from inclusions, void growth and void coalescence, Figure 2.2. The voids nucleate at second phase particles by either particle failure or decohesion from the matrix, and the voids grow due to the plastic straining and the hydrostatic stress state of the surrounding material. The void nucleation at a particle is strongly dependent on the bonding of the particle to the matrix. If the bonding is weak, as for manganese sulphides in steel, nucleation will occur at low stresses and strain. If the bonding is strong, as for carbides in steel, nucleation will first take place after large plastic straining, Bonora (1999). Fracture will occur when the
Figure 2.2: Ductile fracture due to void growth. The voids are initiated at second phase particles. When the voids have grown sufficiently and the strain localises between the voids, necking of the ligaments (between the voids) occurs and eventually the ligaments fracture.

Today, the list of different fracture models/criteria is long. Atkins (1997) discusses several different fracture criteria and damage models and has an extensive reference list. The ductile failure criteria and models were by Atkins (1997) divided into a group of four consisting of the empirical models, the void growth models, the continuum damage mechanics (CDM) and the porosity based material models. In the following, a few of the criteria and the models will be discussed briefly to give an overview of this wide field of research. For more comprehensive details reference is to be made to the specific papers.

2.3.1 Empirical Criteria

There are numerous empirical criteria for ductile failure and most of them are just simple maximum stress, strain, plastic work/volume criteria or combinations of these. A simple and a common fracture criterion in metal forming is the critical through thickness strain. When the through thickness strain reaches a critical value, typical of the material and the thickness, fracture will occur. The plastic work/volume is simply the integral \( \int \sigma_{eq} d\varepsilon_{eq} \), where \( \sigma_{eq} \) is the equivalent von Mises stress. When the plastic work/volume reaches a critical value, fracture is assumed to occur. The damage criterion by Cockeroff and Latham (1972) is a modified
work/volume model and is based on the first principal stress component, \( \sigma_1 \), giving

\[
D = \int \sigma_1 d\varepsilon_{eq}
\]  

(2.14)

Oh et al. (1979) modified the Cockcroft and Latham criterion to include the stress ratio:

\[
D = \int \frac{\sigma_1}{\sigma_{eq}} d\varepsilon_{eq}
\]  

(2.15)

The modified Cockcroft and Latham criterion has proved to be able to predict fracture for applications with negative and low triaxialities, which means bodies somehow subjected to a compressive stress state. Brozzo et al. (1972) modified the Cockcroft and Latham criterion to include an explicit dependence on the hydrostatic stress. Fracture is predicted to occur when the integrated product, \( D \), of the equivalent plastic strain increment and a function of the hydrostatic stress reaches a critical value, \( D_{cr} \), according to

\[
D = \int \frac{2\sigma_1}{3(\sigma_1 - \sigma_H)} d\varepsilon_{eq}
\]  

(2.16)

The critical value \( D_{cr} \) is to be considered as a material parameter. The criterion of Norris et al. (1978) is, as most of the empirical criteria, based on the hydrostatic stress and on the plastic strain, according to

\[
D = \int \frac{1}{1 - c_N \sigma_H} d\varepsilon_{eq}
\]  

(2.17)

where \( c_N \) is a material parameter. The criterion must be reached or exceeded in a zone of the size \( r_c \), which is a characteristic material parameter. Atkins (1981) proposed a damage indicator which is a modified version of the criterion by Norris et al. and takes the form

\[
D = \int \left(1 + \frac{1}{2 \frac{d\varepsilon_{eq}}{d\varepsilon_{eq}}}\right) \frac{1}{1 - c_A \sigma_H} d\varepsilon_{eq}
\]  

(2.18)

with \( \varepsilon_\theta \) and \( \varepsilon_z \) denoting respectively the circumferential and the axial strain of cylindrical test pieces in compression (or \( \frac{d\varepsilon_{eq}}{d\varepsilon_{eq}} \) in the general case). The parameter \( c_A \) is for calibration.

The general fracture strain model of Johnson and Cook (1985) includes the effects from the triaxial stress state, the strain, the strain rates and the temperature. The damage is given by

\[
D = \int \frac{d\varepsilon_{eq}}{\varepsilon_f}
\]  

(2.19)
where $\varepsilon_f$ is the equivalent strain to fracture, under the current conditions of strain rate, temperature and triaxial stress state. Fracture will take place when $D = 1.0$. The expression for the strain at fracture is given by:

$$
\varepsilon_f = \left[ D_1 + D_2 \exp \left( D_3 \frac{\sigma H}{\sigma_{eq}} \right) \right] \left[ 1 + D_4 \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right] [1 + D_5 T^*] \quad (2.20)
$$

where $D_1 \ldots D_5$ are calibration constants, $\dot{\varepsilon}_0$ is typically set to $1.0 s^{-1}$ and $T^*$ is the homologous temperature defined as

$$
T^* = \frac{T - T_{room}}{T_{melt} - T_{room}} \quad (2.21)
$$

If the strain rate ratio and the homologous temperature are equal to zero, the expression in the first brackets is similar to the Rice-Tracey void growth model (Rice and Tracey (1969), see Equation 2.25). To avoid problems when $\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} < 1.0$, Camacho and Ortiz (1997) modified the strain rate sensitivity term so that the Johnson and Cook damage criterion can be written as

$$
\varepsilon_f = \left[ D_1 + D_2 \exp \left( D_3 \frac{\sigma H}{\sigma_{eq}} \right) \right] \left[ 1 + \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right] D_4 [1 + D_5 T^*] \quad (2.22)
$$

Due to the relatively low velocity in ship collisions and ship groundings, the strain rate is low compared to other high-speed impact problems, like ballistic penetration. The adiabatic heating and the temperature dependence in the formula will therefore not contribute to material softening and will therefore not be used in the following low-speed impact simulations.

### 2.3.2 Void Growth Models

Many studies have been made on micromechanics to predict ductile failure, of which the analyses by McClintock and Rice-Tracey are the two classical void growth models. McClintock (1968) modelled in 2-D a series of uniformly distributed cylindrical holes with the same radius subjected to a remote stress field. The growth of the voids was found to be described by

$$
\frac{\dot{R}}{R} = \frac{\dot{\varepsilon}_{eq} \sqrt{3}}{2(1 - n) \sigma_{eq}} \sinh \frac{(1 - n) \sigma_H \sqrt{3}}{\sigma_{eq}} \quad (2.23)
$$

where $R$ and $\dot{R}$ are respectively the radius and the growth rate of the radius for the cylinders. The parameter $n$ is the hardening exponent in the strain hardening law, according to $\sigma =
2.3 Ductile Fracture Models

In a simplified version of the McClintock model, a function $D$ describes the damage due to void growth according to

$$D = \int \frac{\sigma_H}{\sigma_{eq}} d\varepsilon_{eq}$$

(2.24)

When $D$ has reached the critical value, $D_{cr}$, void coalescence occurs and the specimen fails. Rice and Tracey (1969) studied the growth of one single spherical void subjected to a remote stress and strain field. They derived a model for the void growth rate in a stress-strain field, where they assumed that void coalescence occurs when the void radius has reached a critical value. The void growth was modelled according to

$$\ln \left( \frac{R}{R_0} \right) = \int_{\varepsilon_{nuc}}^{\varepsilon_{coa}} \alpha_{RT} \exp \left( \frac{3\sigma_H}{2\sigma_{eq}} \right) d\varepsilon_{eq}$$

(2.25)

where $R$ and $R_0$ are the current and the initial void radii, $\varepsilon_{nuc}$ and $\varepsilon_{coa}$ are the strains at nucleation and at void coalescence respectively, and $\alpha_{RT}$ is a material constant. Rice-Tracey found $\alpha_{RT}$ to be 0.283 for mild steel, whereas other authors have later found $\alpha_{RT}$ to be 0.427 (Atkins (1997)). Void coalescence occurs when the void growth ratio exceeds a critical value $R/R_0 > (R/R_0)_c$. The parameter $\alpha_{RT}$ is not critical as it appears as a proportional parameter and has to be found from experiments together with the parameter $(R/R_0)_c$. The Rice-Tracey void model is often formulated as a damage parameter, (Hancock and Mackenzie (1976):

$$D = \int \alpha_{HM} \exp \left( c_{HM} \frac{\sigma_H}{\sigma_{eq}} \right) d\varepsilon_{eq}$$

(2.26)

With the parameter $c_{HM}$ equal to 3/2, the model is similar to the Rice and Tracey void growth model. However, many experimental results have shown that with $c_{HM}$ equal to 2.0 better agreement is achieved. $\alpha_{HM}$ is a calibration constant.

The damage work model described by Chaouadi et al. (1994) uses the plastic work per unit volume, in which they include the volume change due to the void growth process. They use the Rice-Tracey model to describe the void growth, which can be related to the volume change and the damage work, $dW_D$, and is described as

$$dW_D = \left[ 1 + \frac{3\alpha_c\sigma_H}{\sigma_{eq}} \exp \left( \frac{3\sigma_H}{2\sigma_{eq}} \right) \right] \sigma_{eq} d\varepsilon_{eq}$$

(2.27)

Fracture is said to occur when the damage work reaches a critical value $W_{D,\text{crit}}$ which is a characteristic material parameter. Here, the factor $\alpha_c$ is an experimental fitting constant.
LeRoy et al. (1981) proposed a ductile fracture model by examining the void growth and the accumulation of damage during tensile loading in carbon steel, giving the formula:

$$ D = \int (\sigma_1 - \sigma_H) d\varepsilon $$

(2.28)

In all of the above described criteria, both empirical and void growth models, the damage is not coupled with the constitutive material law. This is clearly a simplification of describing the damage behaviour in the material, nevertheless, due to the simplicity this is very common in all kinds of applications.

### 2.3.3 Continuum Damage Mechanics

Two kinds of approaches of coupled constitutive law and ductile failure are mainly used today: the porosity-type model as proposed by Gurson and the continuum damage mechanics (CDM) approach first proposed by Kachanov (1958). The CDM model proposed by Lemaitre (1985) is a coupled model of elasto-plasticity and isotropic damage. The damage evolution is based on a continuous damage variable $D$, which in Lemaitre’s damage model is a linear function of the plastic strain. Damage takes into account the progressive degradation of the material properties and the loss of stiffness due to the irreversible process of void nucleation and growth. If it is assumed that microcracks and voids are scattered in an isotropic way, then the damage can be represented by a scalar parameter $D$ and thus orientation independent.

The damage variable is based on the effect of damage on the elastic properties:

$$ D = 1 - \frac{E_{eff}}{E_0} $$

(2.29)

or equally described as loss of the load carrying area:

$$ D = 1 - \frac{A_{eff}}{A_0} $$

(2.30)

where $E_{eff}$ and $A_{eff}$ are the current effective Young’s modulus and area respectively. $E_0$ and $A_0$ are the initial Young’s modulus and area. In the framework of CDM, it is assumed that a damage dissipation potential $F_D$ exists, and in case of plasticity damage, the total dissipation work potential is given as

$$ F = F_p(\tilde{\sigma}_{ij}, H) + F_D(Y, \varepsilon_{eq}, D) $$

(2.31)

where $Y$ is the internal variable associated with damage. $F_p$ is the dissipation potential associated with the plastic deformation, which is a function of the actual stress $\tilde{\sigma}$ and the
isotropic hardening stress $H$, which is a function of the hardening variable $r$. The plastic dissipation potential for a standard plasticity with the von Mises yield criterion then becomes

$$F_p(\sigma_{ij}, H, D) = \sqrt{\frac{3}{2} s_{ij} s_{ij}} - H(r) - \sigma_y$$  \hspace{1cm} (2.32)

Lemaitre made a hypothesis of strain equivalence:

$$\varepsilon = \frac{\bar{\sigma}}{E} = \frac{\sigma}{(1-D)E}$$  \hspace{1cm} (2.33)

which states that the strain behaviour of the damaged material is represented by the constitutive equations of the virgin material ($D = 0$). The effective stress, $\bar{\sigma}$, is the stress calculated over the section that effectively resists the forces:

$$\bar{\sigma} = \frac{\sigma}{1 - D}$$  \hspace{1cm} (2.34)

The evolution of the damage is given by

$$\dot{D} = -\lambda \frac{\partial F_D}{\partial Y}$$  \hspace{1cm} (2.35)

where $\lambda$ is the plastic multiplier and $Y$ is the strain energy density release rate:

$$Y = \frac{\sigma_{eq}^2 f\left(\frac{\sigma_H}{\sigma_{eq}}\right)}{2E(1-D)^2}$$  \hspace{1cm} (2.36)

with the triaxiality function defined as

$$f\left(\frac{\sigma_H}{\sigma_{eq}}\right) = \frac{2}{3}(1 + \nu) + 3(1 - 2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2$$  \hspace{1cm} (2.37)

with $\nu$ as Poisson’s ratio. The expression of the damage potential $F_D$ is found mostly from phenomenological considerations. Lemaitre proposed a damage potential according to

$$F_D = \frac{S}{s_0 + 1} \left(\frac{-Y}{S}\right)^{s_0 + 1} (1 - D)^{-1}$$  \hspace{1cm} (2.38)
where $S$ and $s_0$ are material constants and where $s_0$ is often taken to be 0. The damage evolution in the Lemaitre damage model is thus given by

$$
\dot{D} = \begin{cases} 
0 & r \leq r_D \\
\frac{Y}{S(1-D)} \dot{\varepsilon} & r > r_D 
\end{cases} 
(2.39)
$$

where $\dot{D}$ and $r_D$ are the damage rate and the damage threshold respectively. The hardening variable $\dot{r}$ is found from

$$
\dot{r} = \dot{\varepsilon}_{eq}(1 - D) 
(2.40)
$$

When the damage variable reaches a critical value $D_{cr}$ which is considered as a material parameter, it is assumed that a macrocrack occurs in the material.

Several experimental observations have shown that the growth of microvoids results in a nonlinear damage accumulation with plastic straining. Chandrakanth and Pandey (1994) and Tai and Yang (1986) proposed nonlinear damage models for specific materials, and more recently Bonora (1997) proposed an isotropic nonlinear damage model that can describe the damage evolution for different materials. The damage law given by Bonora is as follows:

$$
dD = \alpha \frac{(D_{cr} - D_0)^{\frac{1}{\alpha}}}{\ln \varepsilon_f - \ln \varepsilon_{th}} f \left( \frac{\frac{\sigma_{eq}}{\sigma_{eq}}} {D_{cr} - D} \right) \left( \frac{\alpha - 1}{\alpha} \right) \frac{dD}{dr} 
(2.41)
$$

Here $D_{cr}$ and $\varepsilon_f$ are respectively the critical value of the damage variable and the uniaxial strain for which ductile fracture occurs. The uniaxial damage strain threshold $\varepsilon_{th}$ indicates when void nucleation is initiated. The damage exponent, $\alpha$, characterises the nonlinearity of the damage evolution, and the triaxiality function is the same as in Lemaitre’s model, Equation 2.37. The parameter $D_0$ defines the initial damage in the reference volume and is mostly set to zero.

From experiments on spheroidised carbon steel (LeRoy et al. (1981)), it is known that the damage growth is very slow initially for growth of the first population of voids and, after a certain plastic straining, the damage increases rapidly. This is partly due to necking formed in the specimen as the triaxiality rises with necking. Tai (1990) and Bonora (1997) describe the different damage stages in the metals, and in Figure 2.3 three different damage evolutions with varying damage exponents $\alpha$ according to Bonora’s damage model (Equation 2.41) are shown.

In metals with a low $\alpha$, as pure copper, it has been experimentally observed that a few voids nucleate when the threshold strain is reached and with further straining only a few more voids nucleate, while the existing voids grow, causing the damage accumulation to grow.
slowly. When the strain reaches the $\varepsilon_{cr}$, the larger voids coalesce rapidly and fracture occurs. Mild steel typically has an $\alpha$ value of about 0.2 and is characterised by voids nucleating at the threshold strain, and at increasing strain more voids are initiated while the existing voids grow. In different aluminium alloys ($\alpha = 0.8$) a large number of voids nucleate at the threshold strain and with increasing strain the void size is almost kept constant while more new voids nucleate.

In different experiments several materials have shown an anisotropic damage behaviour and several damage models have been developed that includes the tensorial damage, e.g. Murakami (1987), Chow and Wang (1987) and Lemaitre et al. (2000).

### 2.3.4 Porosity Based Models

Oyane (1972) stated that ductile fracture occurs when the volume fraction of voids in the material reaches a critical value. He derived a modified von Mises yield criterion to describe the porous plasticity. In a simplified model the damage can be described according to

$$D = \int_{\varepsilon_{vi}}^{\varepsilon_{ef}} \left( 1 + \frac{\sigma_H}{a_0\sigma_{eq}} \right) d\varepsilon_{eq}$$  \hspace{1cm} (2.42)

where $a_0$ is a material constant. Gurson (1977) proposed a continuum model for porous plasticity based on a material containing microcavities in a plastic stress field. It was based on the study of the evolution of a single spherical void in a ductile and incompressible matrix, which was considered as rigid perfectly plastic. The yield condition is described as

$$F(\sigma_{eq}, \sigma_{kk}, f, \sigma_y) = \left( \frac{\sigma_{eq}}{\sigma_y} \right)^2 + 2f \cosh \left( \frac{3\sigma_m}{2\sigma_y} \right) - (1 + f^2)$$  \hspace{1cm} (2.43)
where \( f \) is the ratio of the void to the total volume and \( \sigma_{eq} \) and \( \sigma_H \) are the equivalent von Mises stress and the hydrostatic pressure respectively. \( \sigma_y \) is the yield stress of the matrix material.

The porosity \( f \) reduces the stress carrying capacity for macroscopic stresses and when the porosity \( f \) reaches 1, the yield surface has been reduced to a point. When the porosity \( f \) is zero, the yield function is reduced to the standard von Mises yield function. The porosity evolution is assumed to be governed by growth of existing voids for which the volume fraction void can be determined by assuming plastic incompressibility of the matrix material. Tvergaard and Needleman (1984) modified the Gurson model, hereafter denoted GTN, in order to include multivoid arrays and to fit relevant experimental data:

\[
F(\sigma_{eq}, \sigma_{kk}, f, \sigma_y) = \left( \frac{\sigma_{eq}}{\sigma_y} \right)^2 + 2q_1 f^* \cosh \left( q_2 \frac{3\sigma_m}{2\sigma_y} \right) - 1 - (q_3 f^*)^2
\]

(2.44)

where \( q_1, q_2 \) and \( q_3 \) are calibration constants. To account for the rapid growth of the porosity during the coalescence of the voids Tvergaard and Needleman defined the porosity evolution process as

\[
f^* = \begin{cases} 
  f & f \leq f_c \\
  f_c + K(f - f_c) & f > f_c
\end{cases}
\]

(2.45)

where

\[
K = \frac{f^*_f - f_c}{f_f - f_c}
\]

(2.46)

\( f_c \) is the value at which the coalescence process is initiated, \( f_f \) is the porosity value at fracture and \( f^*_u \) is the ultimate porosity value. The void volume grows from nucleation of new voids as well as from expansion of existing voids:

\[
\dot{f} = \dot{f}_{\text{nucleation}} + \dot{f}_{\text{growth}}
\]

(2.47)

The nucleation of voids is assumed to be initiated at a certain strain level, Chu and Needleman (1980), according to

\[
\dot{f}_{\text{nucleation}} = A_N \dot{\varepsilon}^p
\]

(2.48)

where

\[
A_N(\varepsilon_{eq}^p) = \frac{f_N}{S_N \sqrt{2\pi}} \exp \left[ \frac{1}{2} \left( \frac{\varepsilon_{eq}^p - \varepsilon_N}{S_N} \right)^2 \right]
\]

(2.49)
2.4 Fracture Models Used in Simulations of Ship Collisions and Groundings

$S_N$ and $\varepsilon_N$ are respectively the standard deviation and the mean value.

The damage mechanics model proposed by Rousselier (1987) is a combination of the damage mechanics models and the porosity models. This model is derived from the free energy and the plastic potential and it allows a more general formulation than Gurson’s. The yield condition is found according to

$$F(\sigma_{eq}, \sigma_{kk}, f, \sigma_y) = \frac{\sigma_{eq}}{(1 - f)\sigma_y} + \frac{\sigma_y}{\sigma_y} f D \exp \left( \frac{\sigma_{kk}}{3(1 - f)\sigma_1} \right) - 1$$

(2.50)

2.4 Fracture Models Used in Simulations of Ship Collisions and Groundings

As earlier mentioned the most commonly used fracture criterion in ship collision and grounding analysis has been the equivalent plastic strain. However, as previously discussed the strain at fracture depends on the stress state, which in turn is influenced by various parameters, such as geometry, loading, local yielding, bending, welding, boundary and so on, and thus often varies for each structural member in the ship’s structure. In addition the fracture strain is also often mesh-dependent, and to overcome this the critical plastic strain values for various element sizes are often given as in Figure 2.4. Simulations of tensile tests with different mesh discretisation are used to derive the critical plastic strain value for a specific element size. The curve in Figure 2.4 is taken from a collision simulation between two VLCC’s by Kitamura (2001a).

The plastic strain criterion has been very common due to its simplicity and as it is often the only implemented fracture criterion in finite element codes. By using the plastic strain the effects of the stress state on the fracture strain will not be included and thus the results will be less sensitive to the structural arrangements. A few other fracture criteria have been used in numerical simulations of accidental loading of ships and offshore structures. Kitamura (1997) used a formula according to

$$D(t) = \int_0^t G(\alpha_t) \frac{1}{\varepsilon_f(t)} \frac{d\sigma(t)}{dt} dt \leq 1.0$$

$$G(\alpha_t) = \min \left[ \frac{\alpha_t}{\alpha_{t1}}, 1.0 \right]$$

(2.51)

$$\alpha_{t1} = \tan^{-1}\left( \frac{2}{3} \right) \quad \alpha_t = \tan^{-1}\left( \frac{\sigma_{eq}}{\sigma_H} \right)$$

where $G(\alpha_t)$ is the pressure coefficient and $\varepsilon_f(t)$ is the equivalent breaking strain at time $t$. When the cumulative strain factor $D(t)$ exceeds 1.0, the plate is assumed to have fractured. As seen in Equation 2.51, the stress triaxiality is accounted for in the $G(\alpha_t)$ function.
Lehmann and Yu (1998) discussed a simple failure criterion designed for simulations of ship collisions and groundings. They used a rupture index $I_R$ defined as

$$I_R = f \left( \frac{\sigma_H}{\sigma_{eq}} \right)^m \varepsilon_v$$

where $\varepsilon_v$ is the maximum effective strain and $m_L = 2n + 1$, with the hardening coefficient $n$. $f(\frac{\sigma_H}{\sigma_{eq}})$ is a triaxiality function defined according to Equation 2.37. The critical value of $I_R$ is found from simulations of simple tensile tests and when it is exceeded, fracture is assumed to have occurred. This model is based on the CDM model by Lemaitre and experimental data analysed by Bridgman’s formula, see Equations 2.9 and 2.12.

Another approach has been proposed by Germanischer Lloyd AG in the framework of the CrashCoaster project (Vredevelt and Feenstra (2001)). The criterion is based on measurements of the through thickness strain on damaged plating from actual ship collisions and grounding events. Both the mean value of the measured uniform through thickness strain and the necking strain are used in the evaluation, and average values of 5.6% and 54% are found for the uniform strain and for the necking strain respectively. A computational failure strain $\varepsilon_f$ for different element sizes can be determined as

$$\varepsilon_f(l_e) = \varepsilon_g + \frac{\varepsilon_n t}{l_e}$$

where $\varepsilon_g$ and $\varepsilon_n$ are the uniform through thickness strain and the necking strain respectively with the values $\varepsilon_g = 0.056$ and $\varepsilon_n = 0.54$. The plate thickness is denoted $t$ and $l_e$ is the element size in the strain direction.
2.5 Proposal of a Fracture Criterion

Many of the empirical, void growth models, CDM as well as the porosity based fracture models, include a triaxiality function \( f\left(\frac{\sigma_H}{\sigma_{eq}}\right) \) in the damage model. Wierzbicki and Werner (1998) showed that the Cockcroft and Latham criterion can be expressed in the form of \( \int f\left(\frac{\sigma_H}{\sigma_{eq}}\right) d\varepsilon \) and therefore also includes a description of the void growth due to the hydrostatic stress state. The derivation of the triaxiality function was based on experimental observations by Kudo and Aoi (1967), who performed axial compression of solid cylinders, the so-called upsetting test. They observed that the points of failure in the principal strain space form a straight line with a slope of -1/2. These results were later confirmed and extended by Kuhn and Dieter (1977) and Thomason (1990). The experimental failure line suggests a fracture locus in the principal strain given by

\[
\varepsilon_1 + \frac{1}{2} \varepsilon_2 = k
\]

where \( k \) is the intercept of the \( \varepsilon_1 \) axis in a fracture forming diagram (FFLD). By use of the \( J_2 \) flow theory, assumption of plane stress state in the \((1,2)\)-plane, \( \sigma_{3j} = 0 \), and with negligible shear components, \( \sigma_{12} \approx 0 \), the triaxiality function for the Cockcroft and Latham damage criterion can be written as (Wierzbicki and Werner (1998))

\[
f\left(\frac{\sigma_H}{\sigma_{eq}}\right) = 2 \frac{1 + \frac{\sigma_H}{\sigma_{eq}} \sqrt{12 - 27\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2}}{3 \frac{\sigma_H}{\sigma_{eq}} + \sqrt{12 - 27\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2}}
\]

The formula is only valid for plane stress and in low triaxialities, and at \( \frac{\sigma_H}{\sigma_{eq}} > \frac{2}{3} \) the mathematical description of the formula fails. Nevertheless, the triaxiality function (Equation 2.55) describes that material subjected to homogenous compression (i.e. \( \frac{\sigma_H}{\sigma_{eq}} < -\frac{1}{3} \)) will not fracture, which has been experimentally observed. The triaxiality functions for some of the earlier described criteria and models are plotted in Figure 2.5.

As it is seen, the Cockcroft and Latham criterion describes that fracture will not occur in homogenous hydrostatic compression, while for larger negative values the damage accumulation will grow for the other criteria. The triaxiality function in the CDM models, Equation 2.37, is quadratic and therefore the damage accumulation rate will incorrectly increase for larger negative values. However, most of the void growth, CDM and porosity models are developed for positive triaxialities as they are derived to model the void growth and not ductile shear fracture in the material. Gänser et al. (2000) with success used the Rice-Tracey criteria in
negative triaxialities in an upsetting test of an aluminium alloy, where they used a coefficient of 2 inside the exponential function (Equation 2.26), which had been experimentally observed by Arndt (1997). Bao and Wierzbicki (2002) carried out several tests on aluminium alloys at different triaxialities and drew a failure strain curve for triaxialities varying from -1/3 to 1. They also studied several criteria for the different experimental setups.

For positive triaxialities, void growth criteria and especially the Rice-Tracey criterion are able to model void growth correctly. However, the model predicts that damage can occur also for triaxialities equal and lower than -1/3, where fracture will not occur in practice. The void growth damage models can be modified to have a cutoff at a triaxiality of -1/3, to avoid damage accumulation increasing in pure hydrostatic compression. Gänsler et al. (2000) showed by FE-calculations that the Rice-Tracey criterion predicts fracture initiation reasonably well for upsetting tests, however, as they only post-processed the surface elements, they would not detect failure in the centre of the specimen. Without a cutoff in the damage function at a triaxiality of -1/3, elements in the centre of the specimen would probably fail. Nevertheless, based on the results of Gänsler et al., a Rice-Tracey damage function with cutoff seems to be able to predict ductile failure in aluminium for a large range of triaxialities.

The Cockcroft-Latham criterion is believed to be able to predict ductile shear fracture correctly, while the Rice-Tracey criterion will model fracture due to void growth correctly. A combined Rice-Tracey and Cockcroft-Latham criterion (RTCL) is a natural combination that will switch between the two triaxiality functions at a triaxiality of 1/3 and thus cover the whole range of triaxialities. Only one parameter has to be determined to calibrate the RTCL criterion as the criteria by Rice-Tracey and Cockcroft-Latham will give the same damage value in a uniaxial tensile test with a constant triaxiality of 1/3. The criterion is then given as
\[ D = \int f \left( \frac{\sigma_H}{\sigma_{eq}} \right)_{RTCL} \, d\varepsilon \] 

where

\[
f \left( \frac{\sigma_H}{\sigma_{eq}} \right)_{RTCL} = \begin{cases} 
0 & \text{for } \frac{\sigma_H}{\sigma_{eq}} \leq -\frac{1}{3} \\
\frac{1}{2} \left[ 1 + \frac{\sigma_H}{\sigma_{eq}} \sqrt{12 - 27 \left( \frac{\sigma_H}{\sigma_{eq}} \right)^2} \right] & \text{for } -\frac{1}{3} < \frac{\sigma_H}{\sigma_{eq}} < \frac{1}{3} \\
\frac{1}{1.65} \exp \left( \frac{3\sigma_H}{2\sigma_{eq}} \right) & \text{for } \frac{\sigma_H}{\sigma_{eq}} \geq \frac{1}{3}
\end{cases}
\] 

\[ (2.56) \]

\[ (2.57) \]

2.6 Constitutive Material Models with Fracture

In the empirical and the void growth models, the constitutive law can be described by using e.g. the von Mises flow rule. This means that the damage and the constitutive relations are uncoupled, whereas this is not the case in the damage mechanics and the porous plasticity. Fracture initiation can be studied for the uncoupled models by most FE-programs by means of a post processor. However, for simulating effects due to fracture, such as crack propagation, the criteria have to be implemented in the FE-code.

Several of the previously described fracture criteria and models have been implemented by the author into the explicit finite element program LS-DYNA\(^2\) for brick and shell elements, by use of a user-defined material model. This will allow the models and the criteria to be studied and compared for different structures subjected to various loadings and stress states.

The constitutive material model for the empirical and the void growth criteria is based on the \( J_2 \) flow theory with isotropic hardening by means of the radial return algorithm, Ortiz and Popov (1985). The implementation follows the description by Hallquist (1998) and is given in Appendix B. The implemented material model has different ways of describing the strain hardening by either bilinear elasto-plasticity, power law hardening or by an arbitrary user-defined hardening curve. Strain rate effects can be included in the simulations by use of a plastic strain rate dependent scaling law (Cowper and Symonds (1957)) or by using a rate dependent plasticity option, see Section 3.5. For shell elements the through thickness

---

\(^2\)In Appendix A the main principles of the explicit finite element method as well as the main shell elements used in the thesis with the commercial finite element code LS-DYNA are briefly described.
strain is calculated by an iterative algorithm to be able to simulate the thickness reduction and the necking in sheets.

The empirical and the void growth fracture criteria are implemented into LS-DYNA as normalised ductile failure damage indicators $D_i$, following the definition by Fischer et al. (1995). For the Rice-Tracey criterion the normalised damage indicator, $D_i$, becomes

$$D_i = \frac{1}{1.65\varepsilon_0} \int \exp \left( \frac{3\sigma_H}{2\sigma_{eq}} \right) d\varepsilon_{eq}^p$$

(2.58)

where $\varepsilon_0$ is the uniaxial damage strain and when $D_i = 1$ the critical damage state is reached. This is a very convenient way to define damage in FE-programs as the damage state can easily be determined in the post-processing. The fracture criteria are coupled with the element kill option in LS-DYNA, which removes the element from the calculation when the damage indicator reaches unity. This means that crack propagation can, at least roughly, be simulated.

The CDM models by Lemaitre and Bonora have also been implemented into LS-DYNA (see Appendix B.1.4) following the description by Berstad et al. (1999) for shells and solid elements. The GTN model, coded by Simonsen and Li (2003), has been implemented for bricks by the author. These models are also coupled with the element kill feature in LS-DYNA in order to simulate cracks and crack propagation.

One of the major problems with different damage models is that numerical simulations can show an inherent mesh sensitivity, especially shell elements, since close to the point of failure the damage will tend to localise in regions as narrow as possible within the mesh size. Unless a finite material length is included in the damage model, the damage will localise in an unphysical narrow band and the fracture prediction will be mesh-dependent. In the nonlocal damage theory the parameters influencing the strain softening are controlled, which leaves the strains and stresses in the constitutive material law unchanged, Bazant and Pijaudier-Cabot (1988). The damage volume is averaged over the reference volume element controlled by the material length scale by using a weighting function, Tvergaard and Needleman (1995). The mesh size sensitivity on failure is reduced, leading to results which converge as the mesh is refined. A nonlocal algorithm is available in LS-DYNA which can be used for the CDM models and the porosity model implemented with the user-defined material models. The uncoupled damage indicators can also be used with the nonlocal algorithm. However, as the damage is not coupled with the constitutive law, the mesh sensitivity can only be marginally reduced.

A more practical way to overcome the mesh dependence for the damage models, especially when shell elements are used, is to use a critical damage value versus element size curve, Figure 2.6(b). As the element size for the same material and thickness usually varies for simulations of large structures, i.e. ships and oil rigs, it would be very time-consuming to give the correct fracture value for each element size. An additional option is therefore
<table>
<thead>
<tr>
<th>Criterion/model</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>McClintock (1968)</td>
<td>( D_i = \frac{1}{\varepsilon_0} \int \frac{\sigma_H}{\sigma_{eq}} d\varepsilon_{eq} )</td>
</tr>
<tr>
<td>Rice and Tracey (1969)</td>
<td>( D_i = \frac{1}{\beta_{6690}} \int \exp \left( \frac{3\sigma_H}{2\sigma_{eq}} \right) d\varepsilon_{eq} )</td>
</tr>
<tr>
<td>Norris et al. (1978)</td>
<td>( D_i = \frac{1}{\varepsilon_0} \int \frac{1}{(1-c_N\sigma_H)} d\varepsilon_{eq} )</td>
</tr>
<tr>
<td>Damage Work - Chaouadi et al. (1994)</td>
<td>( D_i = \int \left[ 1 + 3a_H \frac{\sigma_H}{\sigma_{eq}} \exp \left( \frac{3\sigma_H}{2\sigma_{eq}} \right) \right] \frac{\sigma_{eq}}{W_0} d\varepsilon_{eq} )</td>
</tr>
<tr>
<td>LeRoy et al. (1981)</td>
<td>( D_i = \frac{1}{W_0} \int (\sigma_1 - \sigma_H) d\varepsilon_{eq} )</td>
</tr>
<tr>
<td>Brozzo et al. (1972)</td>
<td>( D_i = \frac{1}{\varepsilon_0} \int \frac{2\sigma_1}{3(\sigma_1 - \sigma_H)} d\varepsilon_{eq} )</td>
</tr>
<tr>
<td>Cockcroft and Latham (1972)</td>
<td>( D_i = \frac{1}{\varepsilon_0} \int \frac{\sigma_1}{\sigma_{eq}} d\varepsilon_{eq} )</td>
</tr>
<tr>
<td>( \text{RTCL (see Section 2.5)} )</td>
<td></td>
</tr>
</tbody>
</table>
|                                             \( f \left( \frac{\sigma_H}{\sigma_{eq}} \right)_{\text{RTCL}} = \begin{cases} 
0 & \text{for } \frac{\sigma_H}{\sigma_{eq}} \leq -\frac{1}{3} \\
\frac{1 + \frac{2\mu}{\sigma_{eq}} \sqrt{12 - 27 \left( \frac{2\mu}{\sigma_{eq}} \right)^2}}{3 \frac{2\mu}{\sigma_{eq}} + \sqrt{12 - 27 \left( \frac{2\mu}{\sigma_{eq}} \right)^2}} & \text{for } -\frac{1}{3} < \frac{\sigma_H}{\sigma_{eq}} < \frac{1}{3} \\
\frac{1}{1.65} \exp \left( \frac{3\sigma_H}{2\sigma_{eq}} \right) & \text{for } \frac{\sigma_H}{\sigma_{eq}} \geq \frac{1}{3} \end{cases} \) |                                                                                     |
| Johnson and Cook (1985)                             | \( \varepsilon_f = [D_1 + D_2 \exp \left( D_3 \frac{\mu}{\sigma_{eq}} \right)] \left[ 1 + \left( \frac{\varepsilon}{\varepsilon_0} \right) \right]^{D_4} \left[ 1 + D_5 T^* \right] \) |
| CDM - Lemaitre (1985)                               | \( dD = \frac{\partial_x \left( \frac{\mu}{\sigma_{eq}} \right)}{2 K_0} d\varepsilon_{eq} \) |
| CDM - Bonora (1997)                                 | \( dD = \alpha \frac{(D_{cr} - D_0)^\frac{1}{2}}{\ln f - \ln e_{eq}} \int f \left( \frac{\mu}{\sigma_{eq}} \right) (D_{cr} - D)^{\left( \alpha - 1 \right)} d\varepsilon_{eq} \) |
| GTN - Tvergaard and Needleman (1984)                 | \( F(\sigma_{eq}, \sigma_h, f, \sigma_y) = \left( \frac{\sigma_{eq}}{\sigma_y} \right)^2 + 2q_1 f^* \cosh \left( q_2 \frac{3\sigma_{eq}}{\sigma_y} \right) - 1 - (q_3 f^*)^2 \)}
Chapter 2. Ductile Fracture in Metals

Figure 2.6: (a): The damage value at the experimental point of fracture. (b): The damage index just before the point of fracture, when the critical damage value versus element length option was used.

implemented into LS-DYNA by the author, where it is assumed that the element aspect ratio at the start of the simulation is nearly 1:1. The initial area of a specific element is calculated in the initialisation of the simulation and the square root of the element area will thus give a reference element length. The critical damage values to the element sizes are given into the finite element program as a user-defined curve, and at the initiation of the simulation the damage value for a specific element is found from interpolation of the curve points. In Figure 2.6 an example is given of a simulation of five tensile test specimens with varying element sizes. The left figure shows the critical damage value at the experimental point of fracture, and the left figure shows the damage indicator value at the same deflection where the above described option was used. Figure 2.7 shows the force-deflection curve obtained from the tensile test simulation of the specimens in Figure 2.6 as well as the critical damage value versus the element size. It is seen that the specimens fracture at the same strain independent of the element size.

The different material and damage models have also been implemented into an MPP\(^3\) (Massively Parallel Processing) version of LS-DYNA.

\(^3\)In an MPP simulation the problem is divided into several CPU’s to reduce the calculation time. An MPP system can consist of very large numbers of processors (200 or more) that are loosely coupled. This means that each CPU has its own memory and devices and runs its own operating system.
2.7 Summary of the Different Fracture Criteria/Models

The Cockcroft-Latham model has been shown to model ductile fracture for low triaxialities and is believed to be able to predict ductile shear fracture correctly, while the Rice-Tracey criterion will model fracture due to void growth correctly. With the combined Rice-Tracey and Cockcroft-Latham criterion (RTCL) the whole range of triaxialities is covered. Only one parameter has to be determined to calibrate the RTCL criterion as the criteria by Rice-Tracey and Cockcroft-Latham will give the same damage value in a uniaxial tensile test with a constant triaxiality of 1/3.

