Application of Conditional Waves as Critical Wave Episodes for Extreme Loads on Marine Structures

Jesper Skjoldager Dietz
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Preface

This thesis is submitted as a partial fulfilment of the requirements for the Danish PhD defence. The work was performed at the Section of Maritime Engineering, Department of Mechanical Engineering, Technical University of Denmark, during the period of May 2001 to July 2004. The project was supervised by Professor Peter Friis-Hansen and Professor Jørgen Juncher Jensen, whose help during the project is greatly appreciated.

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Jesper Skjoldager Dietz
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Executive Summary

Application of conditional waves as critical wave episodes for extreme loads on marine structures has been studied. A conditional wave episode represents normally a short wave sequence with specified properties at a given time instant and location. Often waves of large amplitudes are sought. A most likely wave conditional on a given crest amplitude may be generated, however, this type of wave sequence may not always be critical in a structural point of view if the vertical bending moment (VBM) amidships is considered.

The most likely response wave represents a more critical wave sequence as the wave is established with information on the vessel considered. This type of wave is generated by conditioning on a given linear response, after which the most likely response wave is generated by introducing information from the linear transfer function. The wave profile derived may subsequently be applied to a non-linear time domain simulation. It is thus assumed that the linear model is a good predictor of the non-linear extremes.

One of the main objectives of the present thesis is to establish statistical methods by application of the conditional wave to predict short- or long-term response statistics. Simulation time thus reduces significantly.

The main aspects covered in this thesis are:

Probabilistic methods for ship responses are reviewed. Focus on short- and long-term ship response statistics are made.

A review of existing models for conditioning a wave process is made, here first and second order models are compared. Models for conditioning of ship responses is additionally studied, with establishment of a new model on Most Likely Response Waves, (MLRW).

Ship responses due to the Most Likely Waves (MLW) are studied. Time domain simulations can either be performed on basis of the deterministic MLW or by the MLW embedded in a stochastic sea way. This profile is referred to at the Conditional Random Wave (CRW).

A model for estimation of short-term response statistics by application of the CRW is presented. Upcrossing rates of the VBM amidships are compared to results of brute force simulations, where fine agreements are obtained for both hogging- and sagging-induced responses in the sea state and for the operational profile selected.
However, by application of the deterministic MLW, the peak responses are found to be biased with a factor of $1.1 \rightarrow 1.4$ as compared to results from the CRW model. The analyses performed indicate furthermore that the responses are sensitive to the zero-upcrossing period selected and the wave spectrum applied. The location of the MLW crest relative to the hull girder is also important and should be considered carefully.

Ship Responses by application of MLRW are examined and a new model for estimation of short-term response statistics is given. Extreme value response distributions are generated by application of the deterministic MLRW and the MLRW with the random background wave included. The latter profile is referred to as the Conditional Random Response Wave (CRRW). Figure 1 shows upcrossing rates derived on basis of the MLRW and CRRW approaches and compared to results of brute force simulations where very good agreements are obtained.

![Figure 1: Upcrossing rates on the basis of the MLRW and the CRRW approaches. Comparison is made to results of brute force simulations.](image)

By application of the proposed approaches, CPU time is reduced significantly. The upcrossing rates from the MLRW model is derived on the basis of 10 constrained simulations of approximately 100 sec. each.

The MLRW approach has been coupled to the Model Correction Factor Method, Ditlevsen & Arnbjerg-Nielsen (1991). An advanced model can be established from either the CRRW model or from brute force simulations. The advanced model can hereafter be predicted by the MLRW model which is correlated to bias factors, which are a function of several stochastic variables. These are the operational and the response level considered. The bias factors are considered independent of the actual sea state as the MLRWs calculated practically are identical for all combinations of $H_s$ and $T_z$ given the same response level. Bias factors to the simplified model are found between $0.98 \rightarrow 1.16$ as compared to the CRRW model.

Hydroelastic responses are studied by application of the MLRW, and models for predicting the effect of whipping are given. Figure 2 shows upcrossing rates derived on basis of the MLRW and CRRW approaches assuming a flexible hull girder. Fine agreement with results from brute force analyses is obtained for both hogging- and sagging-induced responses.
Figure 2: Upcrossing rates on the basis of the MLRW and the CRRW approaches. Comparison is made to results of brute force simulations.

The MLRW model is found to capture most of the non-linear effects for sagging-induced responses. Bias factors to the simplified model are established and found between 0.95 → 1.12 as compared to results of the CRRW model. Bias factors are found up to 1.26 as hogging-induced responses are considered.

It is therefore recommended that the CRRW approach is used to establish short-term response statistics as the model practically captures all the non-linear effects. The model correction factor method and derived bias factors are recommended as long-term response statistics are sought.

The simulated responses in the present thesis are obtained by application of the non-linear time domain strip theory program ShipStar, Xia et al. (1998). The models derived are implemented in this program and are therefore made available for further use among other naval architects.

The major part of the analyses is performed for a Panmax container ship.
Anvendelse af betingede bølgeprocesser som kritiske bølgeepisoder til bestemmelse af ekstreme belastninger på maritime konstruktioner er blevet studeret. En betinget bølge er normalt karakteriseret som en kort bølgesekvens med en række specifikerede egenskaber til et bestemt tidspunkt og sted. Ofte er bølgeprofiler med høje bølgeamplituder søgt. Den mest sandsynlige bølge betinget på en give bølgeamplitude kan uden større problemer simuleres, dog er denne type bølge ikke nødvendigvis den mest kritiske fra et strukturelt synspunkt, hvilket for skibe ofte er det vertikale midtskibsøjningsmoment.


Et af hovedformålene for dette studie er at etablere statistiske modeller til bestemmelse af kort- og langtidsstatistik ved anvendelse af betingede processer, hvorved simuleringstiden reduceres kraftigt.

Hovedemnerne, som er behandlet i denne rapport, er:

Sandsynlighedsteoretiske modeller for bestemmelse af kort- og langtidsstatistik for skibs-respons.

Et overblik over eksisterende modeller til bestemmelse af betingede bølgeprocesser gives, hvor blandt andet første- og andenordens bølgemodeller er sammenlignet. Metoder til anvendelse af betingende processer for skibsresponser er yderligere præsenteret med introduktion af en ny model for “Most Likely Response Waves”, (MLRW).

Skibsresponser for en givet mest sandsynlige bølge er studeret. Analyserne indikerer, at det genererede responser er afhængigt af den valgte bølges nul-opkrydsningsperiode og det anvendte bølgespektrum. Tids-domæne simuleringer kan enten foretages på baggrund af det deterministiske MLW bølge profil eller hvor MLW bølgen er overlejet med en stokastisk


![Figure 3: Udkrydsningsrater genereret på basis af MLRW og CRRW modellerne. Sammenligning er foretaget med resultater fra "brute force" simuleringer.](image)

Ved at anvende de fremsatte modeller reduceres CPU tid kraftigt. Udkrydsningsrater er genereret på baggrund af 10 betingede simuleringer med længde på ca. 100 sek.


**Figure 4:** Udkrydsningsrater genereret på basis af MLRW og CRRW modellerne. Sammenligning er foretaget med resultater fra “brute force” simuleringer.

MLRW modellen ses at fange de fleste ikke-linære effekter. Bias faktorer til den simple model er etableret. De sagging inducerede responser skal modelkorrigeres i et interval fra 0.95 til 1.12 i forhold til CRRW metoden. Bias faktorer op til 1.26 skal anvendes på de hogging inducerede responser.

Det anbefales derfor at CRRW modellen anvendes til at etablere kort-tidsstatistik for skibsresponser da denne model fanger flere ikke-lineære effekter. Modelkorrektions faktor metoden med tilhørende bias faktorer bliver mere interessant for en lang-tidsanalyse siden beregningsstiden kan reduceres kraftigt.

De simulerede processer er frembragt ved anvendelse af det ikke-lineære tids-domæne stripe-teori program ShipStar, Xia et al. (1998). De etablerede modeller er implementerede i dette program og er derved gjort brugbare for andre skibsingeniører verden over.

Hovedparten af analyserne er lavet for et Panmax containerskib.
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Contents

Preface i

Executive Summary iii

Synopsis (in Danish) vii

Contents x

1 Introduction 1

1.1 Background .................................................. 1

1.2 Motivation and Objective of the Present Thesis .................. 4

1.3 Organisation and Contributions of This Thesis ................... 5

2 Probabilistic Methods for Ship Responses 7

2.1 Introduction .................................................. 7

2.2 Ocean Environment ........................................... 8

2.2.1 Introduction .............................................. 8

2.2.2 Sea State Characterisation ................................ 9

2.2.3 Short-Term Wave Climate .................................. 10

2.2.4 Directional Wave Elevation Spectrum ....................... 13

2.2.5 Joint Distribution of Wave Height and Frequency ............. 14

2.3 Response Spectrum ........................................... 15
### 2.3 Transfer Function
- 2.3.1 Transfer Function ........................................... 15
- 2.3.2 Linear Response .............................................. 16

### 2.4 Time Series Generation
- 2.4.1 Irregular Wave Elevation .................................... 17
- 2.4.2 Irregular Response ............................................ 17

### 2.5 Short-Term Response Statistics
- 2.5.1 The Weibull Distribution .................................... 18
- 2.5.2 The Rayleigh Distribution .................................... 19
- 2.5.3 The Generalised Extreme Value Distribution .............. 19
- 2.5.4 Peak Distribution ............................................. 20

### 2.6 Extreme Value Statistics
- 2.6.1 Extreme Value Distributions .................................. 21

### 2.7 Long-Term Response Statistics
- 2.7.1 Long-Term Response ........................................... 24
- 2.7.2 Operation of the Vessel and Effect on Long-Term Responses ....... 25
- 2.7.3 Extreme Value Analysis ....................................... 26
- 2.7.4 Long-Term Wave Climate ..................................... 27

### 3 Conditioning on Ship Responses
- 3.1 Introduction ..................................................... 29
- 3.2 Most Likely Wave Models ...................................... 30
  - 3.2.1 The New Wave Profile, Tromans et al. ....................... 30
  - 3.2.2 The Most Likely Wave Profile, Friis-Hansen & Nielsen ....... 34
- 3.3 Second Order Models ............................................ 36
  - 3.3.1 Second Order Most Likely Wave Profile ...................... 36
## Contents

3.4 Evaluation of Most Likely Wave Models ........................................... 39
   3.4.1 Effect of Zero-Upcrossing Period ........................................... 39
   3.4.2 Effect of Applied Wave Spectrum ........................................... 41
3.5 Existing Models for Extreme Ship Response Estimation ......................... 43
   3.5.1 Critical Wave Episodes ................................................... 44
   3.5.2 Most Likely Wave Approach ............................................. 44
   3.5.3 Most Likely Extreme Response, MLER .................................. 45
3.6 New Model for Extreme Ship Response Estimation ............................ 47
   3.6.1 Most Likely Response Wave Model, MLRW ............................. 47
3.7 Evaluation of the MLRW/CRRW Model .......................................... 51
   3.7.1 Discussion of the MLRW Model ........................................... 51
   3.7.2 Effect of Zero-Upcrossing Period ........................................ 52
   3.7.3 Effect of Applied Wave Spectrum ........................................ 54
3.8 Concluding Remarks ................................................................. 55

4 Most Likely Wave Analyses ............................................................. 57
   4.1 Introduction ................................................................. 57
   4.2 Time Domain Simulation Using a Constrained Signal ...................... 59
   4.3 Parameter Study, MLW Responses ........................................... 61
      4.3.1 Effects of used Wave Spectrum ....................................... 61
      4.3.2 Selection of the Critical MLW Profile ................................ 63
      4.3.3 MLW Responses, Changing the Velocity ............................ 68
      4.3.4 MLW Responses, Changing the Heading Angle, $\beta$ ............. 69
      4.3.5 Discussion of Responses due to the Deterministic MLW Profile .. 70
   4.4 MLW Responses, Short-Term Response Statistics .......................... 71
4.4.1 The Constrained Signals ........................................ 72
4.5 Extreme Value Distributions ..................................... 73
4.5.1 Selection of Peak Responses ................................. 75
4.5.2 Response Distribution, Part One ............................ 75
4.5.3 Response Distribution, Part Two ............................ 79
4.6 Concluding Remarks ............................................. 85

5 Most Likely Response Wave Analyses .............................. 87
5.1 Introduction ..................................................... 87
5.2 Parameter Study, Most Likely Response Waves ............... 88
5.3 CRRW, Short-Term Response Statistics ....................... 93
5.3.1 CRRW Responses ........................................... 95
5.3.2 Calculation Procedures .................................... 97
5.3.3 Effect of Vessel’s Speed .................................... 101
5.3.4 Effect of Bow Quartering Seas ............................. 104
5.4 Model Correction Factor Method ................................ 104
5.4.1 Bias Factors for the MLRW Model ........................ 105
5.4.2 Correction Factors to the Linear Model .................... 106
5.5 Discussion on the MLW and MLRW Approaches ............... 108
5.6 Concluding Remarks ............................................. 108

6 Hydroelastic Responses .............................................. 111
6.1 Introduction ..................................................... 111
6.2 Brute Force Simulation ......................................... 112
6.2.1 The Wave- and Whipping-Induced Responses ............ 113
6.3 Parameter Study, Hydroelastic Responses ..................... 116
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.3.1 The Slamming Problem - MLRW</td>
<td>116</td>
</tr>
<tr>
<td>6.3.2 MLRW, Hydroelastic Responses</td>
<td>118</td>
</tr>
<tr>
<td>6.3.3 Selection of Peak Responses, MLRW / CRRW Approach</td>
<td>123</td>
</tr>
<tr>
<td>6.3.4 Effect of Changed Linear Constrained Response and Forward Speed</td>
<td>123</td>
</tr>
<tr>
<td>6.4 Hydroelastic Responses, Short-Term Statistics</td>
<td>125</td>
</tr>
<tr>
<td>6.4.1 Hydroelastic Response Analyses of Application of the CRRW</td>
<td>125</td>
</tr>
<tr>
<td>6.4.2 Short-Term Response Statistics - Hydroelastic Responses</td>
<td>127</td>
</tr>
<tr>
<td>6.4.3 Short-Term Response Distributions of Wave- and Whipping-Induced Responses</td>
<td>128</td>
</tr>
<tr>
<td>6.4.4 Model Correction Factor Method - Hydroelastic Responses</td>
<td>130</td>
</tr>
<tr>
<td>6.5 Short-Term Response Distribution - Approximative Solution</td>
<td>131</td>
</tr>
<tr>
<td>6.5.1 Establishment of Peak Distributions - Brute Force Simulations</td>
<td>131</td>
</tr>
<tr>
<td>6.5.2 Selection of Peak Responses</td>
<td>132</td>
</tr>
<tr>
<td>6.5.3 Peak Distributions</td>
<td>133</td>
</tr>
<tr>
<td>6.6 Correlation between Wave- and Whipping-Induced Maxima</td>
<td>135</td>
</tr>
<tr>
<td>6.7 Concluding Remarks</td>
<td>137</td>
</tr>
<tr>
<td>7 Long-Term Response Statistics by Application of CRRW</td>
<td>139</td>
</tr>
<tr>
<td>7.1 Introduction</td>
<td>139</td>
</tr>
<tr>
<td>7.2 Long-term contribution factor and CRRW Approach</td>
<td>139</td>
</tr>
<tr>
<td>7.3 MLRW, Long-Term Response Statistics</td>
<td>140</td>
</tr>
<tr>
<td>7.4 Concluding Remarks</td>
<td>144</td>
</tr>
<tr>
<td>8 Conclusions and Suggestions for Further Work</td>
<td>145</td>
</tr>
<tr>
<td>8.1 Conclusions</td>
<td>145</td>
</tr>
<tr>
<td>8.2 Suggestions for Further Work</td>
<td>148</td>
</tr>
</tbody>
</table>
# Contents

References ............................................. 151

## A Ship Responses and Load Theory ............................................. 155

A.1 Introduction ............................................. 155

A.2 Non-Linear Time Domain Strip Theory ............................................. 156

  A.2.1 The Hydrodynamic Force ............................................. 156

  A.2.2 Hydroelastic Equation of Vertical Motion ............................................. 157

A.3 Modifications to the Existing ShipStar Code ............................................. 158

  A.3.1 Implemented Functions in the Existing Code ............................................. 158

  A.3.2 Using ShipStar ............................................. 159

## B The Panmax Container Ship ............................................. 165

B.1 The Panmax Vessel ............................................. 166

B.2 Body Plan ............................................. 167

B.3 Weight Distribution ............................................. 167

B.4 Modelling of the Hull Girder ............................................. 168

B.5 Linear Transfer Functions ............................................. 169

## C Estimation of Wave Spectra ............................................. 171

C.1 Introduction ............................................. 171

C.2 Full-Scale Measurements ............................................. 171

  C.2.1 The Recorded Signals ............................................. 172

C.3 Sea State Estimation ............................................. 173

  C.3.1 Estimated Significant Wave Height and Zero-Upcrossing Period . . . 174

## D Most Likely Wave ............................................. 177
Contents

E Additional Results by Application of the MLW 181

E.1 Response Distribution, JONSWAP Spectrum 183

E.1.1 Response Distribution, Part One 183

E.1.2 Response Distribution, Part Two 185

E.2 Most Likely Wave, Hydroelastic Responses 187

E.2.1 Effect of MLW Amplitude and Forward Speed 187

List of PhD Theses Available from the Department 189
Chapter 1

Introduction

1.1 Background

Several important issues as the overall safety, the economy, the surrounding environment and the operational performance of the vessel are affected by the loads and corresponding motions. Therefore, reliable estimation of hydrodynamic forces, wave-induced motions and the corresponding ship responses is important to the overall design of a ship.