Several tests have, however, to be conducted to determine the constants \( D_1 \ldots D_5 \) in the Johnson and Cook criterion, Equation 2.22. However, when these constants have been found, the model is believed to be useful in many impact problems. The Johnson-Cook criterion coupled with a CDM model has also been shown to be a useful tool in simulations of projectile penetration of thick plates, Børvik et al. (2001). The variations of the CDM models are today great as well as the application areas for the models. The CDM model by Bonora has been used on skin core debonding and core crack propagation which are the dominating mechanisms in the collapse modes of sandwich structures, Berggreen et al. (2003). The reference volume element (RVE) includes several cells in the core material and the idea is to reduce progressively the stiffness of the RVE as material between the cells fractures. When the fracture stress in the RVE is reached, the core material flows as perfect elasto-plastic material, where the damage function progressively reduces the stiffness of the RVE until total fracture occurs. The ”plastic” strain is less than 2% and should only simulate the cohesive zone in the RVE. A remarkably good agreement was found between the results of the damage mechanics method and a more classical mixed-mode fracture mechanics method, Berggreen et al. (2003).
MacKenzie et al. (1977) discussed that a certain volume of material is needed to predict failure in a non-uniform field. The aim of the different damage models is to describe a discontinuous state (microcracks and microcavities) by a continuous macroscopic variable, and the mean values in a reference volume are usually sufficient to represent the defect characteristics. The representative volume element in continuum mechanics is the smallest volume in which a density may represent a field of discontinuous properties. The size of the representative volume element depends therefore on the size of cavities, voids or other defects and varies for different materials. In Lemaitre and Dufailly (1987) different length scales for four materials were given, Table 2.2.

<table>
<thead>
<tr>
<th>Material</th>
<th>Volume element size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metals</td>
<td>0.05 - 0.5 mm</td>
</tr>
<tr>
<td>Polymers</td>
<td>0.1 - 1.0 mm</td>
</tr>
<tr>
<td>Wood</td>
<td>1.0 - 10.0 mm</td>
</tr>
<tr>
<td>Concrete</td>
<td>10.0 - 100.0 mm</td>
</tr>
</tbody>
</table>

The main reason for not using more advanced damage models coupled with the constitutive material law is the difficulties in finding the many material parameters for the CDM models and the porosity model by Gurson. The main disadvantage of the GTN model is also that the many parameters cannot be determined directly by simple tensile tests. Some of the parameters can only be estimated on the basis of metallurgical observations, such as the volume fraction of voids. Moreover, the coupling between the damage and the constitutive material laws for the CDM models and the GTN model results in a much more CPU demanding material algorithm.

The criteria by Lehmann and Yu (1998) and Germanischer Lloyd together with the failure strain curve (2.4) have several limitations and are questionable in crash simulations. Most important is that the failure strain curve and the criteria by Germanischer Lloyd do not have the effects of the triaxial stress state included in the models. The damage criterion by Lehmann and Yu (1998) is based on the CDM model by Lemaitre (1985). The criterion includes the triaxiality function, Equation 2.37, which has a quadratic term of the triaxiality and thus the damage accumulation rate will increase for larger negative triaxialities, which is in contradiction with experimental observations. Moreover, the criterion is load path independent, which is contrary to experimental observations.

2.8 Numerical Simulation of Crack Propagation

As seen, there are several possible models/criteria for analysing ductile fracture initiation in large structures. There are some common methods for implementation of the damage
2.8 Numerical Simulation of Crack Propagation

criteria and models in order to simulate crack propagation by a finite element program.

**Remeshing** - A new mesh is built around the propagating crack front. This is a very CPU demanding option as a new stiffness/topology matrix has to be created frequently.

**Stress/Stiffness reduction** - When an element has reached the failure criterion, the stress or the stiffness is removed or decreased over several time steps until the stiffness is completely removed. The material softening/stiffness reduction can also follow a damage evolution law and can be a convenient way of simulating ductile fracture. In simulations of impact, where contact is important, the element will however still be active in the contact algorithm, which can cause un-physical results.

**Node release** - Each adjacent element has completely independent nodes. Groups of nodes are initially constrained to move together, and when the average failure variable of the adjacent element reaches the critical value of the chosen criterion, the constraints are eliminated and the element separates.

**Element kill option** - When an element has reached the failure criterion value, the element is deleted from the calculation. This will often cause elastic stress waves in the structure as the stiffness is suddenly reduced. The element kill algorithm can be coupled with the stiffness reduction and is sometimes a useful method in simulations of fracture initiation and crack propagation.

**Mesh-free methods** - There are alternative numerical methods for simulating crack propagation, so-called mesh-free methods (the Reproducing Kernel Particle Method, see Hao et al. (2000)), where problems with distorted elements and mesh discretisation are somewhat avoided.

In the following finite element simulations the element kill option was used due to the simplicity and cost effectiveness. The element kill algorithm for shell elements is more complex. When the damage value in a through thickness integration point reaches the critical value, the stiffness is set to zero, and as soon as the damage for all integration points has reached the criterion, the element is deleted. This can be very important when an element is subjected to bending, as it typically is in progressive collapse/buckling of structures.
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Chapter 3

Experiments with Simple Components

Several small-scale experiments with various materials have been conducted by the author to assess the accuracy and effectiveness of the ductile fracture criteria presented in the previous chapter for a large range of stress and strain states.

3.1 Study of Ductile Fracture Initiation in a Ship Plate

The present section deals with an experimental and numerical study of steel material subjected to multiaxial stress states. The influence of different stress states on the ductile failure strain has been studied by Bridgman (1952) and Clausing (1969) and in the 70’s by e.g. Hancock and Mackenzie (1976) and by MacKenzie et al. (1977). They used grooved axisymmetric tensile specimens and showed how the failure strain varied as a function of the stress triaxiality, $\frac{\sigma_H}{\sigma_{eq}}$. Notched tensile tests are very useful for studying materials in multiaxial stress states as the failure initiation mostly occurs in a controlled region. In this study the same procedure as that described by Hancock and Mackenzie (1976) was used to determine the influence of the triaxiality on a 40 mm thick steel plate from the ship ‘Sea Empress’, which was obtained from Reading University.

The tanker ‘Sea Empress’, which was loaded with about 130,000 tonnes of light crude oil, grounded off the Middle Channel Rocks in the approach to Milford Haven on 15 February 1996, Figure 3.1. The grounding resulted in approximately 73,000 tonnes of crude oil lost in sea. The ship was constructed with a single hull and a piece from keel plate was cut out near the fracture site to be studied with the emphasis on ductility for different stress states.
3.1.1 Test Specimens and Test Method

Tensile tests with circumferentially notched specimens, with varying notch radius, were carried out to study the influence of the triaxial stress state on the failure strain. The 40 mm thick keel plate from the ‘Sea Empress’ had a dimension of about 400 x 100 mm, see Figure 3.2. Rectangular pieces of 70x13x13 mm were cut out both parallel and perpendicularly to the rolling direction, allowing three cylindrical in-plane specimens to be made through the thickness. All specimens had the same diameter at the minimum cross section and the same outer diameter and only the notch radius varied. The tensile specimens had a diameter, \( d \), of 6 mm and a gauge length of 30 mm. The specimens with grooves had an outer diameter of 12 mm and the smallest diameter, \( d \), of 6 mm. Three different notch radii, \( R \), were machined, 1.5 mm, 3 mm and 10 mm, see Figure 3.2(b).

The experiments were performed in a 100kN Instron machine with controlled displacement and a constant loading velocity of 1 mm/min. The load and displacement were continuously recorded and the tests were stopped numerous times to measure the diameter change at the minimum cross section.

3.1.2 Metallographic Study

The fracture surface from the steel piece showed a ‘black line’ in the middle of the thickness. It was not clear if the line was created during the fracture process or if it was a change in the metal structure. When the tensile test pieces were sawn out, the ‘black line’ was again clearly visible in the centre. A Rockwell hardness test was performed to measure the hardness variation over the thickness and a value of 94 HRB was found in a narrow band in the middle of the thickness. The rest of the cross section had a uniform hardness of about 83 HRB.
3.1 Study of Ductile Fracture Initiation in a Ship Plate

Figure 3.2: (a): The keel plate divided into samples which allowed three specimens in the thickness. (b): Tensile test specimens without (upper) and with (lower) a groove.

A metallographic study showed that the microstructure was a matrix of mainly ferrite and pearlite, and in the 'black line' much smaller grain sizes with a higher percentage of pearlite were found, Figure 3.3. This will, of course, give harder material properties, which also the hardness test and later the tensile tests showed.

This effect comes from the different stages of the manufacturing process, where the steel was either ingot or continuously cast. In ingot casting molten steel is poured into a large mould where it cools and solidifies to form an ingot, whereas for continuous casting molten steel is poured into a reservoir at the top of the casting machine. It passes at a controlled rate into a water cooled mould where the outer shell of the steel solidifies. The steel is drawn down into a series of rolls and water sprays to ensure that it is both rolled into shape and fully solidified at the same time.

Impurities, such as oxides of aluminium, calcium and iron, tend to move into the centre and to the top of the tundish bath. As the molten steel solidifies there will be a higher concentration of impurities in the centre of the finished product, e.g. a slab which is a long, thick, flat piece of steel, with a rectangular cross section. Moreover, when the molten steel starts to solidify, the iron with the smallest percentage of carbon (according to a phase diagram) will first start to crystallise and as a result the fraction of carbon will increase in the rest of the liquid. This means that in the very centre of the slab the carbon fraction is higher than in the rest of the slab. This gives smaller grains and a higher percentage of pearlite according to an iron-carbon phase diagram.

The cause of the band texture is the rolling process and the phenomenon is also called rolling texture. As the plate is rolled, the impurities in the slab become thinner following the thickness reduction of the plate. For thin plates this band of smaller grains and impurities will be insignificantly small, however, for thick plates (as the keel plate studied here) the line will be detectable. In most cases, this will not influence the structural behaviour as the
Figure 3.3: The microstructure of the 40mm mild steel plate. (a): The grains in the black line (x50). (b): The grains in the surrounding structure (x50) (c): The grains in the black line (x200). (d): The grains in the surrounding structure (x200)
3.1 Study of Ductile Fracture Initiation in a Ship Plate

3.1.3 Experimental Results

In Figure 3.4 the true stress (load divided by the current cross section area) is plotted against the true logarithmic strain, $2 \ln(d_0/d)$, where $d_0$ is the start diameter and $d$ is the current diameter at the minimum section (in the necking area) for the smooth tensile tests. The true logarithmic strain, found from $2 \ln(d_0/d)$, will hereafter be called the Bridgman strain, $\varepsilon_{br}$, to avoid confusion with the true plastic strain obtained from the finite element simulations. As seen in Figure 3.4 for the two outer specimens, both parallel with and perpendicular to the rolling direction, the stress-strain curves nearly coincided. The centre specimens had higher strain hardening curves than the outer specimens and also exhibited a more brittle behaviour. It should be noted that the centre specimens had a minimum diameter of 6 mm and therefore about 20% of the cross section area were grains from the "black line". For the outer specimens the failure strain in the two directions varied insignificantly, however, the brittleness of the centre specimens was more sensitive to the rolling direction.

The ductile behaviour of the centre specimens was of course influenced by the centre line in the specimens, while the outer test had a nicely shaped cup and cone fracture pattern. The centre specimens had a V-shaped pattern, as the centre line was more brittle and failed earlier than the surrounding material.

By recalling Equations 2.9 and 2.12, the triaxiality values for circumferentially notched tensile tests are calculated from the initial value of $d_0/(4R_n)$ using Bridgman’s approximation, Equation 2.9. Thus, with a smaller notch radius, higher stresses are achieved and the influence of the stress triaxiality on the failure strain can clearly be seen in the results. As the stress-strain curve for the centre specimens in Figure 3.5(b) shows, there is a considerably scatter due to the material discontinuity in the centre. In the following, only the outer specimens will be treated, as the material properties are uniform and isotropic.
The yield stress of the material (not including the centre "black line") was found to be around 348 MPa and the material hardening curve could be described by a power law according to

\[ \sigma_y = C \varepsilon_{eq}^n \]  

where the material parameters \( C \) and \( n \) were found from the tensile tests to be 850 MPa and 0.2 respectively. Young’s modulus \( E \) was found to be about 212 GPa and Poisson’s ratio \( \nu \) was assumed to be 0.3.

The Bridgman strain at failure was calculated at the point where the stress-strain curve starts to decrease, by use of Equation 2.12. At this point the voids link up in the cross section and therefore lead to a reduction in the load carrying area, which corresponds to a first formation of a crack by hole coalescence. The triaxiality versus the Bridgman strain to failure initiation is plotted in Figure 3.6.

Bridgman assumed that the strain was uniform in the minimum cross section, however, Clausing (1969) showed that the strain in the minimum cross section was non-uniform, which was also shown later by several finite element simulations, i.e. Mirza et al. (1996). Clausing added a nonlinear term to Bridgman’s analysis (Bridgman assumed linear radial displacement as a function of radius), which he determined experimentally.

### 3.1.4 FE-simulation without Fracture

The present finite element model was made of four-node axisymmetric constant strain elements. The implemented user-defined material model was used in the simulations, with a power law describing the strain hardening with \( C = 850 \) MPa and \( n = 0.2 \). Different element
sizes and aspect ratios were investigated where the elements were concentrated in the region with high stress fields and strain gradients, see Figure 3.7. With an element aspect ratio of 1:3 (length to width) a correct fracture pattern was obtained and the aspect ratio was kept close to unity even after large plastic deformations. The following results were achieved with twenty elements in width at the smallest cross section and with an aspect ratio of 1:3 (length to width), Figure 3.7. Element formulations and sizes as well as material laws have previously been investigated by Besson et al. (2001) for the same type of experiments.

In Figure 3.8 the effective stress against the Bridgman strain is plotted for experiments and finite element simulation. The effective stress and the Bridgman strain for the FE-simulation were calculated in the same manner as for the experiments by using Equations 2.9 and 2.12. Failure was not included in the numerical model and the curves therefore continued after the failure strain obtained in the experiments. Good correlation of the stress levels between the experiments and the simulations was achieved showing that the material parameters were well chosen.

The triaxiality is 1/3 in the beginning for a smooth tensile test and as a neck starts to form the triaxiality rises in the centre, whereas it almost stays at the same level on the surface of the groove. This effect is plotted in Figure 3.9(a) for the finite element simulation, where it is seen that at a Bridgman strain of a little less than 0.2 the specimen starts to neck. This is in agreement with the analysis by Consideré (1885), which gives a strain of $\varepsilon_{eq} = n$ at the moment of necking when the material strain hardening curve is described by a power law, Equation 2.1.

Bridgmans triaxiality approximation is thus only valid until necking occurs. When a neck has formed in a smooth or large-radius specimen, the strain distribution in the smallest cross section is no longer uniform. The centre of the specimen is subjected to higher straining, Figure 3.9(b).

For smaller notch radii, the maximum strain was not in the centre of the cross section, but...
Figure 3.7: Finite element mesh of the notched specimens, with initial element aspect ratio of 1:1. (a): 10 mm notch radius. (b): 3 mm notch radius. (c): 1.5 mm notch radius.

Figure 3.8: Effective stress versus effective strain. The marks are from the experiments and the lines from the finite element simulations.
3.1 Study of Ductile Fracture Initiation in a Ship Plate

Figure 3.9: (a): The triaxiality during the loading in a smooth tensile specimen. (b): The true strain in the centre and on the surface of a smooth tensile test. The average analytical strain found from Equation 2.12 is also shown.

on the surface of the notch, Figure 3.10(a). As also seen, the element aspect ratio was still acceptable even though the local strain had reached a value of more than 75%. This analysis clearly showed that the plastic strain is not suitable as a fracture criterion even for tensile tests, as fracture varies with the triaxiality.

Further complexity was added as during the loading the triaxiality in the centre of the notch varied, Figure 3.11(a). The triaxiality for the smaller radius decreased as the notch radius got diffused, while for the smooth test pieces or specimens with large radius, a second phase neck was created and thus the triaxiality increased.

In Figure 3.12 the triaxiality versus the plastic strain in the centre of the specimens obtained from the finite element calculations is shown. At the point of fracture in the experiments the triaxiality curves are cut off at the corresponding Bridgman strain. The triaxiality curve achieved before from Bridgman’s classical analysis (Figure 3.6) is also plotted in Figure 3.12. It is seen that the agreement between the results of the classical Bridgman analysis and the FE-calculation is best for small notch radii. Since the stress triaxiality varied during the entire deformation an average triaxiality was calculated according to

\[
\left( \frac{\sigma_H}{\sigma_{eq}} \right)_{avg} = \frac{1}{\varepsilon_f} \int_0^{\varepsilon_f} \frac{\sigma_H}{\sigma_{eq}} d\varepsilon_{eq} \tag{3.2}
\]

where \( \varepsilon_f \) is the effective plastic strain to fracture. The calculated average triaxiality is given in Table 3.1. The average triaxial values will later be used in the calibration of the fracture criteria and the damage models.
Figure 3.10: Plastic strain distribution in the notched specimen with $R = 1.5$ mm and the smooth tensile specimens. (a): The plastic strain distribution. (b): The triaxial stress state.

Figure 3.11: (a): The triaxiality in the centre during the deformation for the smooth and notched specimen. Bridgman’s analytical triaxiality value is also given. (b): The effective plastic strain in the centre of the cross section during the loading. The Bridgman strain is also shown with the slope 1:1.
3.1 Study of Ductile Fracture Initiation in a Ship Plate

Figure 3.12: The triaxiality as a function of the effective strain until the rupture strain obtained from the experiment. The dash-dotted curve is the curve fit obtained in Figure 3.6.

Table 3.1: The average triaxiality and the effective plastic failure strain obtained in the FE-simulations.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Analytical triaxiality, $\frac{\sigma_H}{\sigma_{eq}}$</th>
<th>Average triaxiality, $\left(\frac{\sigma_H}{\sigma_{eq}}\right)_{avg}$</th>
<th>Plastic failure strain, $\varepsilon_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>0.33</td>
<td>0.51</td>
<td>1.50</td>
</tr>
<tr>
<td>R_n= 10.0 mm</td>
<td>0.47</td>
<td>0.69</td>
<td>1.01</td>
</tr>
<tr>
<td>R_n= 3.0 mm</td>
<td>0.74</td>
<td>1.02</td>
<td>0.62</td>
</tr>
<tr>
<td>R_n= 1.5 mm</td>
<td>1.02</td>
<td>1.27</td>
<td>0.45</td>
</tr>
</tbody>
</table>

3.1.5 FE-simulation with Fracture

Empirical and Void Growth Criteria

The smooth tensile test specimens were used to calibrate the damage indicator, $D_i$, for the different criteria by using the derived average triaxiality values. When fracture occurred in the tests, the damage value at the element in the centre of the smallest cross section was set to be the effective triaxial damage strain $\varepsilon_0$ or the triaxial damage work $W_0$ for the corresponding average triaxiality values. This implies that all damage indicators are normalised so that $D_i=1$ corresponds to crack initiation. The damage values for the investigated criteria are given in Table 3.2. The triaxiality functions for the McClintock, Rice-Tracey and Cockcroft-Latham criteria are plotted in Figure 3.13 after calibration of the damage parameter.

For the Johnson-Cook criterion, the parameters were found by use of a direct curve fit to the experimental data, on assumption of proportional straining and a constant triaxiality ratio.
during each test, where the average triaxiality values were used. Furthermore, the strain rate effects and the temperature were neglected, so that only three parameters, $D_1 \ldots D_3$, had to be determined in the damage function. The least square method was used, which minimised the residual sum between the experimental results and the curve fit according to

$$\mathcal{R}(\varepsilon_f) = \sum_{\beta=1}^{N_d} \left\{ \varepsilon^\text{exp} \left( \frac{\sigma_H}{\sigma_{eq}} \right)^\beta - \left[ D_1 + D_2 \exp \left( D_3 \left( \frac{\sigma_H}{\sigma_{eq}} \right)^\beta \right) \right] \right\}^2$$  \hspace{1cm} (3.3)

The curve fit is plotted in Figure 3.13 and the damage parameters for the Johnson-Cook criterion are given in Table 3.2.

All the tests performed had relatively high triaxialities, ranging from 1/3 initially for the tensile tests to 1.4 for the 1.5 mm notch radius. All of the studied criteria and models were not believed to determine the fracture locus for the four tests, as they may be derived for a special application or material. Figure 3.14 shows the damage evolution for the different fracture criteria for the four test specimens. The damage curves are plotted against the Bridgman strain found from $\varepsilon_{eq} = 2 \ln(d_0/d)$, up to the point where the experiments failed. As the damage indicators were calibrated for the smooth tensile test, these all reached a value of 1.0, where they fractured.

Both the Rice-Tracey criterion and the Johnson-Cook criterion predicted the fracture locus for the different geometries quite accurately, i.e. $D_i \simeq 1$ for specimen geometries at the experimental point of fracture. However, Johnson-Cook should predict the fracture locus, as the damage parameters were calibrated from the experimental results. The reason why Johnson-Cook did not predict fracture at a damage value of exactly 1.0 is that the average triaxiality value was used in the least square method. Nevertheless, it shows the usefulness of using the average triaxiality value, instead of using Bridgman’s formula for calibrating the damage criteria.
Table 3.2: The fracture parameters for the different fracture criteria calibrated to fit the smooth tensile test.

<table>
<thead>
<tr>
<th>Criterion/model</th>
<th>Formula</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>McClintock</td>
<td>$D_i = \frac{1}{\varepsilon_0} \int \frac{\sigma_H}{\sigma_{eq}} d\varepsilon^p_{eq}$</td>
<td>$\varepsilon_0 = 0.72$</td>
</tr>
<tr>
<td>Rice-Tracey</td>
<td>$D_i = \frac{1}{\varepsilon_0} \int \frac{1}{(1 - cN\sigma_H)} d\varepsilon^p_{eq}$</td>
<td>$c = 0.001 \text{ MPa}^{-1}$, $\varepsilon_0 = 1.28$</td>
</tr>
<tr>
<td>Norris</td>
<td>$D_i = \frac{1}{\varepsilon_0} \int \frac{1}{(1 - cN\sigma_H)} d\varepsilon^p_{eq}$</td>
<td>$\varepsilon_0 = 1.86$</td>
</tr>
<tr>
<td>Damage Work</td>
<td>$D_i = \int \left[ 1 + \frac{3\varepsilon_0}{\sigma_{eq}} \exp \left( \frac{3\sigma_H}{2\sigma_{eq}} \right) \right] \frac{\sigma_{eq} d\varepsilon^p_{eq}}{W_0}$</td>
<td>$W_0 = 2380 \text{ MPa}$, $\alpha = 1.0$</td>
</tr>
<tr>
<td>LeRoy</td>
<td>$D_i = \frac{1}{\varepsilon_0} \int (\sigma_1 - \sigma_H) d\varepsilon^p_{eq}$</td>
<td>$W_0 = 640 \text{ MPa}$</td>
</tr>
<tr>
<td>Brozzo</td>
<td>$D_i = \frac{1}{\varepsilon_0} \int \frac{2\sigma_1}{3(\sigma_1 - \sigma_H)} d\varepsilon^p_{eq}$</td>
<td>$\varepsilon_0 = 1.57$</td>
</tr>
<tr>
<td>Cockcroft-Latham</td>
<td>$D_i = \frac{1}{\varepsilon_0} \int \frac{\sigma_1}{\sigma_{eq}} d\varepsilon^p_{eq}$</td>
<td>$\varepsilon_0 = 0.67$</td>
</tr>
<tr>
<td>Johnson-Cook</td>
<td>$\varepsilon_f = \left[ D_1 + D_2 \exp \left( D_3 \frac{\sigma_H}{\sigma_{eq}} \right) \right] \left[ 1 + \left( \frac{\tau}{\tau_0} \right)^3 \right] [1 + D_5 T^*]$</td>
<td>$D_1 = -0.28$, $D_2 = 4.40$, $D_3 = -2.26$, $D_4 = D_5 = 0$</td>
</tr>
</tbody>
</table>

The Cockcroft-Latham, Norris, LeRoy and Brozzo criteria did not capture the decrease in ductility as the triaxiality increased. It is surprising that the LeRoy criterion failed to simulate this, as it is a void growth model. The other three criteria that failed to predict the decrease in ductility were empirical criteria and probably only valid for a certain application. The McClintock and Damage Work gave poor results, nevertheless, predicted less ductility for higher triaxial stress states.

CDM and Porosity Models

The damage evolution in the material was not measured during the present tests and therefore the damage parameters for the CDM models and the porosity model were estimated and/or taken from Bonora (1997). The strain at fracture for Lemaitre’s damage model, Equation 2.39, can be found from proportional loadings with constant triaxiality, Lemaitre (1985), according to

$$\varepsilon_f = \frac{\varepsilon_f(\sigma_H/\sigma_{eq})}{f(\sigma_H/\sigma_{eq})} = \frac{\varepsilon_{f,uni}}{f(\sigma_H/\sigma_{eq})}$$

(3.4)

by remembering the triaxiality function, $f(\sigma_H/\sigma_{eq}) = \frac{2}{3}(1 + \nu) + 3(1 - 2\nu) \left( \frac{\sigma_H}{\sigma_{eq}} \right)^2$, where $\varepsilon_{f,uni}$ is the fracture strain for uniaxial tensile loading, with a constant triaxiality of $1/3$. The
constant $\varepsilon_{f,uni}$ was found to be 1.56 from a least square fit of the experimental results, Figure 3.15(a). The fracture strain function for the damage model by Bonora under proportional loading was found from, Bonora (1997):

$$\varepsilon_f(\frac{\sigma_H}{\sigma_{eq}}) = \varepsilon_{th} \left( \frac{\varepsilon_{f,uni}}{\varepsilon_{th}} \right)^{1/f(\frac{\sigma_H}{\sigma_{eq}})}$$  \hspace{1cm} (3.5)

The threshold strain $\varepsilon_{th}$, at which void nucleation was assumed to initiate, and $\varepsilon_{f,uni}$ were found by a least square fit to be 0.22 and 2.0 respectively. The fracture strain triaxiality curves for Bonora and Lemaitre are given in Figure 3.15(a). Lemaitre’s damage model was clearly not able to describe the ductility dependence of the triaxiality, whereas Bonora’s damage model gave good agreement with the experimental results. However, in view of the curves being based on the average triaxiality, it was not clear how well the two damage models would predict fracture in the simulations of the grooved tensile tests. It is worth noting that both CDM models were not able to describe the increase in the fracture strain for negative triaxial stress states, due to the quadratic term of the triaxial stress ratio in the triaxiality function. The CDM models are, however, developed for ductile fracture due to void growth, and for negative triaxialities shear fracture is the dominating failure mode, so the CDM models should not be used for negative and low triaxialities.

Bonora (1997) gave a table with damage parameters for different materials where mild steel had a critical damage value of 0.1, which was used in the finite element simulations of the experiments. The material parameter $S$ in the Lemaitre CDM model can now be found from integration of Equation 2.39 by the material hardening law, $\sigma = C\varepsilon^n$, and by assuming...
Chapter 3. Experiments with Simple Components

Figure 3.16: (a): The strain hardening curves for the CDM models and the power law curve. (b): A power law fit of Bonora strain hardening curve, with $C=872$ MPa and $n=0.23$.

constant triaxiality:

$$S = \frac{f\left(\frac{\sigma}{\sigma_0}\right) * C^2}{2D \sigma E \left(1 + 2n\right)} \left(\varepsilon_f^{(1+2n)} - \varepsilon_{th}^{(1+2n)}\right)$$  \hspace{1cm} (3.6)

The parameter $S$ was found to be 22.9 MPa. The exponent $\alpha$ in Bonora’s damage function was set to 0.2, in accordance with Bonora (1997). The differences in the two damage functions are shown in Figure 3.15(b). In Bonora’s model, the damage evolution for mild steel was better modelled, as physically few voids initially nucleate and grow slowly causing the damage evolution to increase slowly and when the strain gets close to $\varepsilon_f$, the larger voids coalesce rapidly and lead eventually to failure. The damage parameters for both Lemaitre and Bonora are given in Table 3.3.

In both CDM models the strain hardening is a function of the hardening variable $r$, Equation 2.32, and thus the hardening curves for the CDM models have to be obtained. By using the earlier found power law curve, the hardening curves for the CDM models are found from $r = \varepsilon(1 - D(\varepsilon))$ and $\bar{\sigma} = \frac{\sigma}{1 - D(\varepsilon)}$. Figure 3.16(a). A disadvantage of this method is that when the triaxial stress state is lower than 1/3, the hardening curve will increase rapidly close to the point of fracture. This causes the material to strain harden un-physically in low triaxialities and to avoid this problem, a new power law fit of the calculated strain hardening curve is used, Figure 3.16(b). For the Lemaitre damage model the derived power law parameters were $C=880$ MPa and $n=0.28$, while the parameters for Bonora’s damage model were found to be $C=872$ MPa and $n=0.23$.

The CDM model by Lemaitre underestimated the effect of the triaxial stress state on the strain at failure, Figure 3.17. This was expected due to poor fit of the triaxiality curve to the failure strain. The CDM model proposed by Bonora predicted almost the same failure
Table 3.3: The damage parameters for the CDM and porosity models.

<table>
<thead>
<tr>
<th>Criterion/model</th>
<th>Formula</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDM (Lemaitre)</td>
<td>[ dD = \frac{\dot{\varepsilon}<em>{eq}}{2E} \left( \frac{\sigma</em>{eq}}{\sigma_y} \right) d\varepsilon_{eq} ]</td>
<td>C=880 MPa, n=0.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ S = 25.5 ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ r_d = 0.22, D_c = 0.1 ]</td>
</tr>
<tr>
<td>CDM (Bonora)</td>
<td>[ dD = \alpha \left( \frac{D_{cr} - D_0}{\ln f - \ln \varepsilon^{\theta}} \right) f \left( \frac{\sigma_{eq}}{\sigma_y} \right) (D_{cr} - D) \left( \frac{\alpha - 1}{\alpha} \right) \frac{d\varepsilon}{\varepsilon} ]</td>
<td>C=872 MPa, n=0.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ \alpha = 0.2, D_c = 0.1 ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ r_d = 0.22, \varepsilon_f = 2.0 ]</td>
</tr>
<tr>
<td>Porosity model (GTN)</td>
<td>[ F(\sigma_{eq}, \sigma_{kk}, f, \sigma_y) = \left( \frac{\sigma_{eq}}{\sigma_y} \right)^2 + 2q_1 f^* \cosh \left( q_2 \frac{3\sigma_{eq}}{2\sigma_y} \right) - 1 - (q_3 f^*)^2 ]</td>
<td>[ \varepsilon_n = 0.3, f_n = 0.004 ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ s_n = 0.10, f_0 = 0.0 ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ f_c = 0.055, f_f = 0.2 ]</td>
</tr>
</tbody>
</table>

Figure 3.17: The results from the finite element simulations. (a): Lemaitre’s damage model. (b): Bonora’s damage model.
strain as in the experiments. The differences in the damage accumulation during the loading of the different notched specimens are shown in Figure 3.18.

The main disadvantage with Gurson-Tvergaard-Needleman porosity model (GTN) is the many parameters that cannot be determined directly by simple tensile tests. Some of the parameters can be estimated on the basis of metallurgical observations, such as the volume fraction of voids. The Gurson parameters for the numerical simulations were initially taken from Bonora (1997) and fitted until the round smooth tensile test simulation corresponded with the experiment. The parameters used in the simulation are given in Table 3.3. In Figure 3.19 the results from the simulations with the implemented GTN material model are presented. With the material parameters given in Table 3.3 the damage tends to localise in a narrow band and thus fracture earlier than in the experiments. Nevertheless, it is believed that the parameters could be calibrated so that better agreement between the finite element simulations and the experiments would be achieved.

3.1.6 Discussion

Several damage criteria and models were tested on various triaxial stress states for a thick steel plate, which was taken from the "Sea Empress" that grounded at Milford Haven in 1996. The material properties through the thickness were found to be highly non-uniform as a thin layer in the middle plane of the plate had a much smaller grain size compared to that of the surrounding material. This led to a harder and more brittle behaviour and from studies of the fracture surface it is believed that the fracture initiated in the centre of the plate. This could often be the case due to the manufacturing process of the steel plates, where a high concentration of impurities can be found in the thickness centre.

The finite element simulations of the smooth and grooved round tensile tests showed that the triaxial stress state varied throughout the tests. It is therefore questionable to use the
classical analysis by Bridgman for calculation of the triaxial stress state. By using the average triaxiality during the loading several of the fracture criteria could be calibrated, so that good agreement was achieved between the finite element simulations and the experiments. The void growth model proposed by Rice-Tracey and the empirical model by Johnson-Cook predicted quite accurately the failure initiation for the different specimens. The empirical damage model of Johnson-Cook was calibrated from the tests and by application of the calibration constants the Johnson-Cook and Rice-Tracey models were nearly identical. Due to the high triaxiality in the experiments the RTCL criterion is equal to the Rice-Tracey criterion and therefore not used in the simulations. The other damage criteria failed to predict the point of fracture for the various geometries.

The damage model by Bonora managed to calculate the point of failure for all specimens, whereas Lemaitre’s damage model underestimated the effect of the triaxiality on the fracture strain. Several experiments have also showed (LeRoy et al. (1981), Tai (1990) and Bonora (1997)) a highly nonlinear damage evolution in steel, which Bonora’s nonlinear damage model simulates. The porosity model by GTN has the disadvantage that the parameters cannot be found from simple tensile tests. It is therefore a relatively time-consuming process to calibrate the model so that a correct response is achieved.

It is shown that for simulation of fracture initiation in low carbon steel in high triaxial stress states, the criterion by Rice and Tracey predicted the failure locus well. However, there might be differences in fracture initiation and crack propagation. The next section will investigate the different criteria in crack propagation.
3.2 Study of Crack Propagation in Ship Steel

The present section studies the fracture criteria on thick steel plates subjected to biaxial loading. The main focus will be on how well the different criteria are able to model crack propagation in typical ship steel plates for various stress states. The ”giant” hydraulic bulger at the University of Reading was used for the experiments, where a novel test setup was used to study fracture toughness and crack propagation in biaxially loaded thick metal plates, Figure 3.20.

3.2.1 Introduction

The use of a hydraulic bulger is a well known method for evaluating sheet materials subjected to states of biaxial tension. The hydraulic bulge uses pressurised oil to push a test specimen through an aperture, where different shapes of the aperture create various biaxial loadings of the specimen. A circular shape creates an equal biaxial tension at the top of the dome and by using elliptical shapes unequal biaxial tensions can be achieved. When the plates are bulged they obtain a curvature and are thus subjected to bending, however, these effects are limited and can thus be neglected. It is also obvious that the plates are exposed to a lateral stress component, but since this component is much smaller than the in-plane stresses it can also be disregarded. The main advantage of the hydraulic bulger test compared to tests where mechanical punches are used is the absence of friction between the fluid and the sheet. In mechanical tests the amount and the localisation of the straining are very dependent on the lubrication between the die and the punch.
In uniaxial tensile tests the instability occurs relatively early and as a result the hardening law can usually only be determined up to the point of necking. For the bulger tests the strain that can be introduced before instability is much higher, and therefore the test arrangement is suitable for determining the material hardening law. Hill (1950) showed that the instability strain for an equally biaxially loaded plate would be at least of a magnitude $4/11$, which is often a factor of two greater than the instability strain in uniaxial tensile tests. He used the von Mises stress-strain relationship to get an explicit solution for the bulging process including modelling of the point of instability $\varepsilon_{int}$, which gives the instability strain in an equally biaxially loaded plate ($\varepsilon_1 = \varepsilon_2$):

$$\varepsilon_{int} = \frac{4}{11}(2n + 1)$$

(3.7)

where $n$ is the hardening exponent in the power law description.

For construction of the forming limit diagram (FLD), the bulger is a very suitable tool as it can generate biaxial strain limits in a very large range. It covers the range from 1:1 biaxial straining to uniaxial (1:-1/2) biaxial stress states, by use of a narrow strip tests piece (narrower than the orifice) bulged by a driver sheet which seals the pressurised fluid. Moreover, different fracture criteria can be studied for a large range of biaxialities by use of a hydraulic bulger.

The size of the orifice has to be quite large for the bending and shear effects to be negligible. The ratio of aperture to plate thickness should be kept as large as possible. In practice a ratio around 25:1 is considered sufficiently large.

There are other methods of creating biaxially loaded sheets. The cruciform-shaped specimen loaded by four separate actuators is the traditional way of studying fracture in biaxially loaded thick plates. However, a drawback is that the specimen often has to be thinned down in the centre, so that the specimen is expensive to produce. Pressurised pipes are another method of introducing biaxial loading. By using vacuum inside the tube instead of pressure, the loading is changed from tension-tension (2:1) to compression-compression. By twisting or pulling the tube, the loading combinations are unlimited. The major drawback of the pipe method is the production of the tubes. If plates are used, they have to be bent and welded, which will change the material properties and not allow a fair comparison with bulger or cruciform tests.

### 3.2.2 Test Specimens and Experimental Setup

To study fracture initiation in biaxially loaded steel plates, the plates were bulged in two differently shaped orifices, one circular and one elliptical. The circular die had a diameter of 960 mm and the ellipse had a major axis of 960 and a minor of 480 mm, see Figure 3.21. Both dies had a bending radius at the rim of 125 mm. Five bulging tests were performed
Chapter 3. Experiments with Simple Components

Figure 3.21: (a): The ellipse-shaped orifice to the "giant bulger" at Reading University. (b): The backing plate of stainless steel glued onto the plate specimen.

with two thicknesses of 12mm and 18mm. The plates were made of mild steel EN 10025 S275 and the plates had starter cracks at the centre allowing crack propagation to be studied. Four of the plates with starter crack had an initial crack length of 170 mm and one plate had an initial crack length of 50mm. The starter cracks were about one millimetre wide and were manufactured by a laser. The crack tips therefore had a small radius, which creates a larger plastic field around the crack tip than if it had been generated by fatigue. A single 12mm plate without a starter crack was tested in the circular orifice, however, as the pressure reached the safety limit of the bulging machine the test was stopped.

The plates were grinded on one side to attain a good surface finish for the sealing. Backing plates (510mm long, 100mm wide and 1.2 mm thick) in stainless steel, were together with 5 mm thick rubber plates glued on the pressure side of the plates with starter cracks to seal off the crack and therefore leakage was avoided. The plates were rigidly clamped at the periphery and bulged into a domed shape with pressurised oil. The flow rate (about 2 litres/min) and the pressure were logged during the tests. Several large-strain strain gauges were also mounted at the centre and close to the rim, see Figures 3.22 and 3.23. The gauges were used to validate the FE-simulations as well as to study the biaxiality during the test. The top and the bottom crack tip propagations were visually marked at every 5mm and were recorded by the data acquisition equipment. The test was also taped on video to allow fracture progress to be studied afterwards.

To find the material properties normal round tensile tests were conducted for each plate with a thickness of 12mm and 18mm in three directions, 0°, 45° and 90° to the rolling direction. The tensile tests were performed in an Instron machine at a loading rate of 5mm/min. An attached extensometer was used to measure the elongation in the specimen.

To measure the damage parameters tensile test specimens with rectangular cross section (see Figure 3.23(b)) were tested in an MTS machine with a 100 kN load cell at a loading velocity
3.2 Study of Crack Propagation in Ship Steel

Figure 3.22: The placement of the strain gauges on the bulger plates. (a): Strain gauges on the 12 mm plates with a 170 mm long starter crack. (b): Strain gauges on the 12 mm plates without and with a 50 mm long starter crack.

Figure 3.23: The placement of the strain gauges on the bulger plates. (a): The strain gauge placement on the 18 mm bulger plate. (b): Tensile test specimen with rectangular cross section.
of 5 mm/min. These specimens were also machined in three directions - 0°, 45° and 90° to the rolling direction. The elongation was measured by an extensometer with a gauge length of 50 mm, and during the test the specimens were unloaded several times to allow the smallest cross section to be measured manually. A strain gauge was initially mounted in the centre of the gauge length on the specimens, and when a clear visible neck was developed additional strain gauges were mounted to achieve additional damage parameters after necking. The damage values obtained from the extensometer were only valid up to the point of necking as all straining took place in the necking area afterwards.

Three-point-bend (TPB) tests were also conducted for both thicknesses. The tests were performed in an Instron machine at a constant loading velocity and the crack propagation distance was recorded together with the load and deformation. The dimensions of the three-point-bend test specimens are seen in Figure 3.24.

The normal procedure for producing the crack tip in TPB specimens is to use a very sharp knife, after which a natural fatigue crack is created by small loading/unloading cycles of the specimen. The procedures for determining the energy release rate $J$ are regulated in the standards, such as ASTM (1996). The definition of the energy release rate is

$$J = -\frac{d\Pi}{dA}$$

where $\Pi$ is the potential energy and $A$ is the crack area. In general the $J$ integral for a variety of configurations can be divided into elastic and plastic components:

$$J = \frac{K_I^2}{E'} + \frac{\eta_p U_p}{Bb}$$

where $K_I$ is the stress intensity factor and $E' = E$ for plane stress and $E' = E/(1 - \nu^2)$ for plane strain. $U_p$ is the plastic energy absorbed by the specimen and $B$ and $b$ are the specimen width and the ligament length in front of the crack. The dimensionless constant $\eta_p$ varies for different configurations and for a cracked plate in pure bending $\eta_p = 2$.

The starter cracks in the specimens used here were sawed to the specified crack depth. This will, however, give an incorrect fracture toughness value when analysed by classical fracture mechanics, as the crack tip is blunted and thus gives a higher loading force and a larger plastic field. However, for comparison with the finite element model, this causes no problems, as the crack tip in the finite element model can be built in the same way.

### 3.2.3 Experimental Results

**Tensile Tests**

The purposes of the tensile tests were to determine the material and damage properties and to investigate whether the material could be considered as isotropic. The load deflection
3.2 Study of Crack Propagation in Ship Steel

Figure 3.24: The dimensions for the three point bend test specimens.

curve for the different directions and plates showed that the material was nearly isotropic and had nearly the same material properties for all plates and thicknesses. The yield stress and Young’s modulus for both plate thicknesses were found to be about 310 MPa respectively 210,000 MPa. A small variation in the material properties was however found, but to be able to compare the results this difference was neglected. A power law with the coefficient $C=805$ MPa and the hardening exponent $n=0.23$ was found to describe the material strain hardening. An average true fracture strain of 1.00 was found from the following

$$
\varepsilon_f = \ln \left( \frac{A_0}{A_f} \right)
$$

(3.10)

where $A_0$ is the original cross section area and $A_f$ is the fracture cross section area of the test specimen. The fracture surfaces had a cup and cone pattern with small shear lips, and two of the typically fractured tensile test specimens are shown in Figure 3.25(a).

The true stress-strain curve is given in Figure 3.25(b) and shows both the true stress-strain curve recalculated from the nominal stress-strain curve, as well as the curve calculated from the applied load and the current minimum cross-sectional area. The true stress and strain pairs were found from the nominal stress and strain pairs $(\sigma_{\text{nom}}, \varepsilon_{\text{nom}})$ according to

$$
\sigma = \sigma_{\text{nom}} (1 + \varepsilon_{\text{nom}})
$$

(3.11)

$$
\varepsilon = \ln(1 + \varepsilon_{\text{nom}})
$$

(3.12)
Figure 3.25: (a): The fracture surfaces with small shear lips for the rectangular tensile test. (b): True stress-strain curve for a 12mm plate.

The curves follow each other as expected until the point of necking, which occurs when $\varepsilon = n$, where the recalculated engineering stress-strain curve drops.

Due to the nucleation, growth and coalescence of voids during large plastic straining, the material exhibits a progressive reduction of the load carrying area, resulting in loss of material strength and stiffness. The damage evolution in the material was measured during the tests according to the procedure described by Bonora (1999). The stiffness reduction was measured from the repeated loading-unloading ramps in tensile tests. The damage could be estimated from the change in stiffness according to Equation 2.29, which gives $D = 1 - E_{\text{eff}}/E_0$. There are several other methods for estimating the damage effects on the material properties (see e.g. Lemaitre and Dufailly (1987)), and even though this method of direct measurement of the stiffness loss is very time consuming and requires accuracy in the preparation of the specimens, it is the simplest and probably the most accurate method.

The strain gauges, which were initially mounted on the specimens, revealed that no damage occurred during the 1% of straining, whereas the extensometer gave a relatively large increase of the damage value. The damage value obtained from the extensometer was thereafter constant until a strain of about 10% was reached and the damage again slowly increased. The reason for the large increase in damage found by the extensometer during the 1% of straining was probably the extensometer being stabilised during the first loading-unloading cycles. The damage was therefore believed to be initiated at a threshold strain of 0.1. When a neck was developed additional strain gauges were mounted, however, as the stress state was no longer uniaxial and as severe texture on the necking surface of the specimen was developed, the damage measurements from the additional mounted strain gauges could not
be used.

When the threshold strain was reached, it is assumed that many voids nucleated and with further straining the voids grew slowly together with the nucleation of a few more voids. When the strain reached the critical strain, the larger voids coalesced and the material eventually fractured, which means that just before the critical strain was reached the damage increased rapidly. This is a typical damage behaviour for low carbon steel as found by Bonora (1997), Mirza et al. (1996) and Chandrakanth and Pandey (1994).

**Three-Point-Bend Tests**

The three-point-bend tests were unloaded and reloaded when the crack had propagated 1mm on the surface to mark the crack propagation distance on the force-deflection curve, see Figure 3.26(a). As it can be seen, the force reached a high peak before the crack started to propagate and a considerable plastic field was developed due to the blunt sawn crack tip. Because of this, the elastic part of the energy release rate were incorrect, (at least in the initiation phase) and only the plastic part of the energy release rate in Equation 3.9 was used in the construction of the crack growth resistance curves, see Figure 3.26(b). The effect of the thickness can clearly be seen on the curves and agrees with the theory that energy release rates are lower in plane strain than in plane stress conditions. The 18mm plate is closer to plane strain at the crack tip than the 12mm plate.

The specimens were cooled with liquid nitrogen and brittly broken open to study visually the crack fronts and it was revealed that the crack fronts were shaped as a thumbnail (see Figure 3.27(a)), which means that the crack propagated further in the centre of the specimens, again a well-known feature in fracture toughness testing.

The necking profiles (the through thickness strain) along the crack were measured for both thicknesses and are shown in Figure 3.27(b). The 12mm plate experienced a larger thickness reduction than the 18mm plate. The reason for the negative strain is the deformation at the contact point with the denting cylinder.

**Bulger Tests**

For the plates with starter cracks, the crack tips started to propagate after the plates had achieved a relatively bulged shape. When the crack opening was critical, i.e. when the backing plate started to bulge into the crack opening, the test was stopped and the plates were unloaded.

A thicker plate gave a higher pressure during the loading, while the plate thickness had little influence on the volume at fracture. Moreover, with a longer starter crack, less volume was needed for the crack to start propagating, Figure 3.28. As seen the 18 mm plates had a
Figure 3.26: (a): The force-deflection curve for the three-point-bend tests. (b): The J-R curves for the 12 mm and 18 mm thick plates for a TPB test. The crack extension is measured on the specimen surface.

Figure 3.27: (a): Thumbnail-shaped crack tip. The black crack surface is from the ductile fracture process, while the blank crack surface is from the brittle fracture process produced by breaking open in liquid nitrogen. In the left specimen the crack front is visualised by a thin black line. (b): The necking profile along the crack.
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Figure 3.28: Results from the bulging. (a): Pressure versus volume. The dots indicate the initiation of crack propagation on the plate surface. (b): Crack propagation distance on the plate surface versus volume.

knuckle on the pressure versus volume curve. This was probably due to initial bending of the plate around the rim.

Some of the strain gauges recorded strains up to approximately 10% where they either debonded or failed, however, most of the gauges were not subjected to strains larger than 3-4%. The in-plane strain ratio (radial to tangent strain components) varied during the tests, where of course the crack prevented the stress and strain field from being uniform. Figure 3.29 shows the strain components as well as the in-plane strain ratio for the plate in the circular diaphragm with the 50 mm crack. The material in front of the crack was subjected to much more straining and as the crack propagated, the radial strain component decreased. Eventually, the radial strain component vanished as a localised neck was formed in front of the crack tip. In a localised neck the straining will tend to take place perpendicularly to the neck direction and through the thickness.

Figure 3.30 shows the final crack propagation for the 12mm plate with a 170mm crack tip. The material around the crack tips was flame cut out and the cracks were opened up after the pieces had been cooled by liquid nitrogen. This allowed the crack front to be studied, as the ductile fracture region from the bulging test could clearly be distinguished from the brittle fracture caused by the extremely low temperature of the liquid nitrogen. The crack surface revealed that the crack fractured earlier in the centre of the plate thickness, see Figure 3.32. This indicates that there was no plane stress field during the fracture propagation and that the same V-shaped crack tip could be found in all thickness and aperture shapes.

Considerable necking was found on the crack surfaces and just in front of the crack tip. The thickness around the crack was measured and it was found that the thickness reduction was negligible at the starter crack, but almost directly at the start of the crack surfaces a thickness reduction of 25% was found, Figure 3.34. The through thickness strain continued to increase closer to the final crack tip location, since the plate had been stretched before
Figure 3.29: The 12mm plate with 50mm crack length. (a): Strain components. SG# is the strain gauge number. (b): Strain ratio (radial over tangential).

Figure 3.30: The bulged 12mm plate with a 170mm starter crack. (a): The crack propagation of the two crack tips. (b): The crack propagation of the upper crack tip.
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Figure 3.31: Crack tip from the 12 mm thick plate with a 50 mm starter crack bulged in the circular aperture. The V-shaped crack tip can clearly be seen on the crack surface. The brittle crack surface in front of the crack is lighter due to ice crystals formed on the brittle crack surface when the specimen still was very cool.

Figure 3.32: Crack tip from the 18 mm thick plate bulged in the circular aperture. The V-shaped crack tip can be seen on the crack surface.
the crack reached this area. The 12 mm thick plates had more necking at the crack than the 18mm plates, however, the necking profile seemed to be little influenced by the different (circular and elliptical) biaxial strainings. The through thickness strain from the TPB tests was about 65% less for both thicknesses, Figure 3.27(b), and since the crack propagation distance in the TPB test was small the necks were probably not allowed to be fully developed.

A 12mm thick plate without a starter crack was also tested, but the pressure reached the safety limit of the machine and therefore the test was stopped. The maximum pressure was 18 MPa at an added volume of 54.5 litres, Figure 3.35(a). The mounted strain gauges failed at strain values from 5% (rim) to 10% (pole) corresponding to an added volume of about 12 litres. Figure 3.35(b) shows the strain ratio (radial/tangential) at the pole and close to the rim. The pole was equally biaxially loaded from the start of the loading, whereas at the rim the biaxial strain ratio changed as the plate was initially bent around the rim and changed to an equally biaxially loaded state.

3.2.4 Calibration of Fracture Criteria and Tensile Test Simulations

A finite element model was made to simulate the rectangular tensile tests. Both a model with shell elements and one with solid elements were used to calibrate the fracture properties. Several different mesh sizes were used to study the mesh dependence.

Only the gauge length of the specimens was modelled of which the free ends were constrained to move in the thickness and the width directions. One end was totally fixed while the opposite end was loaded with a linear increasing velocity to avoid spurious noise. The material strain hardening law found earlier from tensile tests, with $C=805$ and $n=0.23$, was
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Figure 3.34: (a): Comparison of the final necking profiles (through thickness strain) for the plates with a 170mm starter crack. The strains were measured at x=30mm perpendicularly to the crack direction. (b): The through thickness strain along the cracks. The final crack length were about 60mm for the 12mm plate with 170mm starter crack and 90 mm for the remaining plates.

Figure 3.35: (a): The pressure versus added volume curve for the uncracked 12mm plate in the circular orifice. (b): The biaxial strain ratio (radial/tangential) at the pole and at the rim for the 12mm uncracked steel plate.
used in the simulations. Young’s modulus and the yield stress were 210,000 MPa and 310 MPa respectively and Poisson’s ratio was assumed to be 0.3.

**Simulation of Tensile Test with Solid Elements**

Simulations without fracture were performed to find the maximum plastic strain and the critical fracture values for the different uncoupled fracture criteria at the experimental point of fracture. The finite element simulation with solid elements showed reasonably good agreement with the experimental results for an element length of 1mm, Figure 3.36. With a larger element size than 2mm, the finite element results were too stiff after the point of necking, due to a limited number of elements for description of the necking profile. Moreover, when very small elements were used, they became too distorted and lost accuracy. By using a different aspect ratio with a smaller element length in the loading direction, better results were achieved. However, as the purpose of these simulations was to find an appropriate criterion for different biaxial loadings, the aspect ratio was kept as close as possible to 1:1:1 for solid elements and 1:1 for shell elements.

Only the criterion by Rice and Tracey together with the two CDM models were used in the following simulations. The criterion by Johnson-Cook would demand more experiments at different triaxialities to determine the damage parameters. However, the Johnson-Cook damage model could have had the same parameters as in the Rice-Tracey criterion, but then the criterion would have been identical to the Rice-Tracey criterion as the strain rate and the temperature effects were negligible in all experiments. As the triaxial stress states were positive in all experiments, the RTCL criterion would also give the same results as the Rice-Tracey criterion and was therefore not considered. The other criteria that are uncoupled from the constitutive material law were not believed to achieve more accurate results than the Rice-Tracey criterion and were therefore disregarded.
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Figure 3.37: FE-simulation of the rectangular tensile test specimens with brick elements. (a): The true stress-strain curves with the Rice-Tracey damage criterion. (b): The triaxiality in the centre of the smallest cross section and the average triaxiality during the loading.

The maximum plastic strain in the centre of the FE-specimen at the experimental point of fracture was found to be 1.13 and the triaxial damage strain parameter, \( \varepsilon_0 \), in the Rice-Tracey model was found to be 1.32 by use of the same method as in Section 3.1.5. In Figure 3.37 the true stress-strain curves and the triaxiality in the centre of the smallest cross sections are given for the simulation by the Rice-Tracey damage criterion. The average triaxiality was found to be 0.47 by use of Equation 3.2.

Damage was earlier found to be initiated at a threshold strain of 0.1, but the critical damage value \( D_{cr} \) and the damage exponent \( \alpha \) in Bonora’s model could not be derived from the experiments and were thus taken from Bonora (1997) to be 0.1 and 0.2 respectively. The constant \( \varepsilon_{f,uni} \) for Lemaitre’s CDM model can be found from proportional loadings with constant triaxiality, according to Equation 3.4, and was found to be 1.25. The same parameter for the damage model by Bonora under proportional loading was also found from Equation 3.5 to be 1.25. The material parameter \( S \) in the Lemaitre CDM model was obtained from Equation 3.6 as 14.4 MPa. The damage curves for Bonora and Lemaitre together with the experimental results found by Equation 2.29 are shown in Figure 3.38. The damage parameters for both Lemaitre and Bonora are given in Table 3.4.

The strain hardening curves for the Lemaitre and Bonora damage models had to be obtained. The hardening curves for the two CDM models were found from \( r = \varepsilon_0(1 - D(\varepsilon)) \) and \( \tilde{\sigma} = \frac{\sigma}{1 - D(\varepsilon)} \) following the procedure in Section 3.1. The least square method was used to fit the hardening curves to a power law and the curves are shown in Figure 3.38.
Figure 3.38: (a): The damage functions for Lemaitre and Bonora. (b): The strain hardening curves for the damage models.

Figure 3.39: Simulation of the tensile test with the different damage models.

Table 3.4: Damage parameters and power law coefficients for the damage models.

<table>
<thead>
<tr>
<th>Fracture Model</th>
<th>Damage Parameters</th>
<th>Power law coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rice-Tracey</td>
<td>$\varepsilon_0=1.32$</td>
<td>$C=805\text{MPa}, ; n=0.23$</td>
</tr>
<tr>
<td>Lemaitre</td>
<td>$\varepsilon_{th}=0.10$, $D_{cr}=0.10$, $S=14.4$ MPa</td>
<td>$C=866\text{MPa}, ; n=0.29$</td>
</tr>
<tr>
<td>Bonora</td>
<td>$\varepsilon_{th}=0.10$, $D_{cr}=0.10$, $\varepsilon_f=1.25$, $\alpha=0.2$</td>
<td>$C=840\text{MPa}, ; n=0.27$</td>
</tr>
</tbody>
</table>

Figure 3.39 gives the force-deformation curves for the simulations by the different damage models and good agreement with the experiments is achieved.
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Tensile Test Simulation with Shell Elements

The simulations of the tensile tests with shell elements were performed with several element sizes to investigate the influence of mesh discretisation. Five different element sizes formed part of the simulations and the same procedure and material properties as before were used to find the critical fracture value, however, only the Rice-Tracey criterion was applied to the simulations. A disadvantage of shell elements is the mesh dependence when a tensile specimen has obtained a well developed neck. The results become very sensitive to the mesh discretisation. In a localised neck the stress state is highly triaxial and as shell elements are based on plane stress \((\sigma_{33} \equiv 0)\), the accuracy is lost. Besides, shell elements are not able to model the out-of-plane shear that develops in the necking region.

Figure 3.40 clearly shows how small elements (element side length \(<\) thickness) tended to develop a localised neck too rapidly, while the largest elements did not develop a localised neck at all. However, the dimensions of the specimen used here were not suited for simulation of the tensile test with element sizes larger than about 4mm, as at least three quadratic elements should normally be used in the width to avoid a too stiff response.

In most finite element simulations of various plate structures the element sides are of the size of \(t\) up to 100\(t\) where \(t\) is the plate thickness. In the automotive industry, which is the leading industry within crash simulations, a typical element side length is about 6-10 times larger than the thickness, giving a typical element length of 6-10mm. With an element size of five times the thickness or larger a localised neck will usually not be developed in the numerical simulation (if a power law is used for the strain hardening description). Due to the absence of a localised neck, the triaxiality will remain almost constant at a value of 1/3 throughout the calculation, which means that the damage value in the Rice-Tracey criterion is nearly the same as the plastic failure strain value for the specific element size.

Due to the element size sensitivity the same element size should always be employed in the actual simulation of the plate structure as in the simulations of the tensile test where the material properties are calibrated. However, if the option described in Section 3.1, where the critical damage values versus the element lengths are applied, the results will become less mesh-dependent.

The triaxial stress state after localisation for the 1mm element size was not correctly described, which of course also affected the damage evolution. This was due to the elements in front of the crack tip becoming very distorted. It is noteworthy that the maximum stress triaxiality that can be obtained by use of shell elements is 2/3. In Figure 3.41 the critical fracture values \(\epsilon_0\) for the Rice-Tracey criterion are shown with different element sizes. The curve was derived from numerical simulations of the tensile test with various element sizes. At the point of fracture in the experiments the maximum damage values for the elements sizes were found at the corresponding point in the numerical simulations.

A practical way of achieving better results in simulations of tensile tests like these would be to modify the strain hardening curve after the equivalent plastic strain equal to the power
Figure 3.40: Simulation of tensile test with shell elements with various element side lengths for a 12mm thick plate. (a): Force-deflection curve for five element sizes. (b): Triaxiality curve.

Figure 3.41: The critical damage value $\varepsilon_0$ for the Rice-Tracey criterion at varying element sizes.
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Figure 3.42: The FE-element model of the three-point-bend tests. (a): The total model with the denting and supporting cylinders. (b): A close-up of the mesh around the crack tip of an element size of 0.5mm.

law coefficient $n$. For small elements the curve tangent should be increased to achieve a better result and the opposite for larger element sizes. The calibrated parameters should then be used for the same element size in the actual finite element simulation. However, as seen in Figure 3.40 the results for larger elements are within reasonable agreement with the experimental results, and to simplify the derivation of the critical damage values for various element sizes, it is recommended that the same strain hardening curve is used for all element sizes.

3.2.5 Simulation of Three-Point-Bend Tests

The three-point-bend tests were simulated of solid elements with various element side lengths in the fracture area. The indenting and the support cylinders were modelled as rigid, and the support cylinders were totally fixed. The three-point-bend tests were simulated by the Rice-Tracey, Lemaitre and Bonora fracture models with the material properties found from the tensile test simulations with solid elements, Table 3.4. Figure 3.42 shows the geometry with an element side length of 0.5mm.

The machine-made starter crack generated a higher force in the experiments than a natural sharp starter crack would have required. The finite element models were therefore also modelled with a 1mm wide starter crack, except in the simulations with element side lengths of 2.5mm where the crack width was the same as the element side length.

Only the element size of 0.25 mm was able to simulate the crack propagation correctly. The material properties used were the average values taken from the different plates and thicknesses, and this was probably the reason why the curves did not match totally. Nevertheless, the trend of a rapidly decreasing force-deflection curve after the maximum force should at
least be correctly predicted. Generally, with larger elements, higher forces were achieved for
the crack to begin propagating because the elements had to be deformed more to reach the
damage state, Figures 3.43 and 3.44. With the Rice-Tracey model the force-deflection curve
matched the experimental results for an element size of 0.25mm. The CDM model by Bonora
with an element length of 0.25mm showed reasonably good agreement with the experiments.
To achieve more accurate results with larger elements, the stress strain curves as well as
the fracture criteria values would have to be adjusted. The reason for the small differences
between the three damage models before the point of fracture was probably differences
in the strain hardening curves.

The linear CDM model proposed by Lemaitre was not able to predict the deformation at
the maximum force and was also not able to simulate the reduction in the force when the crack
started to propagate. The reason for the CDM model by Lemaitre not being able to simulate
the fracture process was the high triaxiality value of 2.5 in front of the crack tip. It was
previously shown that the CDM model by Lemaitre is not capable of predicting the damage
process at high triaxialities, see Section 3.1.5. The high triaxiality value of 2.5 is also in
accordance with the classical slip-line analysis by McClintock (1971) for a three-point-bend
test.

Figure 3.45 shows the simulation of the TPB test by the Rice-Tracey fracture model with an
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Figure 3.45: The FE-element model of the TPB-test with the Rice-Tracey fracture criterion and with an element size of 0.5mm. (a): Contour plot of the damage indicator. Red: \( D_i = 1.0 \), Blue: \( D_i = 0.0 \). (b): Contour plot of the damage indicator in the centre cross section of the specimen. The thumbnail shaped crack front can be seen just above the high damage field. (c): The triaxial stress state. Red: \( \frac{\sigma_{hh}}{\sigma_{eq}} = 2.5 \), Blue: \( \frac{\sigma_{hh}}{\sigma_{eq}} = -2.5 \)

The minimum element size for describing the crack propagation with solid elements is 0.25mm, and it was previously mentioned that in order to model the fracture correctly the length scale must be around 0.1-0.2 mm for steel. It is believed that with element sizes of 0.1 mm the same results would almost be achieved, but the results would probably never converge if even smaller elements were used. A nonlocal algorithm is often used in simulations where the element sizes are smaller than the macrolength scale of the material, which will lead to less mesh sensitive results.

The plastic crack growth resistance curves for the simulations by the Rice-Tracey and the Bonora damage models and element sizes of 0.25mm are compared with the experimental results in Figure 3.46. The \( J \) values in the FE-simulations are calculated in the same manner as in the experiments, Equation 3.9. The \( J - R \) curves from the simulation by the Rice-Tracey damage model nearly coincide with the \( J - R \) curves obtained from the experiments, while the damage model proposed by Bonora resulted in slightly lower curves. The \( J - R \) curves obtained by Lemaitre’s CDM model are not shown due to the poor agreement with the results.

The TPB test was only simulated with solid elements, since with shell elements of 1 mm and larger the number of elements were too few to be able to describe the crack propagation correctly. It was also of no interest to simulate the three-point-bend test with shell element sizes less than 1mm, as the element size to the thickness ratio would then have been to small for practical applications.
3.2.6 Simulation of Bulger Tests

The bulger tests were simulated with both solid and shell elements. Solid elements were used to be able to describe the out-of-plane stress state at the crack tip, as it is well known that a plane stress assumption is not valid at a crack tip in a thick plate, which is seen in Figure 3.45. Moreover, based on the results of the simulations of the TPB tests, very small elements are needed in modelling of crack propagation. The small elements made the simulations of the crack propagation in the bulger test very CPU demanding and to reduce the calculation time, symmetry conditions were used in a plane perpendicular to the crack. Often in simulations of TPB tests a second symmetry plane parallel to the crack is used, located in the crack centre, but as a symmetry plane could influence the direction of the crack propagation the symmetry condition along the crack was not used. Lemaitre’s damage model was clearly not capable of simulating crack propagation, due to the high triaxiality in front of the crack tip, and was therefore disregarded for the simulations of the bulger experiments.

The bulger dies were modelled with shell elements and the friction coefficient between the plates and the dies was estimated to be 0.3, Figure 3.47. A control volume was modelled at the back of the plate where the internal volume was increased at a linearly increasing volume rate by using the airbag option in LS-DYNA. This generated a pressure at the back of the plate and created a loading similar to that in the experiments, which made it possible to simulate decreasing pressure at increasing volume.

Simulations with Solid Elements

To describe the through thickness stress and strain distribution, six and eight solid elements were used in the thickness direction for the 12 mm and 18 mm plates respectively, and three
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Figure 3.47: The FE-element model of the aperture. (a): Circular aperture in shell elements with the control volume modelled with the coarse mesh. (b): Elliptical aperture in shell elements.

times as many elements were used through the thickness in the crack area. The starter cracks were modelled with the same width as in the experiments (1.0mm). In Figures 3.48 and 3.49 the undeformed 12mm circular plate and the deformed 18 mm elliptical plate are shown.

To increase the time step size in the simulations, the smallest elements were mass-scaled, which means that the density was increased for these elements. The total increase in mass was below 2% of the total mass and the effects from the added mass were therefore assumed to be negligible on the global behaviour. The simulations were conducted on an MPI-SUN machine where each simulation used about 300 CPU hours.

The cracks started to propagate from one of the corners of the starter crack. As the cracks propagated the crack front had the same shape of a thumbnail as in the TPB simulations and not a V-shaped crack front as observed in the experiments, Figure 3.50. It is believed that the mild steel plates used in the experiments had the same impurities in the centre of the thickness as the 40mm thick steel plate in Section 3.1. This most likely caused the V-shaped crack front in the experiment due to the brittler behaviour of the centre material. The reason why the TPB test did not have the same V-shaped crack front could be due to less straining in the TPB specimen compared to the large biaxial straining in the bulging tests. When the crack surfaces of the specimens were studied, deep cracks into the material could be observed in the middle plane of the plate, which confirms that the material was not totally homogenous.

The pressure-volume curves were generally slightly higher in the simulations than in the experiments, and the initiation of crack propagation was slightly delayed when the Rice-Tracey criterion was used. Nevertheless, the results still encouragingly show that a simple criterion as the Rice-Tracey damage model can predict crack propagation in thick plates.
Figure 3.48: The FE-element solid model of a plate with a 50mm crack. (a): Overall geometry. The symmetry plane is used in the model. (b): The transition from large to smaller elements.

Figure 3.49: The FE-element solid model of the 18mm thick plate in the ellipse shaped orifice with a 170 mm starter crack. (a): The deformed plate. (b): Crack propagation.
subjected to various biaxial loadings. The impurities in the thickness centre most likely also caused the plate to fracture earlier than if the plate had been homogenous as assumed in the current model. The numerical calculation by Bonora’s CDM model failed to simulate the crack propagation. Even though the elements were very small, probably even smaller elements would be needed to achieve more accurate results.

The through thickness strains in the vicinity of the crack are compared in Figure 3.52. The Figure shows the final through thickness strain variation perpendicular to the crack at a distance of 30 mm from the initial crack tip location for the plates with a starter crack of 170mm for both the numerical and the experimental results. The nominal through thickness strain and the through thickness strain at the crack obtained in the finite element simulations are in good agreement with the experimental results, however, the strain gradients were slightly steeper in the experiments than in the FE-simulations. The finite element simulation also showed that the necking profile is insensitive to the two different biaxial loading ratios.

The novel test set up with the giant bulger made it possible to study crack propagation in biaxially loaded plates. However, as already found in the experiments the biaxial strain ratio will not be uniform because of the crack and as the crack propagates the ratio will change. In Figure 3.53 the in-plane strain ratio (radial over tangential) is plotted just in front of the propagating crack for the 18 mm thick plates in both the circular and the ellipse-shaped dies at four different crack extensions. The shape of the orifices clearly influenced the strain biaxiality ahead of the propagating crack, and at the crack tip the radial strain vanished or
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Figure 3.51: Comparison of experimental and numerical results obtained by the Rice-Tracey model. (a): The pressure-volume curves. The thick curves show the numerical results, while the thin lines represent the experimental results. The circles indicate the start of crack propagation in the experiments while the triangles indicate the initiation in the numerical simulations. (b): The crack extension versus the added volume. The curves without markers represents the numerical results.

Figure 3.52: Comparison of the final through thickness strain for the experimental and the finite element simulations when the cracks had propagated about 90mm. (a): The curves show the strain variation perpendicular to the crack at a distance of 30 mm from the initiation point. (b): Comparison of the necking profile of the 12mm plate with a 50mm starter crack along the crack.
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Figure 3.53: Biaxial strain ratio (radial over tangential) in front of the crack tip for 18 mm thick plate in the circular and elliptical die. (a): The strain ratio after 1mm of crack propagation. (b): The strain ratio after 7-8mm of crack propagation. (c): The strain ratio after 22-24mm of crack propagation. (d): The strain ratio after 38-41mm of crack propagation.

became compressive. In Figure 3.54 the upper thickness layer is removed from the FE-model in front of the crack tip to illustrate the triaxiality distribution in 3-D. The triaxial stress state distributions and therefore also the damage distributions ahead of the crack tip were not noticeably influenced by the strain biaxiality, see Figure 3.55.
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Figure 3.54: The triaxiality distribution in the 18mm thick steel plate bulged in the ellipse-shaped orifice. The upper layer of the thickness has been removed just in front of the propagating crack tip. (a): Half model. (b): The crack tip.

Figure 3.55: Comparison of the simulations of the circular and the elliptical die with a 12mm plate and a 50mm starter crack. The figure shows results at four different crack propagation distances. (a): Damage distribution ($D_i$). (b): Triaxiality distribution.

Simulations with Shell Elements

To be able to simulate large plate structures, such as ships and oil rigs, with propagating cracks shell elements must be used. The shell element sizes used in the simulation of the
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The results of the numerical simulation of the 12mm thick plate with a 50 mm starter crack. (a): The pressure-volume curves. (b): The crack extension versus volume.

Bulger tests were 1mm, 2mm, 4mm, 8mm, 12mm, 24mm and 50mm. The RTCL criterion together with a critical damage value curve for different element sizes was used, with the curve obtained from the tensile test simulations with shell elements, Figure 3.41. In Figure 3.56 the results for the 12mm thick plate with a 50 mm starter crack are shown. The pressure-volume curve obtained from the numerical calculations agreed well with the experiments up to the point of fracture. However, in the simulation with a 1mm element size the plate fractured prematurely and the elements in front of the crack were unphysically stretched, see Figure 3.57(b). This was due to plastic localisation, which was also expected in view of the too early localisation in the simulations of the tensile tests with the same element size, Figure 3.40(a).

The simulation with an element size of 2mm had an excellent agreement with the experimental results. The fracture initiation in the experiment and the finite element simulation took place at the same added volume and the simulated crack propagation distance was in agreement with the experiment.

For larger elements the point of fracture occurred at a greater added volume than in the experiments, resulting in too high pressures which increased with the element size. This clearly indicates that when shell elements are used, whose element side length is larger than the plate thickness, the critical damage values for crack initiation and crack propagation are not the same.

To obtain better results for the bulging simulations the critical damage values \( \varepsilon_0 \) for the different elements sizes were calibrated, so that the same maximum pressure as in the experiments was obtained in the finite element simulations. A comparison of the critical damage value \( \varepsilon_0 \) obtained from the calibration procedure of tensile tests and the damage value calibrated to fit the maximum pressure in the bulging test is shown in Figure 3.59. As the mesh with 1 mm elements was unable to reach the maximum pressure, it was excluded from the
Figure 3.57: Simulation with shell elements of the 12mm thick plate with a 50 mm long starter crack. (a): The deformed plate with a 1mm element size in the crack propagation area. (b): The damage distribution ($D_i$) at the crack tip. The elements in front of the crack tip are unphysically stretched.

Figure 3.58: Simulation with shell elements of the 12mm thick plate with a 50 mm long starter crack. (a): The deformed plate with a 12mm element size in the crack propagation area. (b): The damage distribution ($D_i$) at the crack tip.
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Figure 3.59: Comparison of the critical RTCL damage value $\varepsilon_0$ obtained from the calibration procedure of tensile tests and the damage value calibrated to fit the crack propagation.

following simulations. Moreover, as the mesh with the 2mm elements showed good agreement with the experimental results, the critical damage value was the same as found in the calibration procedure of the tensile tests.

In the simulations with the new calibrated elements the correct maximum pressure was achieved, but, for the larger elements the crack propagated too fast compared to the experiments, Figure 3.60. The through thickness strain distributions in the plates for the different element sizes were identical some distance away from the crack, Figure 3.61. In the nearest vicinity of the crack larger elements could of course not achieve the maximum through-thickness strain. Nevertheless, the larger elements did manage to capture the necking profile. In Figure 3.62(a) the necking profiles for the 2mm and 12mm meshes are compared with the 12mm thick plate with a 50mm starter crack. The thickness reduction value is plotted in the centre of the elements, and the first value for the 12mm element size is therefore given at a distance of 6mm from the crack location. Excellent agreement between the two curves was obtained.

The strain biaxiality in front of the propagating crack in the simulations with shell elements is compared with the simulation with solid elements. The same results were obtained a few elements away from the crack tip, but at the crack tip the large shell elements were not able to simulate the bump in the biaxiality curve. Moreover, the elements in front of the propagating crack in the finite element simulation with a mesh size of 2mm were slightly distorted and thus also gave an incorrect result.

Shell elements cannot describe the stress state at the crack tip. The maximum triaxiality that shell elements can achieve is 2/3, which occurs in an equally biaxially loaded plate, as for shell elements $\sigma_{33} \equiv 0$ giving $(\frac{\sigma_{\text{eq}}}{\sigma_{\text{eq}}})_{\text{max}} = \frac{2}{3}$. As found previously from simulations with solid elements, the maximum triaxiality value at the crack tip was almost 2.5. In Figure 3.63(a) the triaxial stress state at the crack tip is shown from the bulging simulations with shell elements. The triaxiality actually decreases closer to the crack, instead of increasing. This definitely raises the question of using a void growth criterion for simulations of crack
Figure 3.60: The results of the numerical simulation of the 12mm thick plate with a 50 mm starter crack. (a): The pressure-volume curves. (b): The crack extension versus volume.

Figure 3.61: Contour plot of the through thickness strain at a crack propagation distance of 50mm for the 12mm plate with a 50 mm starter crack. (a): 2 mm element size. (b): 12 mm element size.
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Figure 3.62: Comparison of the 2mm mesh and the 12mm mesh. (a): The final necking profile (through thickness strain) perpendicular to the crack orientation at a distance of 50mm from the starter crack. (b): The biaxial strain ratio in front of the crack tip at a crack extension of 18mm.

propagation with shell elements. Further studies on this topic have to be conducted, but a possibility is to use the through thickness strain as a crack propagation criterion, as the necking profile in crack propagation was not affected by different loading ratios. However, the through thickness strain varied for different thicknesses.

By using different critical damage values for varying element sizes, differences in the damage states will be given. In Figure 3.63(b) the damage indicator is plotted for the 2mm and 12mm meshes at a crack propagation distance of 18mm.

3.2.7 Discussion

The novel test setup to study crack propagation and the fracture toughness characteristics in thick biaxially loaded steel plates was used for five different configurations with varying plate thicknesses, biaxial strain ratios and starter crack lengths. Several tensile tests in various directions for the five plates were employed to determine the material properties with a variation of 5%. An average value was used for the yield stress and for the power law coefficients.

Three-point-bend fracture tests were performed for the 12mm and 18mm thick plates. In the numerical simulations of the TPB tests with solid elements, it was found that a very small element size of 0.25mm was needed to achieve the same force-deflection curves as in the experiments. This element size corresponds to the characteristic material length for metals (see Table 2.2). The CDM model by Lemaitre was not able to model the fracture propagation, due to the high triaxiality in front of the crack tip, while the Rice-Tracey and the Bonora models showed good correlation with the experiments.
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Figure 3.63: Comparison of the 2mm mesh and 12mm mesh in front of the crack tip at a crack extension of 18mm. (a): The triaxiality. (b): The damage indicator distribution.

The bulger experiments were simulated with small solid elements of 0.25mm by means of the Rice-Tracey and the Bonora damage models. The modified damage model originally proposed by Rice and Tracey (1969) has shown to be able to predict ductile fracture propagation in thick steel plates subjected to biaxial loading. Nevertheless, the element size had to be of the same size as the macrolength scale of the material. If larger elements are to be used, the material properties have to be adjusted to be able to simulate correctly the crack propagation process. If smaller elements are to be used the forces and pressures would most likely be lower, but by using a nonlocal damage algorithm, the results would converge with a refinement of the elements.

In the simulations of the bulging experiments with shell elements, it was found that the initiation and the propagation damage values were not the same by application of the Rice-Tracey damage criterion. Moreover, if the critical damage value was calibrated so that the correct maximum pressure was achieved in the numerical simulations, the crack propagated too fast for the largest elements compared to the experiments. As shell elements are unable to achieve triaxiality values larger than 2/3 it is questionable to use a void growth criterion for simulations of crack propagation. Another approach which distinguishes between fracture initiation and crack propagation is necessary when shell elements are employed. Further complexity is added as the plates can have achieved various damage states before fracture initiation, which influences the crack propagation process.

However, in most finite element simulations of accidental loading on large complex plate structures, the element size is many times greater than the plate thickness. Therefore, the effects of applying the same criterion to fracture initiation and crack propagation are likely to be relatively small. The critical damage value should probably most often be calibrated for initiation, as it is of vital importance to be able to predict fracture initiation in crash simulations. And by using the same critical damage value for propagation the energy absorption capability will be slightly overestimated.
3.3 Study of Fracture Initiation in Ship Steel

The present section studies fracture initiation on 5mm thick steel plates subjected to biaxial loading. The "giant" hydraulic bulger at the University of Reading was used for the tests with plates made of mild steel without an initial crack. The critical plastic strain, RTCL and the two CDM models were used to demonstrate their validity to predict fracture initiation in two different biaxial stress states.