Figure 1.1: Bow flare slamming in heavy sea on a sea trail from Rotterdam to Halifax. Photograph by Jesper S. Dietz.

An important task within design and operation of ships or offshore structures will therefore be to improve the motion behaviour. To obtain safer structural designs and operations, several parameters should be considered. Some of these are the operational scenario, which
Chapter 1. Introduction

contains information on the expected lifetime of the vessel, various loading conditions and operational profiles, which include information on the vessel’s speed and heading relative to the waves. Prediction of the environmental parameters is similarly important, this data is normally obtained by scatter diagrams which describe the joint probability distribution of the significant wave height and the zero-upcrossing period.

In practice, the design of a trading vessel is normally based on safety requirements, which are specified by ship rules and given by the various classification societies and international maritime organizations. These semi-empirical rules have been generated over several years and are to some extent based upon experience. Anyhow, they are easy to use for the naval architects around the world.

As new ship designs enter the arena, they are normally incorporated in the rules and regulations by the classification societies. Variations and complexities within the worldwide fleet suggest, however, that direct simulations should be used more often as severe non-linear responses occur.

In more detailed analyses of the loads and motions of ship structures, it seems reasonable to distinguish between the different load components that affect the hull girder. The most important parameters in the structural design of ship structures are the vertical shear forces and the corresponding bending moments. Remaining and less important are components as the lateral shear forces and bending moments, torsional moments and the longitudinal compression and tension forces. Torsional moments are though important for open structures like a containership. Below the main contributors to the total hull girder load are listed.

1. Still water loads at zero speed. These loads are a result of the loading condition in still water, i.e. the difference between the weight of the vessel and the corresponding buoyancy distribution. These load components may change dramatically during loading or unloading and may in extreme cases cause the hull girder to collapse.

2. Dynamic loads. The hydrodynamic problem is normally dealt with in two parts. Forces and moments of the first part correspond to the situation where the rigid hull girder is retained from oscillating with incident regular waves. The wave excitation loads are composed of the Froude-Kriloff and diffraction forces and moments. Forces and moments of the second part correspond to the situation where the vessel is forced to oscillate with different wave excitation frequencies. No incident waves are present, and the corresponding hydrodynamic loads are identified as added mass, damping and restoring terms. Assuming linearity in the forces given by part one and two, these may be added to form the total hydrodynamic forces.

3. Slamming- and whipping-induced loads. A slamming event and the associated generated impact occur as the bottom or bow flare of the vessel enters the water surface after a series of large heave and pitch motions. Slamming events introduce shock-like forces in the bow regions and corresponding transient whipping-induced vibrations. For container ships, the transient whipping responses are often introduced by bow flare slamming.
4. Green water on deck. The problem arises when the ship, usually in harsh weather conditions, encounters waves that exceed the freeboard and wet the deck. The term 'green water' refers to the shipped water from the wave crests encountering the bow of the vessel. In some cases the amount of water can be so large that it causes damage to deck equipment, plating, structures or the cargo being transported.

5. Springing-induced loads. Springing is the resonant two-node hull vibration due to unsteady pressure field and mostly expected for high-speed vessels or hulls with low rigidity. Many times this phenomenon has been reported for large tankers, bulk carriers etc. The fatigue life of the vessel can be dramatically reduced due to the springing vibration.

6. Quasi-static effect of forward speed. Sinkage and trim are introduced with forward speed, which affect forces and moments on the hull girder and represents one of the minor contributions. Change in the wave pattern is generated with increased forward speed.

Figure 1.2: Harsh weather on the Tasmanian Sea.

Establishing design loads from direct calculations by means of a long-term description of the response requires analyses over a multiple range of sea states and operational profiles. These may be summarised to obtain the desired long-term response distribution. If a linear analysis is performed, the long-term responses are generated using the linear transfer functions, while time domain simulations are generally necessary if non-linear analyses are required. The latter case leads to time-consuming simulations, which are practically impossible from a designer’s point of view. On the importance of non-linear loading, and with the knowledge from linear theory which over/under predicts wave-induced responses, a more efficient and practicable method is therefore wanted for obtaining more reliable short- or long-term response statistics.
Chapter 1. Introduction

During the past decades, there has been a continuous interest in accurate estimation of the non-linear hydrodynamic forces. Three-dimensional (3-D) time domain theories have been developed and implemented in software packages as SWAN, Vada & Helmers (1992) or LAMP, Lin et al. (1996). However, the 3-D theories have not seen the same success as traditional strip theories, which is mainly due to long CPU time and to some extent difficulties with the theory and implementation.

Simplified approaches are available for predicting wave-induced loads and responses. These theories are formulated from the linear strip theory with non-linear modifications to the hydrodynamic force. One such non-linear time domain strip theory code by, Xia et al. (1998), on which the present study is based upon. The time domain code, which is called ShipStar, takes into account non-linearities in wave- and slamming-induced rigid-body motions and structural responses of ships such as heave, pitch and the vertical bending moment. An general view of existing non-linear hydrodynamic theories is for example given by Wang (2000).

The non-linear contributions are generated within few areas of the scatter diagram. Sagli (2000) applied this idea and studied the non-linear vertical bending moment amidships for the S-175 container ship. By performing calculations within only nine sea states, an error of 2% on the life time extreme was found. The method was extended further. The sea states have a certain probability of occurrence, and each is identified by a contour line in the \( (H_s, T_p) \)-plane, where \( H_s \) is the significant wave height and \( T_p \) the mean peak period. This approach makes it possible to estimate the extreme load effects on the vessel in a practical and accurate manner, Videiro & Moan (1999), Sagli (2000) and Baarholm & Moan (2001).

1.2 Motivation and Objective of the Present Thesis

In the past decade, there has been a continuous interest in how limited time series information may be used in an optimal way to predict extreme value ship response statistics. In order to reduce the required number and length of simulated time series, Tromans et al. (1991) introduced the so-called “New Wave”, which is an expected wave profile established conditional on a given wave crest, \( a \), at time zero. The method was later extended to include the instantaneous wave frequency, Friis-Hansen & Nielsen (1995).

Taylor et al. (1995) showed how the “New Wave” or the Most Likely Wave (MLW) embedded in a stochastic seaway could be used to estimate the extreme response of a Jack-Up in a severe sea state in a robust and efficient manner. For dynamically responding structures, the extreme response does not always correspond to the extreme input surface elevation, and the extreme response might very well correspond to a combination of the local extreme wave with unfavourable background structural memory effects. The result of their work was a technique for estimating the probability distribution of extreme response in a given period using short constrained simulations. Moreover, it was found that extreme response on the basis of the MLW itself provided results in the lower tail of the distribution, which
1.3 Organisation and Contributions of This Thesis

indicates that the random background wave for dynamically responding structures is not to be neglected.

Torhaug (1996) introduced the Critical Wave Episode (CWE) model. The model is based on identification of wave candidates from the linear model that are likely to produce extreme responses. The wave sequences or critical wave episodes are typically a few wave cycles long, to be used as input for short non-linear simulations.

Adegeest et al. (1998) made another development and proposed the Most Likely Extreme Response method (MLER). By use of the amplitude and phase information from a set of linear transfer functions, it is possible to derive the underlying wave profile, which causes the MLER. Successively, a non-linear calculation can be carried out with the derived wave train as input. Their work results in one response value, which represents the most likely extreme response. Promising sag/hog ratios and corresponding good agreement between measurements were found.

In the present work, the feasibility of applying most likely response wave profiles to prediction of extreme wave responses of ship structures is developed further. The main objective is to formulate models for estimation of ship response statistics by application of conditional process whereas significant CPU time can be saved, especially on a long-term assessment. It is furthermore of interest to study the critical wave episodes that generate these extreme responses and to identify if effects of for example whipping may be captured efficiently.

1.3 Organisation and Contributions of This Thesis

This thesis is composed as follows. Chapter 1 outlines the basic ideas and the main objective of the current work.

Chapter 2 introduces some basic probabilistic concepts. Modelling of the ocean environment is introduced, here the most common formulations are given. Short-term and extreme value statistics are presented. Long-term response statistics and the effect of operational restrictions are briefly touched.

Chapter 3 gives an overview of the most interesting models of fast and accurate estimation of extreme ship response statistics using conditional approaches. The main theories are presented in addition with a new model for generating Most Likely Response Waves (MLRW) for estimation of ship response statistics. The new model is evaluated and compared with other theories.

Chapter 4 discusses the possibilities of application of the Most Likely Wave (MLW) and the Conditional Random Wave (CRW) profiles to prediction of extreme ship response statistics. A model for applying the MLW to estimation of extreme ship response statistics is presented and compared to results from brute force simulations. The shape of the critical wave profiles relative to the instantaneous position of the hull girder is further studied.
Chapter 5 introduces application of MLRW and Conditional Random Response Wave (CRRW) profiles to prediction of extreme ship response statistics. The current model allows short- and long-term response statistics to be generated in a fast and efficient manner. The MLRW and CRRW approaches have been coupled to the Model Correction Factor Method and bias factors for the stochastic model are established. Good agreement between the results of the CRRW approach and brute force simulations is found for the selected operational profiles and the selected harsh sea states.

Chapter 6 discusses results by application of the MLRW to an elastic hull girder where these conditional wave profiles have been applied to study the slamming problem. A model for generating short-term response statistics is presented and compared to results obtained from brute force simulations. The model derived allows the contribution of whipping to be examined from both hogging- and sagging-induced responses. The correlation between the wave- and whipping-induced bending moment has furthermore been studied and methods for establishment of peak distributions are given.

Chapter 7 discusses application of the MLRW and CRRW models with respect to establishment of long-term response statistics. Existing models are compared and discussed.

Finally, in chapter 8, conclusions and recommendations for future work are given.

The models derived are implemented in the non-linear time domain strip theory program ShipStar, Xia et al. (1998). This hydrodynamic program has been applied throughout all the simulations. A brief introduction and additional information on application of the models derived are given in Appendix A. The vessel considered is described in Appendix B.
Chapter 2

Probabilistic Methods for Ship Responses

2.1 Introduction

Application of probabilistic tools is playing an increasing role in engineering and physical sciences. This is mainly because of a growing realisation that many random phenomena observed are well described using stochastic prediction methodologies. Probabilistic approaches have therefore become an important component in modern design of for example marine structures.

Ship response analyses may either be performed in the frequency domain or the time domain. Frequency domain analyses are significantly faster than time domain simulations. The analyses within the frequency domain are performed on the basis of a linear transfer function and a selected wave spectrum, after which the response spectrum can be generated. By use of the statistical properties of the process, i.e. the spectral moments, statistical values may easily be generated.

For time domain analyses both the wave and corresponding response is described, and are given as to discrete time increments. The statistical uncertainty of the spectral moments will be reduced by making the time series longer.

Normally, time domain simulations are performed for non-linear cases. These analyses are time consuming, especially for three-dimensional (3-D) cases. Application of conditional waves or responses to time domain simulations introduces new, fast and very useful approaches for estimation of non-linear extreme value statistics. In this chapter probabilistic methods for a stochastic process are introduced. Some basic concepts are given, in addition to short-term and extreme value statistics. Approaches on how to obtain long-term responses statistics are briefly discussed.
2.2 Ocean Environment

2.2.1 Introduction

The constant changes of the wind velocity and direction have a strong influence on the typical wave height and period. The short-term changes in the wind-generated waves, though sufficiently small, allow the term a sea state to make sense.

The statistical description of the wave elevation is normally well characterised by a Gaussian distribution with zero mean value and variance denoted by $m_0$. It is assumed that the wave process may be considered stationary within a limited window of time (normally 30 minutes to 3 hours) and ergodic. The individual limited windows of time are called sea states. A sea state is described by a wave spectrum that contains information on the distribution of wave energy for different wave frequencies and directions. The Gaussian assumption is only acceptable for deep-water and moderate waves. For very shallow water, the wind-generated waves change shape, and the wave process cannot be considered Gaussian anymore. However, for ship structures, this non-Gaussianity appears to be of minor importance.

A sea state is often identified by the significant wave height, $H_s$, and a corresponding mean zero-upcrossing period, $T_z$. The surface of the sea has been divided into the so-called Marsden areas, where each of these geographical squares contains information on the joint distribution of $H_s$ and $T_z$. The wave data has been obtained from either wave buoys or visual observations.

For ships and other offshore installations which are operating around the world, the wave climate experienced during their lifetime can simply be described by a summation of all the various short-term sea states. A long-term model is thus introduced. If the sequence effect as previously mentioned is ignored, it is common to present the long-term variation in a so-called scatter diagram. Here the joint distribution of the wave spectral parameters, which refers to $H_s$ and $T_z$, is described for each of the individual short-term Gaussian processes.

The functional accuracy of the scatter diagrams is highly correlated with the number of observations included. The majority of the scatter diagrams only include up to a thousand observations of short-term sea states, whereas others include up to 100,000 observations, making these diagrams much more useful since the rarer events are better represented.

There are various different wave loading situations of great importance to the overall safety of marine structures. For bottom supported offshore structures the ultimate failure situation with very large waves and strong winds is of primary interest. For trading ships this ultimate failure situation is of similar interest, where a combination of several large waves often becomes very critical. It should be mentioned that ships have the possibility of avoiding these extreme weather episodes by weather routing and thus minimising the probability of the ultimate failure scenario. The fatigue process may lead to an ultimate failure if the installation is not kept under control by inspection and repair. For ships the ultimate failure due to fatigue is often avoided since “leak before failure” normally provides the owner a last call.
2.2 Ocean Environment

2.2.2 Sea State Characterisation

Provided that the sea surface is stationary, the sea state is characterised by the significant wave height $H_s$ and the mean zero-crossing period $T_z$. The definition of $H_s$ is in part based on an old visual observation classification system of the waves by experienced seamen, where their observations have turned out to be a good fit of the average height of the upper one third of the waves within one sea state. A reliable system for estimation of the sea state seems of significant value, and an efficient tool for many shipowners to avoid to damage their vessel is by changing speed or course in time.

The characteristic mean zero-crossing period $T_z$ is based on the concept of a generalisation of the period of a pure sinusoidal wave by applying linear wave theory, the wave elevation of a long-crested irregular sea propagating along a positive x-axis can be written as a large number of phase shifted sinusoidal waves of different heights and periods. The irregular surface necessarily moves up and down below the mean water level. The time between these crossings fluctuates around an average value denoted by $T_z$.

![Figure 2.1: A satellite prediction of the significant wave height, the North Atlantic Ocean on 5 Dec. 2002.](image)

Today, advanced satellite systems make it possible to observe the roughness of the ocean and provide the trading ships with valuable information on the sea state in a specific area and the expected development. Figure 2.1 shows an example of a sea state prediction for a specific region seen by the eye of a satellite.

Prediction of Wave Spectra

For shipmasters who operate large vessels, an accurate method for estimation of the sea state concerned is of great importance, especially during night when it is extremely difficult
to predict visually the actual sea state concerned. Appendix C outlines an approach to estimation of wave spectra from a continuously recorded strain signal amidships. Good agreement between satellite measured and calculated wave spectra is obtained from the derived model.

### 2.2.3 Short-Term Wave Climate

The wave spectrum is an important environmental input for estimation of marine structure responses. The wave spectrum is traditionally given for a fully developed sea. The wave spectrum may be divided into two parts as described by Torsethaugen (1993). The main part is represented by a purely wind-generated part, which may be generated from the Pierson Moskowitz or for example the JONSWAP spectrum. The second part consists of a mixture of swell- and wind-generated waves. The Torsethaugen wave spectrum therefore becomes a two-peaked formulation where both swell and wind waves are included.

#### The Gamma Spectrum

For response analyses of marine structures, the Pierson Moskowitz spectrum is often used. The spectrum $S_\xi(\omega)$ is a special case of the Gamma spectrum, Gran (1992), which is given as

$$S_\xi(\omega \mid h_s, T_z) = A\omega^{-\xi} \exp(-B\omega^{-\kappa}), \quad \omega > 0 \quad (2.1)$$

where $\xi$ controls the power of the tail, $\kappa$ describes the steepness in the lower frequency part. $\omega$ represents the wave frequency. The values $A$ and $B$ are given through $H_s$ and $T_z$ as

$$A = \frac{H_s^2 \kappa}{16} \left(\frac{2\pi}{T_z}\right)^{\xi-1} \frac{\Gamma\left(\frac{\xi-1}{\kappa}\right)}{\Gamma\left(\frac{\xi-3}{\kappa}\right)} \quad (2.2)$$

$$B = \left(\frac{2\pi}{T_z}\right) \kappa \frac{\Gamma\left(\frac{\xi-1}{\kappa}\right)}{\Gamma\left(\frac{\xi-3}{\kappa}\right)} \quad (2.3)$$

where $\Gamma(\cdot)$ is the Gamma function. The $n$’th spectral moment $m_n$ of the Gamma spectrum is given as

$$m_n = \frac{AB^{-(\xi-1-n)/\kappa}}{\kappa} \Gamma\left(\frac{\xi-1-n}{\kappa}\right) \quad (2.4)$$

The dimensionless Gamma spectrum is illustrated in Figure 2.2 for different values of $\kappa$ and with $\xi = 5$. The spectrum is given as

$$\frac{S_\xi(\omega) 2\pi}{H_s^2 T_z} = \frac{\kappa}{16} \left(\frac{\omega T_z}{2\pi}\right)^{-\xi} \frac{\Gamma\left(\frac{4}{\kappa}\right)}{\Gamma\left(\frac{2}{\kappa}\right)^2} \exp \left[-\left(\frac{\Gamma\left(\frac{4}{\kappa}\right)}{\Gamma\left(\frac{2}{\kappa}\right)}\right) \left(\frac{\omega T_z}{2\pi}\right)^{-\kappa}\right] \quad (2.5)$$
2.2 Ocean Environment

Figure 2.2: Dimensionless gamma spectrum for selected values of $\kappa$ and $\xi = 5$.

For $\kappa = 4$ and $\xi = 5$, the gamma spectrum corresponds to the Pierson Moskowitz wave spectrum.

**The Pierson Moskowitz Spectrum**

The Pierson Moskowitz (PM) wave spectrum was proposed in 1964 and is based on a large number of wave measurements. The spectrum can be derived from the gamma spectrum and reduces to

$$S_\zeta(\omega) = \frac{H_s^2 T_z}{8\pi^2} \left( \frac{\omega T_z}{2\pi} \right)^{-5} \exp \left[ - \frac{1}{\pi} \left( \frac{\omega T_z}{2\pi} \right)^{-4} \right]$$  \hspace{1cm} (2.6)

The mean period, $T_1$, is given by the spectral moments as

$$T_1 = 2\pi \frac{m_0}{m_1}$$ \hspace{1cm} (2.7)

and the zero-upcrossing period, $T_z$, is

$$T_z = 2\pi \sqrt{\frac{m_0}{m_2}}$$ \hspace{1cm} (2.8)

where the spectral moment of order $n$ is obtained as

$$m_n = \int_0^\infty \omega^n S_\zeta(\omega) d\omega, \quad n = 0, 1, 2, 3, ...$$ \hspace{1cm} (2.9)

The following relation exists between the zero-upcrossing period, the mean period and the peak period, $T_p$:

$$T_1 = 1.086 T_z$$ \hspace{1cm} (2.10)
\[ T_p = 1.408 T_z \]  
(2.11)

The PM wave spectrum is considered useful for a fully developed sea, Bishop & Price (1979). It is important to realise that the PM spectrum is less narrow-banded than the JONSWAP spectrum.

**The Generalised JONSWAP Spectrum**

Another formulation of wave frequency spectra in marine applications is the JONSWAP spectrum (J0int North Sea WAve Project) for a fully developed or developing sea. It is formulated as a modification of the PM spectrum for a developing sea state in a fetch limited situation. In offshore industry the JONSWAP spectrum is used for extreme load analysis. The spectrum has a peakedness factor \( \gamma \), which indicates the energy concentration around the peak frequency \( \omega_p \). The higher value of \( \gamma \), the more energy around the peak frequency. The JONSWAP spectrum as a function of the wave frequency \( \omega \) is in Bishop & Price (1979) given as

\[ S_\xi(\omega) = \frac{\alpha g^2}{\omega^5} \exp \left[ -\beta \left( \frac{\omega_p}{\omega} \right)^4 \right] \gamma^\gamma \exp \left[ -\left( \frac{\omega - \omega_p}{\sigma} \right)^2 \right] \]  
(2.12)

where \( \alpha \) is a spectral parameter, \( g \) is acceleration of gravity, \( \beta \) is a spectral parameter given as 1.25, \( \sigma \) is a spectral parameter where \( \sigma = 0.07 \) for \( \omega < \omega_p \) and \( \sigma = 0.09 \) for \( \omega > \omega_p \).

The relation between the spectral parameter, \( \alpha \), the significant wave height, \( H_s \), the peakedness parameter, \( \gamma \), and the peak period \( T_p \), is given as

\[ \alpha = 5.061 \frac{H_s^2}{T_p^4} (1 - 0.287 \ln(\gamma)) \]  
(2.13)

The peakedness parameter \( \gamma \) is varied between 1 and 6 and approximately normal distributed with a mean of 3.3. The ratio between the zero-upcrossing and peak period is given by the peakedness factor as

\[ \frac{T_p}{T_z} = -0.01698 \gamma + 1.30301 + \frac{0.12102}{\gamma} \]  
(2.14)

For \( \gamma = 1.0 \), the PM and JONSWAP wave spectra are identical. The difference between the PM and JONSWAP spectrum with \( \gamma = 3.3 \) is illustrated in Figure 2.3.
2.2 Ocean Environment

2.2.4 Directional Wave Elevation Spectrum

By introducing a spreading function $D_{\beta}(\beta)$, the directional wave spectrum is introduced. This form of spectrum is of interest in short-crested sea, where the waves tend to spread according to a dominant wind direction, i.e. the wave heading has to be taken into account. Usually, the same spreading function is used for all frequencies:

$$S_{z, \beta}(\omega, \beta) = S_{z}(\omega)D_{\beta}(\beta)$$  \hspace{1cm} (2.15)

Several approaches to the spreading function have been proposed. One of the most common is a cosine type function, as given by Bishop & Price (1979):

$$D_{\beta}(\beta) = A_{n}\cos^{n}(\beta)$$  \hspace{1cm} (2.16)

where the wave direction relative to the wind direction, $\beta$, is given in the interval $-\pi/2 \leq \beta \leq \pi/2$. The coefficient $A_{n}$ is obtained by use of the gamma function $\Gamma(\cdot)$:

$$A_{n} = \left[ \int_{-\pi/2}^{\pi/2} \cos^{2}(\beta)d\beta \right]^{-1} = \frac{2^{n-1}}{\pi} \frac{\Gamma(n/2)\Gamma(n/2 + 1)}{\Gamma(n)}$$  \hspace{1cm} (2.17)

A typical example of the directional wave spectrum is given in Figure 2.4.
Figure 2.4: A directional wave spectrum using the JONSWAP spectrum with $H_s = 5.0$ m and $T_z = 10.0$ sec. Eq. (2.17) has been applied with $n = 2$. The legend represents contour lines corresponding to response levels of $S_{\zeta, \beta}(\omega, \beta)$.

### 2.2.5 Joint Distribution of Wave Height and Frequency

Application of a joint model of a wave amplitude $r$ and frequency $\omega$ can be formulated from a normal process $x(t) = r \cos(\omega t + \epsilon)$, where $\epsilon$ is the phase. The process is required to be stationary, which implies that the phase should be uniformly distributed. The wave amplitude may be defined by the envelope of the wave elevation as $r = \sqrt{x(t)^2 + \hat{x}(t)^2}$, where $\hat{x}(t)$ is the Hilbert transform of $x(t)$, Cramer & Leadbetter (1967). It follows that the amplitudes may be represented by a Rayleigh distribution, cf. Eq. (2.36), given both $x(t)$ and $\hat{x}(t)$ are Gaussian variables.

**The Longuet-Higgins Model**

The Longuet-Higgins model, Longuet-Higgins (1975) and Longuet-Higgins (1983) provides a joint distribution of wave amplitude and period. The model is established on the assumption of a slowly varying amplitude process $A(t)$ and phase process $\Theta(t)$. The obtained joint density function is derived from the representation $X = A \cos(\hat{\omega} t + \Theta)$. $\hat{\omega} = m_1/m_2$ represents the point of gravity of the spectrum. $m_1$ and $m_2$ are spectral moments of order one and two. Further details are given in Madsen et al. (1986) and Ditlevsen (2002). The Longuet-Higgins density $f_{R,T}(r, t)$ is given as

$$f_{R,T}(r, t) = \frac{4\sqrt{2} \delta}{(1 + \delta) \sqrt{1 - \delta^2}} r^2 \exp(-r^2) \Phi \left[ \frac{r \delta \sqrt{2}}{\sqrt{1 - \delta^2} \left( 1 - \frac{1}{t} \right)} \right] \frac{1}{t^2}, \quad r \geq 0, t \geq 0 \quad (2.18)$$

and the corresponding marginal densities of $f_R(r)$ and $f_T(t)$ as

$$f_R(r) = \frac{4}{1 + \delta} r \exp(-r^2) \Phi \left( \frac{\delta \sqrt{2}}{\sqrt{1 - \delta^2}} r \right), \quad r \geq 0 \quad (2.19)$$
where $\delta = m_1 / \sqrt{m_0 m_2}$ is the spectral width parameter. $\Phi(\cdot)$ is the standard normal distribution function and $\varphi(\cdot)$ the corresponding probability density function, respectively. Figure 2.5 shows contour curves of the Longuet-Higgins density distribution, $f_{R,T}(t)$. The distribution has been plotted for the PM spectrum with $H_s = 4.0$ m and $T_z = 7.0$ sec. Additionally, are found results from simulations, where there is good agreement between the model and the simulated data (o). The zero crossing periods and corresponding maxima are collected from a sampled wave signal with $H_s = 4.0$ m and $T_z = 7.0$ sec.

![Graph showing the joint density of $f_{R,T}(r,t)$]

Figure 2.5: Contour plots of the Longuet-Higgins density, Eq. (2.18), using the PM spectrum with $H_s = 4.0$ m and $T_z = 7.0$ sec.

## 2.3 Response Spectrum

The total response in a given seaway may be described on the assumption that the vessel responds linearly to the wave action. The irregular response becomes a superposition of the response to all the regular wave components.

### 2.3.1 Transfer Function

The transfer function $\Phi_\delta(\omega \mid v, \beta)$ provides information on the response to a sinusoidal wave of a given frequency $\omega$ and a unit wave amplitude. The transfer function may either be obtained from calculations based on the equation of motion assuming linear theory or from traditional towing tank experiments. The transfer function is given for a specified ship's velocity $v$ and heading angle $\beta$. 
2.3.2 Linear Response

The response spectrum \( S_{\eta, \zeta}^e(\omega_e) \) of a given process \( \eta \) is obtained from the transfer function and a wave spectrum:

\[
S_{\eta, \zeta}^e(\omega_e | h_s, t_z, v, \beta) = |\Phi_{\eta}^e(\omega_e | v, \beta)|^2 S_{\zeta}(\omega_e | h_s, t_z, v, \beta)
\]  

(2.21)

where \( \omega_e \) is the encounter frequency and \( \beta \) is the wave heading. \( \Phi_{\eta}^e(\omega_e | v, \beta) \) is the transfer function and \( S_{\zeta}(\omega_e) \) the wave spectrum, both given as a function of the encounter frequency. There is a unique relation between the frequency of encounter, \( \omega_e \), the wave frequency, \( \omega \), the vessel’s velocity, \( v \), and the heading angle \( \beta \).

\[
\omega_e = \omega - \frac{\omega^2 v}{g} \cos(\beta)
\]  

(2.22)

It is important to note the difference between the wave spectrum experienced by the vessel, \( S_{\zeta}^e(\omega_e) \), and the estimated wave spectrum for the sea, \( S_{\zeta}(\omega) \). For head sea, these are related through

\[
S_{\zeta}^e(\omega_e) = S_{\zeta}(\omega) \frac{d\omega}{d\omega_e}
\]  

(2.23)

The \( n \)’th spectral moment, \( m_n \), of the process is given as

\[
m_n = \int_0^\infty \omega_e^n S_{\eta, \zeta}^e(\omega_e | h_s, t_z, v, \beta) d\omega_e
\]  

\[
= \int_0^\infty |\omega - \frac{\omega^2 v}{g} \cos(\beta)|^n S_{\eta, \zeta}(\omega | h_s, t_z, v, \beta) d\omega
\]  

(2.24)

The present formulation avoids problems with the frequencies in for example following seas. It is furthermore important to take care in calculation of higher order spectral moments due to the possibility of a divergent integral. For the PM spectrum, the spectral moments are divergent for \( n \geq 4 \).

2.4 Time Series Generation

Consider a zero-mean stationary Gaussian process \( Z(t) \) with the spectral density \( S(\omega) \). The process has the spectral representation:

\[
Z(t) = \int_0^\infty [\cos(\omega t) dV(\omega) + \sin(\omega t) dW(\omega)]
\]  

(2.25)

where \( V(\omega) \) and \( W(\omega) \) are zero-mean independent standard normal random variables with variance equal to \( E[dV^2(\omega)] = E[dW^2(\omega)] = S(\omega)d\omega \). It is not possible to generate samples of \( Z(t) \) from Eq. (2.25) because it involves an uncountable set of random variables in the process \( V(\omega) \) and \( W(\omega) \). Anyhow, simulations may be performed on approximations of \( Z(t) \) that involve a finite number of random variables. Two simulation techniques which are based on an approximative spectral representation of \( Z(t) \) is given in the following.
2.4 Time Series Generation

2.4.1 Irregular Wave Elevation

Linear superposition of long-crested sinusoidal wave components generates an irregular wave train. The ocean surface may be seen as statistically stationary within a given area and during a limited period of time. The wave elevation $Z(x, t)$, as given in many textbooks, Jensen (2001):

$$Z(x, t) = \sum_{n=1}^{N} c_{\zeta, n} \cos(k_n x - \omega_n t + \theta_{\zeta, n})$$  \hspace{1cm} (2.26)

where $N$ is large and the expected value of amplitude $c_{\zeta, n}$ is given by

$$c_{\zeta, n} = \sqrt{2S_\zeta(\omega_n) \Delta \omega}$$  \hspace{1cm} (2.27)

For the individual wave components, $k_n = \frac{\omega_n}{g}$ is the wave number obtained from the dispersion relation for deep water waves. $g$ is the acceleration of gravity. $\omega_n$ is the wave frequency and $\theta_{\zeta, n}$ is a uniformly distributed random phase angle between $0 - 2\pi$. $S_\zeta(\omega_n)$ represents the wave spectrum. A directional spectrum may additionally be introduced.