3.3.1 Introduction

During the bulging of a 12mm thick mild steel plate without a starter crack, the pressure reached the safety limit of the machine and the test was stopped. Additional bulging tests with 5mm thick mild steel plates were thus conducted to study fracture initiation in biaxially loaded mild steel plates. The same test setup and material as in Section 3.2 were employed, but the plates were oiled and pickled so the surface finish of the plates was of good quality and grinding was not needed. Since the plates were rolled to much smaller thicknesses and differently treated than the plates in Section 3.2, new tensile tests had to be conducted to derive the material properties for the 5mm plates.

3.3.2 Tensile Tests

Experiments

Tensile tests were manufactured from the plates covering three orientations, 0°, 45° and 90° to the rolling direction. The dimensions of the test specimens are given in Figure 3.64(a). The tensile tests were conducted in a 100 kN MTS machine with a loading displacement rate of 5mm/min. An extensometer was attached to the specimens to measure the extension of the specimen gauge length.

The stress-strain curves showed that the material was nearly isotropic, 3.64(b). The yield stress, $\sigma_y$, and Young’s modulus, $E$, were found to be about 290 MPa and 210 GPa respectively. By using the least square method, a power law with the coefficient $C = 730$MPa and the hardening exponent $n=0.20$ was found to describe the material strain hardening.

Numerical Simulations

As the stress and strain distributions at the pole of a bulged plate are uniform over a relatively large area, the numerical simulations would be almost insensitive to the mesh discretisation. Therefore, to attain the damage parameters for the various fracture criteria and damage
models, tensile test simulations with solid elements were conducted, since simulations with shell elements would be influenced by the necking process.

The solid elements used in the tensile test simulations had an element side length of 1mm in the width and thickness directions of the specimen. In the loading direction an element side length of 0.5mm was used to avoid problems with distorted elements after large plastic deformations, see Figure 3.65. Only the gauge length of the specimens was modelled and the symmetry conditions along the specimen were used to reduce the calculation time.

The damage parameters obtained in Section 3.2 were used again, however, as the strain hardening curve and the maximum plastic strain in the specimen were not identical, new values of the $S$ parameter in Lemaitre’s damage model and the $\varepsilon_f$ value in Bonora’s damage model had to be derived. Moreover, new power law constants for both CDM models had to be found. The same procedure as described in Section 3.1.5 was applied and the new derived values are given in Table 3.5. The results from the simulations of the tensile tests are plotted in Figure 3.66(a). The curves show the engineering stress and strain calculated in the same manner as in the experiments. Good agreement with the experiments was achieved. In Figure 3.66(b) the three normalised damage evolution curves are plotted against the normalised strain.

**Table 3.5: Damage parameters and power law coefficients for the damage models**

<table>
<thead>
<tr>
<th>Fracture model</th>
<th>Damage parameters</th>
<th>Power law coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical plastic strain</td>
<td>$\varepsilon_{cr}=0.96$</td>
<td>$C=730\text{MPa}$, $n=0.20$</td>
</tr>
<tr>
<td>RTCL</td>
<td>$\varepsilon_0=1.12$</td>
<td>$C=730\text{MPa}$, $n=0.20$</td>
</tr>
<tr>
<td>Lemaitre</td>
<td>$\varepsilon_{th}=0.10$, $D_{cr}=0.10$, $S=9.39\text{MPa}$</td>
<td>$C=749\text{MPa}$, $n=0.21$</td>
</tr>
<tr>
<td>Bonora</td>
<td>$\varepsilon_{th}=0.10$, $D_{cr}=0.10$, $\varepsilon_f=1.19$, $\alpha=0.2$</td>
<td>$C=744\text{MPa}$, $n=0.21$</td>
</tr>
</tbody>
</table>

Figure 3.64: (a): Tensile test specimen for the 5mm thick plates. (b): The tensile test results.
3.3 Study of Fracture Initiation in Ship Steel

3.3.3 Bulging Tests

Experiments

Two bulging tests were conducted with crack-free plates and both test plates fractured after the plates had achieved a large bulge, Figure 3.67(a). For the plate bulged in the circular die the crack started at the pole and instantly propagated in shear fracture (45° through the thickness) in two directions oriented in the rolling direction, see Figure 3.67(b). For the plate tested in the elliptical orifice the crack was also initiated at the pole and was immediately extended in two directions following the major ellipse axis.

The plate in the circular die reached a maximum pressure of 8.91 MPa at an added volume of 71.4 litres. The pressure then slightly dropped and at the point of fracture the pressure was 8.88 MPa at an added volume of 73.3 litres. The pressurised area in the elliptical die was smaller than for the circular die, which of course generated higher pressures at a lower volume for the plate to fail. The fracture pressure for the ellipse was 13.2 MPa at a volume of 25.18 litres, Figure 3.68.

Finite Element Simulations

As earlier mentioned the stress and strain distributions at the pole were almost uniform over a large area, and therefore the numerical simulations were nearly unaffected by the element side lengths. Both CDM models were modelled with element side lengths of about...
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Figure 3.66: Simulation of tensile test with shell elements with various element side lengths for a 12mm thick plate. (a): Engineering stress-strain curve for RTCL, Lemaitre and Bonora damage models. (b): The normalised damage curves for RTCL, Lemaitre and Bonora damage models.

Figure 3.67: The 5mm thick plate bulged in the circular orifice. (a): The bulged shape. (b): The crack.
3.3 Study of Fracture Initiation in Ship Steel

Figure 3.68: The results of the bulging experiments with 5mm thick mild steel plates.

10mm, whereas the simulations by the RTCL criterion were modelled with element sizes of 5mm, 10mm, 15mm and 30mm to verify that the stress, strain and damage distribution were nearly uniform at the pole and therefore insensitive to the mesh discretisation. All plates, for both the circular and elliptical dies, were made of Belytschko-Tsay shell elements with five integration points through the thickness. As the material properties were not perfectly homogenous imperfections were introduced. The element thickness varied randomly according to a Gauss distribution with a mean value of 5.0 mm and a standard deviation of 0.01 mm.

The die and support structures were modelled as rigid and the friction coefficient between the plate and the die was estimated to be 0.3. The airbag option in LS-DYNA was used to pressurise the plate by an incompressible fluid at a constantly increasing volume rate.

The response of the plate bulged in the circular orifice was slightly stiffer in the experiments than in the finite element simulations, nevertheless, the same maximum pressure was obtained in the numerical simulation but at a higher added volume, Figure 3.69(a). The simulations by the critical plastic strain failure criterion and Lemaitre’s damage model overestimated the added volume at failure, whereas the CDM model by Bonora predicted failure slightly too early. The RTCL criterion managed with reasonable agreement to predict the added volume at fracture. As the experimental response was stiffer, which could be due to improper calibration of the bulger machine, the fracture could have occurred at a lower volume in the experiments.

In the finite element simulation of the plate bulged in the elliptical orifice, the CDM models and the RTCL criterion predicted the point of fracture, while the critical plastic strain criterion again overestimated the added volume at failure, Figure 3.69(b).

The simulations of the bulging process in the circular die are almost insensitive to the discretisation of the elements if, of course, a sufficient number of elements are used to describe the bulging process. In Figure 3.70 the damage indicator for an element size of 10mm and 30mm is plotted immediately before the point of fracture. As it is seen the damage field was
uniform and unaffected by the element size. The same was valid for the plates bulged in the elliptical die, but immediately before the point of fracture a localised neck was developed and thus the element size could influence the point of fracture. Nevertheless, the added volume at the point of fracture varied very little in the simulations of the elliptical die with various element sizes.

The triaxiality was uniform over the circularly bulged plates, with a value of 2/3. In the elliptically bulged plates, the triaxiality varied as shown in Figure 3.71 by a maximum value of 0.65. The through thickness reduction at the pole at the point of fracture for the numerical simulation of the circular plate was 50% and for the plate in the elliptical die the reduction was 45%.

### 3.3.4 Discussion

The RTCL criterion predicted fracture with reasonable accuracy, whereas the critical plastic strain criterion overestimated the added volume at fracture. The CDM models by Bonora and Lemaitre were not calibrated for the 5mm mild steel plates and the damage parameters were therefore assumed to be the same as in the previous section. It is thus likely that with correct damage parameters, better results could be achieved for both CDM models. It is also believed that since the triaxiality in the two bulging tests was relatively low (max 2/3), the difference between the two CDM models was small.
3.3 Study of Fracture Initiation in Ship Steel

Figure 3.70: The RTCL damage indicator $D_i$ at an added volume of 79 litres (immediately before fracture). Red indicates a damage state above 0.9. (a): 10mm mesh. (b): 30mm mesh.

Figure 3.71: The triaxiality distribution. (a): The circular die. (b): The elliptical die.
Aluminium has proved to be a highly cost-effective building material for weight critical structures. Compared to steel aluminium has low density, it can be extruded in complex geometries and has good corrosion resistance in many environments. Aluminium is today a common material in shipbuilding, especially for high-speed passenger ferries and for superstructures. It is therefore of importance to study fracture initiation and crack propagation and which of the fracture criteria can be used in simulations of ship collisions and grounding of ships built of aluminium. Aluminium plates of a thickness of 10.5 mm were tested at different triaxialities in order to study five different fracture models. The plates were of heat-treated aluminium alloy EN 6082 (T6), which is a common alloy in shipbuilding as the alloy has relatively high strength and at the same time is suitable for welding. In the following section the experimental setups and results will first be presented and thereafter the experiments will be compared to the numerical simulations of the tests.

### 3.4.1 Test Specimens and Experimental Setup

In this study three different fracture models were investigated for different stress states. The RTCL damage model was investigated together with the two CDM models by Lemaitre and Bonora, where the damage evolution is included in the constitutive material law. A similar study has recently been made by Bao and Wierzbicki (2002) in a comparative study on various ductile fracture criteria in aluminium. The chemical composition of the aluminium alloy as follows

<table>
<thead>
<tr>
<th>Weight %</th>
<th>Al</th>
<th>Si</th>
<th>Fe</th>
<th>Cu</th>
<th>Mn</th>
<th>Cr</th>
<th>Mg</th>
<th>Zn</th>
<th>Ti</th>
<th>Others Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alloy 6082</td>
<td>Bal</td>
<td>0.7-1.3</td>
<td>0.5 max</td>
<td>0.1 max</td>
<td>0.40-1.00</td>
<td>0.25 max</td>
<td>0.06-1.20</td>
<td>0.2 max</td>
<td>0.1 max</td>
<td>0.15 max</td>
</tr>
</tbody>
</table>

Tensile specimens with rectangular cross section were machined in three directions - 0°, 45° and 90° to the rolling direction. The tensile tests were carried out on an MTS machine at a 100 kN load cell with a loading velocity of 5 mm/min. The elongation was measured by an extensometer with a gauge length of 50 mm and during the test the specimens were unloaded several times to allow the smallest cross section to be manually measured. A strain gauge was initially mounted in the centre of the gauge length on every specimen and when a clear visible neck was formed, additional strain gauges were mounted to achieve additional damage values.

Round tensile tests were also performed with a circumferential groove with various radii to achieve different triaxial stress states. The grooved specimens had a diameter of 6 mm at
the smallest cross section and the two groove radii were 2 mm and 4 mm, which should give triaxiality values of 0.9 and 0.65 respectively for the 2 mm and 4 mm radii, according to Bridgman’s analysis, Equation 2.9. The gauge length for the smooth tensile test was 60 mm with a diameter of 8mm. The elongation was measured by an extensometer and during the test, the diameter at the smallest cross section was measured numerous times.

A commonly used experiment for studying the onset of fracture in compressive stress states is the so-called upsetting test. Short solid cylindrical test specimens are compressed between parallel plates, and due to friction between the specimens and the compressing plates, the cylinders acquire a barrel shape. Tensile stresses are developed at the equator, which eventually causes fracture on the specimen surface. The upsetting tests were performed in a 100 kN MTS machine with test cylinders of a diameter of 10mm and three different heights, \( h \), 7mm, 10mm and 15mm. The deformation was measured by an extensometer and the test was unloaded several times to inspect the cylinder surface for cracks. The tests were conducted with three different ratios of initial diameter, \( d_0 \), to height, \( h/d_0 \), of 0.7, 1.0 and 1.5, which in the following will be denoted H7, H10 and H15 respectively.

To investigate fracture initiation in biaxially loaded plates, the 10.5 mm thick aluminium plates were bulged in orifices with different shapes, as also described in Section 3.2.2. Several large-strain gauges were mounted in the centre and close to the rim, see Figure 3.73. The gauges were used to study the biaxiality during the test as well as to validate the FE-simulations. The plates were continuously loaded until fracture where, of course, oil leakage occurred.
3.4.2 Experimental Results

Rectangular Tensile Tests

The purposes of the tensile tests were to determine the material properties and to investigate whether the material could be considered as isotropic. The load-deflection curve for the different directions showed that the material was almost isotropic in-plane. However, the calculated $R_L$-values (Lankford parameters) showed a transverse anisotropy. The $R_L$-values are found from the rectangular tensile test specimens by the width ratio $\ln\left(\frac{w_0}{w}\right)$ over the thickness ratio $\ln\left(\frac{t_0}{t}\right)$, where $w_0$ and $w$ is the initial and the actual width respectively and where $t_0$ and $t$ are the initial and the actual thickness respectively, according to:

$$R_L = \frac{\ln\left(\frac{w_0}{w}\right)}{\ln\left(\frac{t_0}{t}\right)}$$

which gave a value of about 0.6 for the three directions (i.e. nearly in-plane isotropic), see Figure 3.74(a). This phenomenon is typical of cold-worked thick aluminium plates. However, the anisotropy was assumed to have small effect on the experimental results, and as the implemented material model is only valid for isotropic material behaviour, the alloy will be treated as isotropic in the following.
3.4 Study of Fracture Initiation in Aluminium

Figure 3.74: (a): The $R_L$-value in the $0^\circ$, $45^\circ$ and $90^\circ$ direction. (b): True stress-strain curve.

The transition from elastic to plastic state was smooth without any distinct yield point. The yield stress $\sigma_y$ was thus estimated to be 290 MPa. Young’s modulus $E$ was found to be around 70,000 MPa and after yielding low strain hardening was observed. A power law with the coefficients $C = 425$ MPa and the hardening exponent $n=0.085$ was found by a least square fit to describe the material strain hardening. The material showed some ductility where the fracture strain varied for the different directions. For $0^\circ$ and $90^\circ$, a true fracture strain of respectively 0.5 and 0.3 was found, Equation 3.13.

The fracture surfaces showed large shear fracture lips, where some of the typical fracture surfaces from the tensile test are shown in Figure 3.75(b).

The true stress-strain curve is given in Figure 3.74(b), which shows both the recalculated engineering stress-strain curve (Equations 3.11 and 3.12) and the curve calculated from the applied load divided by the actual minimum cross-sectional area. The curves follow each other as expected until necking, where the recalculated engineering stress-strain curve drops.

Due to the nucleation, growth and coalescence of voids during large plastic straining, the material exhibits a progressive reduction of load carrying area resulting in loss of material strength and stiffness. The damage was estimated from the change in Young’s modulus according to Equation 2.29. The damage evolution was found to increase rapidly at low strains and with further straining the damage evolution slows down. This is in accordance with other experimental results for different aluminium alloys, Bonora (1997). The damage process for aluminium can be described as a large number of small voids nucleating at second phase particles at low strains, and at further straining the void size remains almost constant while more voids nucleate. When the strain gets close to the fracture strain, larger voids will coalesce rapidly and eventually the specimens will fracture.

The critical damage can be estimated by $D_{cr} = 1 - A_f/A_0$, Bonora (1999), which gives a
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Figure 3.75: (a): Damage curve. (b): The fractured tensile test specimen.

Table 3.7: The derived material properties for the aluminium alloy.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus:</td>
<td>70,000 MPa</td>
</tr>
<tr>
<td>Yield stress:</td>
<td>290 MPa</td>
</tr>
<tr>
<td>Power law coefficients:</td>
<td>$C=450$ MPa , and $n=0.085$</td>
</tr>
<tr>
<td>Experimental failure strain:</td>
<td>$\varepsilon_f=0.3$</td>
</tr>
<tr>
<td>Damage parameters:</td>
<td>$D_{cr} = 0.28$, $\alpha = 0.44$, $\varepsilon_{th}=0.0014$</td>
</tr>
</tbody>
</table>

value of $D_{cr} = 0.28$, where $A_0$ and $A_f$ are the initial area and the fracture area respectively. This method is questionable\textsuperscript{1}, but since the specimens only developed a small diffuse neck, the method was accepted. The damage exponent $\alpha$ and the threshold strain $\varepsilon_{th}$ were found from plotting the double logarithmic values of the damage and the plastic strain, see Figure 3.76. From the linear curve fit of the damage points the damage exponent was found to be $\alpha = 0.44$. The threshold strain was found to be $\varepsilon_{th} = 0.0014$ obtained from

$$\varepsilon_{th} = \varepsilon_f \exp(- \exp(m/\alpha)) \quad (3.14)$$

where $m=0.74$ is the constant in the logarithmic curve fit, Figure 3.76. The material data found from this identification procedure is given in Table 3.7.

Grooved Tensile Tests

As the aluminium alloy was relatively brittle compared to steel, the method of manually measuring the diameter change for the grooved tensile tests was not appropriate. The specimens

\textsuperscript{1}The damage is a measurement of the area reduction due to void growth or stiffness reduction and not a measurement of the geometrical change of the specimen.
failed early without any considerable deformation and it was therefore difficult to achieve enough accurate data. An extensometer should be mounted on the specimens to measure the diameter change continuously, but such an extensometer was not available when the experiments took place. Nevertheless, even though the results cannot be used to calibrate the fracture criteria for different triaxial stress states, the data can be used to compare the FE-models and the experiments.

**Upsetting Test Results**

The upsetting test specimens were compressed and unloaded numerous times allowing visual cracks on the surface to be detected. The end areas of the specimens had rough surfaces to increase the friction between the steel fixture and the aluminium specimens. As the upsetting specimens were compressed, they lost their cylindrical shape and the cross sections became slightly ellipse shaped, due to the slightly anisotropic material. All fractured specimens had a 45° crack at the equatorial diameter, see Figure 3.77(d), where the cracks were developed on the surface of the minor ellipse axis. The force-deformation curves for the three different heights are shown in Figure 3.77.

The visual detection of the cracks made it hard to obtain accurate results. If the specimens were removed from the setup and studied in a microscope, small cracks were found earlier at the periphery which eventually grew to cracks that could be observed by the eye. In Table 3.8 the deformations are given at the point of fracture, which was defined as when a crack could visually be observed on the cylinder surface.

**Bulger Test Results**

The two bulger tests were carried out with crack-free plates, and both test plates fractured after considerable straining. For the circular die, which was oriented in the test rig with the
Figure 3.77: Force-deformation for the upsetting tests. (a): H7. (b): H10. (c): H15. (d): Deformed specimens, with 45° cracks on the surface.

Table 3.8: The deformation at fracture for the upsetting tests.

<table>
<thead>
<tr>
<th>Model</th>
<th>H7</th>
<th>H10</th>
<th>H15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deformation at fracture</td>
<td>3.2-3.55 mm</td>
<td>5.0-5.24 mm</td>
<td>5.7-6.3 mm</td>
</tr>
</tbody>
</table>
rolling direction vertically, the crack started at the pole and instantly extended vertically in two directions to the orifice, see Figure 3.78(a). The plate bulged in the elliptical orifice was also oriented with the rolling direction in the major axis of the die. There, the crack started again at the pole and propagated instantly in both directions following the rolling path, and halfway to the orifice both crack tips diverted creating a crack pattern shaped as a H, see Figure 3.78(b). Both plates had large shear lips on the cracked surfaces at the pole, and the crack seemed to have propagated in shear fracture (45° through the thickness), where the crack angle through the thickness varied randomly along the cracks. When the crack was initiated the elastic energy in the system was released so that a stress pulse was generated, which caused the large cracks in the relatively brittle aluminium alloy. Pieces of the aluminium were shot away from the plate during the fracture process and penetrated deeply into the safety window made of polycarbonate, which gives an idea of the accumulated energy in the system.

The plate in the circular die reached a pressure at the point of fracture of 10.7 MPa at an added volume of 55.6 litres. The pressurised area in the elliptical die was smaller than for the circular die, which of course generated higher pressures at a lower volume for the plate to fail. The fracture pressure for the ellipse was 14.9 MPa at a volume of 19.6 litres, Figure 3.79(a).

The strain gauges showed that the plate was subjected to almost isotropic straining but the gauges failed at strains of about 10%, Figure 3.79(b). The strain ratio (radial over tangential) for the circular die varied between 0.98 to 1.04, whereas for the elliptical die the strain ratio (minor axis over the major axis strain component) varied from 2.0 to 2.5. The strain ratios varied slightly along the radius of the plates.

### 3.4.3 Simulation of Tensile Tests

Finite element simulations of the tensile tests were conducted for which the FE-model was made of 8-node solid elements. In the FE-models, only the gauge length (75mm) of the tensile test specimens was modelled. One end was totally fixed while the opposite end was loaded with a linearly increasing velocity. Symmetry was used in the length-width plane to reduce the CPU time. The displacement was measured over a length of 50 mm.

As the material showed some anisotropy, especially for the fracture strain, the specimens transverse to the rolling direction were chosen as reference for the finite element simulations. They were a little less ductile and it was believed that for the upsetting tests and the bulger tests, the specimens would fracture in the “weakest” direction, which was confirmed by the bulging experiments. The material properties found from the 90° tensile tests were used for the simulation and to simplify the constitutive material model the material was modelled as isotropic. For the tensile and bulger simulations, this simplification most likely did not influence the results, however, it might have affected the results of the upsetting simulations.
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Figure 3.78: Fractured aluminium plates. (a): Circular die. (b): Elliptical die. (c): Fracture at the pole. (d): The T-shaped crack in the ellipse.
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Figure 3.79: (a): Pressure-volume curves for the aluminium bulger tests. (b): The biaxial strain state during the bulging for both the circular and the elliptical die.

The specimens were simulated without fracture to determine the critical damage value for the RTCL damage model at the experimental point of fracture. The identification process of the material and the damage parameters followed the procedure described in Section 3.1.5. The maximum plastic strain in the centre of the specimens was also found, as the failure strain found in the experiments was an average value (recalling Figure 3.9(b)). The engineering stress-strain curve obtained from the simulation matched the experimental results well, but the true stress-strain was a little too high compared to the experiments. The experimental test specimens experienced more necking than in the FEM, nevertheless, the earlier derived power law coefficient was accepted to model the material hardening curve. This could be due to uncertainties in the manual measuring at the smallest cross section. The average triaxiality value was found from Equation 3.2 to be 0.38 and by assuming proportional loading the damage evolution for Lemaitre and Bonora could be derived.

Equations 3.4 and 3.5 were used to find the fracture strain at a constant triaxiality of 1/3 for the Lemaitre and Bonora damage models, by use of the least squares method.

For the Lemaitre damage model the material constant $S$ had to be determined. By assuming proportional loading and as the failure strain and the critical damage are known, $S$ could be directly derived from Equation 3.6 and was found to be 1.65 MPa. For both CDM models the failure strains were found to be $\varepsilon_f=0.37$.

On the assumption that the void nucleation and growth followed Bonora’s damage evolution law, Bonora’s analysis gave an $\alpha$ of 0.44, $\varepsilon_{th} = 0.0014$ and a critical damage value of 0.28, when the earlier obtained plastic fracture strain was applied. The damage curves for Lemaitre and Bonora’s models are shown in Figure 3.80. Note the pronounced difference for Bonora’s damage curves for steel and aluminium, Figure 3.80(a). The material hardening curves for both Lemaitre and Bonora had to be calculated, as the damage is implicitly included in the power law description. The hardening curves were found from $r = \varepsilon(1 - D(\varepsilon))$ and $\sigma = \sigma/(1 - D(\varepsilon))$ and new power laws were fitted to the calculated curves, giving $C = 590$
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Figure 3.80: The damage functions for Lemaitre and Bonora.

Table 3.9: The derived material properties

<table>
<thead>
<tr>
<th>Model</th>
<th>Yield stress</th>
<th>Power law</th>
<th>Damage parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>RTCL</td>
<td>290 MPa</td>
<td>$C = 500$ MPa, $n = 0.085$</td>
<td>$\varepsilon_0 = 0.364$</td>
</tr>
<tr>
<td>Lemaitre</td>
<td>290 MPa</td>
<td>$C = 500$ MPa, $n = 0.225$</td>
<td>$\varepsilon_{th} = 0.0014$, $D_{cr} = 0.28$, $S = 1.65$</td>
</tr>
<tr>
<td>Bonora</td>
<td>290 MPa</td>
<td>$C = 500$ MPa, $n = 0.225$</td>
<td>$\varepsilon_{th} = 0.0014$, $D_{cr} = 0.28$, $\varepsilon_f = 0.37$, $\alpha = 0.44$</td>
</tr>
</tbody>
</table>

MPa and $n = 0.225$ for Lemaitre’s damage model and $C = 585$ MPa and $n = 0.16$ for Bonora’s damage model. The tensile tests were simulated by the RTCL, Lemaitre and Bonora damage models and in Table 3.9 the material properties for the material and the fracture models are given.

The finite element simulations showed good agreement with the experimental results, Figure 3.81.

The circumferentially grooved tensile tests were simulated with axisymmetric elements with 20 elements through the thickness. The elements had an initial aspect ratio of 1:2 (length to width) to assure that the elements were kept as quadratic as possible throughout the deformation. The models were loaded with a low linearly increasing velocity to avoid any dynamic effects. The material and the damage properties were the same for the three damage models as found earlier.

The strain at failure in the numerical model for the smooth specimens varied for the three different damage models, but they were in agreement with the experimental results, Figure 3.82. However, for the grooved specimens the strain at failure was highly underestimated by Bonora’s damage model, while the damage model by Lemaitre slightly overestimated the failure strain.

As seen in Figure 3.83 the effect of the triaxiality in the grooved specimens causes the damage
3.4 Study of Fracture Initiation in Aluminium

Figure 3.81: Fractured aluminium plates. (a): Engineering stress-strain. (b): True stress-strain.

Figure 3.82: Notched aluminium tensile test specimens. (a): Finite element mesh. (b): True stress-strain.
Figure 3.83: Grooved aluminium tensile test specimens. (a): The damage evolution in the centre of the smallest cross section. (b): The triaxiality in the centre of the smallest cross section for the RTCL damage model.

function in Bonora’s damage model to increase too rapidly and therefore the specimen failed far too early. The problem with Bonora’s damage model will be discussed later.

In Figure 3.83 the damage functions for the three damage models are plotted. As it is seen there are large differences in the damage evolution for the notched specimens between Bonora’s damage model and the two other damage models. The triaxiality versus the plastic strain for the RTCL specimen is also shown in Figure 3.83.

In the simulation of the bulger tests shell elements were used. The rectangular tensile specimens were therefore also simulated with shell elements. The same material properties were used for the simulations with shell elements as in the simulations with solid elements.

The same damage parameters for the CDM models that were used for the tensile test simulations with solid elements were used the simulations with shell elements. Fracture occurred slightly earlier for the CDM models earlier localisation in the thickness direction as is expected for shell elements. Nevertheless, good agreement was achieved when the earlier material and damage properties were used in the simulations of the bulger experiments.

3.4.4 Simulation of Upsetting Tests

The upsetting test specimens were modelled with 4-node axisymmetric elements. The compression plates were modelled as rigid, and a contact definition was applied between the cylinder and the contact surfaces with a friction coefficient of 0.6. The friction coefficient was found from a simple sliding test between aluminium and steel. Upsetting tests are very sensitive to the friction coefficient between the specimen and loading plates and it was therefore crucial that the correct friction coefficient was used.
3.4 Study of Fracture Initiation in Aluminium

Figure 3.84: Tensile test simulations with shell elements. (a): Engineering stress-strain. (b): True stress-strain.

Figure 3.85: The force-deformation curves for the upsetting tests. (a): H7. (b): H10. (c): H15.

The compression behaviour of cylinders with aspect ratios \( h/d_0 \), 0.7 (H7), 1.0 (H10) and 1.5 (H15), was analysed with different mesh sizes. The results were found to be relatively insensitive to the mesh discretisation. In Figure 3.85 the FE-results are compared with the experimental data.

The force deflection curve from the RTCL criterion agreed well with two of the experimental results, however, the onset of fracture was slightly delayed compared to the experimental observations. It is most likely that, as the material was slightly anisotropic, the experimental tests failed earlier. In the experiments, specimen H15 became very distorted in the axial direction and this is probably the reason for the poor agreement with the numerical results. The two other specimens also became ellipse-shaped, and thus the circumferential strain distribution was not uniform, and the higher strains most likely caused the specimens to fail earlier than if the specimens had remained cylindrical. This was also observed by performing additional upsetting tests where the upsetting cylinders were taken through the thickness with varying cylinder diameters.

The simulations by the CDM models by Lemaitre and Bonora gave poor results compared to the experimental data - both in the prediction of the force-deflection curve, as well as
Chapter 3. Experiments with Simple Components

Figure 3.86: (a): The triaxial stress state evolution during the deformation for the different models. (b): The triaxiality in the H7 specimen.

the final deformation at failure. This was nevertheless expected as the triaxial stress state is negative (i.e. compressive) in the upsetting tests and in both CDM models triaxiality functions decrease for increasing negative triaxialities, Equation 2.37 should be recalled. This is clearly in contradiction with experimental evidence. In Figure 3.86 the triaxial stress state evolution is plotted during the compression of the different specimens and fracture models. The surface at the equator first exhibited negative triaxiality and as the specimens deformed to a barrel shape, the triaxiality became larger and eventually positive, Figure 3.86. However, at the centre axis of the specimen the triaxial stress state was in pure compression during the whole test.

The CDM models are developed for ductile fracture due to void growth, and in upsetting tests the dominant fracture mechanisms are shear fracture. It is therefore questionable to use CDM models in compressive fracture test simulations.

Gänsler et al. (2000) found that the Rice-Tracey criterion worked well in upsetting tests, but they did not have the fracture criterion implemented in the FE-program. The results were achieved from post-processing and it is therefore possible that they did not observe fracture in the centre of the specimens.

3.4.5 Simulation of Bulger Tests

The finite element models for the circular and the elliptical dies were made of Belytschko-Tsay shell elements and had about 30000 nodes respectively 23000 shell elements, giving an element size of 5x5mm. In order to introduce imperfections into the model, the element thickness was defined by a random Gauss distribution with a mean value of 10.5 mm and a standard deviation of 0.01 mm. This should account for small changes in the material properties as well as for the thickness variations in the plate.
3.4 Study of Fracture Initiation in Aluminium

![Figure 3.87: Comparison of experimental and simulation results. (a): The volume-pressure curves for the circular and the elliptical die. (b): The biaxial strain ratio at the top of the pole for the circular and the elliptical die.](image)

The die and support structures were modelled as rigid, where the friction coefficient between the plate and the die was estimated to be 0.3. The airbag option in LS-DYNA was used to pressurise the plate by an incompressible fluid with constantly increasing volume.

For the simulations of the circular and the elliptical orifices, the Lemaitre and the RTCL gave almost the same results and agreed well with the experimental results, both for pressure-volume as well as for the biaxial strain ratio, Figure 3.87. The maximum pressure was a little higher in the experiments compared to the finite element calculations. The biaxial strain ratio in the circular orifice was almost constant 1:1, which was achieved both in the experiment and in the numerical simulation. In the elliptical orifice the biaxial strain ratio, defined as the minor axis strain component over the major axis strain component, varied during the loading and was slightly higher in the FE-simulation compared to the experimental result. The elliptical axis dimensions had a ratio of 1:2 (minor:major), which created a strain ratio of about 2:1 to 3:1.

In the simulation of the circular specimen the cracks propagated in two directions following the element orientation, whereas in the experiment, the cracks only went in one direction following the rolling direction. This difference can be ascribed to the fact that the material had small anisotropic material behaviour in the plane, whereas the finite element material was modelled as isotropic. The crack pattern for the elliptically shaped specimen was identical to the finite element simulation and the experiment.

By application of the CDM model proposed by Bonora, the plate is predicted to fracture at 50% of the maximum experimental pressure. The damage evolution curve in Bonora’s model describes probably correctly the void nucleation and growth for a triaxiality of 1/3. Due to the relatively high triaxiality (∼2/3, Figure 3.90) in both bulged plates the damage increases too rapidly and the plate therefore fails too early. The nucleation of voids is probably more sensitive to plastic straining than to the triaxial stress state. It is known from experiments
Figure 3.88: The fractured specimens by application of the RTCL model. (a): The circular die. (b): The elliptical die. It should be noted that the fracture pattern is identical to the experimental fracture pattern in Figure 3.78(b).

Figure 3.89: FE-simulation results for the different damage models and die shapes. (a): Plastic strain at the top of the pole versus the added fluid volume. (b): Normalised damage value at the top of the pole versus the plastic strain at the pole.
3.4 Study of Fracture Initiation in Aluminium

of aluminium that at small strains a large number of small voids nucleate. It is also proved that the void growth rate is increased with increasing triaxiality. However, it is likely that the nucleation of voids is not affected by the triaxiality to the same extent as the void growth. Moreover, the number of second phase particles, at which the voids nucleate, will not increase in number due to high triaxiality.

Therefore, for the bulger simulations, where the triaxiality is twice the triaxiality in tensile tests, the Bonora damage function will grow with a factor of four (remembering Equation 2.37) and with the large increase in damage at low strains, the specimens will fail too early, see Figure 3.89(b).

As seen in Figure 3.90 the triaxiality was kept constant at a value of 2/3 for the circular diaphragm and at a slightly lower value for the elliptical diaphragm of about 0.65. The equivalent plastic strain at failure was therefore nearly the same for the same damage model in the simulations of the circular and elliptical dies, Figure 3.89(b).

As the material was assumed to be isotropic and the geometry was axisymmetric, the bulging experiments with the circular die were simulated with axisymmetric elements, where the die and support frames were modelled as rigid, Figure 3.91. This was done to investigate whether the stress state could be considered as plane stress during the bulging process, and thus differences in the results would be achieved between the use of shell elements and solid elements. Ten axisymmetric elements were used through the thickness to describe the stress and strain distribution of the bulged plate. The same friction coefficient as for the shell element model was used between the plate and the die with a coefficient of 0.3. The plate was loaded with a uniform pressure as the airbag option could not be used for axisymmetric simulations. This also meant that the volume could not be calculated directly and therefore the models were compared with the maximum deflection at the pole.

The deflection-pressure curve obtained from the axisymmetric simulation gave results identical to those of the shell element model of the circular die. In Figure 3.92 the plastic strain, the damage and the triaxial stress state are shown for the RTCL model. The triaxiality through the thickness was almost constant before fracture over the whole bulge.
Figure 3.91: (a): The axisymmetric FE-element model of the circular bulging test. (b): Comparison of shell element and axisymmetric finite element models.

Figure 3.92: The axisymmetric FE-element model for the RTCL material model just before fracture. (a): Plastic strain. (b): Damage. (c): Triaxiality.
3.4 Study of Fracture Initiation in Aluminium

3.4.6 Discussion

Various experimental tests were performed with the aluminium alloy 6082 T6 to study the fracture initiation for different stress states. A combined Rice-Tracey and Cockcroft-Latham criterion proved to be able to predict fracture initiation for the different experimental setups. The CDM model proposed by Lemaitre worked well for the positive triaxial stress state, whereas the triaxial function decreased for increasing negative triaxiality for the upsetting test and predicted fracture too early.

Damage mechanics has successfully been used in prediction of fracture limit diagrams (e.g. Lemaitre (1984) and Tang et al. (1998)). The different tests performed in this study had various strain paths, which can be used in the construction of FFLD diagrams. The maximum in-plane strain pairs up to fracture for the RTCL criterion and the Lemaitre damage model are plotted in 3.93. In bulk metal forming it is well known that the slope of the fracture strain pair in a FFLD-diagram follows approximately a straight line with a slope of $\tan^{-1}(-1/2)$ in the second quadrant ($\varepsilon_1 > 0, \varepsilon_2 < 0$), Kudo and Aoi (1967). The same slope was found from the numerical simulations of the upsetting tests by application of the RTCL criterion, Figure 3.93(a). In sheet metal forming the slope is steeper with an angle of approximately $\tan^{-1}(-1)$ and the same was found from the bulger and the tensile test simulations when the RTCL criterion was applied. A slope of $\tan^{-1}(-1)$ means a constant thickness strain at fracture, since $\varepsilon_1 + \varepsilon_2 = \varepsilon_3$, which also the simplified McClintock criterion predicts, Equation 2.24 (Atkins (1997)).

The CDM model by Bonora was unable to predict failure in most applications with the derived damage parameters for aluminium. The parameters were found from experiment and agreed well with the damage parameters for aluminium in Bonora (1997). It is therefore believed that the nonlinear damage model by Bonora does not work for high damage exponents,
\( \alpha \), as the void nucleation is probably more affected by the strain than the triaxiality.
3.5 Strain Rate Effects on Ductile Fracture

Several studies (e.g. the classical work by Marsh and Campbell (1963) and recently Pussegoda et al. (1996), Mirza et al. (1996), Jones (1989)) have shown the effects of high strain rates on different materials. It was found that the material characteristics vary when subjected to high loading rate and especially the plastic flow and the fracture toughness are sensitive to the strain rate. It is thus of vital importance that a constitutive material model can predict the strain rate effects correctly in dynamic problems, such as vehicle crashworthiness and ballistic impacts. A ship collision is indeed a dynamic event and the structural response will be affected by dynamic effects, such as the material strain sensitivity, inertia effects and dynamic frictional forces. In the following section a few visco-plastic models are discussed, of which two have been implemented into LS-DYNA and are demonstrated on a simple tensile test.

3.5.1 Introduction

Many metals are sensitive to the strain rate and normally the apparent yield strength increases with increasing strain rate. There are many different constitutive material models for the strain rate sensitive behaviour of materials in the literature, and for structural engineers one of the most famous models was proposed by Cowper and Symonds (1957):

\[
\frac{\sigma_d}{\sigma_0} = 1 + \left( \frac{\dot{\varepsilon}}{C_{cs}} \right)^{1/q}
\]  

(3.15)

where \(\sigma_d\) is the dynamic flow stress at the uniaxial strain rate, \(\dot{\varepsilon}\), and \(\sigma_0\) is the static flow stress. The strain rate is defined as

\[
\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}
\]  

(3.16)

\(C_{cs}\) and \(q\) are material constants. The Norton model is a common model for metal formers and metallurgists:

\[
\sigma_y = C\varepsilon^n\dot{\varepsilon}^{m_N}
\]  

(3.17)

where \(m_N\) is a material parameter and \(\dot{\varepsilon}\) is the strain rate. As seen in Equation 3.17 the Norton model is an extended power law model that includes strain rate effects. Johnson and Cook (1983) proposed a model for the von Mises tensile flow stress with material strain rate and temperature dependence according to

\[
\sigma_y = [A_{j\varepsilon} + B_{j\varepsilon}\varepsilon^n][1 + C_{j\varepsilon}\ln \dot{\varepsilon}^*][1 - T^m]
\]  

(3.18)
where $\dot{\epsilon}^* = \frac{\dot{\epsilon}}{\epsilon_0}$ is the dimensionless plastic strain rate and $T^*$ is the homologous temperature, Equation 2.21. The five parameters $A_{j_c}, B_{j_c}, n, C_{j_c}, m$ are material constants.

Cowper and Symonds (1957) found $C_{cs} = 40.4$ s$^{-1}$ and $q = 5$ to be a reasonable estimation for scaling the initial flow stress for mild steel, and for aluminium alloys the values $C_{cs} = 6500$ s$^{-1}$ and $q = 4$ are often used. While a strain rate of 40.4 s$^{-1}$ will double the initial yield stress for mild steel, a strain rate of 6500 s$^{-1}$ is required to double the yield stress for an aluminium alloy. However, if these coefficients were used for the whole stress-strain range, it would result in a far too large value of ultimate stress. Many studies have also found that several aluminium alloys are more or less insensitive to strain rate effects.

One of the major problems in conducting strain rate tests is that the strain rate changes during the loading. The strain rate should preferably be constant during the loading, which means that the test machine has to grip the test specimens at a certain loading speed and increase the velocity throughout the deformation with the same rate as the elongation rate. As a neck starts to develop all deformation will take place in the necking area and the material in the necking region will thus exhibit even larger strain rates. It is therefore hard to achieve accurate measurement results and the parameters will typically be average values.

A strain rate option is implemented by the author into the different user-defined material models in LS-DYNA, where the dynamic yield stress is achieved by simply scaling the static yield stress according to Equations 3.15 and 3.16. An alternative approach has also been implemented into the material models to avoid spurious noise in the numerical solution due to the elastic part in the strain rate (Equation 3.16) and is described in detail in Appendix B.1.3. The alternative approach is a rate dependent plasticity option which is based on the equivalent plastic strain rate, where the dynamic yield stress is computed from

$$
\sigma^d_y(\dot{\epsilon}^p_{eq}, \dot{\epsilon}^p_{eq}) = \sigma_y(\dot{\epsilon}^p_{eq}) + \sigma_0 \left( \frac{\dot{\epsilon}^p_{eq}}{C_{cs}} \right)^{\frac{1}{q}}
$$

(3.19)

where $\sigma_0$ is the initial static yield stress and the plastic strain rate is found by

$$
\dot{\epsilon}^p_{eq} = \frac{\epsilon^p_{eq}}{\Delta t}
$$

(3.20)

where $\Delta t$ is the current time step increment. A disadvantage of this option is that the plastic strain increment has to be solved iteratively by a Newton-Raphson method, which considerably slows down the calculation speed. Nevertheless, the results are drastically improved when the rate dependent plasticity option is used instead of scaling the yield stress directly, as in Equation 3.15, see Figure 3.94. It should be noted that the two strain rate options do not give the same stress level when the same coefficients are used. In the following simulations only the rate dependent plasticity option will be used.
3.5 Strain Rate Effects on Ductile Fracture

3.5.2 Strain Rate Tests by ISSC

The 2003 ISSC V.3 Committee has performed both static and dynamic tensile tests with several thicknesses of mild steel at a strain rate range of 1 s\(^{-1}\) to 100 s\(^{-1}\). A power law hardening curve with the coefficient \(C=800\ \text{MPa}\) and \(n=0.25\) was found to describe the static hardening curve. To be able to calibrate the Cowper-Symond constants in the rate dependent plasticity model, an average dynamic stress value \(\sigma_d^{\text{avg}}\) was used, according to

\[
\sigma_d^{\text{avg}} = \frac{1}{n} \int_0^n \sigma^d d\varepsilon
\]

where \(n\) is the hardening exponent in the power law. The dynamic stress-strain curves for a 7 mm thick tensile test specimen are shown in Figure 3.95 together with the derived average dynamic stress values. The least square method was then used to minimise the residuals of the strain rate model:

\[
\mathcal{R}(\sigma_d^{\text{avg}}) = \sum_{\beta=1}^{N_\beta} \left[ \sigma_d^{\text{exp}}(\dot{\varepsilon}_\beta) - \left( \sigma_{\text{avg}} + \sigma_0 \left( \frac{\dot{\varepsilon}_\beta}{C_{cs}} \right)^{1/q} \right) \right]^2
\]

The strain rate constants determined from the identification procedure were found to be \(C_{cs} = 3600\ \text{s}^{-1}\) and \(q = 5.5\). It can be argued that the average dynamic stress value should be used to calibrate the strain rate coefficients instead of the maximum load, as the energy dissipation in crashworthy simulations is the most vital parameter. In car crashes with occupants where the maximum deceleration is to be found, the ultimate dynamic stress is often used since the human body can only withstand a certain g-force level. In Figure 3.95(b) the dynamic yield values are plotted together with the curve fits of the values.
Chapter 3. Experiments with Simple Components

Figure 3.95: (a): The stress-strain curves for a 7mm thick plate at various strain rates together with the average stress value. (b): The least square fit of the average dynamic stress, the yield stress and the ultimate stress values to the Cowper-Symond model, Equation 3.15.

3.5.3 FE-simulation of ISSC Tensile Test

The tensile specimens were made of shell elements with the same dimensions as in the experiments. Figure 3.96(a) shows the mesh that was used in the finite element calculations. The RTCL criterion was applied together with the rate dependent plasticity option as well as the parameters found above.

Reasonable agreement was obtained between the numerical calculations and the experiments. The stresses at low straining for the highest strain rates were about 5-7% lower in the finite element simulations than in the experiments, and when the strains had almost reached the failure strain, the FE-stresses were higher than the experimental stresses. This was of course because the static strain hardening curve was scaled according to the corresponding strain rate. The strain at fracture in the FE-simulations also agreed well with the experiments.

3.5.4 Discussion

From the identification process the strain rate parameters were found to be $C_{cs} = 3600\,s^{-1}$ and $q = 5.5$ and there is a great difference in the $C$ parameter as compared with the value for mild steel in Cowper and Symonds (1957). In simulations of e.g. ship collisions the use of the original values ($C_{cs}=40.4\,s^{-1}$ and $q=5$ ) may result in a far too stiff response, which will of course result in less penetration depth. By using a rate dependent plasticity algorithm, instead of scaling the yield stress directly according to the original Cowper-Symond model, spurious noise is avoided.
3.5 Strain Rate Effects on Ductile Fracture

![Image of mesh and stress-strain curve]

Figure 3.96: (a): The mesh of the 7mm thick tensile test specimen (b): Comparison of the experimental and finite element results.

When the rate dependent plasticity option is used, the deformation of the specimens will not localise to the same extent as in static simulations. When a neck has formed, all straining will take place in the necking area and thus exhibit a higher strain rate and as a consequence larger stresses. The localisation is therefore somewhat prevented by the strain rate effects and the specimens can be subjected to larger deformations before the maximum static fracture strain is reached. It is often observed in finite element simulations of dynamic tensile tests that larger deformations are achieved for higher strain rates, if the effect of strain rate on the fracture strain is not accounted for.
3.6 Concluding Remarks

Several different fracture criteria and models were calibrated and tested on various applications. The void growth model by Rice-Tracey have shown to be able to predict fracture initiation and crack propagation for positive triaxial stress states. The triaxiality function in the modified Rice-Tracey criterion gives a good approximation of the growth of the voids and thus the ductile fracture.

There is experimental evidence that for triaxial stress states in homogenous compression $\sigma_{eq} < -\frac{1}{3}$ fracture will not occur and to avoid that fracture occurs in triaxial stress states lower than -1/3, a cut-off of the damage function can be included when the first principal stress becomes negative in a plane-stress state. The coupled Rice-Tracey and Cockcroft-Latham (RTCL) criterion will cover the whole spectrum of triaxialities, as the Cockcroft-Latham triaxiality function has shown to model ductile shear fracture in negative and low positive triaxial stress states correctly, whereas as previously mentioned the Rice-Tracey criterion is used in the tensile stress states.

The more general empirical criterion by Johnson-Cook is partly based on the Rice-Tracey criterion, where the exponential triaxiality function can be calibrated so that the damage process can be better modelled for different materials. If only a single tensile test is performed for calibration of the damage parameters, the constants in the Rice-Tracey triaxiality function can be assumed to be a good estimate of describing the void growth. The Johnson-Cook damage model also includes the strain rate and the temperature dependence. The material strain rate sensitive is important in impact simulations of ships, but as the strain rate is relatively low, the adiabatic heating will be insignificant and thus the temperature dependence in the Johnson-Cook damage model can be neglected.

The CDM models are assumed to model the loss of stiffness due to void growth better than the uncoupled damage functions. The CDM model by Bonora has proved to model accurately ductile fracture for positive triaxialities in steel. However, for aluminium, where a higher value of the damage exponent is used, the damage function will predict fracture far too early in high triaxialities. In the damage model by Lemaitre, where the damage is a linear function of the strain, the ductility will not be correctly described due to the difficulties of modelling the effect of triaxial stress state on the fracture strain. The strain at fracture in the Lemaitre model is often overestimated at high triaxialities for the metals tested here. Moreover, both CDM models are not able to predict fracture for negative triaxialities, due to the quadratic triaxiality function and should therefore not be used in compression tests. Almost all material properties in the CDM models can be determined by tensile tests, where also the damage evolution is measured. An additional FE-simulation is needed where the material hardening law derived from the tensile test is used, to find the average triaxiality and the maximum plastic strain at the experimental point of fracture. Thus, no fitting of the parameters is needed to achieve the correct material and damage parameters.

The modified Gurson porosity model proposed by Needleman and Tvergaard (1984) is quite widely used today in small-scale simulations of fracture initiation and crack propagation. The
3.6 Concluding Remarks

GTN model has been shown to describe accurately the loss in load carrying capacity due to void growth. The major drawback of the GTN models is the many material parameters to be determined, which makes the model very hard to use for practical engineering problems.

In the simulations of the bulging experiments with shell elements, it was found that the initiation and the propagation damage values were not the same. Moreover, if the critical damage value was calibrated so that the correct maximum pressure was achieved in the numerical simulations, the crack propagated too fast for the largest elements compared to the experiments. The Rice-Tracey criterion was used for both initiation and propagation, and as shell elements cannot achieve higher triaxialities than 2/3 it is likely, when shell element are used, that the Rice-Tracey criterion is unappropriate for predicting crack propagation. This should be further investigated, as there is a need for a general criterion for to prediction/modelling of both the fracture initiation as well as the crack propagation.

For simulations of accidental loading of large structures with several different materials and thicknesses, the material and the damage models have to be relatively simple with only a few parameters to be determined. The RTCL model has proved to predict correctly fracture initiation in various materials and stress states and should be useful in accidental loadings of large structures. The Rice-Tracey and the Johnson-Cook models with a damage cut-off at high negative triaxialities are also suitable for these kinds of simulations. The CDM models are more CPU demanding, due to the Newton-Raphson iteration and more damage parameters to be determined.
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Chapter 4

Verification of the RTCL Criterion on Large Scale Experiments

The present chapter is devoted to validation of the RTCL fracture criterion by simulations of several different large-scale experiments of various collision or grounding scenarios. Several experiments of ship collisions and groundings have been carried out since the early 1960’s, and some of these have been selected for the validation of the two finite element fracture criteria for typical ship structures at various loadings.

4.1 Introduction

In the previous chapter it was found that the combined ductile fracture criteria of Rice-Tracey and Cockcroft-Latham, called RTCL, and the Johnson-Cook criterion were capable of predicting fracture initiation in various stress/strain states and for different materials. The element sizes were, however, very small, often many times smaller than the plate thickness. For large plate structures, like as ships and oil rigs, shell elements larger than the plate thickness must be used.

The mesh discretisation must be small enough to capture stress and strain concentrations, but at the same time not too small to avoid impractical calculation times. A rule of thumb is that the calculation time increases by a factor of about 8 if the element side length is reduced by half for the same structure. A reliable crashworthiness analysis can be recognised by the mesh being capable of smoothly representing the deformed structure. It was previously mentioned that in the car industry element sizes of 6 to 10 mm are used. The typical element sizes in typical ship collision simulations are today about 100x100 mm up to 1000x1000 mm in the collision area (Kitamura (2001a), Kitamura (1997) and Lee and Kim (2001)), giving an element size to thickness ratio $l_{el}/t$ of about 10 to 60, where $l_{el}$ and $t$ are the element side length and the plate thickness respectively.
To be able to describe stress and strain concentrations as well as fracture initiation, the element side length $l_{el}$ should only be a few times larger than the plate thickness. This is, of course, dependent on the geometry. Large unstiffened plates may have larger elements and will probably still be able to describe the stress and strain distribution, whereas to capture accurately the stress and strain field in e.g. stiffeners the ratio should probably be very small. When a crack has been initiated the problem will be very localised and as shown earlier, very small elements are needed to describe the stress and strain field correctly around the crack tip.

In the following simulations of large-scale experiments, the element length to thickness ratio was all around five. However, as the different members was of varying thickness, the element size to thickness ratio could not be constant. The critical damage value versus the element size for the RTCL criterion was found for the different structural members described in Section 2.6.

The damage parameters were calibrated by tensile test simulations in which about the same element sizes as in the large-scale simulations were used. With an element size around five times the thickness, a localised neck will usually not be developed in the numerical simulation (if a power law is used for the strain hardening description), and the strain at failure will therefore often correspond to the failure strain. Due to the absence of a localised neck, the triaxiality in the simulated specimens will be almost constant at a value of $1/3$ throughout the calculation, which means that the damage value in the RTCL criterion is nearly the same as the true failure strain value for the specific element size.
4.2 Experimental and Numerical Simulation of Double Hull Stranding

Amdahl and Kavlie (1992) conducted model tests simulating a grounding scenario of a rock indentation into two different double bottom structures. The two geometries had approximately a 1:5 scale of a typical double bottom and the two structures differed by the stiffening configuration.

4.2.1 Experimental Setup

The first structure had a relatively large number of floors and girders and longitudinal flat bar stiffeners on the inner and outer bottom plating. The plate thickness of the inner and the outer bottoms was 4 mm and the double bottom height was 500 mm. The thickness of the floors and girders was 3 mm. The strain hardening curves for the different members were given in the experimental description and fitted to power law curves. The geometries are shown in Figure 4.1 and the derived material parameters are given in Table 4.1.

![Figure 4.1](image)

Figure 4.1: The dimensions for the stranding test by Amdahl and Kavlie (1992). (a): The geometry for Model 1. (b): The geometry for Model 2.
Table 4.1: The material parameters for Model 1

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness</th>
<th>Yield Stress</th>
<th>Power law coefficients</th>
<th>Experimental Fracture Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer/Inner Side shell</td>
<td>4 mm</td>
<td>320 MPa</td>
<td>$C=770$ MPa $n=0.21$</td>
<td>$\varepsilon_f=0.30$</td>
</tr>
<tr>
<td>Girder</td>
<td>3 mm</td>
<td>335 MPa</td>
<td>$C=820$ MPa $n=0.22$</td>
<td>$\varepsilon_f=0.26$</td>
</tr>
<tr>
<td>Floors</td>
<td>3 mm</td>
<td>335 MPa</td>
<td>$C=820$ MPa $n=0.22$</td>
<td>$\varepsilon_f=0.26$</td>
</tr>
<tr>
<td>Longitudinals</td>
<td>3 mm</td>
<td>330 MPa</td>
<td>$C=750$ MPa $n=0.22$</td>
<td>$\varepsilon_f=0.29$</td>
</tr>
</tbody>
</table>

The second model had the same overall dimensions and plate thickness as the first structure, but with fewer longitudinal girders. The outer and inner bottoms together with the three middle floors had secondary flat bar stiffeners. A power law stress-strain curve was fitted to the given material hardening curves for the different structural members and given in Table 4.2.

Table 4.2: The material parameters for Model 2

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness</th>
<th>Yield Stress</th>
<th>Power law coefficients</th>
<th>Experimental Fracture Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer/Inner Side shell</td>
<td>4 mm</td>
<td>320 MPa</td>
<td>$C=750$ MPa $n=0.20$</td>
<td>$\varepsilon_f=0.31$</td>
</tr>
<tr>
<td>Frames</td>
<td>3 mm</td>
<td>330 MPa</td>
<td>$C=820$ MPa $n=0.22$</td>
<td>$\varepsilon_f=0.25$</td>
</tr>
<tr>
<td>Stringer decks</td>
<td>3 mm</td>
<td>330 MPa</td>
<td>$C=820$ MPa $n=0.22$</td>
<td>$\varepsilon_f=0.25$</td>
</tr>
<tr>
<td>Longitudinals</td>
<td>3 mm</td>
<td>275 MPa</td>
<td>$C=720$ MPa $n=0.22$</td>
<td>$\varepsilon_f=0.33$</td>
</tr>
<tr>
<td>Frame Stiffener</td>
<td>3 mm</td>
<td>275 MPa</td>
<td>$C=720$ MPa $n=0.22$</td>
<td>$\varepsilon_f=0.33$</td>
</tr>
</tbody>
</table>

4.2.2 Finite Element Model

The shape of the indenter was modelled as a truncated cylindrical cone. The dimensions of the indenter were estimated to have a semi-apex angle of 60 degrees with a diameter of 420 mm at the top.

The double bottom structures were modelled with a mesh size of about 15x15mm in the deformation area, giving an element to thickness ration of about 4 to 5, Figure 4.2. The user defined material models with the RTCL fracture criterion were used in the tensile test simulations where the strain hardening followed the power law descriptions in Table 4.1 and 4.2. Simulations of tensile tests with varying element sizes were conducted to achieve a curve of the damage value $\varepsilon_0$ against element size for each material, Figure 4.3. As the elements were relatively large, necking barely occurred and thereby the triaxiality was almost constant value of $1/3$ throughout the tensile test simulation.
Imperfections following a sinusoidal curve were assigned to the floors and girders with an amplitude of $0.1t$. A randomly varying Gauss distribution of the element thicknesses with a standard deviation of $0.01t$, where $t$ is the actual plate thickness, was assigned to the outer bottom shell. The plating was modelled with Belytschko-Tsay shell elements, while the stiffeners were modelled with fully integrated elements to avoid hourglassing problems. If the number of Belytschko-Tsay elements in width of a structural member are three or less it is possible for a certain loading that zero-energy modes may occur and to prevent this fully integrated elements was used (see also Section A.3). To be able to model the out-of-plane strain distribution five integration points through the thickness were used. The total number of elements for Model 1 and Model 2 was about 188,000 and 217,000 respectively.

The friction coefficient between the indenter and the structural members was assumed to be 0.3. The structures were indented at a low velocity to avoid dynamic effects.

### 4.2.3 Results

The force-deflection curves obtained from the finite element simulations are shown in Figure 4.4. Reasonable agreement with the experimental results was achieved. In Model 1 the outer bottom plating fractured at an indentation of 140 mm. After 190 mm the cone came in contact with the floors and girders and the force increased rapidly, but in the experiments the cone came first in contact with the floors and girders after a deformation of 205 mm. The estimated dimensions of the cone were therefore not correct, nevertheless, the results were in good agreement with the experiments.
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Figure 4.3: The critical damage value $\varepsilon_0$ as a function of the element size. (a): Model 1. (b): Model 2.

In Model 2 the initial stiffness of the plating was slightly too stiff giving a higher force in the finite element simulations. At a distance of 150mm the plating fractured and the crushing force dropped considerably. In the experiments the plating probably also fractured at the same indentation distance, but the force did not drop. Again the crushing force was slightly too large at the end of the deformation, probably due to incorrect cone dimensions.

The deformed structure of Model 2 at an indentation of 250mm is presented in Figure 4.5.

Figure 4.4: The force-deflection curves from the simulations and the experiments. (a): The results for Model 1. (b): The results for Model 2.

4.2.4 Discussion

Relatively good agreement with the experimental results was achieved. An interesting observation is that compared to Model 1 the outer bottom failed after greater indentation depths
in Model 2. The structure in Model 1 was stiffer and thus the strain tended to localise at an earlier stage and led to earlier fracture. However, at an indentation distance of 400 mm 15% more energy was dissipated in Model 1 (480 kJ) than in Model 2 (408 kJ).

In Figure 4.6 the dissipated energy percentage for the different structural members for the two models is shown at an indentation distance of 400 mm. In Model 1 most energy was absorbed by the floors and girders, but in Model 2 more energy was absorbed by the outer plating than in the floors and girders. The energy dissipated through friction was in both simulations around 8%.

![Figure 4.5: The deformation of Model 2](image-url)

![Figure 4.6: The dissipated energy percentage for the various structural members. (a): Model 1. (b): Model 2.](image-url)
4.3 Collision of Ship’s Side with a Bridge Pier

To estimate the impact load, Nagasawa et al. (1981) carried out a series of small-scale experiments concerning collisions of ship’s sides with a bridge pier. Three transversely framed models and one longitudinally framed model were used in the numerical simulations of the tests.

4.3.1 Experimental Setup

Nagasawa used two cylindrical steel indenters with a radius of 300 respectively 450mm for modelling the bridge pier. The ship’s side models were typically transversely and longitudinally framed and compared to actual vessels the scale was about 1:11. Figure 4.7 shows the different configurations of the side structures.

![Figure 4.7: The geometries for experiments of a ship collision with a bridge pier. The figure shows the indenter (top), the transversely stiffened geometries (middle) and longitudinally stiffened geometry (bottom) (Nagasawa et al. (1981)).](image)

The main dimensions for the transverse framed side models were:

- Side height: 800 mm
- Frame depth: 45 mm
4.3 Collision of Ship’s Side with a Bridge Pier

- Spacing between frames: 55 mm

The principal dimensions of the test specimens for the longitudinal model were:

- Side height: 800 mm
- Frame depth: 80 mm
- Spacing between frames: 319 mm
- Longitudinal depth: 35 mm
- Longitudinal spacing: 66.7 mm

The structures were made of mild steel with an yield stress of 250 MPa. The thickness of the members and the radius of the indenter vary in the tests according to Table 4.3.

Table 4.3: The thickness of the structural members and the indenter radius for the tests.

<table>
<thead>
<tr>
<th>Model</th>
<th>Side plating</th>
<th>Deck and bottom</th>
<th>Transverse frames</th>
<th>Longitudinals</th>
<th>Cylinder radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse 1</td>
<td>1.2 mm</td>
<td>1.2 mm</td>
<td>1.6 mm</td>
<td>-</td>
<td>450 mm</td>
</tr>
<tr>
<td>Transverse 2</td>
<td>1.6 mm</td>
<td>1.6 mm</td>
<td>2.3 mm</td>
<td>-</td>
<td>450 mm</td>
</tr>
<tr>
<td>Transverse 3</td>
<td>1.2 mm</td>
<td>1.2 mm</td>
<td>1.6 mm</td>
<td>-</td>
<td>300 mm</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>1.6 mm</td>
<td>1.6 mm</td>
<td>1.6 mm</td>
<td>1.2 mm</td>
<td>450 mm</td>
</tr>
</tbody>
</table>

4.3.2 Finite Element Simulation

The side structures were modelled with a mesh size of about five times the thickness giving an element length of 5.5mm. The material hardening law was assumed to follow a power law description with the coefficients $C= 800$ and $n=0.22$. As the structures were made of mild steel the logarithmic fracture strains were assumed to be 0.30 (corresponding to a nominal strain of about 0.35). All elements in the deformation area were of the same mesh size and had the same material properties, and therefore only a single value was given for the critical damage parameter. The RTCL criterion was used and it was assumed that the fracture value was equal to the fracture strain, giving $\varepsilon_0=0.3$, as necking would not occur in the tensile test simulations with the chosen element size.

Only a quarter of the structure was modelled, as the geometries were symmetric in two planes. The indenters were modelled as rigid and loaded with a slowly increasing velocity. The friction coefficient was set to 0.3 between the indenting cylinder and the structural members.
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Small imperfections were assigned to the deck structures with an amplitude of $0.1t$. To avoid hourglassing in longitudinal stiffeners they were modelled with fully integrated elements. The side, decks and frames were modelled with the Belytschko-Tsay shell elements. All shell elements had five integration points through the thickness. The total number of elements for the transversely stiffened models and the longitudinal model was about 52,000 and 51,000 respectively.

4.3.3 Results

The deformation of the longitudinally stiffened side structure at four different stages of the crushing is shown in Figure 4.8. The deck plating folded in nice regular patterns under the cylinders without fracturing, whereas the transverse frames buckled in a complex mode and developed large cracks. Good agreement between the experimental crushing forces and the forces in the numerical simulations was obtained during the first 100mm of deformation, see Figure 4.9. At deformations of about 120-150mm cracks were developed in the side plating in the finite element simulation, but the cracks in the side plating propagated relatively slowly, which probably caused the slightly higher forces in the numerical simulation compared to the experimental crushing forces.

The damage and the triaxial stress distribution in the transversely stiffened Model 3 at an indentation of 120mm is shown in Figure 4.10. As seen cracks are developed at the deck/side plating intersection. The dissipated energy at a deformation of 200 mm in the transversely stiffened Model 2 was 70% higher compared to the transversely Model 1, which is surprising as the mass increase in Model 2 was only 30%.

The energy dissipation of the different structural members is shown in Figure 4.11, just before fracture of the shell plating and at the final deformation of 0.2 metres.

4.3.4 Discussion

The experimental test setup by Nagasawa et al. (1981) was simulated by the RTCL damage model. Good agreement was achieved between the numerical and the experimental results for the first 100 mm of deformation. However, in the simulations the cracks at the side plating probably propagated too slowly compared to the experiments. In the previous chapter it was found that the critical damage value for fracture initiation and crack propagation was not the same, which could explain the differences in the results.

This is especially of importance when the structure is subjected to a uniform stress and strain field, as the side plating in the experiments by Nagasawa. When stresses and strains are localised, the damage will increase rapidly and thus the extra external energy needed to propagate the crack due to a too high critical damage value will often be of less importance compared to the energy dissipated in the large plastic deformations.
4.3 Collision of Ship’s Side with a Bridge Pier

Figure 4.8: The deformation of the longitudinally stiffened side structure.
Figure 4.9: Calculated and measured crushing forces for the large-scale experiments by Nagasawa et al. (1981). (a): The longitudinally stiffened model. (b): The transversely stiffened Model 1. (c): The transversely stiffened Model 2. (d): The transversely stiffened Model 3.
4.3 Collision of Ship’s Side with a Bridge Pier

Figure 4.10: The deformed transversely stiffened model 3. The model has been mirrored along the centre plane for illustrative purposes. (a): The damage distribution ($D_i$) of the model. (b): The triaxial stress state distribution in the structure.

Figure 4.11: The dissipated energy in the four simulated models at a crushing distance of 0.15 metre.
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4.4 Ship Grounding by NSWC

A large-scale test was performed to simulate grounding of an oil tanker of a size of 30,000 to 40,000 DTW. The test was executed by the Naval Surface Warfare Center in Virginia, USA.

4.4.1 Experimental Setup

The testing facility was an assembled typical ship’s bottom structure in scale 1:5 mounted onto a railway carriage. It was released from the top of a hill to run down over an artificial rock with mounted load cells for measuring of the horizontal and vertical loads, see Figure 4.12(a). The test specimen was about 7 m long, 2.5 m wide and its double bottom height was about 0.4 m. The bottom structure was hit by the rock tip about 50 mm below the inner bottom and left the structure with a penetration of the inner bottom that was equal to the height of the double bottom. This should ensure that rupture of the inner bottom shell would occur as the rock passed through the whole structure. The main data of the grounding tests is given in Table 4.4.

![Figure 4.12: (a): The experimental setup of the test by NSWC. (b): The structural arrangement of the conventional double bottom structure on scale of 1:5 by NSWC (Rodd and MacCampbell (1994)).](image)
Table 4.4: *Main scantlings used in the numerical simulation of the NSWC test (*) : assumed values*.

<table>
<thead>
<tr>
<th>Weight of test vehicle:</th>
<th>223 tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact speed:</td>
<td>12 knots</td>
</tr>
<tr>
<td>Rock tip radius:</td>
<td>0.17 m</td>
</tr>
<tr>
<td>Rock semi-apex angle, α</td>
<td>45 deg</td>
</tr>
<tr>
<td>Material:</td>
<td>ASTM A569</td>
</tr>
<tr>
<td>Yield strength, σ_y:</td>
<td>283 MPa</td>
</tr>
<tr>
<td>Ultimate strength, σ_u:</td>
<td>345 MPa</td>
</tr>
<tr>
<td>Power law coefficients*: C = 600 MPa, n=0.24</td>
<td></td>
</tr>
<tr>
<td>Experimental fracture strain*: ε_f :</td>
<td>30%</td>
</tr>
</tbody>
</table>

Four different double bottom geometries were tested, but only the conventional double bottom was considered in the present finite element simulation. The structure had transverse and longitudinal webs, transverse bulkheads and longitudinal stiffeners on the inner and outer bottom plating, see Figure 4.12(b). The main scantlings for the model are given in Table 4.5.

Table 4.5: *Main scantlings used in the numerical simulation of the NSWC test*.

<table>
<thead>
<tr>
<th>Length: 7.32 m</th>
<th>Width: 2.54 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>DB spacing: 0.38 m</td>
<td>Pitch angle: 3.38 deg</td>
</tr>
<tr>
<td>Plate thickness inner/outer bottom: 3.0 mm</td>
<td>Plate thickness transverse bhd: 3.0 mm</td>
</tr>
<tr>
<td>Plate thickness longitudinal web: 2.3 mm</td>
<td>Plate thickness transverse web: 3.0 mm</td>
</tr>
<tr>
<td>Thickness of longitudinal stiffener: 3.0 mm</td>
<td>Thickness of vertical stiffener: 3.0 mm</td>
</tr>
<tr>
<td>Thickness of cut-out reinforcement: 4.8 mm</td>
<td></td>
</tr>
</tbody>
</table>

4.4.2 Finite Element Simulation

The mesh size (element side length) in the deformation area was again about five times the thickness of the bottom plating. The stiffeners and the cut-out hole reinforcements were modelled with fully integrated elements and the remaining structure was made of Belytschko-Tsay elements. The shell elements in the outer bottom shell and the longitudinal web had five integration points through the thickness and the rest of the elements had three integration points through the thickness. All structural members in the double bottom had the same material properties and were modelled by the user-defined material model, which includes the rate dependent plasticity option where the Cowper-Symonds parameters, Equation 3.19, were assumed to be $C_{cs}=4000s^{-1}$ respectively $q=5$. The Rice-Tracey fracture criterion was used and the critical damage values $ε_0$ for different element sizes were found by simulation of tensile tests, Figure 4.13.
The double bottom structure was given an initial velocity of 12 knots (=6.2 m/s) and a heel angle of 3.38 deg, which corresponds to the experiment. The rock was modelled as rigid and fully fixed. The test wagon was simulated with rigid bodies on the side of the bottom structure and was fixed in the vertical direction to avoid rotation around the transverse axis. The total mass of the test structure was 224.5 tons.

A friction coefficient of 0.3 between the structural members and the rock was used. Self contact was also included in the model.

4.4.3 Results

Rodd (1996) gave the experimental results and Figure 4.14 shows both the measured and the calculated forces. The bumps on the force curves came from a transverse bulkhead in the double bottom structure passed the rock. The horizontal forces as well as the vertical forces were in very good agreement with the experimental results, except when the rock passed the bulkhead where 50% higher vertical forces were obtained in the experiment compared to the numerical results. The horizontal force peaks were slightly higher in the numerical calculation.

4.4.4 Discussion

The energy dissipated due to fracture in a complex grounding simulation (as the one simulated here), is relatively low compared to the energy dissipated in large plastic deformation. Therefore, the problem with too high critical damage values for simulating crack propagation is not crucial as in the simulations of Nagasawa’s experiments, where the stress and strain fields were more uniform.
4.4 Ship Grounding by NSWC

Figure 4.14: Comparison of the experimental and the calculated forces. (a): The horizontal forces. (b): The vertical forces.

Figure 4.15: (a): The dissipated energy for the different structural members in the FE-simulation of the NSWC grounding test. (b): The percentage energy a single member has absorbed of the total dissipated energy.
In Figure 4.15 the energy for the different structural members is given during the 'grounding' event. The grounding event was very complex with large plastic deformations and fracture in the plating. Figure 4.16 shows the finite element model during the grounding simulation. The outer bottom plating fractured almost during the whole process, while the inner bottom fractured first after a deformation of 2.7 metres.
Figure 4.16: The numerical simulation of the grounding test by NSWC. (a, b) The FE-model and the artificial rock. One side of the inner bottom plating is removed for illustrative purposes. (c, d, e, f) The FE-model after a deformation of 5 metres.
4.5 ISSC Benchmark of Ship Grounding

This section describes a benchmark problem by Ito (2001a) with all the data given by ASIS (Association of Structure Improvement of Shipbuilding Industry, Japan). The experimental setup should simulate a grounding event of a 319 metres long VLCC of a deadweight of 272,000 tons.

4.5.1 Test Setup

The test setup was constructed to simulate ship grounding where the structure investigated was a double bottom model on a scale of 1:3, Ito (2001a). The lower end of the test structure was welded to a thick plate and the sides were stiffened by rigid H-beams. The side parts of the upper end, which were initially hit by the rock model, were stiffened with flat bars. The wedge (the rock model) was made of thick plates and was assumed to be rigid compared to the double bottom structure. The tip angle of the wedge was 90 degrees.

The loading velocity of the rock model was 0.76 mm/s to ensure a quasi-static response from the test setup.

The material hardening curves for the different plates were given in the experimental description and transformed into true stress-strain curves and fitted to a power law description. The members in the double bottom structure had the same Young’s modulus $E$ of 210 GPa and a Poisson’s ratio $\nu$ of 0.3. The other derived material properties for the structural members are given in Table 4.6.

Table 4.6: The material parameters for the structural members of the ISSC grounding benchmark test.

<table>
<thead>
<tr>
<th>Structural Member</th>
<th>Thickness</th>
<th>Yield Stress</th>
<th>Power Law Coefficients</th>
<th>Experimental Fracture Strain $\varepsilon_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner and outer bottom shell</td>
<td>7.0 mm</td>
<td>295 MPa</td>
<td>$C=720$ MPa $n=0.24$</td>
<td>0.34</td>
</tr>
<tr>
<td>Floor plating</td>
<td>5.0 mm</td>
<td>315 MPa</td>
<td>$C=720$ MPa $n=0.23$</td>
<td>0.33</td>
</tr>
<tr>
<td>Longitudinal Stiffener Web</td>
<td>4.5 mm</td>
<td>315 MPa</td>
<td>$C=720$ MPa $n=0.23$</td>
<td>0.33</td>
</tr>
<tr>
<td>Longitudinal Stiffener Flange</td>
<td>7.0 mm</td>
<td>295 MPa</td>
<td>$C=780$ MPa $n=0.24$</td>
<td>0.34</td>
</tr>
<tr>
<td>Floor stiffener</td>
<td>4.5 mm</td>
<td>315 MPa</td>
<td>$C=720$ MPa $n=0.23$</td>
<td>0.33</td>
</tr>
</tbody>
</table>

4.5.2 FE-model

The bottom structure was somewhat simplified in order to reduce the modelling and the calculation time. Only the double bottom structure and the indenter were modelled, whereas the rigid beams and the thick plate at the bottom were simulated with fixed boundary
conditions. The indenter was modelled as rigid. The welds in the double bottom structure as well as the cut-outs in the floor for the stiffeners were not considered in the finite element model. Contact was modelled between all structural members, including self contact, with a friction coefficient of 0.3.

Most of the bottom structure was modelled with the underintegrated Belytschko-Tsay elements with five integration points through the thickness. To prevent hourglassing (zero energy modes) the floor stiffeners and the flanges of the bottom stiffeners were modelled with fully integrated shell elements with five integration points through the thickness.

The typical shell element size for the whole ship structure was about 25x25 mm, which was about five times the thickness. This gave a total number of 82,500 shell elements. As the element size for a structural member was the same over the whole structure, only one tensile test simulation with the same element size as in the double bottom structure was needed for each member to calibrate the damage value $\varepsilon_0$.

The simulated time span for the calculations was 0.2 second, and the total deformation
of the indenter was 1.25 m. This ensured that the dynamic effects in the simulation were insignificant and loading could be considered as quasi-static, which gave a CPU time on an HP8000 Unix machine of 30 hours.

4.5.3 Results

The force-penetration curves obtained from the finite element simulation and the experiments are shown in Figure 4.18. Very good agreement was achieved with the experimental results.

Most of the damage and deformation took place under the wedge, leaving the rest of the structure nearly intact. After 400 mm the wedge hit the floor plating and the contact force increased rapidly and soon after the floor plating fractured. After 500 mm the wedge shoulders entered the plate and the force settled. The wedge came in contact with more material resulting in severe damage and deformations of the bottom structure, until the maximum force of 4800 kN was reached after a deformation of 900 mm. The bottom plating mostly ruptured in front of the wedge edge by tearing. As the penetration continued, the damaged material was pushed to the side by the wedge. The penetration of the double bottom at four different stages is illustrated in Figure 4.19.

Most of the dissipated energy was, of course, absorbed by the bottom plating, but more than 20% of the dissipated energy was caused by the frictional forces.

4.5.4 Discussion

Excellent agreement was achieved between the numerical simulations and the experimental results. The problem is very complex with tearing, folding and self-contact of the deck plates as the wedge penetrates the structure.
Figure 4.19: The penetration of the wedge at the penetration depths of 100mm, 700mm, 1100mm and 1300mm respectively.

Figure 4.20: The percentage of the total energy dissipated by the structural members and friction.
In the benchmark description the radius of the wedge tip toe was specified to be sharp. In a
finite element simulation contacts between edges can cause problems and the results can be
very mesh dependent. The tip toe will only hit one or a few shell elements and therefore most
of the deformation will occur in this element. Larger elements must have a lower damage
value than smaller elements to attain the same force in a penetrating wedge simulation.

The load case was also quite critical as the plates could fail in shear fracture close to the
stiffeners. Shell elements do have a limitation in simulating out-of-plane shear failure and
instead they fail in a Mode 1 tearing.
4.6 ISSC Benchmark of Ship Collision

The experimental setup described in the present section should simulate a ship-ship collision of a double hull tanker, where the bulbous bow was assumed to strike between two adjacent transverse webs. The tests were used in a benchmark problem described by Ito (2001b) and the data was given by ASIS (Association of Structural Improvement of Shipbuilding Industry, Japan).

4.6.1 Test Setup

Two tests were performed, a quasi-static and a dynamic test. The test setup was constructed to simulate a ship-ship collision where the structure investigated was a double hull structure on a scale of 1:2, see Figure 4.21. The double hull models used in the static and dynamic tests were constructed by the same specification. The main scantlings of the model are given in Table 4.7.

![Figure 4.21: (a): The collision scenario for the ASIS test described by Ito (2001b). A bulbous bow of the colliding ship was assumed to strike at the stringer deck between the two adjacent transverse webs of a collided double hull tanker. (b): The experimental setup for the collision tests, Ito (2001b).](image)
Chapter 4. Verification of the RTCL Criterion on Large Scale Experiments

Table 4.7: Main scantlings used in the simulations of the ship collision benchmark by ISSC.