The process $Z(x, t)$ can moreover be modelled as a vector process

$$Z(x, t) = \sum_{n=1}^{N} a_{\zeta, n} [V_n \cos(k_n x - \omega_n t) + W_n \sin(k_n x - \omega_n t)]$$  \hspace{1cm} (2.28)

where $N$ is large and $a_{\zeta, n}$ are coefficients determined from the wave spectrum. $V_n$ and $W_n$ are independent standard normal random variables. The amplitude $a_{\zeta, n}$ is given as

$$a_{\zeta, n} = \sqrt{S_\zeta(\omega_n) \Delta \omega}$$  \hspace{1cm} (2.29)

The wave process $Z(x, t)$ is Gaussian distributed with zero mean, variance $\sigma_\zeta^2 = \int_0^{\infty} S_\zeta(\omega) d\omega$, continuous in time and differentiable.

The model from Eq. (2.26) is used extensively in many engineering applications because it depends on only $N$ random variables, the random phases $\theta_{\zeta, n}$, rather than on $2N$ random variables as for the model given in Eq. (2.28). The latter model is Gaussian since it is a linear transformation of Gaussian random variables.

The model from Eq. (2.26) does not follow a Gaussian distribution when $N < \infty$ because $\sum_{n=1}^{N} \sqrt{2S_\zeta(\omega_n) \Delta \omega}$ is finite.

2.4.2 Irregular Response

A linear irregular response $H(x, t)$ is obtained as a sum of the individual responses from each of the sinusoidal wave components. The linear response becomes

$$H(x, t) = \sum_{n=1}^{N} a_{\eta, n} [V_n \cos(k_n x - \omega_n t + \epsilon_{\eta, n}) + W_n \sin(k_n x - \omega_n t + \epsilon_{\eta, n})]$$  \hspace{1cm} (2.30)
where $N$ is large and $a_{\eta,n}$ is given as

$$a_{\eta,n} = \Phi_{\eta}(\omega_n \mid v, \beta)\sqrt{S_{\xi}(\omega_n) \Delta \omega}$$  \hspace{1cm} (2.31)

$\epsilon_{\eta,n}$ is the phase angle associated with the transfer function $\Phi_{\eta}(\omega_n)$. The above formulation of the irregular wave elevation or irregular response may also be formulated on the basis of the encounter frequency.

The above method is only to be used for linear calculations. Non-linear irregular responses are found by solving the hydrodynamic problem for each of the individual time steps.

### 2.5 Short-Term Response Statistics

Maxima in time series may be described by a number of different probability density functions. The Weibull distribution is one of the most widely used lifetime distributions in reliability engineering and often used to describe natural non-linear phenomena. A special case of the Weibull distribution is the Rayleigh distribution. The Rayleigh distribution is an exact distribution for the peaks in a narrow-banded Gaussian process.

#### 2.5.1 The Weibull Distribution

The three-parameter Weibull probability density distribution is given as, Ochi (1990):

$$f_R(r) = \frac{\beta}{\alpha} \left( \frac{r - \delta}{\alpha} \right)^{\beta-1} \exp \left[ - \left( \frac{r - \delta}{\alpha} \right)^\beta \right]$$  \hspace{1cm} (2.32)

and the cumulative distribution function as

$$F_R(r) = 1 - \exp \left[ - \left( \frac{r - \delta}{\alpha} \right)^\beta \right]$$  \hspace{1cm} (2.33)

for both Eq. (2.32) and Eq. (2.33) the location parameter $\delta$ is given as $\delta \leq r \leq \infty$. $\alpha$ is the scale parameter, and $\beta$ the shape parameter. The mean value and standard deviation are given as

$$\mu = \delta + \alpha \Gamma \left( 1 + \frac{1}{\beta} \right)$$  \hspace{1cm} (2.34)

$$\sigma = \alpha \sqrt{\Gamma \left( 1 + \frac{2}{\beta} \right) - \Gamma^2 \left( 1 + \frac{1}{\beta} \right)}$$  \hspace{1cm} (2.35)

where $\Gamma(\cdot)$ is the Gamma function.
2.5.2 The Rayleigh Distribution

A special case of the Weibull distribution is the Rayleigh distribution with $\beta = 2$. The Rayleigh probability function is given as, Ochi (1990):

$$f_R(r) = \frac{2r}{\alpha^2} \exp \left[ - \left( \frac{r - \delta}{\alpha} \right)^2 \right]$$

(2.36)

and the cumulative distribution as

$$F_R(r) = 1 - \exp \left[ - \left( \frac{r - \delta}{\alpha} \right)^2 \right]$$

(2.37)

The mean value and standard deviation are given as

$$\mu = \alpha \Gamma \left( \frac{3}{2} \right)$$

(2.38)

$$\sigma = \alpha \sqrt{1 - \frac{\pi}{4}}$$

(2.39)

2.5.3 The Generalised Extreme Value Distribution

The generalised extreme value distribution without upper bound, Ochi (1990), is given as

$$F_R(r) = \exp \left[ - \left( 1 - \left( \frac{\kappa(r - \alpha)}{\beta} \right)^{1/\kappa} \right) \right], \quad \kappa \neq 0$$

(2.40)

for $r > \beta/\kappa + < \alpha$ (when $\kappa \leq 0$) and $r < \alpha + \beta/\kappa$ (when $\kappa > 0$), where $\kappa$, $\beta$ are scaling parameters and $\alpha$ a location parameter relative to $r$. For $\kappa \equiv 0$ the Gumbel distribution is obtained, which is mostly used for extreme value predictions. The probability distribution function is given as

$$f_R(r) = \frac{1}{\beta} \exp \left[ - \left( \frac{r - \alpha}{\beta} \right) \right] \exp \left[ - \exp \left( - \left( \frac{r - \alpha}{\beta} \right) \right) \right]$$

(2.41)

and the cumulative distribution as

$$F_R(r) = \exp \left[ - \exp \left( - \left( \frac{r - \alpha}{\beta} \right) \right) \right]$$

(2.42)

The mean value is given as

$$\mu = \alpha + C \beta$$

(2.43)

where $C$ is Euler’s constant, $C \simeq 0.577215$, and the standard deviation is given as

$$\sigma = \beta \sqrt{\frac{\pi^2}{6}}$$

(2.44)
2.5.4 Peak Distribution

Given a stationary zero-mean Gaussian wave process, the response of a linear system is also a Gaussian zero-mean process. For narrow-banded processes the peak distribution follows the Rayleigh distribution as given in Eq. (2.36). The peak distribution changes according to the response process, and the narrow-banded assumption does not always hold. The amplitudes of a stationary zero-mean Gaussian distributed response process are described in general by the Rice distribution, e.g. Cramer & Leadbetter (1967).

\[
f_R(r) = \frac{\epsilon}{2\pi} \exp \left( -\frac{1}{2} \left( \frac{u}{\epsilon} \right)^2 \right) + \sqrt{1 - \epsilon^2} u \exp \left( -\frac{1}{2} u^2 \right) \Phi \left( \frac{\sqrt{1 - \epsilon^2}}{\epsilon} u \right) \frac{1}{\sqrt{m_0}} (2.45)
\]

where \( \Phi(\cdot) \) is the normal distribution and \( u \) is given as

\[
u \equiv \frac{r}{\sqrt{m_0}} \tag{2.46}
\]

\( \epsilon \) is the bandwidth parameter given as

\[
\epsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} \tag{2.47}
\]

where \( m_n \) is the spectral moments of order \( n \). Figure 2.6 illustrates the probability distribution for selected values of \( \epsilon \).

![Figure 2.6: Probability density function of individual maxima for different bandwidths. Used bandwidths are \( \epsilon = 0, 0.25, 0.5, 0.75 \) and 1.0.](image)

Figure 2.6 shows that the Rice distribution is an interpolation between the broad-banded process with \( \epsilon = 1 \) (the normal distribution) and the narrow-banded process with \( \epsilon = 0 \) (the Rayleigh distribution). It is seen that both of these distributions are a function of the first four spectral moments of the response spectrum.
2.6 Extreme Value Statistics

Upcrossing and Peak Rates

For narrow-banded response processes, upcrossings through a mean level is always followed by only one peak. The probability that the peak response exceeds a given level \( r \) becomes equal to the ratio between upcrossing of the level \( r \) and the mean level \( \mu_r \):

\[
P(R > r) = \frac{\nu(r)}{\nu(\mu_r)}
\]

which leads to the probability distribution \( F_R(r) \) of the peaks:

\[
F_R(r) = P(R \leq r) = 1 - \frac{\nu(r)}{\nu(\mu_r)}
\]

The number of peaks within each period of time is given by the spectral moments. The peak rate becomes

\[
\nu_p = \frac{1}{2\pi} \sqrt{\frac{m_4}{m_2}}
\]

For narrow-banded processes, the peak rate may be approximated by the zero-crossing rate, now

\[
\nu_p = \nu_0 = \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}}
\]

2.6 Extreme Value Statistics

The largest value of a random variable that a given structure will experience during a selected period \([0, T]\) or within a given number of observations is called the extreme value. The probability distribution \( F_{T_{\text{max}}}(r) \) for the maximum value may be expressed as

\[
F_{T_{\text{max}}}(r) = P(R_1 < r, R_2 < r, ..., R_N < r)
\]

where \( R_i, \ i = 1, 2, ..., N \) are the extreme peaks within the period \([0, T]\). The number of expected peaks are given through

\[
N = \nu_p T
\]

It follows hereafter that the extreme value distribution of \( N \) independent and identically distributed variables becomes

\[
F_{T_{\text{max}}}(r) \equiv F_{R_{\text{max}}}(r) = [F_R(r)]^N
\]

The corresponding probability density function is obtained as

\[
f_{R_{\text{max}}}(r) = \frac{d[F_R(r)]^N}{dr} = NF_R(r)^{(N-1)}f_R(r)
\]
The Gumbel distribution is widely used for extreme value predictions. The Generalized Extreme Value (GEV) distribution states one important theorem that the maximum of a sequence of observations under very general conditions will be one of three types: Type I Gumbel (light tail), Type II Frechet (heavy tail), and Type III Weibull (bounded tail).

The fundamental assumption within the above derivations is that the peaks are considered statistically independent, and that they hereby may be rearranged in ascending order. Thereby the name order statistics.

The Weibull Distribution

Given the Weibull distribution from Eq. (2.33), the expected largest value, $R_{\text{max}}$, the probable extreme value, $r_p$, and the design extreme value, e.g. Ochi (1990) and Farnes (1990) may be derived as

$$E[R_{\text{max}}] = \delta + \alpha \left[ (\ln N)^{1/\beta} + 0.577215 \frac{(\ln N)^{(1/\beta)}}{\beta} \right]$$

$$r_p = \alpha [\ln(N)]^{1/\beta} + \delta$$

$$r_{\xi,\text{design}} = \alpha \left[ \ln \left( \frac{N}{\xi} \right) \right]^{1/\beta} + \delta, \text{ with } \hat{\xi} = 1 - \xi$$

where $N$ is the number of peaks within a selected time interval $[0, T]$. $\alpha$, $\beta$ and $\delta$ are the fitted Weibull coefficients. $r_{\xi,\text{design}}$ represents an extreme value corresponding to the probability level $\hat{\xi}$ of being exceeded or a probability of $\hat{\xi}$ being below.

The expected largest value represents the value which corresponds to the center of gravity from the probability function. The probable extreme value represents the value corresponding to the peak of the probability density distribution.

The Rayleigh Distribution

Given the Rayleigh distribution from Eq. (2.37), the expected largest value, the probable extreme value and the design extreme value, $r_{\xi,\text{design}}$, e.g. Ochi (1990) may be derived as

$$E[R_{\text{max}}] = \sqrt{2m_0} \left[ \sqrt{\ln N}^{0.2886} \right] \ln N$$

$$r_p = \sqrt{2m_0 \ln N}$$

$$r_{\xi,\text{design}} = \sqrt{2m_0 \ln \frac{N}{\xi}}, \text{ with } \hat{\xi} = 1 - \xi$$
2.6 Extreme Value Statistics

2.6.1 Extreme Value Distributions

An alternative to order statistics is a procedure which is based on Poisson processes, here it is assumed that upcrossings of any given level \( r \) are statistical independent. In the design phase, it is important to determine the first crossing of a critical level \( r \) within a selected time window \([0, T]\) of a given process, \( Z(t) \). The cumulative density function for crossing a selected level \( r \) for a stationary Gaussian process is found as

\[
F_{R_{\text{max}}}(r) = \exp[-\nu_Z(r)T]
\]

(2.62)

where \( T \) is the selected period and \( \nu_Z(r) \) is the mean crossing rate of the selected level \( r \) given as

\[
\nu_Z(r) = \nu_0 \exp\left(-\frac{r^2}{2m_0}\right)
\]

(2.63)

where \( \nu_0 \) is the zero-crossing rate and \( m_0 \) the spectral moment of order zero. The formulation given in Eq. (2.63) is related to a narrow-banded process where the peaks are Rayleigh distributed.

For narrow-banded processes, the upcrossings tend to concentrate in clumps. This will to some extend violate the assumption of independence of the individual outcrossing of the process \( Z(t) \). This may be accounted for by considering a lower or upper envelope process. Cramer & Leadbetter (1967) provided the upcrossing rate for an envelope process

\[
\nu_R(r) = \sqrt{2\pi \epsilon} \frac{r}{\sqrt{m_0}} \nu_0 \exp\left(-\frac{r^2}{2m_0}\right)
\]

(2.64)

A qualified excursion is categorised as an upcrossing through a critical level of the original process. This modification is needed as the envelope process may have excursions above a critical level without any upcrossing through the original process. The opposite way around, the excursions are said to be empty, Ditlevsen & Lindgren (1988).

It becomes now interesting to determined the long-run fraction of qualified envelope excursions of the narrow banded. An estimate is given by Vanmarcke (1975)

\[
r_v(r) = \frac{\nu_Z(r)}{\nu_R(r)} \left[1 - \exp\left(-\frac{\nu_R(r)}{\nu_Z(r)}\right)\right]
\]

(2.65)

where \( r_v(r) \) is found to be kind of an interpolation between \( \nu_R(r) \) and \( \nu_Z(r) \). Additionally studies are made by Ditlevsen (1971) and Ditlevsen & Lindgren (1988). They presented an expression for an extreme value distribution for the lifetime of a structure that applies to
each of the individual short-term periods. For ship responses the period \( T \) should be taken as the duration of a sea state. The cumulative density function of the maximum value of a stationary ergodic Gaussian process becomes

\[
F_{R_{\text{max}}}(r) = \left[1 - \exp\left(-\frac{r^2}{2m_0}\right)\right] \exp\left[-\frac{r_v(r) \nu_R(r) T}{1 - \exp(-\frac{r^2}{2m_0})}\right]
\]  

(2.66)

where \( m_0 \) is the spectral moment of order zero, and \( \nu_R(r) \) the upcrossing rate of an envelope process as given in Eq. (2.64). \( r_v(r) \) is an estimate of the long-run fraction of qualified excursions.

### 2.7 Long-Term Response Statistics

Estimation of lifetime extreme value ship responses is based on a long-term response analysis. The extreme response becomes a function of several parameters, where the operational profile of the vessel, the ocean environment, the current loading conditions, and the decisions made by the captain on board the vessel contribute to each of the individual short-term events.