<table>
<thead>
<tr>
<th>Weight of bow model:</th>
<th>82.5 tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact speed:</td>
<td>9.7 m/s</td>
</tr>
<tr>
<td>Bow radius:</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Height of structure:</td>
<td>1635 mm</td>
</tr>
<tr>
<td>Length of structure:</td>
<td>6000 mm</td>
</tr>
<tr>
<td>Web spacing:</td>
<td>2000 mm</td>
</tr>
<tr>
<td>Width of side plating:</td>
<td>450 mm</td>
</tr>
<tr>
<td>Stringer spacing:</td>
<td>400 mm</td>
</tr>
<tr>
<td>Stringer height:</td>
<td>100 mm</td>
</tr>
<tr>
<td>Web Stiffener spacing:</td>
<td>930 mm</td>
</tr>
<tr>
<td>Web Stiffener height:</td>
<td>120 mm</td>
</tr>
</tbody>
</table>

The side ends of the model were welded to rigid support structures and the bottom plate was fixed to the test bed with bolts. The bow model, which had a radius of 500 mm, was designed to be rigid compared to the side structure. The total weight of the bow model including the load cell and the dummy weight was about 82.5 tons. The strain hardening curves for the different parts of the structure were given and the material properties were derived in the same manner as in Section 3.5. The derived material properties from the identification process for the structural members are given in Table 4.8, and as seen the material properties for the different thicknesses were nearly the same. The damage parameters \( \varepsilon_0 \) were derived for the RTCL criterion with an element size of 25x25mm.

Table 4.8: Material parameters for the collision models in the benchmark test by the 2002 ISSC Committee.

<table>
<thead>
<tr>
<th>Members</th>
<th>Thickness</th>
<th>Yield Stress</th>
<th>Power Law Coeff.</th>
<th>Critical Damage Value</th>
<th>Strain rate Coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side shell</td>
<td>10.0 mm</td>
<td>324 MPa</td>
<td>C=780 MPa, n=0.25</td>
<td>( \varepsilon_0 = 0.41 )</td>
<td>( C_{cs} = 3600s^{-1}, q=5.5 )</td>
</tr>
<tr>
<td>Deck</td>
<td>7.0 mm</td>
<td>314 MPa</td>
<td>C=800 MPa, n=0.25</td>
<td>( \varepsilon_0 = 0.34 )</td>
<td>( C_{cs} = 3600s^{-1}, q=5.5 )</td>
</tr>
<tr>
<td>Web</td>
<td>8.0 mm</td>
<td>324 MPa</td>
<td>C=800 MPa, n=0.25</td>
<td>( \varepsilon_0 = 0.34 )</td>
<td>( C_{cs} = 3600s^{-1}, q=5.5 )</td>
</tr>
<tr>
<td>Stringer</td>
<td>7.0 mm</td>
<td>314 MPa</td>
<td>C=800 MPa, n=0.25</td>
<td>( \varepsilon_0 = 0.34 )</td>
<td>( C_{cs} = 3600s^{-1}, q=5.5 )</td>
</tr>
<tr>
<td>Web stiff.</td>
<td>7.0 mm</td>
<td>314 MPa</td>
<td>C=800 MPa, n=0.25</td>
<td>( \varepsilon_0 = 0.34 )</td>
<td>( C_{cs} = 3600s^{-1}, q=5.5 )</td>
</tr>
</tbody>
</table>

In the quasi-static test, the bow model was smoothly pushed by a hydraulic piston and as the stroke of the piston was 500 mm, the side structure was first deformed about 450 mm, the piston repositioned and thereafter loaded until a total deformation of 900 mm was achieved.

In the dynamic test the bow model fell freely from a height of 4.8 m above the initial position of the outer hull, which gave an impact velocity of about 9.7 m/s. The test was repeated four times until a permanent deformation of about 900 mm was reached.
4.6.2 FE-model

In order to reduce the modelling and the calculation time the finite element structure was simplified. Only the double hull structure and the indenter were modelled and the supporting structure was simulated with fixed boundary conditions. The indenter was modelled as fully rigid. The welds were not separately modelled in the finite element structure, nor were the small slots at the intersection between the side, the web and the deck plating.

In the dynamic test the velocity of the bow model was set to 9.7 m/s just above the undeformed double hull model. As the structure was deformed and the fall height increased the impact velocities for the subsequent drop tests became larger. To simulate this, the gravity load was included in the finite element simulation.

Strain rate effects were not included in the material model for the quasi-static test, however, for the dynamic test the rate dependent plasticity option was used with the Cowper-Symonds parameters $C_{cs} = 3600$ (1/s) and $q = 5.5$, according to Table 4.8.

Contact was modelled between the indenter and all structural members, including self contact, where the friction coefficient was assumed to be 0.3.

To avoid hourglassing, the stiffeners and the flanges were modelled with fully integrated shell elements with five integration points through the thickness. The remaining bottom structure was modelled with the underintegrated Belytschko-Tsay elements with one integration point in plane and five integration points through the thickness.

The typical shell element size for the whole simulated structure was about 25x25 mm, which gave a total number of 34,000 shell elements. As the same element size was used in the whole structure, a critical damage value versus element size curve was not needed.

4.6.3 Results

The force-penetration curve from the finite element simulation of the quasi-static test is shown in Figure 4.22(a).

Unloading of the model was included in the simulation at a distance of 0.45 m. Good agreement between the numerical simulation and the experiments was achieved, even if a slightly higher ultimate force was obtained in the experiments. The deck plating fractured around a deformation of 0.6 metre in the FE-simulation as well as in the experiments, but the force dropped more in the experiments than in the numerical simulation, probably due to the difficulties of simulating a propagating crack. At a distance of little less than 0.8 metres the side plating failed and the force was considerably reduced.

The force-penetration curve from the finite element simulation of the dynamic test is shown in Figure 4.22(b). Good correlation with the experimental result was achieved, however,
again the ultimate force was slightly smaller in the finite element simulation. As it is seen
four drops were needed in the finite element simulation to achieve a deformation of 0.9 m,
which was the same in the experiments.

Figure 4.22: (a): The vertical contact forces in the simulation of the quasi-static test. (b): The vertical contact forces in the simulation of the dynamic test.

The deck plating started to fracture in the dynamic simulation after a denting distance of
0.57 metre, while fracture first occurred in the side plating at a distance of 0.79 metre. In
Figure 4.23 the damage distribution and the plastic strain rate are shown when the cylinder
in the third drop test hit the test specimen. The damage was mostly localised at the side-deck-web intersection where also the deck plating first fractured. Locally, strain rate values
up to 40 1/s were achieved for a loading velocity around 10 m/s. Figure 4.24 shows the
deformations after the drop tests.

Figure 4.23: The deformation at the start of the third impact of the collision test by Ito
(2001b). A part of the side plating is removed for illustrative purposes. (a): The damage
distribution $D_i$. (b): The strain rate distribution.
4.6.4 Discussion

In Figure 4.26 a comparison of the forces and the energy dissipated from the quasi-static test and the dynamic test is shown. There is, as expected, a clear difference in the load levels and the dissipated energy due to both inertia and strain rate effects. About 23% more energy was dissipated in the dynamic simulation compared to the quasi-static simulation, which clearly illustrates the importance of including strain rate effects in crash simulations.

Figure 4.27 shows the percentage of the total energy absorption of the structural members in the different drop tests.
Figure 4.25: The experimental results at a deformation of 800mm (ASIS 1993).

Figure 4.26: Comparison between the dynamic simulation and the quasi-static. (a): The force-deformation curves. (b): The dissipated energy.

Figure 4.27: Percentage of the total energy absorption of the structural members in the different drop tests.
4.7 Concluding Remarks

In the present chapter several simulations of large-scale experiments have been presented. The finite element results correspond well with the experimental results, showing that the RTCL criterion is suitable for predicting fracture initiation of large complex plate structures subjected to accidental loading. However, if several experiments with simple specimens are available with various triaxial stress states and loading rates, the Johnson-Cook criterion has a better opportunity to be calibrated so that better results can be achieved. The RTCL criterion has only one parameter to be calibrated together with the strain rate parameters, whereas the Johnson-Cook criterion has five additional damage parameters. For simulations of practical engineering problems, the calibration possibilities are often limited and the Johnson-Cook criterion will thus not contribute to a more accurate result.

It has also been shown that with an element length about five times the plate thickness good agreement is achieved. If several plates are used with large scatter in the thickness variation, the smallest thickness of a main structural member which will be subjected to large deformation and which is likely to fracture should be used as reference for determination of the element size. Moreover, if the element size varies for the same member, the results will be less mesh sensitive by use of the critical damage value \( \varepsilon_0 \) versus element size curve. It is therefore important to generate a mesh which is as quadratic as possible. It is usually the element length in the loading direction that determines the fracture value. Thus, if the loading direction is known from the start, the aspect ratio of the element will not be any problem.

In the simulation of the experimental test setup by Nagasawa et al. (1981) it was found that the cracks at the side plating probably propagated too slowly compared with the experiments. This was because the critical damage value for fracture initiation and crack propagation was not the same. The effects of a too high damage value for crack propagation are especially important when the structure is subjected to a uniform stress and strain field, as the side plating in the experiments by Nagasawa. When stresses and strains are localised, the damage will increase rapidly and thus the extra external energy needed to propagate the crack due to a too high critical damage value will often be of less importance compared to the energy dissipated in the large plastic deformations.

By disregarding the strain rate effects in dynamic problems such as ship collisions and ship groundings, the stiffness of structural response will be less, and it is therefore of importance to include the strain rate effects when the dissipated energy is to be found. It may be vital in a ship collision simulation that the strain rate effects are incorporated as less kinetic energy will be absorbed by the structure and thus can be decisive for, whether the cargo hold will be flooded or not. However, for comparing the crashworthiness of various structures, the strain rate effects are less important.

Welds were not modelled in the finite element models of the plate structure in the present chapter. As the plate thickness was relatively low and welds are usually stronger than the
parent material, the welds would most likely not have fractured. However, for real ship structures the parent plate can be very thick and therefore the welds can be the weak link. In the finite element simulations welding-induced residual stresses were not modelled for any of the tests and with regard to the global structural and rupture behaviour the effects from the residual stresses and the heat-affected zones were assumed to be negligible.
Chapter 5

Fracture of Welds

In the present chapter failure of welds will be briefly discussed and a few modelling techniques are presented.

5.1 Introduction

In ship structures a large amount of welds exists and about 30% of the man-hours in the ship production are devoted to welding of which as much as 3/4 are fillet welds. The crashworthiness of the ship depends highly on the weld strength and quality. When welds fail during a collision, each structural part can deform independently causing a much greater damage. It is therefore crucial in a collision simulation that the welds are modelled correctly. Existing cracks in the welds will of course weaken the structure and in simulations of ship collisions with older ships these cracks should preferably be accounted for. However, this is impractical as the locations and the sizes of the cracks are most often not known in advance. In large-scale experiments (as in the previous chapter) the structures are not subjected to cyclic loading and thus no fatigue cracks are developed prior to testing. Usually, the welds in a large scale experiment are also manufactured carefully to avoid failure due to erroneous welds, whereas in ship structures it is likely that defects and improperly manufactured welds exist.

Welds may fail from the cracks introduced from fatigue loading due to the dynamic environment in which the ships operates. Fatigue failure is a local phenomenon that depends on the overall structural scantlings as well as very local properties such as the geometry and the quality of the weld. Improper weld geometry is also a common reason for failure of the weld and if the weld location or geometry is not well designed the probability of fatigue failure increases. The stress concentration (the hot spot) at the fillet toe can be the reason for crack initiation during fatigue loading. Current design standards for fillet welds are based
on ensuring safe operation of the vessel under normal operation conditions and the weld may therefore fail instead of the plating during accidental loading.

Welding in aluminium has a major disadvantage compared to steel as aluminium has relatively low fatigue strength. The virgin aluminium is often considered not to have a lower cut-off stress level in the SN-diagram. Moreover, welding of aluminium structures is rather complicated and even with state-of-the-art welding equipment and good workmanship, the welding reduces the fatigue strength significantly. For these reasons fatigue often becomes a major concern in the design of assembled aluminium structures. An additional disadvantage of welding of aluminium is that the heat-affected zone (HAZ) can be several times larger than the plate thickness, depending on the welding process. This reduces the strength of the structure severely, especially when using the heat-treated aluminium alloys for which the material strength in the case of some alloys can be reduced as much as 60%.

5.2 Weld Failures in Ship Structures

A small study was conducted of the damaged car ferry the Prins Richard, which collided with the breakwater at a speed of 12.6 knots, while entering the harbour of Puttgarden on 19 June 2001. The bulbous bow was severely damaged, see Figure 5.1. The damages of the bulbous bow were investigated and the only damages that occurred to the welds were when the welds had been subjected to peeling. Otherwise, the welds were intact and instead the parent material had failed, see Figure 5.2. A more comprehensive study of weld failures was conducted by the MIT-Industry Consortium on Tanker Safety. It was established in 1992 to investigate the damage to a hull in case of a grounding accident. The research was divided into two areas of focus: damage to the hull plating and damage to welds, Masubuchi et al. (1996). From the study of weld failures, it was found that in many actual accidents the welds remain intact. Nevertheless, welds do fail and it is mostly fillet welds that are critical, while fracture of butt welds is not common.

Fillet welds attaching the longitudinal stiffeners have mainly been designed to withstand the shear stresses developed by hogging and sagging of the ship structure. Fillet welds are used for connecting longitudinal girders, transverse frames and various structural members to the bottom and the side plating. The fillet weld size can be several times smaller than the thickness of the mounted structural member to the hull plating. This is in accordance with the classification societies as the welds are designed to withstand the loading created under normal operating conditions. However, the welds may not be strong enough to hold under accidental loadings. The design of a ship and the structural members is to a large extent influenced by the rules of the classification societies, and often the structural details as the fillet weld size are directly based on these rules.

The fillet welds are mainly loaded in four basic ways: tearing loading, web folding, web bending and longitudinal shearing, see Figure 5.3. The most critical loading is the peeling (tearing) situation. Under the normal tensile loading the weld material must withstand the
5.2 Weld Failures in Ship Structures

Figure 5.1: The damage to the bulbous bow of the car ferry the Prins Richard. (a): Side view of the damaged bulbous bow. (b): The bulbous bow from below.

Figure 5.2: Welded plate pieces from the Prins Richard. (a): Typical peeling failure of a fillet weld. (b): The crack propagates besides the butt weld. The butt weld can be seen just above the crack.
5.3 Fracture Models

In LS-DYNA a failure model for welds is implemented where failure can include both ductile and brittle failure. The weld is defined by a set of nodes where a nodal pair moves rigidly in six degrees of freedom until the failure criterion is reached. Five different weld options are implemented: spot, fillet, butt, cross fillet and a general weld.

The ductile failure criterion of the weld is based on the nodal plastic strain and when it exceeds a defined critical value the nodes are released and the rigid body deleted from the calculation. The nodal plastic strain is found from the adjacent elements by integration through the thickness and the obtained average plastic strain values are used for a least square algorithm to generate the nodal plastic strain values.

The brittle failure is based on the force resultants acting on a nodal pair in the weld definition. When they reach the fracture criterion the nodes are released. The forces are averaged over a number of time steps to avoid failure due to spurious noise. Brittle failure of fillet and butt welds occurs when

\[ \sigma_f \leq \beta_{fail} \sqrt{\sigma_n^2 + 3(\tau_n^2 + \tau_t^2)} \]  

(5.1)

where \( \sigma_f \) is the fracture stress and \( \sigma_n \) is the normal stress. \( \tau_n \) and \( \tau_t \) are the shear stresses in the normal and the transverse direction of the weld. \( \beta_{fail} \) is a calibration constant. The
user defines the length between the nodal pairs, the width of the flange and the fillet leg size, together with the fracture stress \( (\sigma_f) \) and \( \beta_{fail} \). LS-DYNA then calculates the stresses \( (\sigma_n, \tau_n, \tau_t) \) from the nodal forces.

This weld model has some limitations and the main disadvantages are that the weld is totally stiff and that the ductility of the weld is based on the parent material. The modelling of the welds is also difficult and time-consuming.

A potentially more accurate method for modelling is by use of shell or brick elements with the material properties from a typical weld. However, the element size has to be very small to capture the stress and strain distribution in the welds as well as to assure that the plate response is not influenced.

### 5.3.1 Simulation of a Fillet Weld

To study the weld option in LS-DYNA a tearing experiment conducted by Wang (1993) was simulated, Figure 5.4(a). The specimen was designed to simulate peeling failure of a T-joint between a stiffener and a plate. The plates were 12.7 mm thick and were assumed to be of mild steel, with \( E=210 \text{GPa}, \sigma_y=250\text{MPa} \) and the power law coefficients \( C=770\text{MPa} \) and \( n=0.2 \). The total length of the specimen was 279.4 mm with a plate width of 76.2 mm. The weld leg size was 5.08 mm and the ultimate strength of the weld material was assumed to be 550 MPa.

The finite element model was made of Belytschko-Tsay shell elements and the influence of the mesh discretisation was also studied. To avoid that the neighbouring nodal pair was deleted from a stress wave generated by a sudden release of a nodal pair, the stresses in the weld were averaged over 20 time steps. Figure 5.4(b) shows the results from the finite element simulations and the experiment. The force-deflection curves are average values of ten output values since the brittle fracture of the welds resulted in spurious noise in the simulation. Reasonable agreement with the experiment was achieved for an element size of 2.5 mm, but the forces in the numerical simulation decreased, whereas a steady tearing force was achieved in the experiment. For an element size of 5 mm a far too stiff response was achieved, whereas for the 1.25 mm element size the welds fractured too early compared to the experiment. Figure 5.5 shows the von Mises stresses before the point of fracture initiation and after a crack propagation of 150 mm. The spurious noise in the model can clearly be seen.

The simulated experiment was very mesh-dependent, which is obvious due to the stress concentration at the crack tip occurring due to peeling of the welds. The parameter \( \beta_{fail} \) could have been calibrated to achieve more accurate results, however if the element side length varies in the same finite element model calibration process would become very time consuming. Moreover, the welds failed due to the brittle fracture option, whereas in the experiment the fracture process of the welds was ductile. The ductility of the welds in the finite element program is based on the plastic strain in the plates, thus the ductile fracture option cannot be used, as the plate thickness was much thicker than the fillet weld size.
Figure 5.4: (a): The test setup of the tearing experiment, Wang (1993). (b): Comparison of the numerical simulation and the experimental results.

Figure 5.5: The von Mises stresses in the simulation of the peeling test. (a): Before fracture of the weld. (b): After fracture of the weld.
5.4 Concluding Remarks

The crashworthiness of the ship may depend on the weld strength and quality. If a weld fails during accidental loading, each structural member can deform independently, which may result in much greater damage. However, with the weld options available in most finite element programs today, it is questionable if additional gain in the accuracy of the results is achieved for large complex plate structures.

For comparison of various crashworthy ship structures, modelling of the welds will not necessarily give any additional information as the welds should be designed to be stronger than the parent material. In studies of crashworthy structures the structural arrangement should determine the energy absorbing capabilities, except if the welds are designed to fail for a certain loading to trigger a deformation mode.
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Chapter 6

Structural Designs

Extensive work has recently been made on improvements of various structural arrangements resulting in various proposals of more crashworthy ship structures. In the present chapter several improved collision resistant structures will be briefly discussed and the crashworthiness of three different side structures for a small coaster will be studied.

6.1 Introduction

In the last decade the regulations for ship designs have been strengthened. The use of double hull structures was enforced by the Oil Pollution Act of 1990, which requires all new constructed oil tankers operating in US waters to be designed with a double hull, see e.g. Figure 6.1. The rules were a direct result of the Exxon Valdez grounding in 1989 in Alaska.

In an accident of an oil tanker with a double hull structure the oil outflow will normally not occur until the inner hull plating ruptures. The point of fracture of the inner hull plating during a collision is directly related to the structural arrangement and the dimensions of the structural members. With a flexible side structure the striking bow can penetrate further than the original position of the inner side without rupture of the inner shell plating. For a stiff side structure the inner side plating can rupture even before the striking bow has reached the inner side. However, the stiffer side may have dissipated more energy than the flexible side structure at the same penetration depth. The determination of a crashworthy ship structure is therefore a very complex problem and, in addition, the energy absorbing capabilities for a specific ship structure change with the striking ship type as well as with the impact location.
6.2 Crashworthy Ship Designs

The criteria for determination of the scantlings in the ship design phase are normally that the structure can withstand the wave loads, global bending moments, vibrations and fatigue. Nevertheless, in the design of ships transporting hazardous materials, the crashworthiness has long been considered and in the past 10 years the structural arrangements of tankers have become an important task for the ship designers. ASIS (The Association for Structural Improvements of Shipbuilding Industry, Japan) carried out a seven-year research starting in 1991 on the prevention of oil spills from crude oil tankers. The aim of the project was to analyse and predict the structural damage to the ship structure in case of a collision or grounding. The joint MIT-Industry Program on tanker safety was another large project with the aim of developing methods for predicting the energy dissipation of tankers in grounding accidents.

By knowing and controlling the structural behaviour of the hull structure during collisions and groundings the ship safety can be improved. The task is to avoid leakage and to maintain the stability of the ship by increasing the energy absorption of the hull structure. However, the improved performance during collisions should not be negatively influenced by building costs, operational costs and payload. More crashworthy structures can be achieved in various ways and they can be divided into two categories depending on the main energy absorbing mechanisms. The first group consists of designs where the energy is mainly absorbed by the main structural members. The second group uses some secondary energy absorbing mechanism, such as wire ropes, foam, concrete or pressure chambers.

Several proposals of new improved crashworthy ship structures have lately been made, see e.g. Roland and Metchkow (1997), Naar et al. (2001), Lee and Kim (2001), Sano et al. (1996) and Kitamura (1997) (Figure 6.2). In the lower right of the figure a few suggestions of improved energy absorbing side and bottom structures are shown, e.g. the hat type, stringer type and tube type. Several sandwich structures have also been proposed with
Figure 6.2: Different crashworthy ship structures available in the literature (see e.g. Roland and Metchkov (1997), Naar et al. (2001), Lee and Kim (2001), Sano et al. (1996) and Kitamura (1997)).
e.g. corrugated plates, concrete, foam or pipes in between the two skins (lower left). The efficiency of new developed crashworthy structures is, however, dependent on the ship’s type and dimensions and of course on the striking ship. The advantages of non-standard structures should therefore be evaluated to assure that improved energy absorption capability is achieved for a particular ship on a particular route.

A comprehensive finite element study of several different side structural configurations for a VLCC was conducted by Kitamura (1997). The side structures differed not only in the structural layout, but also in the material. Lee and Kim (2001) made a comparative numerical study of four different structural arrangements of a side structure for a 310.000 DWT VLCC. The investigated structures were typical side structures, the hat type, the stringer type and the mixed stringer type, see Figure 6.2. A similar study was performed by Naar et al. (2001), where a conventional, two hat types and a sandwich type were investigated. The numerical simulations resulted in a 34% increase in energy absorption capabilities for the hat type structure compared to the conventional type.

Sano et al. (1996) investigated the influence of the connection between the longitudinals and the web frames on the energy absorbing capabilities of the hull structure. By using soft connections the energy absorption up to fracture of the inner shell could be increased by a factor of 1.16.

Underwater bulbous bows to reduce hydrodynamic resistance are now commonly equipped on most modern ships and it is usually the bulbous bow that causes the greatest damages to the struck side structure. Thus, an alternative approach to improving safety which lately has gained a large focus is the so-called buffer bow. Instead of designing an expensive crashworthy ship hull structure that can withstand a collision, the bow structure of the colliding ship can be designed to absorb a large amount of the energy. The economic benefits of designing a buffer bow compared to a crashworthy side structure are remarkable as the structural arrangements can be limited to the bow region of the colliding ship. Generally, the strength of the bow structure must be less than the collided side structure, and at the same time the energy absorbing capability of the bow structure must be as large as possible. A major drawback is the large time horizon to exchange the existing world fleet from conventional bulb structures to the buffer bows and today it is only an incentive for the shipowners to equip their ships with buffer bows. Moreover, many ships are ice-strengthened which requires a stiff bulb and bow to resist the ice loads.

A seven-years research project was launched in 1998 by the Association for Structural Improvements of the Shipbuilding Industry (ASIS, Japan) for the development of buffer bows. Simplified analytical methods (Suzuki et al. (1999)) as well as advanced finite element simulations (Kitamura (2001a)) were used in the evaluation of the structural response of the buffer bow configurations.
6.3 CrashCoaster Project

The CrashCoaster project studies the crashworthiness and damage stability of small Ro-Ro and coaster side structures. From 1998 damage stability regulations, which say that the ships should have a calculated probability of survival of at least 40\% in case of accidental loading, have become mandatory for new building of small ships of a length between 80 and 100 metres. These ships have traditionally been a single compartment type, however, to comply with the new rules the general arrangements have to be drastically changed and an extra transverse bulkhead in the compartment is often needed. Another solution would be to improve the crashworthiness. The IMO resolution A.684(17) states that alternative arrangements in the calculation of damage survivability may be acceptable, if it can be proved that the same degree of safety as represented by the regulation is achieved. The energy absorbing capabilities of the ship structure can thus be included in the damage survivability calculation, as an increased damage stability survival probability can be expected when more crashworthy ship structures are applied. However, the regulations do not include a procedure for evaluation of the crashworthiness effects on the damage survivability.

A method that incorporates the improved energy absorbing capability of new ship structures in the assessment of the damage stability survivability is to be developed within the project. The energy to be absorbed by different struck ship structures (the so-called energy dissipation reference values) for several shipping routes (Figure 6.3) have been generated to validate that the method will comply with the IMO regulations, Lützen (2001). The energy distribution is given as a function of the displacement of the struck vessel and the striking location as well as percentile value of the energy to be absorbed in the ship’s structure.

The new ship structures with believed improved energy absorbing capabilities were designed and calculated by the finite element method. The finite element simulations will later in the CrashCoaster project be validated by large-scale experiments.

6.3.1 Crashworthy Structures in the CrashCoaster Project

The objective of the crash analysis was to make an estimation of the energy absorbing capabilities of various structural designs for the considered ships in the CrashCoaster project. Only a small part of the ship types was analysed and tested. The test sections with supporting structure were designed to represent the behaviour of the complete side structure. The finite element method was used in the analysis of the sections and four of the promising improved side structures are to be tested in full-scale experiments.

Three crashworthy side structures have been analysed in the CrashCoaster project: a conventional ice strengthened ship structure, an X-corrugated steel panel and a sandwich structure with a concrete core, Figure 6.4. The test sections were 5.94 m high and 5.95 m long and the main scantlings of the three test sections are given in Table 6.1. They were modelled with an element size of about 50mm to assure that the correct folding pattern was achieved,
Figure 6.3: The energy to be absorbed midships versus displacement of the struck vessel for various European routes, Lützen (2001). (a): The 25 percentile value of the energy to be absorbed in the ship’s structure. (b): The 90 percentile value of the energy to be absorbed in the ship’s structure.

especially for the X-corrugated panels. All plates, except the stiffener flanges, were modelled with Belytschko-Tsay shell elements with five integration points through the thickness. The stiffener flanges were modelled with fully integrated elements to prevent hourglassing.

Table 6.1: Main scantlings of the three test sections.

<table>
<thead>
<tr>
<th>Ice Strengthened</th>
<th>Corrugated Panel</th>
<th>Concrete/steel Sandwich Panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double hull spacing:</td>
<td>1300mm</td>
<td>360mm</td>
</tr>
<tr>
<td>Outer plating thickness:</td>
<td>10mm</td>
<td>6mm</td>
</tr>
<tr>
<td>Inner plating thickness:</td>
<td>9mm</td>
<td>4mm</td>
</tr>
<tr>
<td>Web thickness:</td>
<td>8mm</td>
<td></td>
</tr>
<tr>
<td>Outer stiffener:</td>
<td>Web: h = 140 mm, t = 5mm</td>
<td>Flange: w = 150 mm, t = 12mm</td>
</tr>
<tr>
<td>Inner stiffener:</td>
<td>Web: h = 200 mm, t = 9mm</td>
<td>Flange: w = 50 mm, t = 9mm</td>
</tr>
<tr>
<td>Weight (without support structure):</td>
<td>50 tons</td>
<td>36 tons</td>
</tr>
</tbody>
</table>
Figure 6.4: The simulated structures. (a): Ice strengthened side structure. (b): Sandwich structure with X-corrugated steel between the faces. (c): Sandwich structure with concrete. (d): The mesh of the back side of the structures.
Chapter 6. Structural Designs

The test sections were made of mild steel and the material properties and damage values were assumed to be the same as in Section 4.4 with the values given in Table 6.2. Strain rate effects were accounted for by use of the rate dependent plasticity option. To generate the critical damage value versus element size curve an experimental fracture strain $\varepsilon_f$ of 0.3 was assumed for all plate thicknesses. The critical damage value $\varepsilon_0$ versus element size curve for the RTCL criterion is plotted in Figure 6.5(a).

Table 6.2: Material parameters used in the numerical simulations.

<table>
<thead>
<tr>
<th>Material</th>
<th>Mild steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield strength, $\sigma_y$:</td>
<td>283 MPa</td>
</tr>
<tr>
<td>Ultimate strength, $\sigma_u$:</td>
<td>345 MPa</td>
</tr>
<tr>
<td>Power law coefficients: C= 600 MPa, $n$=0.24</td>
<td></td>
</tr>
<tr>
<td>Strain rate parameters: $C_s = 4000s^{-1}$, $q = 5.$</td>
<td></td>
</tr>
</tbody>
</table>

The material properties for the concrete were assumed (taken from LSTC (2001)) and are given in Table 6.3. The volumetric strain-pressure curve is plotted in Figure 6.5(b). It was also assumed that the compressive yield strength was pressure dependent and that the yield strength varied with the pressure according to

$$\sigma_y = [3(b_0 + b_1p + b_2p^2)]$$

(6.1)

where $p$ is the pressure and $b_0, b_1, b_2$ are material parameters.

Table 6.3: Concrete material parameters.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density:</td>
<td>1800 kg/m$^3$</td>
</tr>
<tr>
<td>Shear modulus:</td>
<td>5400 MPa</td>
</tr>
<tr>
<td>Bulk modulus:</td>
<td>41000 MPa</td>
</tr>
<tr>
<td>Tensile strength, $\sigma_y$:</td>
<td>2.0 MPa</td>
</tr>
<tr>
<td>Compressive yield parameters :</td>
<td>$b_0 = 1.16E+14$ $b_1 = 4.15E+07$ $b_2 = -0.0519$</td>
</tr>
</tbody>
</table>

The striking bulb was mounted on the bow of the striking ship and was considered as rigid compared to the side structures, Figure 6.6. The breadth and the height of the bulb were 1.5m and 2.5m respectively and the bulb was modelled with rigid shell elements with an element side length of about 150mm. The shell elements describing the bulb geometry should preferably be of the same size as the test sections, however, only an old meshed model of the bulb was available. The underside of the bulb was placed 0.7m above the tank bottom of the test section. The collision velocity of the bulb was kept constant at a speed of 10m/s throughout the simulations.
6.3 CrashCoaster Project

6.3.2 Results

Figure 6.7 shows the results of the simulations with the three test sections. The point of fracture of the outer and the inner side plating is marked by a filled circle and a filled triangle respectively. The structural response of the X-corrugated and the concrete sandwich panels was, as expected, initially much stiffer than that of the ice strengthened structure. In the concrete sandwich panel the outer side plating failed just before the inner side, whereas in the other two test sections, rupture of the outer side plating took place much earlier than of the inner side. The inner side plating of the test sections with X-corrugated and concrete sandwich panels fractured at about the same penetration distance of 0.46 m. However, the total energy dissipated in the concrete sandwich structure at the point of fracture of the inner side plating was about 10% higher (3.2 MJ) than in the case of the X-corrugated panel (2.9 MJ).

In the ice strengthened section, the bulb hit on the web, which of course added extra stiffness to the structural response. If the bulb had been ramming in between two webs, the contact forces would have been much lower. As a result more energy was dissipated than if the bulb would have hit in between two webs. Thus, the dissipated energy at the point of fracture of the inner shell cannot directly be compared with the other two sections. Nevertheless, the inner side shell fractured at a much greater penetration depth (due to the large spacing of
Figure 6.6: (a): The bulb of the striking ship is mounted on the bow and is considered as rigid compared to the side structures. (b): Finite element model of the bulb.

Figure 6.7: (a): The collision forces for the three simulated structures. The filled circles and triangles mark fracture of the outer respectively the inner side plating. (b): The energy dissipated by plastic work and friction.
the double hull in the ice strengthened section), which together with the additional stiffness from the web resulted in a dissipated energy about three times greater (9.7 MJ) than for the other two structures.

In Figure 6.8 the deformation of the three test sections at the point of fracture of the inner shell plating is shown. As seen the deformations of the ice strengthened structure were much larger than for the two panel structures. It is also seen in Figure 6.8 that the supporting structure sustained rather large plastic deformations.

An additional simulation with the ice strengthened structure, but without the web, was conducted to investigate the effect of the web on the dissipated energy. In Figure 6.9 a comparison of the contact forces as well as the absorbed energy is shown for the ice strengthened structure with and without a transverse web. As expected the structure without the transverse web is less stiff but at the same time rupture of the inner side shell occurred at a greater penetration. Therefore, the dissipated energy up to the point of inner side fracture was only about 2% higher when the bulb hit the web.

6.4 Concluding Remarks

As previously mentioned the plastic deformations of the supporting structures were considerable. The supporting structures for the three test structures were designed identically, with the same plate thicknesses. Thus, they did not reflect the stiffness of a complete side structure for the respective test section. Moreover, the relatively small length of the test sections limits the number of webs in the structure to a maximum of two. A sufficiently large part of the ship structure could have been modelled to assure that the boundary conditions would not influence the structural response. For a single compartment ship this might mean that the whole length of the cargo compartment has to be modelled. However, as the simulations in the present chapter should be verified by full-scale experiments, a complete side structure of the different structural layouts would be far too expensive to produce.

In addition, if different crashworthy side structures are to be investigated for a particular ship type, the shipping route with the most frequently occurring vessel types, displacements, movements, headings and speeds should be studied in order to determine the probability distributions of the colliding bow structure and the released energy levels for crushing of the struck ship structure. Furthermore, it should be taken into account that the energy absorption capability of a structure varies with the impact location, both vertically and longitudinally.

By means of numerical simulation procedures and the RTCL criterion it is believed that the point of fracture was predicted with reasonable accuracy in the complex test sections. The crashworthiness for the three different test setups of different structural arrangements of a coaster side structure was investigated, and the building costs, operational costs as well as the payload were said to be equal. The new structural layouts with X-corrugated panel and
Figure 6.8: The deformation of the structures at the point of fracture of the inner side plating. (a): Ice strengthened side structure. (b): Sandwich structure with X-corrugated steel between the faces. (c): Sandwich structure with concrete. (d): Ice strengthened side structure without web.
6.4 Concluding Remarks

Figure 6.9: (a): The collision forces for an ice strengthened structure with and without a transverse web. The filled circles and triangles mark fracture of the outer respectively the inner side plating. (b): The energy dissipated by plastic work and friction for the two simulations of ice strengthened structures.

Concrete/steel sandwich had less energy absorption capabilities at fracture of the inner side shell compared to the conventional ice strengthened structure. This was mainly due to the small distance between the outer and inner hull plating in the two sandwich panels.

It is often stated that nonlinear finite element programs are not suitable in the design phase of a ship. The empirical and the simplified methods can often give a good estimate, but when completely new designs are developed neither the empirical nor the simplified methods will be able to quantify the energy absorption capabilities. An example would be a design with a steel-concrete sandwich structure as considered here, which in most simplified analytical models cannot be modelled (unless a model has been developed for the specific structure).
Chapter 7

Simulations of Full-scale Accidental Loading

Ship-ship collision events involve complex, coupled and highly nonlinear effects of hydrodynamics and large structural plastic deformations, buckling, contact, friction as well as initiation and propagation of cracks. As earlier discussed reliable prediction of crack initiation and propagation has previously lacked in most finite element analyses of ship collisions, whereas tools for modelling other effects have been mature for practical applications for more than a decade.

In the present chapter a study of eight ship-ship collision scenarios between two identical coasters is presented, where the effects of the global ship motion are taken into account. A simulation of an explosion due to hydrogen gases in the cargo hold of a bulk carrier will also be shown.

7.1 Introduction

In the last decade, the use of the finite element method for simulating groundings, collisions and the crashworthiness of ships has increased considerably. TNO (Lenselink and Thung (1992) and Vredeveeldt et al. (1993)) performed full scale collision tests of inland vessels and conducted finite element simulations of the scenarios. They included the fluid-structure interaction in the simulations by use of a system of springs and dampers\(^1\). They obtained good correlation with the experimental results up to the point of fracture of the side plating.

More recently, Kitamura (2001b) presented results from several large-scale finite element simulations of ship-ship collisions, e.g. the coupling between ship collision and horizontal

\(^1\)It is, however, incorrect to model the fluid-structure interaction by use of a system of springs and dampers, since the added mass and the damping effects varies with the velocity.
hull girder bending. He showed that the effect of the horizontal hull girder bending could lead to less energy absorption capability and also earlier collapse of the ship structure. This was, however, dependent on the type of the colliding ship.

Mizukami et al. (1996) conducted finite element simulations of ship collisions where the surrounding water was included in the model in order to simulate the 3-D flow around the ship structures. Simulations like these are still today very CPU demanding but will most likely simulate the ship-water interaction more accurately than when the ship-fluid interaction is solved analytically in closed-form solutions.

In most finite element simulations of ship-ship collisions presented in the literature the struck ship has been at a standstill and been fixed in space. However, statistical data has shown that the struck ship has often a forward velocity at the point of contact and the forward velocity of the struck ship will, of course, have an influence on the damage extent. The effects of the forward motion on the damage of the struck ship in a ship-ship collision will be shown later. This chapter will also show how the global ship motions affect the structural damage and discuss the validity of some of the assumptions often made in the simplified analytical models.

7.1.1 External Dynamics

Analyses of grounding and collision events are usually divided into external dynamics (global ship motions) and internal mechanisms (crushing of the ship structures). The external dynamics is normally solved by numerical solution of the equations of motion or by an integrated approach where the conservation of energy, the momentum and the angular momentum are used for derivation of analytical expressions of the dissipated energy. This has been very common for both simplified analytical methods as well as for large finite element simulations.

The aim of the external dynamic analysis is to estimate how much of the total energy will be dissipated into large plastic deformations and rupture of the shell platings. An uncoupled analytical model for determination of the energy loss in ship-ship collision was developed by Pedersen and Zhang (1998). Here the estimation of the energy loss for dissipation in structural deformations is based on an analytical method where the energy is expressed in closed-form solutions. However, only surge, sway and yaw motions are considered and the influence of the hydrodynamic forces related to decelerations of the ship is approximated by constant added mass coefficients.

A large part of the kinetic energy at the point of impact will be absorbed by structural damage to the vessels and dissipated by friction work as well as generation of waves. By assuming that the undamaged ship structure will remain undeformed, rigid body mechanics can be used for describing the motions of the colliding ships. A ship motions program called MCOL (Ferry (2001)) is implemented into LS-DYNA. The sub-program is a rigid body mechanics program for modelling the external dynamics for ships. The mass as well as the
inertia tensor for each ship is given as input to MCOL in the body-fixed reference frame, Figure 7.1. Each ship has two reference frames, of which the first is a body-fixed coordinate system with its origin at the centre of mass of the ship and the x-axis in the longitudinal direction and a starboard y-axis. An earth-fixed coordinate system is defined in the initial position of the local coordinate position.