#### 2.7.1 Long-Term Response

By a summation of the individual short-term probabilities of exceedance in all potential combinations of sea states and operational profiles, the long-term peak distribution of maximum responses is obtained:

\[
F_{LT}(r) = \int_{H_s} \int_{T_z} \int_{V} \int_{B} F_R(r \mid h_s, t_z, v, \beta) f_{H_s, T_z}(h_s, t_z) \times f_{V,B}(v, \beta \mid h_s, t_z) W_P(h_s, t_z, v, \beta) \, d\beta \, dv \, dt \, dh_s
\]  

(2.67)

where \( h_s \) and \( t_z \) are the significant wave height and zero-upcrossing period. \( v \) and \( \beta \) are the forward speed and wave heading. \( F_R(r \mid h_s, t_z, v, \beta) \) represents the individual cumulative short-term peak response distributions. \( f_{H_s, T_z}(h_s, t_z) \) is the long-term joint probability distribution of \( H_s \) and \( T_z \), which are normally obtained through a scatter diagram. An example is given later on in Table 2.1. \( f_{V,B}(v, \beta \mid h_s, t_z) \) accounts for the effect of manoeuvring in bad weather with respect to the forward velocity and wave heading. Finally, \( W_P(h_s, t_z, v, \beta) \) represents a weight factor of the average number of peaks per unit time in a sea state.

Given that the response distributions of the individual short-term events are Gaussian and fairly narrow-banded, the cumulative probability distributions may be described by the Rayleigh distribution from Eq. (2.37).
2.7.2 Operation of the Vessel and Effect on Long-Term Responses

The operation of a vessel has an important impact on the long-term wave-induced response. The operational profile is more or less completely controlled by the persons on the bridge of the vessel. Especially in rough seas where their decision-making is of high importance. The captain has firstly to make decisions on the route to be sailed given the available weather forecasts. The captain may have to choose between staying in the harbour or selecting a different route. If the vessel is presently located in heavy seas, the vessel may suffer from green water on deck, bottom or bow flare slamming, large roll motions or large vertical and transverse accelerations, which are all effects introducing severe loads on the hull girder. To reduce these loads, the captain has to change course and/or forward speed, which after an acceptable response level is hopefully achieved. For large vessels which are operated during night in heavy seas, it is often difficult to observe green water on deck and slamming events. Several vessels are therefore additionally equipped with an on board system giving the captain indications about the current sea state.

The NordForsk association, NordForsk (1987), has provided reference levels for operational cases where voluntary speed reduction is often introduced. The criteria for speed reduction are all made by the persons operating the vessel and related both to experience and a physical comfort level on board the vessel. Voluntary speed loss is linked to occurrence of green water on deck, slamming, large roll amplitudes, and acceleration levels in different locations on the vessel. The above criteria are moreover highly correlated with the probability of being seasick.

Inclusion of operability limitations in the long-term analysis implies that the probability distributions \( f_V(v \mid h_s, t_z) \) and \( f_B(\beta \mid h_s, t_z) \) should be established. Given that the forward speed and wave heading are independent variables, the distribution function may be rewritten as

\[
f_{V,B}(v, \beta \mid h_s, t_z) \approx f_V(v \mid h_s, t_z)f_B(\beta \mid h_s, t_z)
\]

As indicated in Eq. (2.68), the distribution is established with information on the zero-upcrossing period \( T_z \). The effect of the zero-upcrossing period is considered less important for large vessels. Other models by for example Friis-Hansen (1994) point towards that the significant wave height is the most dominant parameter on forward speed and heading relative to the waves. Eq. (2.68) may therefore be rewritten to

\[
f_{V,B}(v, \beta \mid h_s, t_z) \approx f_{V,B}(v, \beta \mid h_s) = f_V(v \mid h_s)f_B(\beta \mid h_s)
\]

By introduction of the new scatter diagram \( \tilde{f}_{H_s,T_z}(h_s, t_z) \) accounting for weather routing could be introduced, after which Eq. (2.67) may be rewritten to

\[
F_{LT}(r) = \int_{H_s} \int_{T_z} \int_{V} \int_{B} F_R(r \mid h_s, t_z, v, \beta) \tilde{f}_{H_s,T_z}(h_s, t_z) \times f_V(v \mid h_s, t_z)f_B(\beta \mid h_s, t_z)W_P(h_s, t_z, v, \beta) d\beta dv dt_z dh_s
\]
and given that the effect of the zero-upcrossing period is negligible in manoeuvring in bad weather, the Eq. (2.70) reduces to

\[
F_{LT}(r) = \int_{H_s} \int_{T_z} \int_{V} \int_{B} F_R(r \mid h_s, t_z, v, \beta) \hat{f}_{H_s,T_z}(h_s, t_z) \times f_V(v \mid h_s) f_B(\beta \mid h_s) W_P(h_s, t_z, v, \beta) \, d\beta \, dv \, dt \, dh_s \tag{2.71}
\]

Other models of manoeuvring philosophy are available, by for example that by Cramer & Friis-Hansen (1992) or Friis-Hansen (1994).

### 2.7.3 Extreme Value Analysis

An extreme value analysis should be completed under consideration of the wave-induced responses, the operational profile, the duration and occurrence of each sea state. Focus should also be put on the actual loading condition, i.e. the still water response.

The extreme value distribution for a selected response on a voyage is identical to the first upcrossing by a stationary stochastic process of a constant level \( r \).

Within each of the short-term sea states, the distribution function of maxima \( F_{R_{\text{max}}}(r \mid h_s, t_z) \) should be established.

\[
\nu_{\text{voyage}}(r \mid h_s, t_z) = \frac{1}{t_{\text{sea state}}} \left[ 1 - F_{R_{\text{max}}}(r \mid h_s, t_z) \right] \tag{2.72}
\]

The individual sea states are considered independent with the intensity \( \frac{1}{t_{\text{sea state}}} \) and assumed to arrive as points in a Poisson process. \( t_{\text{sea state}} \) is the duration of the sea state. The model from Eq. (2.72) is conditional on the given sea state, and may therefore be extended to include the entire operational profile.

The cumulative distribution of the maximum wave-induced response for a voyage of a given period \([0, T]\) is hereby given as:

\[
F_{R_{\text{ext}}}(r) = \exp[-\nu_{\text{voyage}}(r) T] \tag{2.73}
\]

where the upcrossing rate \( \nu_{\text{voyage}}(r) \) is given as

\[
\nu_{\text{voyage}}(r) = \int_{H_s} \int_{T_z} \int_{V} \int_{B} \frac{1}{t_{\text{sea state}}} \left( 1 - F_{R_{\text{max}}}(r \mid h_s, t_z, v, \beta) \hat{f}_{H_s,T_z}(h_s, t_z) \times f_V(v \mid h_s) f_B(\beta \mid h_s) W_P(h_s, t_z, v, \beta) \right) \, d\beta \, dv \, dt \, dh_s \tag{2.74}
\]

The effect of manoeuvring in bad weather should additionally be introduced as discussed in Section 2.7.2.
2.7 Long-Term Response Statistics

2.7.4 Long-Term Wave Climate

Section 2.2.3 discussed the short-term description of the sea, where it is assumed that the significant wave height and the zero-upcrossing period are constant throughout the entire period or sea state. The parameters describing the sea will vary in the long run, allowing a long-term description to be introduced. The long-term variations are collected in a scatter diagram as illustrated in Table 2.1, where the joint distribution of significant wave height, $H_s$, and mean zero-crossing period, $T_z$, is shown. The observations from the scatter diagram have been provided by Det Norske Veritas (DNV). The values of $H_s$ are given from 1.0 to 15.0 m with length intervals of 1.0 m. The values of $T_z$ are given from 5.0 to 15 sec with length intervals of 1.0 sec.

Table 2.1: Joint frequency of significant wave height and mean zero-crossing period. Data provided by DNV and considered representative of the North Sea.

<table>
<thead>
<tr>
<th>$H_s$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
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<td>0</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
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<td>356</td>
<td>62</td>
<td>12</td>
<td>2</td>
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<td>0</td>
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<td>318</td>
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<td>25</td>
<td>7</td>
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<td>1</td>
<td>0</td>
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<td>27</td>
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<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

$T_z$

By summing the numbers in the columns of Table 2.1, the sample of $H_s$ is obtained. Figure 2.7 (left) shows the cumulative distribution of the $H_s$ data together with the fitted distribution. The current approach is given by Ditlevsen (2002). Other models are additionally available. The data is fitted to a normal distribution function $\Phi\left(\frac{h_s - \alpha}{\beta}\right)$ with a truncation point $\alpha$. $\beta$ is the standard deviation

$$F_{H_s}(h_s) = 2\Phi\left(\frac{h_s - \alpha}{\beta}\right) - 1, \text{ with}$$

$$\alpha = 0.46 \text{ m and } \beta = 2.86 \text{ m}$$

(2.75)
By summing the observations in the rows of Table 2.1, the sample of $T_z$ is obtained. The data is fitted to a log-normal distribution function with mean $\mu$ and standard deviation $\sigma$:

$$ F_{T_z}(t_z) = \Phi\left(\frac{\log(t_z) - \mu}{\sigma}\right), \quad \text{with} \quad \mu = \log(8.7) \text{ sec and } \sigma = 0.18 \text{ sec} $$

(2.76)

From Figure 2.7 it is seen that both the marginal distribution models $F_{H_s}(h_s)$ and $F_{T_z}(t_z)$ fit well to the empirical distributions of the data. Alternatively, the three-parameter Weibull distribution can be applied to estimate the marginal distribution of $H_s$. An other possibility would be to establish the conditional distribution of the zero-upcrossing period, $T_z$ given the significant wave height, $H_s$. The three-parameter Weibull distribution are normally found to fit the conditional distribution $F_{T_z}(t_z|h_s)$ well.

Figure 2.7: Fitted distribution functions of significant wave height $H_s$ and zero-upcrossing period $T_z$ compared to the corresponding empirical distribution functions of one year of data from the North Sea.

Formulations for the joint distribution with on application of the given marginal distributions are present. One is the Nataf model for formulating joint density of vector data on the basis of given marginal distributions of the single components of the random vector, Ditlevsen & Madsen (1996) and Ditlevsen (2002). The joint distribution model of the two random variables $H_s$ and $T_z$ is obtained by setting up the marginal transformations, by which each of the marginal distributions of the data transform into normal distributions, and then applying these marginal transformations to the data. The pairs of transformed data with marginal normal distributions are next assumed to have a bivariate normal distribution with the correlation coefficient $\rho$ estimated from the transformed data. Back transformation of the obtained bivariate normal density function gives a bivariate density model for the original data pairs which is the Nataf model for formulating joint density function. Experience shows that the Nataf model often gives well fitted joint distribution model, Ditlevsen (2002).
Chapter 3

Conditioning on Ship Responses

3.1 Introduction

Estimation of extreme non-linear response statistics may be obtained by performing time-consuming experiments or numerical simulations in an irregular seaway. A significant amount of the data obtained does not provide any specific information on the interesting extreme events, which are located at the tail of the response distribution. Considerable computational or experimental costs may be saved by introducing a constrained process, whereafter the tail of the response distribution may be predicted more accurately. Different techniques for conditioning on a large crest in a stationary process are available and described later in the present chapter. Conditioning of ship responses allows the underlying wave profile to be generated, and by assuming that the derived incident wave is obtained on the basis of a linear extreme response and that the linear approach is a good predictor of the non-linear extreme, a very realistic non-linear extreme response may be obtained by for example introducing the derived wave profile in a non-linear time domain hydrodynamic program.

It is well known that the response of offshore structures are highly correlated to the instantaneous wave height and dominated by transient effects. This is not the case for large ship structures where the maximum response is defined not only by the wave amplitude, but also by the wavelength and transient effects. Torhaug (1996) studied wave profiles that resulted in maximum linear responses. In his study, he identified what he termed “the critical wave episodes” as a series of approximately three peaks. In this chapter existing models for time conditioning of wave sequences are presented. A model named the “Conditional Random Response Waves” model (CRRW) is additionally formulated. The model takes into account the shape of the vessel considered. It is straightforward to implement in most commercial software allowing a fast estimation of non-linear extreme response statistics.


3.2 Most Likely Wave Models

During simulation of a stochastic process, large amplitude responses will occur. If a large number of these wave sequences of the same amplitude are collected and averaged, the most likely profile around a large crest will be found. Figure 3.1 illustrates this idea, here the results of a brute force simulation are shown. Each time a response crest of a prescribed amplitude and time occurred, the response sequence was taken out, and finally an average was made. By considering the peak response given a response amplitude $a$, the problem is now to predict the most likely response value at a given time step $\tau$ away from the crest. Several models, both first and second order, are presented in the literature, where a basic assumption for solving the problem is to introduce the probabilistic information on the response with the definition of the conditional probabilities.

![Figure 3.1](image-url)  

Figure 3.1: The black curves represent the results of the brute force simulation, where the red curve is simply the mean.

3.2.1 The New Wave Profile, Tromans et al.

In the past decade there has been a continuous interest in how limited time series information can be used in an optimal way to predict extreme value ship response statistics. To reduce the required number and length of simulated time series, Tromans et al. (1991) introduced what they termed the “New Wave”. In alternative studies this wave profile was termed as the most likely wave profile. The wave profile $\zeta(t)$ is established conditional on a given wave crest $a$ at time $t_0 = 0$ and is given as

$$
\zeta(\tau)|_a \equiv \hat{E}[Z(t_0 + \tau) \mid Z(t_0) = a, \dot{Z}(t_0) = 0] = a\rho(\tau) \tag{3.1}
$$

where $\hat{E}[]$ is the conditional mean of the process $Z(t)$. $\dot{Z}(t)$ is the first derivative of the process. $\rho(\tau)$ is the autocorrelation function of $Z(t)$, with $t = t_0 + \tau$. The conditional crest passes at time $t_0$ where $\dot{Z}(t_0) = 0$. 
The Autocorrelation Function

The ensemble average of the product of a stochastic process at times $t_0$ and $t_1$ is defined as the autocorrelation function denoted by $\rho(t_0, t_1)$.

$$\rho(t_0, t_1) = E[Z(t_0)Z(t_1)]$$

$$= \frac{1}{N} \sum_{n=1}^{N} Z_n(t_0) Z_n(t_1)$$

(3.2)

where $Z_n(t)$ represents the $n$’th records within the entire ensemble. For a random process $Z(t)$, as given in Eq. (2.26), which consists of many sinusoidal components, the autocorrelation function is given as

$$\rho(\tau) = E \left[ \sum_{n=1}^{N} c_{\zeta,n} \cos \epsilon_n \sum_{j=1}^{N} c_{\zeta,j} \cos(\epsilon_j - \omega_j \tau) \right]$$

$$= \sum_{n=1}^{N} c_{\zeta,n}^2 E \left[ \cos \epsilon_n \cos(\epsilon_n - \omega_n \tau) \right]$$

$$= \frac{1}{2} \sum_{n=1}^{N} c_{\zeta,n}^2 \cos(\omega_n \tau)$$

(3.3)

where $c_{\zeta,n} = c_{\zeta,j}$ is given in Eq. (2.27). An example of the autocorrelation function is shown in Figure 3.2.

![Figure 3.2: The autocorrelation function, $\rho(\tau)$.](image)

The “New Wave” Model

The ocean surface may be considered to be statistically stationary in a given area and over a short period of time (typically 1 to 3 hours). The wave elevation $Z(x, y, t)$ may be modelled
as the sum of a large number \( N \) of sinusoidal wave components.

\[
Z(x, y, t) = \sum_{n=1}^{N} d_n \cos(k_n x \cos \beta_n + k_n y \sin \beta_n - \omega_n t + \epsilon_n) \tag{3.4}
\]

with the expected value of amplitude \( d_n \) given as

\[
d_n = \sqrt{2D_\beta(\beta_n)S_\xi(\omega_n)\Delta \beta \Delta \omega} \tag{3.5}
\]

\( k_n \) is the wave number from the dispersion relation, \( \omega_n \) is the wave frequency, \( \beta_n \) is the direction relative to the mean wave direction and \( \epsilon_n \) is a uniformly distributed phase angle between \( 0 - 2\pi \) independent of each other. \( S_\xi(\omega) \) represents the wave spectrum and \( D_\beta(\beta_n) \) the spreading function as given in Section 2.2.4. The process is Gaussian distributed for \( N \to \infty \) with zero mean, variance \( \sigma_\xi^2 \), continuous in time and differentiable.