The rigid body inertia matrix is modified to include the effects of the added mass due to the surrounding water. MCOL determines the buoyancy forces from a linear restoring force approximation. At each time step LS-DYNA calculates the internal mechanics and computes the resultant forces and moments on the ships (rigid bodies) and passes them to MCOL, which updates the position of the ships and returns the new ship position to LS-DYNA. For a short theoretical description, see Appendix C.
Chapter 7. Simulations of Full-scale Accidental Loading

7.2 Ship-ship Collision Simulations

The most common way of studying the crashworthiness of ship structures is to model the colliding ship as fully rigid where the struck ship is assumed to be at a standstill. Several ship-ship collision scenarios were therefore investigated to study the effect of different collision angles, loading conditions, with/without a forward velocity of the struck ship and where both ships were deformable. The two ships were identical and a typical coaster with a conventional ice-classed ship structure and a length of 85 m has been chosen, see Figure 7.2. In Table 7.1 the main scantlings for the ship are given.

Table 7.1: Main scantlings of the coaster model.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length overall:</td>
<td>89.7 m</td>
</tr>
<tr>
<td>Length between perpendiculars:</td>
<td>85.0 m</td>
</tr>
<tr>
<td>Breadth moulded:</td>
<td>13.6 m</td>
</tr>
<tr>
<td>Depth to main deck:</td>
<td>7.2 m</td>
</tr>
<tr>
<td>Draught:</td>
<td>5.7 m</td>
</tr>
<tr>
<td>Deadweight:</td>
<td>4135 T</td>
</tr>
<tr>
<td>Displacement:</td>
<td>5700 tons</td>
</tr>
<tr>
<td>Speed:</td>
<td>12 knots</td>
</tr>
<tr>
<td>Ice class:</td>
<td>Swedish Finnish 1A</td>
</tr>
</tbody>
</table>

It should be noted that the line drawings of the original ship were not available and thus the fore and especially the aft were estimated from the section drawings. The real ship had a displacement of 5300 tons fully loaded while the modelled ships had a displacement of 5700 tons. Nevertheless, since the following simulations should only demonstrate the finite element capabilities of modelling different collision scenarios, the extra displacement will be of less importance.

The coaster collision was modelled and simulated in various collision scenarios, where the interaction of surrounding water was accounted for in six of the collision simulations by using...
the MCOL program. In the collision scenarios an identical ship rams the coaster amidships at various collision angles, loading conditions and speeds. In Table 7.2 the different collision scenarios are given. A zero degree collision angle corresponds to a head on collision and a 90 degree collision therefore corresponds to a perpendicular collision.

In Coll1 and Coll2 the striking ship speed was kept constant and the struck ship was fixed against translation and rotation. As the velocity was relatively low the inertia effects were insignificant, and thus by keeping a constant velocity of the striking ship the CPU time was reduced without influencing the results. However, if strain rate effects and the effects from the water-structure interaction should have been included in the simulations, the ships should have been given an initial speed. During the collision event the velocity would decrease as the initial kinetic energy was dissipated through e.g. plastic deformations, friction and generation of waves.

In the ballast loading conditions the ships were assumed to have a displacement of 4100 tons at a draught of 4.1 metres.

<table>
<thead>
<tr>
<th>Collision scenario</th>
<th>Forward speed Striking</th>
<th>Forward speed Struck</th>
<th>Def/Rigid Striking</th>
<th>Def/Rigid Struck</th>
<th>Collision Angle</th>
<th>Strain Rate</th>
<th>MCOL</th>
<th>Ballast Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coll1</td>
<td>const 6 m/s</td>
<td>0</td>
<td>rigid</td>
<td>def</td>
<td>90</td>
<td>no</td>
<td>no</td>
<td>none</td>
</tr>
<tr>
<td>Coll2</td>
<td>const 6 m/s</td>
<td>0</td>
<td>def</td>
<td>rigid</td>
<td>90</td>
<td>no</td>
<td>no</td>
<td>none</td>
</tr>
<tr>
<td>Coll3</td>
<td>initial 6m/s</td>
<td>0</td>
<td>def</td>
<td>def</td>
<td>90</td>
<td>yes</td>
<td>yes</td>
<td>none</td>
</tr>
<tr>
<td>Coll4</td>
<td>initial 6m/s</td>
<td>initial 6 m/s</td>
<td>def</td>
<td>def</td>
<td>90</td>
<td>yes</td>
<td>yes</td>
<td>none</td>
</tr>
<tr>
<td>Coll5</td>
<td>initial 6m/s</td>
<td>initial 6 m/s</td>
<td>def</td>
<td>def</td>
<td>90</td>
<td>yes</td>
<td>yes</td>
<td>none</td>
</tr>
<tr>
<td>Coll6</td>
<td>initial 6m/s</td>
<td>initial 6 m/s</td>
<td>def</td>
<td>def</td>
<td>120</td>
<td>yes</td>
<td>yes</td>
<td>none</td>
</tr>
<tr>
<td>Coll7</td>
<td>initial 6m/s</td>
<td>initial 6 m/s</td>
<td>def</td>
<td>def</td>
<td>90</td>
<td>yes</td>
<td>yes</td>
<td>Striking</td>
</tr>
<tr>
<td>Coll8</td>
<td>initial 6m/s</td>
<td>initial 6 m/s</td>
<td>def</td>
<td>def</td>
<td>90</td>
<td>yes</td>
<td>yes</td>
<td>Struck</td>
</tr>
</tbody>
</table>

### 7.2.1 Finite Element Model

The finite element models were somewhat simplified to reduce the modelling and calculation time. Only the double hull structure midships of the struck ship was modelled as deformable, whereas the remaining ship structures were modelled as fully rigid. For the striking ship only the fore end was modelled as deformable. No care was taken to model the welds in the finite element structure as well as the small brackets and slots at the intersection between the different structural members. The cargo was also assumed not to interact with the structural deformations and was therefore not included in the finite element models.

The HP-stiffeners were modelled as flat bars with the same cross-sectional area as the true structure. The stiffeners were also modelled with fully integrated elements to avoid hour-glassing. The remaining structure were made of Belytschko-Tsay elements, and the shell
elements in the contact area had five integration points through the thickness, and the rest of the elements had three integration points through the thickness. The element size in the contact area was about 70x70mm giving about 133,000 and 206,000 shell elements for the struck and the striking ship respectively. Figures 7.3(a) and 7.3(b) show half of the deformable finite element bow structure and the midship sections.

All structural members in the ship were assumed to be made in mild steel, with the material properties given in Table 7.3. The user-defined material model was used and included the rate dependent plasticity option, Equation 3.19. The RTCL fracture criterion was used where the critical damage values \( \varepsilon_0 \) for different element sizes were as presented in Figure 7.4(a) (the same as in Section 4.4).

| Table 7.3: The material properties for the coaster model. (* assumed values). |
|------------------|------------------|
| Material:        | Mild steel       |
| Yield strength, \( \sigma_y \): | 320 MPa       |
| Power law coefficients*: | \( C = 800 \text{ MPa}, n=0.22 \) |
| Experimental fracture strain*, \( \varepsilon_f \): | 30% |
| Strain rate coefficients*: | \( C_{\varepsilon} = 3600 \text{ s}^{-1}, q=5 \) |

The rigid body inertial properties associated with each ship were given as input instead of using the calculated properties obtained from the finite element mesh of the structure. The masses of the rigid bodies (minus finite element mesh) and the inertia tensors of the ships were defined at the ship’s centre of gravity. For each ship the direction of the rigid body was defined by a local coordinate system.

For each ship the inertia matrix, the hydrostatic restoring matrix and the buoyancy reference parameters were defined. Moreover, the added mass matrix at infinite wave frequency as well as the wave damping matrices were given as input to include the effects of the surrounding water. The in-house strip program I-Ship (Michelsen et al. (1993)) was used to derive the various matrices from the hull and the weight definition. Figure 7.4(b) shows the hydrodynamic coefficients for sway derived from I-Ship. In Appendix C all input data for the coaster is given. Due to the limited extent of the deformable length of the finite element model and the adopted simplifications in the motion analysis, no coupling of the structural response with global hull girder bending response could be analysed. However, as the colliding ships were identical (equal size) the hull girder bending effects were believed to be insignificant (Kitamura (2001b)).
Figure 7.3: (a): The finite element model of the deformable bow structure. (b): The finite element model of the deformable side structure.
Contact Forces

Figure 7.5 shows the damage to the bow after a deformation of 2.3 m and 3.8 m in a head-on collision with a rigid wall (collision scenario Coll2). The plating is folded in a repetitive pattern, without any considerable fracture of the side plating. In Figure 7.6(a) the crushing forces for the bulbous bow are shown. The collision forces are divided into the contact forces from the bulb respectively the bow, and is commonly done in simplified analytical methods as the bulb is often much stiffer (longitudinally stiffened) compared to the bow (transversely stiffened). The start of the crushing distance for the two curves is defined as the point of contact for the bulb respectively the bow with the rigid wall. As seen the bulb structure was much stiffer than the upper bow structure. As the upper part of the bow deformed, the active plastified steel volume increased, due to the shape of the bow, causing the contact force to increase. The shape of the bulb was rather uniform and thus resulted in a nearly constant load level.

The collision forces from the Coll1 simulation (a rigid bow structure colliding with a deformable side structure without external dynamics and strain rate effects included) are plotted in Figure 7.6(b). The collision forces are also here divided into bulb and bow contact forces, with the start of the crushing distance defined as the point of contact for respectively the bulb and the bow. The contact forces from the bulb acting on the side structure were higher than the corresponding forces from the bow. Since the both ships were fully loaded the striking ship bulb was at the same height as the inner double bottom plating, resulting in a much stiffer response compared to the upper part of the side structure.

In simplified analytical methods the damages to the colliding and the collided ship structures are typically computed by relationships between the reaction force and the penetration
7.2 Ship-ship Collision Simulations

Figure 7.5: The collision with a rigid wall. (a): Total deformation of 2.3 m. (b): Total deformation of 3.8 m.

Figure 7.6: (a): The contact forces for the bulb and the bow for a collision with a rigid wall. (b): The contact forces for the rigid bulb and rigid bow colliding with a deformable side structure. The filled circles and triangles mark fracture of the outer respectively the inner side plating caused by the bulb. The start of the deformation (crushing distance) for the bow and the bulb is defined from the initial point of contact.
Chapter 7. Simulations of Full-scale Accidental Loading

Figure 7.7: The contact forces for a collision with rigid objects according to simplified analytical methods. From force equilibrium the crushing distance of the bow and the penetration depth is obtained. (a): The contact forces for the bulb and the lower side structure. (b): The contact forces for the bow and the upper side structure.

depth for the two vessels. The bow and the side structure damages are normally analysed separately. Thus, in the analysis of the side structure damage the bow structure is assumed to be rigid and vice versa for the analysis of bow damage. The force equilibrium between the reaction forces of the two colliding structures must at the same time be satisfied. Figure 7.7 shows the force-penetration curves for the contact forces for the $\text{Coll1}$ and $\text{Coll2}$ simulations, where the contact forces are divided into contact forces from the bulb respectively the bow. By comparing the deformation distances at the same load level, the dissipated energy for each structure can be found by integration of the load-deflection curves up to the point of deformation. However, in this manner the coupling between the different structural parts cannot be obtained, which is definitely a limitation to the simplified methods.

In Figure 7.8(a) the perpendicular contact forces to the struck ship obtained from the three different perpendicular collision simulations with fully loaded ships ($\text{Coll1}$, $\text{Coll3}$, and $\text{Coll4}$) are shown. The penetration depth is defined as the shortest distance from the deformed bulb on the striking ship to the longitudinal midsection plane of the struck vessel ($=B/2$). The location of maximum penetration of the bulb can therefore change as the bulb deforms. The point of fracture of the outer side plating is shown by a solid circle for the respective collision scenarios, and the point of rupture of the inner shell is marked by a solid triangle. Less forces were, of course, obtained for the simulation without strain rate effects and external dynamics included ($\text{Coll1}$) compared to simulation $\text{Coll3}$, where the struck ship was at a standstill and both external dynamics and strain rate effects were accounted for in the simulation. The point of fracture of the outer skin took place at a larger penetration depth for $\text{Coll1}$ (no strain rate and surrounding water effects) compared to the other two simulations, whereas fracture of the inner side plating was initiated at a smaller penetration depth. For the other two simulations ($\text{Coll3}$ and $\text{Coll4}$) fracture of the inner shell occurred at the same penetration depth. However, for the simulation where the collided ship had a forward velocity ($\text{Coll4}$)
smaller forces were obtained, with a large drop in the load level after fracture of the outer side plating.

The collision forces perpendicular to the struck ship for three different collision angles with both ships in laden condition and at forward velocity of the struck ship (Coll4, Coll5 and Coll6) are plotted in Figure 7.8(b). The simulation with a collision angle of 120 deg (Coll6) did not penetrate the inner side plating. The other two simulations resulted in fracture of the inner side shell at about the same penetration depths, but the contact forces for the 30 deg collision (Coll5) were much smaller than for the perpendicular collision scenario (Coll4).

The collision forces for simulations of various loading conditions at a perpendicular collision angle with a forward velocity of the struck vessels (Coll7 and Coll8) are plotted in Figure 7.8(c). As expected the contact forces were lower when the striking ship was in ballast condition, since the bulb of the colliding ship hit the side structure above the inner double bottom plating. The point of fracture of the outer side shell took place at a larger penetration depth, since the side structure of the colliding ship was more flexible. However, an interesting observation is that fracture of the inner side shell occurred at a slightly smaller penetration depth compared to the other two simulations.

The collision angle and the speed of the struck ship had little influence on the penetration depths at the point of fracture of both the outer and the inner bottom plating, whereas fracture of the side plating varied much with the loading condition (in Coll4, Coll7, and Coll8). This is not surprising as for the different loading condition simulations the striking ship rammed at various depths with different structural arrangements.

A common assumption in simplified analytical methods is that the ratio between the longitudinal and the transverse contact forces of the collided ship is constant throughout the collision event. Figure 7.9 shows the force ratios for the simulations where the struck ship had a forward velocity. As seen the force ratio varied during the collision events, depending on the collision angle and the loading condition. For the 60 deg collision (Coll5) the ratio is, of course, the highest as one of the velocity components of the striking ship is in the opposite direction of the struck ship, whereas Coll6 (120 deg) had the lowest ratio. Moreover, for most of the simulations except Coll6 (120 deg collision) the contact force ratio increased with penetration depth, until the bulb fractured and was bent (see e.g. Figure 7.13). This is obvious as more material was in contact for larger penetration depths with higher forces as a result. This is clearly in contradiction with the assumption of constant contact force ratio in the analytical methods.

**Ship Motion Results**

The surge and sway motions of the fully loaded ships in the simulations at different collision angles are plotted in Figure 7.10. The sway velocity of the struck ship increased of course as the surge velocity of the striking ship decreased for all scenarios. In Figure 7.10 the rotational velocities (roll and yaw) are also plotted for the same scenarios. As it is seen the
Figure 7.8: (a): The perpendicular contact forces to the struck ship for the three perpendicular collision simulations (Coll1, Coll3, and Coll4). (b): The perpendicular contact forces to the struck ship for 60 deg, 90 deg and 120 deg collision scenarios with fully loaded ships (Coll4, Coll5, and Coll6). (c): The perpendicular contact forces to the struck ship for three different loading condition simulations (Coll4, Coll7, and Coll8).
7.2 Ship-ship Collision Simulations

Figure 7.9: The contact force ratios (longitudinal/transverse force components for struck ship) for the simulations.

Roll angle velocities were larger than the yaw motions for all simulations. The roll motion could also be observed in Figure 7.11 and was a result of the bulb being in contact with the side structure before the bow and of the lowest part of the side structure being much stiffer than the upper part, which resulted in a roll moment. If the collided ship had been struck at the aft or fore part of the ship, the yaw moment of the collided ship would be considerably larger. As earlier mentioned the roll moments are neglected in the analytical methods, which may be a poor assumption that can lead to inaccurate results.

In Coll6 (120 deg) the yaw motion of the striking ship was relatively large compared to the other collision angles, as a result of the ships sliding against each other. Moreover, the forward motion (surge) for both ships only slightly decreased compared to the other two simulations. Figure 7.11 shows the longitudinal centre cross section of the collided ship at several stages in the simulation where the struck ship was at a standstill (Coll3). The colliding ship first hit the struck ship with the bulbous bow and not until after a penetration of about 1.16m the bow of the striking ship came in contact with the collided ship. As seen the damages to the collided ship were severe and the cargo hold would be flooded. Due to the stiffer structure at the bottom of the side structure and that the bulb hit the side structure before the bow, the struck ship attained a roll motion. The damages to the bow and bulb structure of the striking ship were minor.

Energy Balance Results

Most of the dissipated energy in the collision simulations was absorbed by plastic work of the struck ship, Figure 7.12. The kinetic energy of the dry ship decreased due to the decreasing velocity, whereas the kinetic energy due to the added mass slightly increased for both ships in most collision scenarios as the motions of the ships changed from forward velocities to
Figure 7.10: (a): The sway and surge motions in simulation Coll4 (90 deg collision). (b): The roll and yaw motions in simulation Coll4 (90 deg collision). (c): The sway and surge motions in simulation Coll5 (60 deg collision). (d): The roll and yaw motions in simulation Coll5 (60 deg collision). (e): The sway and surge motions in simulation Coll6 (120 deg collision). (f): The roll and yaw motions in simulation Coll6 (120 deg collision).
e.g. sway and yaw, for which the added mass coefficients are much larger. The dissipated friction work varied with the collision scenario, and in the 120 deg collision (Coll6) only about 1.3% of the total initial energy at the point of inner shell rupture was absorbed by friction, whereas in the 60 deg collision (Coll5) about 5.3% was absorbed. The energy dissipated through hydrodynamic damping was less than 1% in all finite element simulations of the different collision scenarios. In Table 7.4 the penetration depths and the energy results at the point of inner shell rupture are given.

For the analysed scenarios the effects of the energy dissipated through hydrodynamic damping and variation of added mass were insignificant. The CPU demanding damping algorithm was thus not necessary and could have been disregarded. However, this may only be valid the short collision time event of a ship-ship collision with two small vessels as the two considered coasters.
7.2 Ship-ship Collision Simulations

Table 7.4: The simulation results at penetration of the inner side shells. (* In Coll6 the inner shell did not rupture. The given values denote the point when the maximum penetration depth was achieved.)

<table>
<thead>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coll1</td>
<td>1.39 m</td>
<td>16.8 MJ</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.7 MJ</td>
</tr>
<tr>
<td>Coll3</td>
<td>1.69 m</td>
<td>21.5 MJ</td>
<td>1.05 MJ</td>
<td>-</td>
<td>25.2%</td>
<td>1.2 MJ</td>
</tr>
<tr>
<td>Coll4</td>
<td>1.71 m</td>
<td>21.6 MJ</td>
<td>5.88 MJ</td>
<td>8.95 %</td>
<td>22.2%</td>
<td>4.8 MJ</td>
</tr>
<tr>
<td>Coll5</td>
<td>1.85 m</td>
<td>24.3 MJ</td>
<td>7.32 MJ</td>
<td>15.5%</td>
<td>25.9%</td>
<td>11.1 MJ</td>
</tr>
<tr>
<td>Coll6*</td>
<td>1.65 m</td>
<td>12.7 MJ</td>
<td>2.43 MJ</td>
<td>5.4%</td>
<td>9.8%</td>
<td>2.9 MJ</td>
</tr>
<tr>
<td>Coll7</td>
<td>1.76 m</td>
<td>20.5 MJ</td>
<td>3.00 MJ</td>
<td>8.1 %</td>
<td>28.2%</td>
<td>6.2 MJ</td>
</tr>
<tr>
<td>Coll8</td>
<td>2.09 m</td>
<td>22.2 MJ</td>
<td>7.52 MJ</td>
<td>18.3%</td>
<td>27.9%</td>
<td>7.7 MJ</td>
</tr>
</tbody>
</table>

Damage Plots

The damages of the Coll4 simulation where both ships had a forward velocity of 6 m/s are shown in Figure 7.13. Half of the finite element models are removed for illustrative purposes. As it is seen severe damages to both the collided and the colliding ship were the result. Due to the forward motion of the struck vessel, the bulb plating buckled on the starboard side, while the plating fractured on the port side.

Figure 7.14 shows a comparison of the side damages at a collision time of 0.75 second for the Coll3, Coll4, Coll5 and Coll5 simulations. The damage extent of a perpendicular collision when the struck ship was at a standstill (Figure 7.14(a)) was much smaller than when the struck ship had forward velocity (Figure 7.14(b)). However, the opening into the cargo hold was greater in the simulation with a standstill struck ship. The collision angle of 30 deg resulted in large damage extent and an opening into the cargo hold, as the side plating was torn away, Figure 7.14(c). The damages achieved in the simulation of a collision angle of 120 deg are shown in Figure 7.14(d).

The bow structure of the striking ship was severely damaged for a collision angle of 30 deg (Coll5). Also for the simulation of a 90 deg collision the damages to the bulb were great, which was shown earlier in Figure 7.13. The damages to the bow structure at a collision time of 0.75 second are shown in Figure 7.15. The bulb plating buckled on the starboard side, while the plating fractured on the port side.

In Figure 7.16 the damages to the side and bow structures are shown for the Coll7 and Coll8 simulation, where either the striking or the struck ship was in ballast condition. When the striking ship hit above the inner bottom plating a large nicely shaped hole through the side structure was created almost without damaging the bow structure of the striking ship. However, when the bulb collided with the double bottom structure the inner side shell plating did not rupture but instead the inner bottom plating did, which would also result in foundering of the ship. More damage to the colliding bow structure was also attained.
Chapter 7. Simulations of Full-scale Accidental Loading

Figure 7.13: The 90 deg collision where both ships had a forward velocity of 6 m/s. (a): Initial location. (b): Penetration of 1.35 m. (c): Penetration of 2.3 m. (d): Penetration of 2.75 m.
Figure 7.14: The damages to the side structures at a collision time of 0.75 sec. (a): Coll3. (b): Coll4. (c): Coll5. (d): Coll6.
Figure 7.15: The damages to the bow structures at a collision time of 0.75 sec. (a): Coll3. (b): Coll4. (c): Coll5. (d): Coll6.
Figure 7.16: The damage to the bow and the side structure for either the collided or the striking ship in ballast condition. (a): Coll7 - side structure. (b): Coll8 - side structure. (c): Coll7 - bow structure. (d): Coll8 - bow structure.
7.2.3 Discussion

Eight different ship-ship collision scenarios of two coasters were studied by use of the RTCL fracture criterion implemented in LS-DYNA. The importance of including material strain rate sensibility and the effects of the motions of the two ships in simulations of ship-ship collision events was shown. For the specific load cases of two colliding coasters, the hydrodynamic damping effects were however insignificant. The penetration depths at the point of fracture varied only slightly for different collision angles and forward speeds for the same loading conditions. Less energy was dissipated at the point of fracture of the inner side when the struck ship had a forward velocity than when it was at a standstill. For different loading conditions the penetration depths as well as the structural absorbed energy varied with the vertical contact location. If the bulb hit the double bottom, the structure was stiffer and more energy was dissipated in plastic deformations.

It was also shown that some of the assumptions in the simplified analytical methods may be improper, e.g. neglecting the roll motion of the ships and assuming constant contact force ratio ("effective contact of friction", i.e. longitudinal over transverse forces of the struck ship). It is also questionable if the complex interaction of deformation between the structural members of the two ships in a collision event, e.g. bending of the bulb, can be described by simplified analytical methods. Nevertheless, the simplified methods have proved to give good results for perpendicular ship-ship collisions with the struck ship at a standstill.

The simulations showed the usefulness of finite element simulations of ship collisions, but the CPU time still limits the number of different simulations that can be made within a reasonable time span. Most of the simulations conducted in the present section covered a collision time span of 0.75 second and each simulation took about five days of CPU on an HP8000 unix machine. The collisions should preferably be continued until the dissipated plastic work is constant, which is often about three to four seconds after the point of impact. However, by using an MPP version of LS-DYNA the calculation speed could be improved by a factor of 20 on a new Linux cluster with 16 or more nodes.
7.3 MV Thor Emilie

The purpose of this section is to investigate the effect of a hydrogen detonation in the cargo hold of the coaster MV Thor Emilie and to determine whether the structural damage can be so severe that the cargo hold is flooded.

7.3.1 Introduction

On 9 February 2000 the coaster MV Thor Emilie left Dunkerque to ship about 2000 tons of zinc bulk to Sardinia. Around 11 o’clock on 17 February an explosion occurred and the ship foundered, The Danish Maritime Authority (2001). The cargo was zinc skimmings, which in contact with water react and evolve hydrogen and toxic gases. The cargo was evenly distributed over the whole hold and reached up to about one metre below the tween deck. Two crew members were at the forecastle on the day of the explosion to carry out maintenance, overhaul and to change packing at the hatches. They were working with needle pistols, scrapers and angle grinders. It was believed that a spark from the hand tools ignited the hydrogen/air mixture, which detonated.

The Danish registered ship MV Thor Emilie was a one-hold bulk carrier with a tonnage of 1655 GT, a length of 75.6 m, a width of 11.8 m and a draught of 6.8 m. It was built in 1975 and had a hatch opening measuring 45x9.3 metres and hatch covers from MacGregor.

7.3.2 Finite Element Simulation

The ship structure of the Thor Emilie was slightly simplified in order to reduce the modelling and the calculation time. Due to the symmetry of the ship, only half of the ship was modelled and only the cargo hold structure and hatch covers were included in the simulations. The cargo hold was made prismatic so that the whole cargo hold had the same geometrical cross-sectional dimensions over the entire length, Figure 7.17. However, the plate thickness varied along the length according to the drawings.

All HP stiffeners in the bottom, side and deck structure were modelled as flat bar stiffeners, but with a correction on the thickness so that the correct cross section area was achieved, Figure 7.18. The knees connecting the side shell structure and the back deck structure were also included. No care was taken to model the welds in the ship structure and the cut-outs for the stiffeners.

Detailed scantlings for the hatch covers were not available. Therefore, the hatch covers were modelled as a sandwich structure and connected to the ship structure by merging the nodes in the corners with the hatch beam and the covers were merged with each other at the nodes in the centre, Figure 7.19.
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Figure 7.17: (a): View of the Thor Emilie hold. (b): FE-model of the hold structure with hatch covers.

Figure 7.18: The decks, webs and stiffeners in the cargo hold.
Material Properties

The material properties for the modelled hull structure reflect ordinary shipbuilding steel properties, that is Young’s modulus $E$ of 210 000 MPa, Poisson’s ratio $\nu$ of 0.3 and a yield stress $\sigma_y$ of 320 MPa.

The material hardening curves were assumed to be described by a power law, with $C = 800$ MPa and $n = 0.22$. Strain rate effects were included in the model by using the implemented rate dependent plasticity option, with the Cowper-Symonds coefficients assumed to be $C_{cs} = 4000 \, s^{-1}$ and $q = 5$. The RTCL fracture criterion was used, where the critical damage value for different element sizes varied according to Figure 4.13.

The water and the zinc bulk were modelled with 3-D fluid elements, Figure 7.20. 3-D fluid elements are often used to model sand or different kinds of granulates such as the present zinc bulk. This is supported up by the observation that “the cargo had the appearance of grey earth and dust with lumps”, The Danish Maritime Authority (2001).

Loads and Boundary Conditions

To perform the simulation of an explosion it was necessary to generate a realistic pressure history in the cargo hold. According to information from the Danish Defence Research
Establishment a hydrogen content in the cargo space above 18% would cause a detonation and with less hydrogen a combustion would take place, which may have evolved into a detonation. The reaction following an ignition may have slow flames, deflagration, where the front moves subsonically, to a detonation with a supersonic combustion flame. As a spark from the hand tools could have caused the ignition of the hydrogen, the simulated explosion was set to start at a point 10 metres from the fore bulkhead in the centre plane and 5 m up from the baseline. The pressure from the explosion acted on the top surface of the zinc bulk and on the side platings located above the zinc, as well as on the deck and on the lower side of the hatch covers. The pressure wave was defined by a load curve, which travelled through the cargo hold at the speed of sound. This meant that all the described surfaces were subjected to the same pressure, but at different times, depending on how far the surface was from the starting point. The pressure curves for 20% hydrogen/air mixture are shown in Figure 7.21 and were taken from Breitung et al. (2000) and Kunz (1998). Here, the first curve simulated how the hydrogen detonated with a short pressure peak in a large space and the second load curve simulated the pressure development in a closed space. The latter was of longer duration due to expansion of the hot gases.

The only prescribed boundary condition was a symmetry plane along the ship’s centre and a restricted vertical displacement of the lower water surface. The whole structure was also subjected to a gravity load.

The ship structure, except for the stiffeners, was modelled with the underintegrated Belytschko-Tsay elements with five integration points through the thickness. To prevent hourglassing for the stiffeners and the beams, they were all modelled with fully integrated shell elements. The solid elements used in the simulation were constant stress elements with one integration point. The typical shell element sizes for the whole ship structure were about 100x100 mm,
which gave a total number of 240,000 shell elements. The water and zinc were modelled with about 160,000 solid elements.

The integration time for the calculations was 0.2 second, which was enough for simulating the pressure wave travelling over the whole cargo space and for the kinetic energy almost to settle at a constant level. This resulted in a CPU time on an HP8000 Unix machine of 32 hours.

### 7.3.3 Simulation Results

Both simulations of the explosion in the cargo hold of the MV Thor Emilie showed large deformations of the side structures with rupture of the side and deck plating. The hatch covers were detached from the ship. The explosion events from the two FE-simulations are shown in Figures 7.22 and 7.23.

The simulation with the second load curve (maximum pressure of 5.3 bars) showed that the whole side shell blew out and the cargo space was totally opened to the surrounding water. The reason for the greater damage with 5.3-bars simulation compared to the 15-bars simulation was the relatively long duration of the pressure peak so that more energy was released for structural damage to the hull.

For the 15-bars simulation the pressure wave hit the top of the zinc bulk first, but due to the large mass inertia of the zinc bulk, no structural damage occurred at the first moment. When the pressure wave then hit the side structure and the hatch covers a millisecond later, large deformations at the side plating above the water surface took place. The deformation
Figure 7.22: Explosion in the cargo hold of the MV Thor Emilie with a maximum pressure of 15 bars for 4 ms. The deformations are shown for time steps, 0 ms, 28 ms, 56 ms, 84 ms, 112 ms and 200 ms.

Figure 7.23: Explosion in the cargo hold of the MV Thor Emilie with a maximum pressure of 5.3 bars for 50 ms. The deformations are shown for time steps, 0 ms, 28 ms, 56 ms, 84 ms, 112 ms and 180 ms.
Figure 7.24: A sequence of the explosion at the aft bulkhead with a maximum pressure of 15 bars for 4 ms. The first picture represents the time at 80 ms and the last at 200 ms.

was larger above the water surface since the mass inertia from the water restricted the lower plating from deforming. Later the side casings and the web girders ruptured at the bottom and caused the whole side structure to bend outwards. Due to the stiffness at the bulkheads, rupture occurred in the side plating close to both bulkheads, which caused water flooding of the cargo hold, see Figure 7.24.

### 7.3.4 Discussion

The explosion damages caused the global longitudinal strength of the ship to be greatly reduced and if the cargo hold did not flood, the hull would most likely eventually break in the middle due to the longitudinal bending moments acting on the hull.

The ship structural model was too ductile compared to the real ship, since welds were not included in the finite element model. Welds are more brittle than the base plating and will probably fail earlier due to the large deformations in combination with weld defects. Moreover, with a high loading velocity, steel becomes much more brittle compared to the ductility found in static experiments performed in a laboratory. Furthermore, since no cutouts for the stiffeners were modelled, they were more rigidly connected to floors and webs and gave most likely a slightly higher global strength.

In a dynamic problem the transverse shear stresses are very important, especially for high uniform impulsive loading, which can lead to shear fracture at the supports. In a finite element problem with large plate fields, the only practical way today of modelling the structure
Figure 7.25: Dissipated energy for the two load cases. (a): The energy for the 15-bars simulation. (b): The energy for the 5.3-bars simulation.

is to use shell elements. However, shell elements cannot model out-of-plane shear fracture. Thus, to be able to simulate out-of-plane shear fracture the ship has to be modelled with solid elements.

The energy dissipated into plastic deformation and the kinetic energy for the two cases are shown in Figure 7.25. For a hydrogen/air mixture of 20% the equivalent mass of TNT was 1736 kg, which, converted into joules, gives a value of about 8 GJ. This is about 50 times more than the FE-simulation gave for the 15 bar load case and five times more than the 5.3-bars load case. The remaining part of the released energy in the hydrogen detonation was converted into heat and a fraction of the total energy was converted into pressure waves. The total energy dissipated by plastic deformations and kinetic energy of the structure was much less than the total energy content of the mixture.
Chapter 8

Conclusion and Recommendations for Further Work

8.1 Conclusion

8.1.1 Fracture Criterion and Damage Models

Several different fracture criteria and models were studied on various applications with varying stress and strain states. The void growth model by Rice and Tracey (1969) has often been used as a basis for various damage criteria, which have proved to be able to predict fracture initiation and crack propagation for positive triaxial stress states. The triaxiality function in the Rice-Tracey criterion gives good approximation of the growth of the voids and thus for modelling ductile fracture. However, the model predicts that damage can occur also for triaxialities equal to and lower than -1/3, which is in contradiction with experimental evidence.

The fracture criterion by Cockcroft and Latham (1972) has been demonstrated to model ductile fracture for low triaxialities. The triaxiality function derived from the Cockcroft-Latham model by Wierzbicki and Werner (1998) also shows that no failure will occur for triaxialities less than -1/3, whereas at triaxiality values above 1/3 the Cockcroft-Latham model gives incorrect results. Thus, the Cockcroft-Latham criterion is believed to be able to predict ductile shear fracture for low and negative triaxialities correctly, while the Rice-Tracey criterion will correctly model fracture due to void growth. A combined Rice-Tracey and Cockcroft-Latham criterion (the so-called RTCL criterion) is a natural combination, which will switch between the two triaxiality functions at a triaxiality of 1/3 and thus cover the whole range of triaxialities. Only one parameter has to be determined to calibrate the RTCL criterion as the criteria by Rice-Tracey and Cockcroft-Latham will give the same damage value in a uniaxial tensile test with a constant triaxiality of 1/3.
Chapter 8. Conclusion and Recommendations for Further Work

The new proposed RTCL fracture criterion was implemented into a commercial finite element program (LS-DYNA) and validated upon several experiments on both steel and aluminium plates covering a wide range of stress states. The fracture criterion was implemented as a user-defined material model with a constitutive material algorithm based on $J_2$ flow theory with a radial return algorithm and isotropic hardening. The RTCL criterion was defined as damage indicator $D_i$:

$$D_i = \frac{1}{\varepsilon_0} \int f \left( \frac{\sigma_H}{\sigma_{eq}} \right)_{RTCL} d\varepsilon$$

where

$$f \left( \frac{\sigma_H}{\sigma_{eq}} \right)_{RTCL} = \begin{cases} 0 & \text{for } \frac{\sigma_H}{\sigma_{eq}} \leq -\frac{1}{3} \\ 1 + \frac{\sigma_H}{\sigma_{eq}} \sqrt{12 - 27 \left( \frac{\sigma_H}{\sigma_{eq}} \right)^2} & \text{for } -\frac{1}{3} < \frac{\sigma_H}{\sigma_{eq}} < \frac{1}{3} \\ 3 \frac{\sigma_H}{\sigma_{eq}} + \sqrt{12 - 27 \left( \frac{\sigma_H}{\sigma_{eq}} \right)^2} & \text{for } \frac{\sigma_H}{\sigma_{eq}} \geq \frac{1}{3} \\ \frac{1}{1.65} \exp \left( \frac{3\sigma_H}{2\sigma_{eq}} \right) & \text{for } \frac{\sigma_H}{\sigma_{eq}} \geq \frac{1}{3} \end{cases}$$

where $\varepsilon_0$ is the uniaxial damage strain and when $D_i = 1$ the critical damage state is reached. This is a very convenient way to define damage in FE-programs as the damage state can easily be determined in the post-processing. The experiments include tensile tests with notched specimens, upsetting tests and hydraulic bulging tests. By use of solid elements with an element size on the material macroscale, which is about 0.1-0.2mm for metals, both fracture initiation as well as fracture propagation were correctly predicted. When larger elements were used in the numerical simulations, the stress field around the crack tip was not accurately described and thus crack propagation could not be modelled correctly.

In the simulations of the bulging experiments with shell elements, the RTCL criterion was successfully used for predicting fracture initiation in various plate thicknesses and stress states for steel and aluminium. For very small elements crack propagation could also be correctly predicted. It was, however, found that the initiation and the propagation critical damage values were not the same. This is because shell elements cannot achieve higher triaxialities than 2/3 as shell elements is based on plane stress. Thus, when shell elements are used it is likely that the different fracture criteria, which includes a triaxiality function, are inappropriate for predicting crack propagation as, cf. the simulations with solid elements, the triaxiality in front of a crack tip is relatively high with values about 2.5. Nevertheless, the RTCL criterion is still a rather good approximation for predicting crack propagation as shown by several simulations of the large-scale experiments. The criterion will be able to model the damage evaluation due to the nominal biaxial stress state around the crack tip.
The limitations of predicting crack propagation by the RTCL criterion were due to the shell element formulations and inaccurate stress field at the crack tip (the element side length decides the crack width).

A rate dependent plasticity algorithm has also been implemented into the finite element program LS-DYNA to account for the material strain rate sensitivity in dynamic simulations. By use of the rate dependent plasticity algorithm instead of scaling the yield stress directly according to the original Cowper-Symonds model, spurious noise is avoided. A procedure for calibration of the material strain rate parameters is outlined in Section 3.5.

There are other fracture criteria available in the literature. The more general empirical criterion by Johnson and Cook (1985), partly based on the Rice-Tracey criterion, makes it possible to calibrate the fracture criterion so that the damage process can be better modelled for different materials. The Johnson-Cook damage model also includes the strain rate and temperature dependence. The strain rate dependencies are important in impact simulations of ships, but as the strain rates are relatively low the effect of adiabatic heating will be insignificant and thus the temperature dependence in the Johnson-Cook damage criterion can be neglected. However, to calibrate the Johnson-Cook criterion several tensile tests with various stress states have to be conducted and often when large structures are simulated only the results from a simple tensile test are available. If only a single tensile test is performed for calibration of the damage parameters, the constants in the Rice-Tracey triaxiality function can be assumed to be a good estimate of describing the void growth, but for negative triaxialities inaccurate results may be achieved. Thus, by using the Johnson-Cook criterion for low impact speeds without calibration of the damage parameters no gain is achieved compared to the RTCL criterion.