The expected mean variation of the stochastic process \( Z(t) \) in the vicinity of a large crest of height \( a \) at time \( t_0 \) is given by Eq. (3.1). The conditional mean process is derived from the conditional probability density function of \( Z(t) \), given that \( Z(t_0) = a \) and \( \dot{Z}(t_0) = 0 \). It follows that

\[
f(Z(t_0 + \tau) | Z(t_0) = a, \dot{Z}(t_0) = 0) = \frac{f(Z(t_0 + \tau), Z(t_0), \dot{Z}(t_0))}{f(Z(t), \dot{Z}(t_0))} \tag{3.6}
\]

The nominator and denominator of the conditional probability density function can be found in Tromans et al. (1991). Hence

\[
f(Z(t_0 + \tau) | Z(t_0) = a, \dot{Z}(t_0) = 0) = \frac{1}{u(\tau)\sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{Z(t_0 + \tau) - a \rho(\tau)}{u(\tau)} \right)^2 \right] \tag{3.7}
\]

which is normal distributed. \( \rho(\tau) \) is the autocorrelation function from Eq. (3.3). \( u(\tau) \) is given as

\[
u(\tau) = \sigma_\xi \sqrt{1 - \rho^2(\tau) - \dot{\rho}^2(\tau) \frac{m_0}{m_1}} \tag{3.8}
\]

The conditional mean and its standard deviation are found as

\[
\hat{E} \left[ Z(t + \tau) | Z(t) = a, \dot{Z}(t) = 0 \right] = a \rho(\tau) \tag{3.9}
\]

\[
\hat{E} \left[ [Z(t + \tau) - a \rho(\tau)]^2 | Z(t) = a, \dot{Z}(t) = 0 \right] = \sigma_\xi^2 \left[ 1 - \rho^2(\tau) - \dot{\rho}^2(\tau) \frac{m_0}{m_1} \right] \tag{3.10}
\]

where \( m_n \) is the spectral moment of order \( n \). It is seen that the conditional mean around a peak or trough reduces to the autocorrelation function scaled by the crest \( a \). Eqs. (3.7), (3.9) and (3.10) show that for increasing values of \( a \), the expected value \( a \rho(\tau) \) of the wave elevation at time \( t_0 \) also increases, while the standard deviation remains constant. It is further seen that the expected value \( a \rho(\tau) \to 0 \) for \( \tau \to \infty \).
Conditioning on the curvature, $\ddot{Z}(t) < 0$ could additionally be interesting to assume a local positive peak at time $\tau = 0$. A formulation to this problem is given by Lindgren (1970). The most likely wave profile conditioned on a peak and curvature at $t_0$ becomes

$$
\hat{E}[Z(t_0 + \tau) \mid Z(t_0) = a, \dot{Z}(t_0) = 0, \ddot{Z}(t_0) = -\alpha] = \frac{a(\rho(\tau)m_0m_4 + \dot{\rho}(\tau)m_0m_2 - \frac{\alpha}{a}(\rho(\tau)m_0m_2 + \ddot{\rho}(\tau)m_0^2))}{m_0m_4 - m_2^2}
$$

(3.11)

where $\rho(\tau)$ is the autocorrelation function from Eq. (3.3), $\alpha = a\overline{\omega}$ and $\overline{\omega}$ is the mean frequency. The fourth spectra moment is often difficult to obtain for wave spectra as $m_4 \to \infty$ for the majority part of the analytical spectra. For the limiting case where $m_4 \to \infty$, the conditional mean wave profile, Eq. (3.11), reduces to the solution by Tromans et al. (1991).

**Embedding the New Wave Model in a Stochastic Wave Profile**

Taylor et al. (1995) showed how the “New Wave” embedded in a stochastic seaway could be used to estimate the extreme response of a Jack-Up in a severe sea-state. For dynamically responding structures, the extreme response does not always correspond to the extreme input surface elevation, and the extreme response might very well correspond to a combination of the local extreme wave with unfavourable background structural memory effects. The result of their work was a technique for estimating the probability distribution of extreme response in a given period using short constrained simulations. Furthermore, it was found that extreme responses on the basis of the isolated “New Wave” itself (that is the conditional mean profile), would underestimate the peak response as compared to a 50 % fractile obtained from a fully derived peak response distribution.

The method by Taylor et al. (1995) states that a random wave surface has an elevation $a_0$ and slope $\dot{a}_0$ at time $t_0$. From this point, $a_0 \times$ New Wave $(a_0\rho(t))$ and $a_0 \times$ New Wave slope $(\dot{a}_0\dot{\rho}(t))$ are subtracted from the original signal to make the slope zero at $t_0$. The next step is to create the required wave elevation $a$ by adding back $a \times$ New Wave $(a\rho(t))$ to the modified signal. By this procedure an extreme crest of height $a$ is established and is virtually indistinguishable from any other large wave crests of height $a$ present in the original wave process. This is an approximation and a probabilistically correct analysis based on Slepian processes can be established. The Slepian model from Eq. (3.15) is a generalisation of the approach formulated by Taylor et al. (1995). However, for large crest values and for $N \to \infty$ of this procedure, high accuracy is obtained.

By adding $\Delta a$ given by Eq. (3.12) to the wave process as defined in Eq. (2.26), the wave process is obtained conditional on a peak of height $a$ at $t_0$, Tromans et al. (1991):

$$
\Delta a(x, \tau) = \frac{a_0 - a}{2} \sum_{n=1}^{N} c_{n}^2 \cos(k_n x - \omega_n t) - \frac{\dot{a}_0}{2\lambda^2} \sum_{n=1}^{N} c_{n}^2 \sin(k_n x - \omega_n t)
$$

(3.12)
where \( c_{i,n} \) is given in Eq. (2.27) and \( \lambda^2 = \frac{1}{2} \sum_{n=1}^{N} c_{i,n}^2 \). An example of the constrained and original signal is shown in Figure 3.3. It is seen how the two signals are significantly different at \( t_0 \), and how they merge again.

3.2.2 The Most Likely Wave Profile, Friis-Hansen & Nielsen

It is well known that large ocean waves tend to have corresponding long frequencies. The formulation by Tromans et al. (1991) does not take this phenomenon into account. However, Friis-Hansen & Nielsen (1995) derived an extension to the NewWave model, where the approach is extended to include conditioning on the random instantaneous wave frequency as well. The wave process may be written as

\[
\zeta(t)_{a,\omega} \equiv \hat{E}[Z(t) \mid Z(0) = a, \dot{Z}(0) = 0, \Omega = \omega]
\]  

(3.13)

where \( \hat{E}[\cdot] \) is the conditional mean of the process \( Z(t) \). Here \( \Omega \equiv \dot{Z}(0)/Z(0) \), where \( \dot{Z}(t) \) is the Hilbert transform and \( \hat{Z}(t) \) its first derivative.

The Most Likely Wave (MLW) can hence be established from a vector process \( Z(t) = (Z(t), \dot{Z}(t)) \) as

\[
\hat{E}[Z(t) \mid (Z(0), \dot{Z}(0)) = (a, 0), (\dot{Z}(0), \ddot{Z}(0)) = (0, \omega a)]
\]  

(3.14)

The model is obtained by use of Slepian model processes, Ditlevsen (1985).

Slepian Model Process

The Slepian model process is obtained by conditioning a Gaussian vector process \( V(t) \) on a set of vectors \( Y \). The model is briefly outlined below, with a detailed description given by for example Ditlevsen (1985). The model is obtained as a linear regression of \( V(t) \) on \( Y \).

\[
[V(t) \mid Y] = \hat{E}[V(t) \mid Y] + \Delta(t)
\]  

(3.15)
where \( \hat{E}[V(t)|Y] \) represents the conditional mean vector function, which is given as, Ditlevsen (1985)
\[
\hat{E}[V(t)|Y] = E[V] + \text{Cov}[V, Y^T]\text{Cov}[Y, Y^T]^{-1}(Y - E[Y])
\]
(3.16)
where \( E[\cdot] \) is the mean and \( \text{Cov}[\cdot] \) is the covariance. The second term, \( \Delta(t) \), of Eq. (3.15) is the residual vector process, which is a zero-mean Gaussian process, with the covariance matrix function given as, Ditlevsen (1985)
\[
\text{Cov}[^\Delta(t_1), ^\Delta^T(t_2)] = \text{Cov}[V(t_1), V^T(t_2)] - \\
\text{Cov}[V(t_1), Y^T(t_2)]\text{Cov}[Y, Y^T]^{-1}\text{Cov}[Y, V^T(t_2)]
\]
(3.17)
For a Gaussian process the covariance matrix of the residual process matches with the conditional covariance matrix, i.e. \( \text{Cov}[^\Delta(t_1), ^\Delta^T(t_2)] = \text{Cov}[V(t_1), V^T(t_2)|Y] \).

The Wave Process

Friis-Hansen & Nielsen (1995) derived the Most Likely Wave (MLW) profile conditioning on both the wave amplitude, \( a \), and the instantaneous wave frequency, \( \omega \), as given in Eq. (3.13). Mathematical details are given in Appendix D, these may be skipped for now as a similar derivation is given in Section 3.6.1. Anyhow, the mean wave profile conditional on both the amplitude and the instantaneous wave frequency reduces to:
\[
\zeta(t)_{a,\omega} \equiv \hat{E}[Z(t) | Z(0) = a, \dot{Z}(0) = 0, \Omega = \omega] = \\
\frac{a}{m_0m_2 - m_1^2} \sum_{n=1}^{N} a_{\zeta,n}^2 [(m_2 - m_1\omega_n) + \omega(m_0\omega_n - m_1)] \cos(\omega_n t)
\]
(3.18)
where \( m_0, m_1 \) and \( m_2 \) are the spectral moments of order zero, one and two. The coefficient \( a_{\zeta,n} \) is given as
\[
a_{\zeta,n} = \sqrt{S_{\zeta}(\omega_n)}\Delta\omega
\]
(3.19)
The conditional variance is similarly found to be
\[
E\left[ (Z(t) - \hat{E}[Z(t) | Z(0) = a, \dot{Z}(0) = 0, \Omega = \omega])^2 | Z(0) = a, \dot{Z}(0) = 0, \Omega = \omega \right] = \\
\frac{m_0}{m_0m_2 - m_1^2} \left[ - m_2^2 + m_0m_2(1 - c(t)^2 - r(t)^2) + \\
2m_0m_1(c(t)\dot{r}(t) + \dot{c}(t)r(t)) - m_0^2(\dot{c}(t)^2 + \dot{r}(t)^2) \right]
\]
(3.20)
with \( c(t) \) and \( r(t) \) given as
\[
c(t) = \frac{1}{m_0} \int_0^\infty S_{\zeta}(\omega) \cos(\omega t) d\omega
\]
(3.21)
\[
r(t) = \frac{1}{m_0} \int_0^\infty S_{\zeta}(\omega) \sin(\omega t) d\omega
\]
(3.22)
By inserting the mean frequency, \( \overline{\omega} = \frac{m_1}{m_0} \), the solution by Friis-Hansen & Nielsen (1995) reduces to the New Wave profile introduced by Tromans et al. (1991) from Eq. (3.9).
Chapter 3. Conditioning on Ship Responses

3.3 Second Order Models

In the design of bottom-supported offshore marine structures the wave-induced response is often determined by exposing the structure to a regular non-linear fifth order Stokes wave. The procedure accounts for non-linear effects, however, the randomness of the wind-generated stochastic sea is poorly presented. Time domain simulations using a non-linear stochastic wave theory, Karunakaran (1993), are possible but they require an extensive amount of CPU time to obtain reliable extreme value predictions. Brute force simulation of non-linear wave pattern is therefore rarely used in design work. Introduction of constrained simulations significantly reduces the calculation time required to obtain reliable extreme value predictions.

3.3.1 Second Order Most Likely Wave Profile

The wave models by Tromans et al. (1991) and Friis-Hansen & Nielsen (1995), which lead to the same most likely wave profile by applying the mean wave frequency $\overline{\sigma} = \frac{m_1}{m_0}$, are based on linear theory. Jensen (1996) presented a general formulation for the conditional mean and its variance, which were derived for a slightly non-Gaussian process. The conditions applied are established given value and slope at a specific point for a slightly non-Gaussian process. Explicit second order formulas for the conditional mean wave profile and wave kinematics are derived for second order Stokes deep water wave theory. The main results are given below together with an evaluation of the most likely wave models. Numerical examples are presented in Section 3.4. Jensen (2004) has extended the second order most likely wave theory to a shallow water with a superimposed current formulation.

The conditional mean value $\hat{E}[G(x)]$ is given as

$$\hat{E}[G(x)] = E[G(x) \mid Z(0) = a, \dot{z}(0) = 0] = \int_{-\infty}^{\infty} g f(g \mid z, \dot{z}) dg = \frac{1}{f(Z, \dot{z})} \int_{-\infty}^{\infty} g f(g, z, \dot{z}) dg$$

(3.23)

where $\hat{E}[(.]$ is the conditional mean. The parameter $x$ may represent a time or space coordinate. $G(x)$ and $Z(x)$ are two correlated slightly non-Gaussian stationary stochastic processes with mean values equal to zero. $\dot{Z}(x)$ is the first derivative of $Z(x)$. The conditional probability density function $f(g \mid z, \dot{z})$ is given as

$$f(g \mid z, \dot{z}) = \frac{f(g, z, \dot{z})}{f(z, \dot{z})}$$

(3.24)

where $f(g, z, \dot{z})$ and $f(z, \dot{z})$ are joint probability density functions.
From a Gram-Charlier series representation of \( f \), the solution of the second order conditional mean wave profile of a crest of height \( Z(0) = a \) and slope \( \dot{Z}(0) = 0 \) becomes, Jensen (1996):

\[
\hat{E}[Z(x)] = \mathbb{E}[Z(x) \mid Z(0) = a, \dot{Z}(0) = 0] = \sigma_z \left\{ \rho(x) \frac{a}{\sigma_z} + \frac{1}{2} \left[ \left( \frac{a}{\sigma_z} \right)^2 - 1 \right] \times \left[ \lambda_{201}(x) - \rho(x) \lambda_{300} \right] - \frac{1}{2} \left[ \lambda_{021}(x) - \rho(x) \lambda_{120} - \dot{\rho}(x) \lambda_{030} \right] \right\}
\]

(3.25)

where \( a \) is the desired wave elevation and \( \sigma_z \) is the standard deviation of the first order wave elevation. The variance of the conditional mean is found to be, Jensen (1996):

\[
\mathbb{E} \left[ (G(x) - \hat{E}[G(x)])^2 \mid Z(0) = a, \dot{Z}(0) = 0 \right] = \sigma_z^2 \left[ 1 - \rho^2(x) - \dot{\rho}^2(x) + \left( \rho^2(x) \lambda_{300} - 2 \rho(x) \lambda_{201} + \lambda_{102}(x) - 2 \rho(x) \lambda_{111} + \rho^2(x) \lambda_{120} \right) \frac{a}{\sigma_z} \right]
\]

(3.26)

where the linear part \( \sigma_z^2 [1 - \rho^2(x) - \dot{\rho}^2(x)] \) is identical to the solution presented by Tromans et al. (1991). The normalised cumulants \( \lambda_{ijk} \) are obtained by Eq. (3.33) as follows.