The CDM models can be expected to model the loss of stiffness due to void growth better than the uncoupled damage functions. The CDM model by Bonora (1997) has been shown to model accurately ductile fracture for positive triaxialities in steel. However, for aluminium, where a higher value of the damage exponent is used, the damage function will predict fracture far too early in high triaxialities. In the damage model by Lemaitre (1985), where the damage is a linear function of the strain, the ductility will not be correctly described due to the difficulties of modelling the effect of triaxial stress state on the fracture strain. The strain at fracture in the Lemaitre model is often overestimated at high triaxialities for the steel and aluminium alloys tested in the thesis. Moreover, both CDM models are not able to model the damage evolution in negative triaxialities, due to the quadratic triaxiality function, and should therefore not be used in compression tests. Almost all material properties in the CDM models can be determined by tensile tests, where also the damage evolution is measured. An additional FE-simulation is needed where the material hardening law derived from the tensile test is used in order to find the average triaxiality and the maximum plastic strain at the experimental point of fracture. Thus, no fitting of the parameters is needed to achieve the correct material and damage parameters. The CDM models are, however, more CPU demanding compared to the RTCL criterion, due to the Newton-Raphson iteration to find the plastic increment, as well as more damage parameters are to be determined.
The modified Gurson porosity model proposed by Needleman and Tvergaard (1984) is quite widely used today in small-scale simulations of fracture initiation and crack propagation. The GTN model has been shown to describe accurately the loss in stiffness due to void growth. The major drawback of the GTN models is the many material parameters that have to be determined, which makes the model difficult to use in practical engineering fracture simulations of large structures, such as accidental loading of ships.

8.1.2 Verification of the RTCL Criterion

For simulations of accidental loading of large complex structures of several different materials and thicknesses, the material and damage models have to be relatively simple and only a few parameters have to be determined. With a $J_2$ material model coupled with the rate dependent plasticity option and the RTCL fracture criterion, a simple, robust and fast material algorithm with few material parameters for simulations of large complex structures subjected to accidental loadings is available. Several simulations of large-scale experiments have been presented where the experimental results were compared with the numerical simulation results. The RTCL model proved to predict correctly fracture initiation in various structural arrangements and stress states. However, as previously discussed the crack propagation was believed to have been less accurately modelled. Nevertheless, the finite element results corresponded well with the experimental results showing that the RTCL criterion is suitable for predicting fracture initiation in simulations of large complex plate structures subjected to accidental loading. Moreover, the effects of applying the same fracture criterion to fracture initiation and crack propagation seemed to be of less importance. The plastic deformation was localised to a very small region (a single element) in front of the propagating crack. Thus, the extra energy dissipated by the element for the crack to propagate was insignificant at the global energy level, but could have influenced the deformation mode. E.g. in the simulation of the experimental test setup by Nagasawa et al. (1981) it was found that the cracks at the side plating probably propagated too slowly compared with the experiments. This was due to the fracture criterion or the critical damage value for fracture initiation and crack propagation should not have been the same. The effects of a too high damage value for crack propagation by use of the same criterion for initiation and propagation are therefore especially important when the structure is subjected to a uniform stress and strain field without any localisation, as the side plating in the experiments by Nagasawa.

With an element length of about five times the plate thickness, good agreement with the experimental results was achieved. The accuracy will most likely increase with decreasing element size, but the number of elements has to be limited to avoid unpractical calculation time spans. If elements much larger than five times the thickness are used the mesh discretisation will probably be too coarse for the folding pattern of a plate in compression and it will be incorrectly described. The accuracy of the numerical calculation of a large structure is therefore often a trade-off between accuracy and calculation time.

If several plates with a large scatter in the thicknesses are used, the smallest thickness of a main structural member which will be subjected to large deformation and is likely to fracture
should be used as a reference for the selection of the element size. Moreover, if the element size varies for the same member the results will be less mesh sensitive by using the critical damage value versus element size curve. When a critical damage value versus element size curve is used it is important to generate a mesh as regular and quadratic as possible, since the loading direction is often not known prior to the simulation and since the element length in the loading direction determines the critical damage value. However, if the loading direction is known from the start (as in simple tensile tests) the aspect ratio of the element will not cause any problem (if of course the ratio is within the accuracy limits of the element type).

By disregarding the strain rate effects in dynamic problems, such as ship collisions and ship groundings, the stiffness of the structural response will be lower, and it is therefore of great importance to include the strain rate effects when the dissipated energy is to be found during dynamic loading. E.g. in a ship collision simulation it can be the difference whether the cargo hold will be flooded or not. However, for comparing the crashworthiness of various structures, the strain rate effects are less important.

### 8.1.3 Weld Failure

A ship structure is mainly constructed of large stiffened plates assembled in blocks which are welded together into sections. The sections are later assembled in the dock to become the final ship. Thus, a large amount of welds exists in the ship structure and as a result the energy absorption capability of the ship depends on the weld quality and strength. If a weld fractures during an accident, the structural member can deform independently and thus cause much greater damage than if the weld would have remained intact. Several studies of damaged ships have, however, shown that it is mainly fillet welds that fracture in a grounding or collision event. Finite element simulations of weld fracture in a large ship structure are rather difficult to simulate accurately due to the small size of the weld compared to the total ship structure. The weld options available in commercial finite element codes will often not contribute to an increase of the accuracy in the numerical simulations. Thus, the welds should be modelled with deformable solid elements (alternatively with elastoplastic beams or shells), but the required small size of the elements would give very small time steps resulting in an unpractical solving time. However, in studies of crashworthiness the structural arrangement should determine the energy absorbing capabilities if the welds are not designed to fail for a certain loading.

### 8.1.4 Crash Simulations

It is often stated that nonlinear finite element programs are not suitable in the design phase of a ship. The empirical and simplified methods will give a good estimate, but when completely new designs are developed neither the empirical nor the simplified method will be able to quantify the energy absorption capability. Thus, for verifying the crashworthiness of a ship structure the finite element method is a very powerful tool.
By use of the RTCL criterion and a sufficiently small element size it is believed that the point of fracture is predicted with reasonable accuracy in the various accidental loading scenarios. The crashworthiness of three different structural arrangements for a small coaster was investigated, for which the building costs, the operating costs as well as the payload were said to be almost equal. The new structural layouts with corrugated panels and the concrete/steel sandwich had less energy absorption capabilities at fracture of the inner side shell compared to a conventional ice strengthened structure. This was mainly due to the small distance between the outer and inner hull plating in the two sandwich panels and because the bulb hit a web in the ice strengthened structure. An optimal crashworthy design should absorb as much energy as possible at the initial stage of the collision and still have a rather long penetration distance before fracture of the inner side shell.

In most finite element simulations of ship-ship collision events the struck ship has been at a standstill and fixed in space. However, the struck ship has, of course, often a forward velocity at the moment of contact. The forward motion of the struck ship will influence the damage extent. Eight different ship-ship collision scenarios were simulated by use of the ship motion sub-program MCOL where the effects of material strain rate sensitivity and surrounding water were investigated. It was shown that some of the simplifications in the simplified analytical methods are questionable, e.g. neglecting the roll motion of the ships and assuming constant contact force ratio (longitudinal over transverse forces of the struck ship). Another limitation of the simplified analytical methods is the problem of describing the complex interaction of the deformation between the structural members of the two ships, e.g. bending of the bulb.

For the specific load cases of two colliding coasters the dissipated energy due to hydrodynamic damping was insignificant and the wave damping effects could have been excluded from the simulations without loss of accuracy. The importance of including material strain rate sensitivity and the effects of rigid body motions in the water in simulations of ship-ship collision events was significant. The penetration depths at the point of fracture varied only slightly for different collision angles and forward speeds when the loading conditions were the same. Less energy was, however, dissipated at the point of fracture of the inner side when the struck ship had a forward velocity than when it was at a standstill. For different loading conditions the penetration depths as well as the structural absorbed energy varied, depending on the contact location depth. If the bulb hit above the double bottom the side structure was more flexible. Thus, more energy was dissipated when the bulb hit the double bottom structure.

### 8.1.5 Crashworthiness Analysis Procedure

The main aspects of conducting reliable ship collision analysis for the design of crashworthy side structures by use of the finite element method will be briefly outlined in the following. Initially, the aim of the simulations has to be determined. If different crashworthy side structures are to be investigated for a particular ship type, the shipping route with the most
frequently occurring vessel types, displacements, movements, headings and speeds should be studied in order to determine the probability distributions of the colliding bow structure and the released energy levels for crushing of the struck ship structure. The crashworthy structures should then be studied for the collision scenarios in accordance with the found data, i.e. a number of collision scenarios should be studied. It is not enough to determine the crashworthiness from a single particular scenario, as the energy absorption capability of a structure varies with the impact location (both vertically and longitudinally) as well as with the shape of the colliding ship.

A sufficiently large part of the ship structure should also be modelled to assure that the boundary conditions do not influence the structural response. For a single compartment ship this might mean that the whole length of the cargo compartment has to be modelled. The struck ship is preferably modelled without forward motion and fixed in space. Thus, the coupling between external dynamics and internal mechanics can be excluded in the simulations. It is therefore also convenient to model the striking ship with a constant velocity to reduce the calculation time. In addition, strain rate effects will not give additional information about the energy absorption capabilities of the structure and may be excluded from the simulation. The velocity of the striking ship should be kept rather low ($<10 \text{ m/s}$) to reduce the inertia effects. The above simplifications are only valid as long as the energy dissipation of different crashworthy structures is investigated, however, if the sections should later be validated by full-scale experiments, the simulations should be conducted as realistically as possible by including water-structure interaction, strain rate effects and rigid body movements with deceleration of the striking ship.

The structures should, of course, be as representative as possible of the real structural layout of the ship, so that only small details as different brackets and slots are excluded from the finite element structure. The mesh size should be as small as possible, uniform and of approximately the same size in the impact areas for the different structures. For stiffeners modelled with three or less elements, fully integrated elements should be used to prevent zero energy modes, which might occur if Belytschko-Tsay elements are used. Five integration points in the thickness of the shell elements should preferably be used. For the striking ship it is often sufficient to model only the bulb, since it often causes the largest damages. Moreover, the bulb can be assumed to be rigid.

In the deformation area contact between all structural members, including self contact, should be defined. The friction coefficient should preferably be found from experiments, however, a friction coefficient of 0.3 is often used, both for static and dynamic friction.

The material properties, including strain hardening curves, should be accurately defined for all materials and thicknesses. It is often convenient to describe the strain hardening curves by a power law, as material hardening curves obtained from tensile tests are most likely not valid after the point of necking. Preferably tensile tests for all materials and plate thicknesses should be simulated with different element sizes to generate a critical damage value versus element size curve for the RTCL criterion. The different tensile test specimens should therefore be of the same thickness as in the ship structure with a sufficient width and
gauge length to assure that the same element size as used in the crash simulations can be fitted. The maximum damage value in the tensile test simulations of several element sizes at the experimental point of fracture will give the critical damage value versus element size curve. The force-deformation curves obtained in the finite element simulations should, of course, correspond to the experimental results.

Finally, the energy absorption capabilities for the different structural layouts should be compared for the most critical collision scenarios.

8.2 Recommendations for future work

Further studies on using a void growth criterion for simulations of crack propagation with shell elements have to be conducted. A possibility is to use the through thickness strain as a crack propagation criterion, as it was found from the bulging experiments that the necking profile in crack propagation was not affected by different loading ratios. However, the necking strain will not be sufficient as a fracture initiation criterion since the necking strain was only constant in crack propagation and also since the stress triaxiality is not accounted for. This should be further investigated, as there is a need for a general criterion to predict/model both the fracture initiation as well as the crack propagation when using shell elements.

Moreover, the effects of excluding the welds in simulations of complex plate structures should be studied. By modelling the welds with deformable solid elements (alternative elasto-plastic beams or shell elements) on a relatively small plate structure where failure of a weld occurs during the loading, the effects of excluding the weld can be studied and preferably an alternative method for simulation of weld failure can be derived.

The derived comprehensive analysis procedure for collision and grounding damage can also be used for development and verification of simplified analytical methods which are typically used in Monte Carlo simulations for generation of energy dissipation reference values. In order to take full advantage of crashworthy side structures with respect to damage survivability, the existing concept of using damage statistics independent of the structural layout has to be abandoned. It must be replaced by the concept of collision energy reference values, since it is not possible to determine the amount of crushing energy with available damage data in accident databases. The main reason is the lack of information on the scantlings of the structures reported and the ship speed and courses of the moment of impact. Therefore, numerical data was generated for the most important European shipping routes (Lützen (2001)). The results were used to carry out Monte Carlo simulations, which determined the probability distributions of energy levels released for crushing of the target ship structures. From these statistics, energy reference values were found. Thus, an important application of the present research is to use the derived numerical procedure for verifying the energy reference values, which reflects the structural layout for given ships on specific routes, derived by probabilistic damage prediction methods.
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Appendix A

Finite Element Method

This chapter discusses the main principles of the explicit finite element method as well as the main shell elements used in the thesis with the commercial finite element code LS-DYNA.

A.1 Solvers

Finite element analysis is today the most commonly used numerical method for solving the physical response of a structure subjected to various loadings and boundary conditions. In a finite element model the structure is discretised into a large number of small elements. They are connected to each other by nodes in which the relations between forces and displacements are described in low or higher order equations, depending of the choice on element type.

There are two generally different types of methods for solving the differential equations which are derived for nonlinear problems: implicit and explicit time integration. The implicit methods lead to a system of nonlinear equations which have to be solved and can be made unconditionally stable so that the accuracy depends on the chosen time steps. For highly nonlinear problems convergence problems may arise by use of the implicit methods. Implicit solvers are thus suitable for simulation of static or slow transient phenomena. In the implicit methods, a global stiffness matrix is computed and applied to the node forces to obtain the displacement increments. They require relatively large time increments and comparatively large computer resources, RAM, as they are solving the whole matrices.

The advantage of the implicit approach is that only a single or a few time steps are needed to solve the problem. The user can also select a suitable time step size for the problem. The disadvantage is that a large numerical effort is required to build, store and factorise the stiffness matrix. The individual time steps are, therefore, very expensive in terms of computational efforts and should be kept as few as possible. The most common FE-codes use the implicit algorithm, such as the programs ANSYS, MARC, COSMOS, ABAQUS and NISA.
In the explicit codes, by using the principle of virtual work, internal and external forces are summed at each node where division by the nodal mass computes the nodal acceleration:

\[
M \ddot{u} + f_{\text{int}} = f_{\text{ext}} \tag{A.1}
\]

The mass matrix is denoted \(M\) and the displacement and its derivatives are denoted \(u\). \(f_{\text{int}}\) and \(f_{\text{ext}}\) are respectively the internal and the external force vectors. The explicit procedure requires no iterations and no tangent stiffness matrix.

The explicit method is based on the central difference approximation of the time derivatives. It is combined with a lumped mass matrix, which leads to a diagonal mass matrix and therefore a coupled system of equations has not to be solved. However, the algorithm is conditional stable where the maximum permissible time step is the time for an acoustic wave to travel through the smallest element. Integration of the nodal acceleration over time advances the solution where the maximum time step size is decided by the smallest element.

\[
\ddot{u}^{(i)} = M^{-1}(f_{\text{ext}}^{(i)} - f_{\text{int}}^{(i)}) \tag{A.2}
\]

\[
\dot{u}^{(i+\frac{1}{2})} = \dot{u}^{(i-\frac{1}{2})} + \frac{\Delta t^{(i+1)} + \Delta t^{(i)}}{2} \ddot{u}^{(i)} \tag{A.3}
\]

\[
u^{(i+1)} = u^{(i)} + \Delta t^{(i+1)} \dot{u}^{(i+\frac{1}{2})} \tag{A.4}
\]

The time step is denoted \(\Delta t\). The superscript \((i)\) refers to the increment number and \((i - \frac{1}{2})\) and \((i + \frac{1}{2})\) refer to the midincrement values. This results in an algorithm which, typically, requires many small relatively inexpensive time steps. Several commercial explicit finite element programs are available, e.g. LS-DYNA, ABAQUS, ADINA and MSC/DYTRAN.

The explicit finite element solvers are considered to be more suitable for simulations of highly nonlinear problems, e.g. crash simulations, compared to the implicit finite element codes. Even though the explicit codes are somewhat less accurate than the implicit codes, the applicability to complex large structures subjected to accidental loading is remarkable.
A.2 Time Step Control

During the solution LS-DYNA loops through the elements to update the stresses and determine a new time step size by taking the smallest value of all elements:

\[ \Delta t^{i+1} = \alpha_f \min\{\Delta t_1, \Delta t_2, ..., \Delta t_N\} \]  

(A.5)

where \( \Delta t \) is the time step and \( N \) the total number of elements. For stability reasons the scale factor, \( \alpha_f = 0.9 \) or less, is multiplied on the smallest found time step.

The time step size for shell elements is given by

\[ \Delta t = \frac{L_s}{c_s} \]  

(A.6)

where \( L_s \) is the characteristic length and \( c_s \) the speed of sound in the material given by:

\[ c_s = \sqrt{\frac{E}{\rho(1 - \nu^2)}} \]  

(A.7)

where \( E \) is Young’s modulus, \( \rho \) the density and \( \nu \) Poisson’s ratio. The characteristic length is e.g. given by

\[ L_s = \frac{A_s}{\max(L_1, L_2, L_3, L_4)} \]  

(A.8)

where \( A_s \) is the element area and \( L_1, L_2, L_3 \) and \( L_4 \) are the lengths of the element sides.

In some simulations it is possible to increase the density (mass scaling) and thus achieve larger time step increments, leading to shorter CPU time. This must, however, be done with great caution since the inertia effects may be affected.

A.3 Element Types

Shell elements are the main element type in crashworthiness simulations and the advantage of using 3-D elasto-plastic shell elements in finite element simulation is that the solution includes membrane bending effects, history dependence and also handles contact-friction
problems quite easily. Thus, with shell elements it is possible to simulate large complex structures with high accuracy.

The shell elements may differ in shape (quadratic and triangular) and the integration rule. Fully integrated four-node quadrilaterals use four integration points in-plane, whereas under-integrated elements only use one integration point in the plane. Thus, in under-integrated shell elements the strains are evaluated at the element centre, and this allows the element to exhibit zero energy modes (hourglassing) where the elements can deform without dissipating energy. Fully integrated elements do not have these hourglassing problems, but they may lock in shear which will cause the element to exhibit excessive stiffness. Several options are usually implemented into the FE-codes, which prevents the hourglassing modes in under-integrated shell elements from being developed. To reduce the problem further in the numerical simulations in this thesis, when the number of elements in the width is less than four (except in the tensile tests simulations) fully integrated elements are used (element type 16 in LS-DYNA), and elsewhere the underintegrated Belytschko-Tsay elements are used (element type 2).

The number of integration points through the thickness should preferably be five (or more) for elasto-plastic materials subjected to bending, in order to describe accurately the through thickness behaviour. However, the calculation time increases with the number of integration points. Therefore, in the large collision simulations the total number of integration points should be kept low to keep the calculation times reasonable, however, at least three integration points should be used through the thickness.

In the Belytschko-Tsay element, the geometry of the shell is assumed to be perfectly flat and thus warping is not considered. In general, the Belytschko-Tsay is a very efficient element, although it may give incorrect results for twisted beam problems where warping is considerable. This and other simplifications make the element very cost effective, but less accurate than other shell element algorithms. However, due to the low cost of the element, it is the most widely used shell element in metal forming and crash simulations.
Appendix B

Implemented Material Model

B.1 Constitutive Model

The material model was based on the $J_2$ flow theory with isotropic hardening and used a radial return algorithm. The implementation follows the description in Hallquist (1998) and is suitable for explicit finite element solvers.

The stresses are updated elastically:

$$\sigma_{ij}^* = \sigma_{ij}^n + C_{ijkl} \Delta \varepsilon_{kl}$$  \hspace{1cm} (B.1)

where $\sigma_{ij}^*$ and $\sigma_{ij}^n$ are the trial stress tensor and the stress tensor from the previous time step. $C_{ijkl}$ is the elastic tangent modulus matrix and $\Delta \varepsilon_{kl}$ is the incremental strain tensor. The trial elastic deviatoric stress, $s_{ij}^*$, is defined as

$$s_{ij}^* = \sigma_{ij}^* - \frac{\sigma_{kk}^*}{3}$$  \hspace{1cm} (B.2)

If the von Mises yield function is satisfied, the trial stresses are returned and the next time step is calculated. If, however, the yield function is violated an increment in plastic strain is computed and the stresses are scaled back to the yield surface.

The von Mises yield condition is given by

$$\phi = J_2 - \frac{\sigma_{yy}^2}{3}$$  \hspace{1cm} (B.3)
where the second stress invariant, $J_2$, is defined in terms of the deviatoric stress components as

$$J_2 = \frac{1}{2} s_{ij} s_{ij}$$  \hspace{1cm} (B.4)

and the yield stress, $\sigma_y$, is a function of the effective plastic strain, $\varepsilon_{eq}^p$, and the plastic hardening modulus, $E_p$:

$$\sigma_{y,n+1} = \sigma_{y,n} + E_p \Delta \varepsilon_{eq}^p$$  \hspace{1cm} (B.5)

The effective plastic strain is defined as

$$\varepsilon_{eq}^p = \sum \Delta \varepsilon_{eq}^p$$  \hspace{1cm} (B.6)

and the plastic tangent modulus is defined in terms of the tangent modulus, $E_t$, as

$$E_p = \frac{E E_t}{E - E_t}$$  \hspace{1cm} (B.7)

A plastic strain increment is computed:

$$\Delta \varepsilon_{eq}^p = \frac{\left(\frac{3}{2} s_{ij}^* s_{ij}^*\right)^{1/2} - \sigma_{y,n}}{3G + E_p}$$  \hspace{1cm} (B.8)

where $G$ is the shear modulus. The trial stress is scaled back and the updated stress is found according to

$$\sigma_{ij}^{n+1} = \frac{\sigma_{y,n+1}}{\frac{3}{2} (s_{ij}^* s_{ij}^*)^{1/2}} s_{ij}^* - \frac{1}{3} \sigma_{kk}^*$$  \hspace{1cm} (B.9)

### B.1.1 Plane Stress Plasticity

When shell elements are used, a plane stress algorithm is employed to satisfy the yield criterion. The plane stress plasticity must be solved iteratively since the through thickness strain component is not known from the shell kinematics. First, an elastic increment is made:

$$\Delta \varepsilon_{33} = -\frac{\nu(\Delta \varepsilon_{11} + \Delta \varepsilon_{22})}{1 - \nu}$$  \hspace{1cm} (B.10)
B.1 Constitutive Model

Then the trial stresses are computed and if the yield criterion is fulfilled, the trial stresses are returned by $\sigma_{33} = 0$ and the next time step is computed. However, if the stress state is plastic, the plane stress condition will not be fulfilled and a new strain increment through the thickness is calculated by using secant iteration.

$$\Delta \varepsilon_{33}^{i+1} = \Delta \varepsilon_{33}^{i} - \frac{\Delta \varepsilon_{33}^{i} - \Delta \varepsilon_{33}^{i-1}}{\sigma_{33}^{i} - \sigma_{33}^{i-1}} \sigma_{33}^{i-1}$$ \hspace{1cm} (B.11)

where

$$\sigma_{33}^{i+1} = \sigma_{33}^{i} - \frac{3G\Delta \varepsilon_{eq}^{p(i)}}{\left( \frac{3}{2} \frac{s_{ij}^{p(i)} s_{ij}^{p(i)}}{G_{ij}} \right)^{2} G_{33}}$$ \hspace{1cm} (B.12)

and where the starting values are obtained from the initial elastic estimate and by assuming a fully plastic increment:

$$\Delta \varepsilon_{33}^{1} = -(\Delta \varepsilon_{11} - \Delta \varepsilon_{22})$$ \hspace{1cm} (B.13)

The iterations proceed until the normal stress, $\sigma_{33}$, is sufficiently small, and after convergence the stress update is performed as described above.

B.1.2 Material Hardening

The material hardening can be described by a tangent modulus, $E_t$, or by a hardening curve, either by a user-defined effective stress versus equivalent plastic strain curve or a power law, described by

$$\sigma_{y} = C(\varepsilon_{yp} + \varepsilon_{eq}^{p})^{n}$$ \hspace{1cm} (B.14)

where $C$ and $n$ are the material strength coefficient respectively the hardening coefficient, and $\varepsilon_{yp}$ is the elastic strain to yield.
Appendix B. Implemented Material Model

B.1.3 Strain Rate Effects

Strain rate effects can be accounted for by using the Cowper-Symonds formula, which scales the yield stress according to

$$\sigma_y(\varepsilon_{eq}^p, \dot{\varepsilon}) = \sigma_y^s(\varepsilon_{eq}^p) \left[ 1 - \left( \frac{\dot{\varepsilon}}{C_{cs}} \right)^{\frac{1}{p}} \right]$$

(B.15)

where $C_{cs}$ and $p$ are user-defined input constants and $\dot{\varepsilon}$ is the strain rate defined as

$$\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$$

(B.16)

This approach usually causes spurious noise in the numerical solution due to the elastic part in the strain. Therefore, another approach was also implemented which is based on the equivalent plastic strain rate and more suitable for rate dependent plasticity. A drawback is that the plastic strain increment has to be solved iteratively. The dynamic yield stress is then computed from

$$\sigma_y(\varepsilon_{eq}^p, \dot{\varepsilon}_{eq}) = \sigma_y^s(\varepsilon_{eq}^p) + \sigma_0 \left( \frac{\dot{\varepsilon}_{eq}}{C_{cs}} \right)^{\frac{1}{p}}$$

(B.17)

where $\sigma_0$ is the initial yield stress and where the plastic strain rate is found by

$$\dot{\varepsilon}_{eq}^p = \frac{\varepsilon_{eq}^p}{\Delta t}$$

(B.18)

A Newton-Raphson iteration was used to find the plastic strain increment from

$$f(\Delta \varepsilon_{eq}^p) = \sigma_{y,n} - \sigma_{eq,n+1}^s + \sigma_0 \left( \frac{\Delta \varepsilon_{eq}^p}{C_{cs}\Delta t} \right)^{\frac{1}{p}} + (3G + E_p)\Delta \varepsilon_{eq}^p = 0$$

(B.19)

B.1.4 Continuum Damage Model

Lemaitre’s Linear Damage Model

The CDM model proposed by Lemaitre (1985) was implemented in LS-DYNA and follows the implementation described by Berstad et al. (1999). The effective stresses are updated elastically:

$$\tilde{\sigma}^* = \frac{\sigma_n}{1 - D_n} + C_{ijkl}\Delta \varepsilon_{n+\frac{1}{2}}$$

(B.20)
where $\dot{\sigma}^*$ and $\sigma_n$ are the effective trial stress tensor and the stress tensor from the previous time step. $C_{ijkl}$ is the elastic tangent modulus matrix and $\Delta \varepsilon$ is the incremental strain tensor. The effective trial elastic deviatoric stress, $\tilde{s}^*_{ij}$, is defined as

$$\tilde{s}^*_{ij} = \dot{\sigma}^* - \frac{1}{3}\tilde{\sigma}^*_{kk}$$  \hspace{1cm} (B.21)

If the von Mises yield function is satisfied, the trial stresses are returned and the next time step is calculated. If, however, the yield function is violated an increment in plastic strain is computed and the stresses are scaled back to the yield surface.

The von Mises yield condition is given by

$$\phi = \sqrt{\frac{3}{2}S^*_{ij}\tilde{s}^*_{ij} - \sigma_y}$$  \hspace{1cm} (B.22)

and the yield stress, $\sigma_y$, is a function of the effective plastic strain, $\Delta r$, and the plastic hardening modulus, $E_p$:

$$\sigma_{y,n+1} = \sigma_{y,n} + E_p \Delta r$$  \hspace{1cm} (B.23)

where $\Delta r$ is the damage accumulated plastic strain increment and is found by

$$\Delta r_{n+1} = (1 - D) \Delta \varepsilon_{eq,n+1}$$  \hspace{1cm} (B.24)

The effective plastic strain is defined as

$$\varepsilon_{eq,n+1}^p = \varepsilon_{eq,n}^p + \Delta \varepsilon_{eq}^p$$  \hspace{1cm} (B.25)

and the plastic tangent modulus is defined in terms of the tangent modulus, $E_t$, as

$$E_p = \frac{E E_t}{E - E_t}$$  \hspace{1cm} (B.26)

where $G$ is the shear modulus. The damage is calculated:

$$D_{n+1} = D_n + \begin{cases} 0 & r_{n+1} \leq r_D \\ \frac{\dot{\varepsilon}_{eq,n+1} R_{eq,n+1}}{2E} \Delta \varepsilon_{n+1} & r_{n+1} > r_D \end{cases}$$  \hspace{1cm} (B.27)
\[ R_{v,n+1} = \frac{2}{3}(1 + \nu) + 3(1 - 2\nu) \left( \frac{\sigma_{H,n+1}}{\sigma_{eq,n+1}} \right)^2 \]  
(B.28)

\[ \phi = \tilde{\sigma}_{eq,n+1} - \sigma_{y,n+1} = 0 \]  
(B.29)

The above is solved iteratively so that the yield condition is fulfilled. The effective stress is updated according to

\[ \sigma_{ij,n+1} = (1 - D_{n+1})\tilde{\sigma}_{ij,n+1} \]  
(B.30)

**Bonora’s Nonlinear Damage Model**

In Bonora (1997) damage model the damage function from Lemaitre is simply replaced by

\[ D_{n+1} = D_n + \begin{cases} 
0 & r_{n+1} \leq r_D \\
\alpha \left( \frac{D_{cr} - D_0}{\ln \varepsilon_f - \ln \varepsilon_{th}} \right) R_v (D_{cr} - D_n) \left( \frac{n-1}{r_{n+1}} \right) \frac{\Delta \varepsilon_{n+1}}{r_{n+1}} & r_{n+1} > r_D 
\end{cases} \]  
(B.31)

A Newton-Raphson method is used to find the plastic increment so that the yield criterion is fulfilled.
Appendix C

Rigid-Body and Hydrodynamic Model

In the following the rigid-body motion and hydrodynamic model used in the MCOL program is briefly described. For more comprehensive details reference is to be made to Ferry (2001).

C.1 Rigid-Body Motion

The motion of the rigid-body with respect to a body-fixed rotating reference frame with origin in the centre of mass is given by Newton’s law:

\[ \vec{f}_{RG} = m \left( \vec{\dot{v}}_G + \vec{\omega} \times \vec{v}_G \right) \]  \hspace{1cm} (C.1)

\[ \vec{m}_{RG} = I_G \vec{\dot{\omega}} + \vec{\omega} \times (I_G \vec{\omega}) \]  \hspace{1cm} (C.2)

where \( m \) is the mass of the rigid body and \( I_G \) is the inertia tensor in the mass centre of the rigid body. \( \vec{f}_{RG} \) and \( \vec{m}_{RG} \) is the forces and the moments applied to the rigid body. The rigid body equations can be expressed in the body-fixed frame:

\[ M_{RB} \ddot{\vec{y}} + G_{RB} \vec{y} = F_{RB} \]  \hspace{1cm} (C.3)

with \( G_{RB} \) as the skew-symmetrical gyroscopic matrix and the rigid-body inertia matrix defined as

\[
M_{RB} = \begin{bmatrix}
  m & 0 & 0 & 0 & 0 & 0 \\
  0 & m & 0 & 0 & 0 & 0 \\
  0 & 0 & m & 0 & 0 & 0 \\
  0 & 0 & 0 & I_x & -I_{xy} & -I_{xz} \\
  0 & 0 & 0 & I_{xy} & I_y & -I_{yz} \\
  0 & 0 & 0 & -I_{xz} & -I_{yz} & I_z
\end{bmatrix} \]  \hspace{1cm} (C.4)
where \( I \) is the moment of inertia components.

## C.2 Hydrodynamic Model

The forces and moments acting on the ships can be divided into contact forces and moments and hydrodynamic forces and moments due to the surrounding water. The hydrodynamic forces and moments are usually further separated into the inertia forces (added mass) \( F_A \), the restoring forces (buoyancy-gravity) \( F_R \), the viscous forces (drag and lift) \( F_V \) and wave forces \( F_W \).

The gravitational forces and buoyancy forces are written \( W = mg \) and \( B = \rho g \nabla \) where \( g \) is the gravity, \( \rho \) the water density and \( \nabla \) the displacement volume.

The hydrostatic restoring forces are usually expressed as a linear function of displacements relative to a given reference position \( \mathbf{x}_{\text{ref}} \):

\[
\mathbf{F}_H = - \begin{bmatrix} \mathbf{R}^T & 0 \\ 0 & \mathbf{R}^T \end{bmatrix} \mathbf{K} (\mathbf{x} - \mathbf{x}_{\text{ref}}) + \mathbf{F}_{\text{Href}} \tag{C.5}
\]

which are associated with the rotation matrix from the reference water-plane fixed frame \( \mathbf{R}^* \) and the stiffness matrix \( \mathbf{K} \) in the earth fixed frame:

\[
\mathbf{K} = \rho g \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{A}_w & \mathbf{A}_wy_w & -\mathbf{A}_wx_w & 0 \\ 0 & 0 & \mathbf{A}_wy_w & \mathbf{J}_w & -\mathbf{J}_{wx} & 0 \\ 0 & 0 & -\mathbf{A}_wx_w & -\mathbf{J}_{wx} & \mathbf{J}_w & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{C.6}
\]

where \( \mathbf{A}_w \) is the water plane area of the ship, \( x_w \) and \( y_w \) are the body-fixed coordinates of the centre and \( \mathbf{J}_{wx}, \mathbf{J}_w \) and \( \mathbf{J}_{wxy} \) the inertia components of the area.

\( \mathbf{F}_{\text{Href}} \) is the restoring forces and moments in the body-fixed frame corresponding to the reference position \( \mathbf{x}_{\text{ref}} \) described as

\[
\mathbf{F}_{\text{Href}} = \begin{bmatrix} -\sin \theta (mg - \rho q \nabla) \\ \sin \theta \cos \theta (mg - \rho q \nabla) \\ \cos \theta \cos \theta (mg - \rho q \nabla) \\ (z_{\text{Bref}} \sin \theta \cos \theta - y_{\text{Bref}} \cos \theta \cos \theta)(\rho q \nabla) \\ (x_{\text{Bref}} \cos \theta \cos \theta + z_{\text{Bref}} \sin \theta)(\rho q \nabla) \\ (-y_{\text{Bref}} \sin \theta - x_{\text{Bref}} \sin \theta \cos \theta)(\rho q \nabla) \end{bmatrix} \tag{C.7}
\]
C.2 Hydrodynamic Model

\[ [x_{Bref}, y_{Bref}, z_{Bref}]^T \] is the position of the centre of buoyancy in the body-fixed frame with the origin at the centre of mass.

The acceleration inertia forces are assumed to be essentially the results of inertia of the fluid and are evaluated using potential flow theory (i.e. no interaction between inertia and viscous forces). The inertial hydrodynamic forces for a ship are expressed in the body-fixed frame as (Imlay (1961))

\[
F_A = -M_A \dot{y} - G_A y
\]  

For a surface ship \( M_A \) varies strongly with the radiated wave frequency. \( M_A \) therefore only represents the constant added mass and inertia matrix of the ship at infinite wave frequency and the wave effects will be included with wave in a single memory term. \( M_A \) is expressed as

\[
M_A = \begin{bmatrix}
X_u & X_v & X_w & X_p & X_q & X_r \\
Y_u & Y_v & Y_w & Y_p & Y_q & Y_r \\
Z_u & Z_v & Z_w & Z_p & Z_q & Z_r \\
K_u & K_v & K_w & K_p & K_q & K_r \\
M_u & M_v & M_w & M_p & M_q & M_r \\
N_u & N_v & N_w & N_p & N_q & N_r 
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\]  

and \( G_A \) is the skew-symmetric gyroscopic matrix.

During a transient motion the ship generates waves that produce hydrodynamic damping and variation of the added mass. It results in forces and moments written in the body-fixed frame as

\[
F_w = -\int_0^t G(\tau)(y(t-\tau) - y(0))
\]  

with

\[
G_z = \frac{2}{\pi} \int_0^\infty C(\omega) \cos(\omega \tau) d\omega
\]  

The matrix \( C(\omega) \) contains the hydrodynamic damping coefficients depending on the waves pulsation \( \omega \).

The motion of the ship is finally written in the body-fixed frame according to

\[
M \ddot{y} + G(y)y = [F_W + F_H + F_V](y,x) + F_C
\]
with the total mass matrix as

\[
M = M_{RB} + M_A = \begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\] (C.13)

and the total gyroscopic matrix as

\[
G = G_{RB} + G_A = -\begin{bmatrix}
0 & S(M_{11}v + M_{12}\omega) \\
S(M_{11}v + M_{12}\omega) & S(M_{21}v + M_{22}\omega)
\end{bmatrix}
\] (C.14)

### C.3 Hydrodynamic Data for the Collision Simulations

The following data was used in the collision simulations in Section 7.2 for the fully loaded ships.

\[
M_{RB} = \begin{bmatrix}
5.70E+06 & 0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 \\
0.00E+00 & 5.70E+06 & 0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 \\
0.00E+00 & 0.00E+00 & 5.70E+06 & 0.00E+00 & 0.00E+00 & 0.00E+00 \\
0.00E+00 & 0.00E+00 & 0.00E+00 & 3.00E+08 & -1.00E+07 & -3.00E+08 \\
0.00E+00 & 0.00E+00 & 0.00E+00 & -1.00E+07 & 3.43E+09 & -1.00E+07 \\
0.00E+00 & 0.00E+00 & 0.00E+00 & -3.00E+08 & -1.00E+07 & 3.43E+09
\end{bmatrix}
\] (C.15)

\[
K = \rho g = \begin{bmatrix}
0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 \\
0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 \\
0.00E+00 & 0.00E+00 & 0.00E+00 & 1.06E+07 & 0.00E+00 & 0.00E+00 \\
0.00E+00 & 0.00E+00 & 0.00E+00 & 1.95E+07 & 0.00E+00 & 0.00E+00 \\
0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 & 5.77E+08 & 0.00E+00 \\
0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00 & 0.00E+00
\end{bmatrix}
\] (C.16)

\[
\begin{bmatrix}
x_b \\
y_b \\
z_b \\
W \\
B \\
z_{0\text{ref}} \\
\phi_{\text{ref}} \\
\theta_{\text{ref}}
\end{bmatrix} = \begin{bmatrix}
0.00E+00 \\
0.00E+00 \\
-5.00E-01 \\
5.59E+07 \\
5.59E+07 \\
0.00E+00 \\
0.00E+00 \\
0.00E+00
\end{bmatrix}
\] (C.17)
C.3 Hydrodynamic Data for the Collision Simulations

\[
M_A = \begin{bmatrix}
1.14 E+05 & 0.00 E+00 & 0.00 E+00 & 0.00 E+00 & 0.00 E+00 \\
0.00 E+00 & 2.02 E+06 & 0.00 E+00 & 2.40 E+06 & 0.00 E+00 \\
0.00 E+00 & 0.00 E+00 & 7.08 E+06 & 0.00 E+00 & 2.91 E+06 \\
0.00 E+00 & 2.40 E+06 & 0.00 E+00 & 2.61 E+07 & 0.00 E+00 \\
0.00 E+00 & 0.00 E+00 & 2.91 E+06 & 0.00 E+00 & 2.90 E+09 \\
0.00 E+00 & 2.60 E+06 & 0.00 E+00 & 1.00 E+07 & 0.00 E+00 \\
0.00 E+00 & 2.90 E+09 & 0.00 E+00 & 1.24 E+09 & 0.00 E+00
\end{bmatrix} \tag{C.18}
\]

In Figure C.1 the damping coefficients are given for the coaster.

![Graphs showing damping coefficients for various Coasters.](a) (b)

**Figure C.1:** Damping coefficients for the coaster.
<table>
<thead>
<tr>
<th>Year</th>
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<th>Title</th>
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