The standard deviation \( \sigma_z \) of the first order wave elevation matches the wave spectral density \( S_z(\omega) \).

\[
\sigma_z^2 = \int_0^\infty S_z(\omega) d\omega \tag{3.27}
\]

and the variance of the wave slope equals

\[
\sigma_z^2 = \int_0^\infty \omega^2 S_z(\omega) d\omega \tag{3.28}
\]

The correlations \( \rho(x) \) and \( \dot{\rho}(x) \) are given as

\[
\rho(x) = \frac{\mathbb{E}[Z(0)Z(x)]}{\sigma_z^2} \simeq \sum_{i=1}^n v_i \cos(k_i x) \tag{3.29}
\]

\[
\dot{\rho}(x) = \frac{\mathbb{E}[\dot{Z}(0)Z(x)]}{\sigma_z \sigma_z} \simeq \sum_{i=1}^n \bar{k}_i v_i \sin(k_i x) \tag{3.30}
\]

where the unit spectral density is given as

\[
v_i = \frac{S_z(\omega_i) \Delta \omega_i}{\sigma_z^2} \tag{3.31}
\]

and the non-dimensional wave number

\[
\bar{k}_i = \frac{k_i}{k_m} \tag{3.32}
\]
where \( k_m = \frac{1}{2} \left( \frac{2\pi}{T} \right)^2 \) is the mean wave number. \( k_i \) denotes the wave number and is related to the frequency \( \omega_i \) by \( \omega_i^2 = k_i g \). The normalised cumulants \( \lambda_{ijk} \), which link the wave elevation \( Z(x) \) and the wave slope \( \dot{Z}(x) \) at \( x = 0 \):

\[
\lambda_{ijk} = \frac{E[Z(0)^i \dot{Z}(0)^j G(x)^k]}{\sigma_z^i \sigma_{\dot{z}}^j \sigma_g^k}
\]

(3.33)

From Jensen (1996) it follows that

\[
\lambda_{201}(x) = \frac{E[Z(0)^2 Z(x)]}{\sigma_z^3} = \hat{\sigma}_z \sum_{i=1}^n \sum_{j=1}^n \left[ \min(\bar{k}_i, \bar{k}_j)(2 + \cos(k_i x)) \times \cos(k_i x) - \max(\bar{k}_i, \bar{k}_j) \sin(k_i x) \sin(k_j x) \right] v_i v_j
\]

(3.34)

\[
\lambda_{300} = \frac{E[Z(0)^3]}{\sigma_z^3} = 3\hat{\sigma}_z \sum_{i=1}^n \sum_{j=1}^n \min(\bar{k}_i, \bar{k}_j) v_i v_j = \lambda_{201}(0)
\]

(3.35)

\[
\lambda_{021}(x) = \frac{E[\dot{Z}(0)^2 Z(x)]}{\sigma_{\dot{z}} \sigma_z} = \hat{\sigma}_{\dot{z}} \sum_{i=1}^n \sum_{j=1}^n \left[ \min(\bar{k}_i, \bar{k}_j) \times (\bar{k}_i \bar{k}_j \sin(k_i x) \sin(k_j x) + 2k_i^{-2} \cos(k_i x)) + \max(\bar{k}_i, \bar{k}_j) \bar{k}_i \bar{k}_j (2 - \cos(k_i x)) \cos(k_i x) \right] v_i v_j
\]

(3.36)

\[
\lambda_{120} = \frac{E[Z(0)^2 \dot{Z}(0)^2]}{\sigma_z^3} = \hat{\sigma}_z \sum_{i=1}^n \sum_{j=1}^n \left[ 2\min(\bar{k}_i, \bar{k}_j) \bar{k}_i^2 + \max(\bar{k}_i, \bar{k}_j) \bar{k}_i \bar{k}_j \right] v_i v_j = \lambda_{021}(0)
\]

(3.37)

\[
\lambda_{030} = \frac{E[\dot{Z}(0)^3]}{\sigma_{\dot{z}}^3} = 0
\]

(3.38)

The skewness, \( \lambda_{030} \), of the wave slope is zero due to vertical symmetry in the Stokes wave profile.

The normalised cumulants \( \lambda_{ijk} \) to be used for calculation of the variance are obtained as

\[
\lambda_{102}(x) = \frac{E[Z(0) Z(x)^2]}{\sigma_z^3} = \lambda_{201}(x)
\]

(3.39)

\[
\lambda_{111}(x) = \frac{E[Z(0) \dot{Z}(0)^2 Z(x)]}{\sigma_z^3 \sigma_{\dot{z}}^2} = \hat{\sigma}_z \sum_{i=1}^n \sum_{j=1}^n \left[ \bar{k}_j \min(\bar{k}_i, \bar{k}_j) \sin(k_j x) \times (1 + \cos(k_i x)) + \bar{k}_i \max(\bar{k}_i, \bar{k}_j) \sin(k_j x) \cos(k_i x) \right] v_i v_j
\]

(3.40)

The wave kinematics is additionally given by Jensen (1996).
### 3.4 Evaluation of Most Likely Wave Models

The first and second order conditional mean wave elevations have been simulated for selected values of $T_z$ and from different wave spectra. It is interesting to note that the two average wave numbers $k_m$ and $k_0$ from Eq. (3.32) are related to the bandwidth parameter $\epsilon$:

$$\epsilon = \sqrt{1 - \left(\frac{k_m}{k_0}\right)^2}$$  \hspace{1cm} (3.41)

By inspection of the equation that generates the second order conditional mean profile, it is found that the wave profile only depends on the parameters $k_m\sigma_\zeta$, $\epsilon$ and $u$, of which the first parameter is a measure of the average slope of the waves.

#### 3.4.1 Effect of Zero-Upcrossing Period

Figures 3.4 to 3.6 show the linear and second order conditional wave profiles for selected values of $T_z$ as a function of both time and space. From the figures it is seen that the second order waves are represented by steeper crests and shallower troughs than those obtained from the linear analyses. Moreover, Jensen (1996) found that the present theory agreed well with measurements by Jonathan et al. (1994). The effect of changing the zero-upcrossing period is seen, for shorter zero-upcrossing periods the wave profile as expected becomes much steeper with corresponding shorter wavelength between the first two wave peaks. It is furthermore observed that trough values are deeper for the mean most likely wave profile plotted as a function of time due to the lesser dispersion in time compared to dispersion in space, Tromans et al. (1991).

![Figure 3.4: Linear and non-linear MLW plotted as a function of time (left) and space (right). Both for $T_z = 8$ sec.](image-url)
Figure 3.5: Linear and non-linear MLW plotted as a function of time (left) and space (right). Both for $T_z = 10$ sec.

Figure 3.6: Linear and non-linear MLW plotted as a function of time (left) and space (right). Both for $T_z = 12$ sec.

Figure 3.7 shows the variance of the conditional wave profile for both the linear and the second order most likely wave theory. The numerical results are obtained from Eq. (3.26) with $G(x) = Z(x)$. The results from Figure 3.7 have been normalised by the linear variance $\sigma^2$. Far away from $t = 0$ and $x = 0$ the normalised variance will converge towards 1.

In Figure 3.7 a significant conditional variance is found to start before the first trough. The phenomenon is observed for the results plotted both as a function of time and space. Second order terms are found to increase the conditional variance close to $x$ and $t$ equal to zero, which presumably is a consequence of the non-linear second order waves being more broad-banded compared to the linear wave profile.
3.4 Evaluation of Most Likely Wave Models

Figure 3.7: Normalised variance for the conditional mean wave profiles as functions of time and space. Both first and second order results are shown, both for the PM spectrum with \( H_s = 10 \) m and \( T_z = 10 \) sec. The legend applies to both subplots.

3.4.2 Effect of Applied Wave Spectrum

The effect of applying a different wave spectrum is observed by comparison with the results obtained by the PM wave spectrum. Figure 3.8 shows the effect of applying the JONSWAP wave spectrum. The first and second order MLW profile becomes steeper with corresponding shorter period between the conditional most likely wave crest and the first following peak, which is found to be larger in wave amplitude. The first following trough by applying either the PM or the JONSWAP spectrum is seen to be almost identical. The transient phase is found to be longer, which is observed in Figure 3.9 where it is seen that the variance converges later towards zero, as an effect of the JONSWAP spectrum being more narrow-banded.

Figure 3.8: Linear and non-linear MLW plotted as a function of time (left) and space (right). Both for \( T_z = 10 \) sec. and by use of the JONSWAP spectrum.
Figure 3.9: Normalised variance for the conditional mean wave profiles as functions of time and space. Both first and second order results are shown, both for the JONSWAP spectrum with \( H_s = 10 \) m and \( T_z = 10 \) sec.

Torsethaugen (1993) calculates a double-peaked (swell + wind) spectrum from the model

\[
S_\zeta(\omega) = S_\zeta,s(\omega) + S_\zeta,w(\omega) \tag{3.42}
\]

where \( S_\zeta,s(\omega) \) and \( S_\zeta,w(\omega) \) are modified JONSWAP spectra for swell and wind peak, respectively. The energy is divided between the two peaks according to empirical parameters, which peak is primary depends on the parameters applied in the analysis. The empirical parameters are found for classes of \( H_s \) and \( T_p \), originating from a dataset consisting of 20000 spectra divided into 146 different classes of \( H_s \) and \( T_p \). (Data measured at the Statfjord field in the North Sea in a period from 1980 to 1989). The range of the measured \( H_s \) and \( T_p \) for the dataset is given from 0.5 to 11 metres and from 3.5 to 19 sec., respectively. See Torsethaugen (1993).

Figure 3.10 shows an example of the Torsethaugen wave spectrum with \( H_s = 10 \) m and \( T_p = 10 \) sec.

Figure 3.10: The Torsethaugen wave spectrum with \( H_s = 10 \) m and \( T_p = 10 \) sec.
Figure 3.11 shows the linear and second order MLW generated from a wind dominated two-peaked wave spectrum.

Figure 3.11: Linear and non-linear MLW plotted as a function of time (left) and space (right). Both for $T_p = 10$ sec. and by use of the Torsethaugen spectrum.

### 3.5 Existing Models for Extreme Ship Response Estimation

The continuous development of non-linear hydrodynamic methods has resulted in more accurately determined responses of offshore and ship structures. For the more extreme sea states where the structures are exposed to severe loads, non-linear effects are pronounced and have to be included in the analysis. It becomes important to have reliable and fast models for performing these analyses.

Seakeeping analyses of ship responses as the vertical bending moment, motions or accelerations of a vessel can either be performed in the frequency domain or in the time domain. The linear frequency analysis is based on the transfer function which is given for an individual combination of speed and wave heading. Given the linear transfer functions, a short- or long-term analysis may be performed. The response is considered linear in wave amplitude. A time domain analysis requires a stochastic encounter wave for the selected vessel, which the responses may be calculated for the given sea state and operational profile. The analyses are performed to get the non-linear responses and may either be performed from a linear or non-linear input wave. Subsequently, the short-term response statistics are obtained by simulating a significantly long wave history where critical wave events eventually will occur. The brute force method is very time-consuming, especially for 3-D codes.

Approaches to reduction of simulation time are found in the literature. In the following, some of the more interesting approaches are illustrated together with the most likely response wave approach.
3.5.1 Critical Wave Episodes

Torhaug (1996) introduced the Critical Wave Episode (CWE) model. The model is based on identification of wave candidates from the linear model that are likely to produce extreme responses. The wave sequences or critical wave episodes are typically a few wave cycles long, to be used as input for short non-linear simulations.

The critical wave episodes are identified from a priori knowledge of the vessel’s behaviour in waves with different speeds and wave directions. Linear time domain simulations are obtained relatively easily and inexpensively if a complete set of transfer functions for the response in question is available. The linear transfer functions become a search tool for selection of critical wave episodes. The model is based on the assumption that the linear model is a good identifier for the non-linear responses. This means, that a wave pattern that leads to an extreme linear response likely also will lead to a non-linear response. This approach allows additional information on the ship behaviour in waves which are not constrained in the linear transfer functions to be included.

It is possible to select critical wave episodes directly from the simulated surface elevation, given some main properties of the response process. It is known that wave episodes with wavelengths close to the ship’s length and wave heights above some specified level may well be candidates for critical wave episodes, if the relevant response is for instance the midships bending moment.

Before short-term or long-term response statistics can be calculated for a given storm and operational profile, a number of $n_H$ response realisations have to be simulated. For each of the realisations it becomes possible to identify $n_E$ critical wave episodes. Each of these will have a certain duration $n_C$, measured in terms of a number of wave cycles. Each of the durations have to be long enough to account for both transients and memory effects within the simulation.

In the estimation of extreme response during a typical storm of a given duration, the necessary simulation time will be proportional to the product of $n_H, n_E$ and $n_C$.

Torhaug et al. (1998) showed furthermore that, for the vertical bending moment of a monohull with a large flare, typically 36 wave cycles ($n_Hn_En_C$) will be sufficient to estimate the hourly maximum vertical bending moment with approximately 5% error.

The approach by Torhaug et al. (1998) requires though that the user have knowledge of the vessel considered and which of the many critical wave episodes to look for.

3.5.2 Most Likely Wave Approach

Application of the MLW approach have shown interesting results for further use. This approach has shown promising results on estimation of short-term response statistics for
bottom-supported offshore structures, Taylor et al. (1995) or Cassidy et al. (2001). Fine agreements with results from brute force simulations have been obtained. The structures are exposed to the MLW of a given crest height whereafter a corresponding extreme response is obtained. Unconditioning is performed on application of the Rayleigh distributed wave peaks, whereafter the extreme value distribution can be generated.

For offshore structures like a Jack-Up, the above approach works fine. As larger floating structures are considered, more caution should be applied as the corresponding response depends on the zero-upcrossing period and the instantaneous wave frequency used. Application of MLW as critical wave episodes are considered within Chapter 4.

Throughout the remaining analyses, the deterministic most likely wave profile is referred to as the MLW. The most likely wave profile with the random background wave included is simply referred to as the Conditional Random Wave (CRW).

### 3.5.3 Most Likely Extreme Response, MLER

Adegeest et al. (1998) presented the Most Likely Extreme Response (MLER). The MLER approach is based on the idea behind the most likely wave formulation. By use of the amplitude and phase information from a set of linear transfer functions, it is possible to derive the underlying wave profile which causes the MLER. Successively, a non-linear calculation can be carried out with the derived wave train as input. Their work results in one response value, which for the linear case represents the most likely extreme response. Promising sag/hog ratios and corresponding good agreement between measurements are found. Further work has been conducted in this area by Pastoor (2002), who introduced the Directional Most Likely Extreme Response (DMLER) approach. A brief description of the MLER and DMLER approaches is given in the following.

**MLER - Definition of Waves**

The incoming irregular wave train is defined by a linear superposition of $N$ regular wave components with different amplitudes, frequencies, headings and phases:

$$Z(t) = \sum_{n=1}^{N} A_n \cos(X k_n \cos \beta_n + Y k_n \sin \beta_n + \epsilon_{w,n} - \omega_n t)$$  \hspace{1cm} (3.43)

where $k = \omega^2/g$ is the wave number. $A_n$ is the expected value of amplitude defined by the wave spectrum. $X$ and $Y$ are defined as earth-fixed coordinates. $\beta$ refers to the wave heading, for which $\beta = 180$ degrees represents head sea conditions. $\omega$ is the wave frequency, where the corresponding encounter frequency $\omega_e$ is obtained by Eq. (2.22). The wave history for the origin of the mean ship-fixed system reduces to

$$Z(t) = \sum_{n=1}^{N} A_n \cos(\epsilon_{w,n} - \omega_{e,n} t)$$  \hspace{1cm} (3.44)
where \( \epsilon_{\omega,n} \) is the random phase angle and \( \omega_{e,n} \) the encounter frequency of the \( n \)’th wave component.

**Derivation of the Most Likely Extreme Response**

From the calculated transfer function, the response spectrum \( S_e^\theta(\omega_e) \) as given in Eq. (2.21) may be calculated, and the related spectral moments \( m_n \) from Eq. (2.24).

The most likely extreme response is obtained by simulating a random set of response amplitudes and phases for a discrete number of frequencies. The response \( H(t) \) is given as

\[
H(t) = \sum_{n=1}^{N} \left[ V_n \cos(\omega_{e,n} t) + W_n \sin(\omega_{e,n} t) \right]
\]

(3.45)

where \( V_n \) and \( W_n \) are Gaussian distributed random variables with zero mean and variance equal to

\[
\sigma_V^2 = \sigma_W^2 = E[V_n^2] = E[W_n^2] = S_{n,e}^\theta(\omega_{e,n}) \Delta \omega_{e,n}
\]

(3.46)

The paper by Adegeest et al. (1998) only very briefly describes the procedure of the conditioning process, where it is stated that the MLW theory is applied. It is specified that the outcome of the conditioning process is a new set of coefficients \( V_n^* \) and \( W_n^* \), and that the conditioning is obtained by use of the mean response frequency \( \overline{\omega} = m_1/m_0 \).

**Calculation of the Underlying Wave History**

The final step in the MLER procedure is to derive the underlying wave profile which causes the most likely extreme response.

The underlying earth-fixed wave profile is found to be

\[
Z(t) = \sum_{n=1}^{N} A_n^* \cos(\epsilon_n^* - \omega_{e,n} t)
\]

(3.47)

where the amplitudes of the incoming \( n \)’th wave component with encounter frequency \( \omega_{e,n} \) are found to be

\[
A_n^* = \frac{\sqrt{V_n^2 + W_n^2}}{|\Phi^\theta_{\eta}(\omega_{e,n})|}
\]

(3.48)

\( \Phi^\theta_{\eta}(\omega_{e,n}) \) is the transfer function. The phase \( \epsilon_n^* \) of the \( n \)’th wave component is derived as:

\[
\epsilon_n^* = \arctan \left( \frac{V_n^*}{W_n^*} \right) - \theta_{n,e,n}^\theta
\]

(3.49)
3.6 New Model for Extreme Ship Response Estimation

where $\theta_{\eta,n}$ is the related phase angles to the applied transfer function.

The process from Eq. (3.47) provides the most likely extreme response wave profile and may subsequently be used in a non-linear analysis. Adegeest et al. (1998) found good agreement between calculations using a 3-D non-linear seakeeping code and model test results.

In the following, the deterministic most likely extreme response wave profile, i.e. without the random background wave included is referred to as MLER.

Other Models

Pastoor (2002) presented the Directional Most Likely Extreme Response (DMLER) model. The model is an extension of the Most Likely Extreme Response (MLER) model by Adegeest et al. (1998). The procedure is based on the linear model for description of a conditional response, after which the underlying wave profile may be generated with information from the linear transfer function for all wave headings.

3.6 New Model for Extreme Ship Response Estimation

For dynamically responding structures, the extreme response does not always correspond to the extreme input surface elevation, and the extreme response might very well correspond to a combination of the local extreme waves with unfavorable background structural memory effects.

3.6.1 Most Likely Response Wave Model, MLRW

In the present model, the feasibility of applying most likely response waves to prediction of extreme wave responses of ship structures is developed further. The ideas by Friis-Hansen & Nielsen (1995) and Adegeest et al. (1998) have been combined, which allows the entire non-linear extreme value distribution to be calculated, given the amplitude and phase information from the linear transfer functions. The MLRW approach is later coupled to the model correction factor method by Ditlevsen & Arnbjerg-Nielsen (1991), which allows short- and long-term response statistics to be calculated effectively.

Original Definition of Waves in ShipStar

The developed procedure is implemented in ShipStar, Xia et al. (1998). ShipStar is a non-linear sea keeping code in the time domain as described in Appendix A. It allows the user to
determine both linear and non-linear loads and responses of monohull ships to regular and irregular waves. The irregular wave train was originally defined by a linear superposition of $N$ regular wave components of different amplitudes, frequencies, heading and phases:

$$Z(x, t) = \sum_{n=1}^{N} c_{\xi,n}^e \sin(k_n^*(x - X_{rec}) + \omega_{e,n}t + \epsilon_n)$$

(3.50)

$k_n^*$ is given as $k_n(-\cos\beta)$, where $k_n$ is the wave number and $\beta$ is the heading angle, with $\beta = 180$ degrees defined as head sea. $X_{rec}$ is the $x$-coordinate of a measured point of the wave relatively to the station considered. The encounter frequency $\omega_{e,n}$ is given as $\omega_n - \frac{\omega_n^2}{\beta}(\cos(\beta))$, $\epsilon_n$ is a uniformly distributed phase angle between $0 - 2\pi$. Finally, $c_{\xi,n}^e$ is given by the wave spectrum as $\sqrt{2S_{\xi}^e(\omega_{e,n})\Delta\omega_{e,n}}$.

The following sections describe the general solution of the most likely response wave approach. The specific solution implemented in ShipStar is explained in Appendix A.

Most Likely Response Waves

The most likely response wave profile is based on conditioning on both the response amplitude, $a$, and the instantaneous response frequency, $\omega$. The response process is described by an envelope process, Cramer & Leadbetter (1967). Similarly as for the MLW model by Friis-Hansen & Nielsen (1995), the present study is based on Slepian model processes, Ditlevsen (1985), see also Section 3.2.2. The objective of the approach by application of most likely response waves is to present a fast and accurate method for estimation of the non-linear extreme response value distribution. The present approach takes into account the memory effects. It allows the entire non-linear response distribution to be generated due to possibility the presence of a random background wave within the MLRW profile.

The deterministic MLRW, $\zeta_c(t)$, is established conditional on a given linear response amplitude $a_\eta$ and instantaneous response frequency $\omega_\eta$ of the process $\eta(t)$:

$$\zeta_c(t) \mid a_\eta, \omega_\eta \equiv \hat{E}[Z(t) \mid H(0) = a_\eta, \Omega_\eta = \omega_\eta, \dot{H}(0) = 0]$$

(3.51)

where $\hat{E}[]$ is the conditional mean of the wave profile process $Z(t)$. $H(t)$ is the response process and $\dot{H}(t)$ is the first derivative. $\Omega_\eta \equiv \dot{H}(0)/H(0)$, where $\dot{H}(t)$ is the Hilbert transform and $\dot{H}(t)$ the first derivative. The wave and response processes are linked through a by a linear transfer function. If $\omega_\eta = \frac{m_a}{m_g}$, the deterministic MLRW becomes identical to the MLER developed by Adegeest et al. (1998). This expression is taken throughout the report, thus MLRW is the same as MLER. The most likely response wave with the random background wave included is referred to as the Conditional Random Response Wave (CRRW) profile. In the following, the model is presented.
3.6 New Model for Extreme Ship Response Estimation

Irregular Wave Elevation

Linear superposition of sinusoidal wave components generates an irregular wave train. The ocean surface may be seen as statistically stationary in a given area and during a limited period of time. The wave elevation \( Z(x, t) \) is given as

\[
Z(x, t) = \sum_{n=1}^{N} a_{\zeta,n}^e \left[ V_n \cos(k_{e,n}x - \omega_{e,n}t) + W_n \sin(k_{e,n}x - \omega_{e,n}t) \right]
\]  

(3.52)

where \( N \) is large. \( V_n \) and \( W_n \) are independent standard normal random variables. For the individual wave components, \( k_{e,n} = k_n \cos(\bar{\theta}) \) with \( k_n = \omega_n^2/g \) being the wave number obtained by the dispersion relation for deep water waves. The coefficients \( a_{\zeta,n}^e \) are determined from the wave spectrum.

\[
a_{\zeta,n}^e = \sqrt{S^e_{\zeta}(\omega_{e,n})} \Delta \omega_{e,n}
\]

(3.53)

The wave process \( Z(x, t) \) is Gaussian distributed with zero mean, variance \( \sigma^2_{\zeta} = \int_0^\infty S^e_{\zeta}(\omega)d\omega \), continuous in time and differentiable.

Irregular Response

A linear irregular response \( H(x, t) \) is obtained as a sum of the individual responses from each of the sinusoidal wave components. The linear response becomes

\[
H(x, t) = \sum_{n=1}^{N} a_{\eta,n}^e \left[ V_n \cos(k_{e,n}x - \omega_{e,n}t + \theta_{\eta,n}^e) + W_n \sin(k_{e,n}x - \omega_{e,n}t + \theta_{\eta,n}^e) \right]
\]

(3.54)

with the Hilbert transform

\[
\hat{H}(x, t) = \sum_{n=1}^{N} a_{\eta,n}^e \left[ -V_n \sin(k_{e,n}x - \omega_{e,n}t + \theta_{\eta,n}^e) + W_n \cos(k_{e,n}x - \omega_{e,n}t + \theta_{\eta,n}^e) \right]
\]

(3.55)

\( a_{\eta,n}^e \) is given as

\[
a_{\eta,n}^e = \Phi_{\eta}^e(\omega_e \mid v, \beta) \sqrt{S^e_{\zeta}(\omega_{e,n})} \Delta \omega_{e,n}
\]

(3.56)

and \( \theta_{\eta,n}^e \) is the phase angle associated with the transfer function \( \Phi_{\eta}^e \).

The initial conditions of the Slepian model process from Eq. (3.15) may be written in terms of a random vector \( \mathbf{Y} = (Y_1, Y_2, Y_3, Y_4) \), where

\[
Y_1 = \sum_{n=1}^{N} a_{\eta,n}^e \left[ V_n \cos(\theta_{\eta,n}^e) + W_n \sin(\theta_{\eta,n}^e) \right] - H(0, 0)
\]

(3.57)
Chapter 3. Conditioning on Ship Responses

\begin{align*}
Y_2 &= \sum_{n=1}^{N} a_{\eta,n}^e \omega_{e,n} \left[ V_n \sin(\theta_{\eta,n}^e) - W_n \cos(\theta_{\eta,n}^e) \right] - \dot{H}(0,0) \\
Y_3 &= \sum_{n=1}^{N} a_{\eta,n}^e \left[ -V_n \sin(\theta_{\eta,n}^e) + W_n \cos(\theta_{\eta,n}^e) \right] - \dot{H}(0,0) \\
Y_4 &= \sum_{n=1}^{N} a_{\eta,n}^e \omega_{e,n} \left[ V_n \cos(\theta_{\eta,n}^e) + W_n \sin(\theta_{\eta,n}^e) \right] - \dot{H}(0,0)
\end{align*}

From Eqs. (3.57) to (3.60) it is seen that the coefficients \( V_n = (V_n, W_n) \) depend on the choice of \( Y \). For \( Y = (0, 0, 0, 0) \) the wave profile will satisfy the given initial conditions, and as the variance of the residual of the conditional random vector \( [V|Y] \) is constant for any \( Y \), the conditional vector becomes

\[
\overline{V}_c = [V|Y = (0, 0, 0, 0)] = V - \hat{E}[V|Y] + \hat{E}[V|Y = (0, 0, 0, 0)] = \\
V - \left\{ E[V] + Cov[V, Y^T]Cov[Y, Y^T]^{-1}(Y - E[Y]) \right\} + \\
\left\{ E[V] + Cov[V, Y^T]Cov[Y, Y^T]^{-1}(Y - E[Y]) \right\}_{Y = (0,0,0,0)} = \\
V - Cov[V, Y^T]Cov[Y, Y^T]^{-1}Y \tag{3.61}
\]

since \( E[Y] = 0 \). The matrix \( Cov[V_n, Y^T] \) becomes

\[
Cov[V_n, Y^T] = a_{\eta}^e \begin{bmatrix}
\cos(\theta_{\eta,n}^e) & \omega_e \sin(\theta_{\eta,n}^e) & -\sin(\theta_{\eta,n}^e) & \omega_e \cos(\theta_{\eta,n}^e) \\
\sin(\theta_{\eta,n}^e) & -\omega_e \cos(\theta_{\eta,n}^e) & \cos(\theta_{\eta,n}^e) & \omega_e \sin(\theta_{\eta,n}^e)
\end{bmatrix} \tag{3.62}
\]

and the inverse of \( Cov[Y, Y^T] \):

\[
Cov[Y, Y^T]^{-1} = \frac{a_{\eta}^e}{m_2m_0 - m_1^2} \begin{bmatrix}
m_2 & 0 & 0 & -m_1 \\
0 & m_0 & m_1 & 0 \\
0 & m_1 & m_2 & 0 \\
-m_1 & 0 & 0 & m_0
\end{bmatrix} \tag{3.63}
\]

CRRW realisations are then obtained by replacing the original random variables \( V_n, W_n \) in Eq. (3.52) with the random constrained coefficients \((\overline{V}_{c,n}, \overline{W}_{c,n}) \equiv [V_n|Y = (0, 0, 0, 0)]\) from Eq. (3.61). For \( H(0, 0) = a_\eta, \hat{H}(0, 0) = 0, \hat{H}(0, 0) = 0 \) and \( \hat{H}(0, 0) = a_\eta \overline{\omega}_\eta \), where \( a_\eta \) is the linear response amplitude and \( \overline{\omega}_\eta = \frac{m_2}{m_0} \) is the mean response frequency, the conditional mean constrained coefficients become

\[
\overline{V}_{c,n} = E[V_n|Y = (0, 0, 0, 0)] = \frac{a_{\eta,n}^e}{m_0m_2 - m_1^2} \times \\
b_\eta(\omega_e \cos(\theta_{\eta,n}^e) - a_\eta \overline{\omega}_\eta(\omega_e m_0 - m_1) \cos(\theta_{\eta,n}^e)) \tag{3.64}
\]

and

\[
\overline{W}_{c,n} = E[W_n|Y = (0, 0, 0, 0)] = \frac{a_{\eta,n}^e}{m_0m_2 - m_1^2} \times \\
b_\eta(\omega_e m_1 - m_2) \sin(\theta_{\eta,n}^e) - a_\eta \overline{\omega}_\eta(\omega_e m_0 - m_1) \sin(\theta_{\eta,n}^e) \tag{3.65}
\]
3.7 Evaluation of the MLRW/CRRW Model

The MLRW conditional on both the amplitude and the instantaneous frequency of the response thus becomes

\[
\zeta_c(x, t) = \sum_{n=1}^{N} a_{c,n}^e \left[ \bar{V}_{c,n} \cos(k_{c,n}x - \omega_{c,n}t) + \bar{W}_{c,n} \sin(k_{c,n}x - \omega_{c,n}t) \right]
\]  

(3.66)

Similarly, the constrained linear mean response is obtained by replacing \(V_n\) and \(W_n\) by \(\bar{V}_{c,n}\) and \(\bar{W}_{c,n}\) in Eq. (3.54).

In summary, the realisations of the CRRW, i.e. the MLRW with the random background wave included are obtained by entering the coefficients \(\bar{V}_n = (V_n, W_n)\) from Eq. (3.61) as a zero-mean Gaussian distributed random vector before the conditioning is performed. The deterministic MLRW profile is obtained by entering the coefficients \(V_n = (V_n, W_n)\) as a zero vector.

3.7 Evaluation of the MLRW/CRRW Model

This section presents linear results obtained from the MLRW model. The analyses are performed for selected values of \(T_z\) and for the PM and JONSWAP wave spectra, all with \(H_s = 10\) m. The constrained response analyses are performed for the vertical bending moment amidships and with the same operational profile (zero speed and head sea) for all the numerical results. The results are plotted as a function of time \(t\) (relative to the midships section) and space \(x\). Non-linear results are given in Chapter 5 in addition with short- and long-term analyses.

The results obtained are based on the Panmax container ship described in Appendix B.

3.7.1 Discussion of the MLRW Model

The presented MLRW model illustrates a new and straightforward method for deriving the deterministic MLRW profile or the CRRW with a simple knowledge of the wave spectrum and a linear transfer function.

The major advantages of the MLRW/CRRW are

1. The entire frequency range in the wave spectrum can be applied.
2. The MLRW is generated from the linear transfer function. The wave profile therefore takes memory and transient effects into account as the wave profile is generated.
3. The method is simple to implement and extremely fast if deterministic MLRW is applied.
4. Since the constrained response process is a function of time only (it relates to a specific point) it avoids the ambiguity of a conditioning on a wave profile in that it is not obvious if a maximum in time also implies a maximum in space.

Thus, one of the important objectives becomes to compare response statistics by application of the MLRW and CRRW models to results from brute force simulations. It will in this way be verified if the linear model is a good identifier for the location of the non-linear responses.

### 3.7.2 Effect of Zero-Upcrossing Period

Figures 3.12 to 3.14 show linear results for selected values of $T_z$. The responses are generated from the PM wave spectrum. The deterministic MLRW (right) and the constrained response (left) are plotted as functions of time $t$. For the three cases, a conditional bending moment response of 3000 MNm is applied at $t = 0$ (left). The effect of a changed zero-upcrossing period is observed. As $T_z$ is increased, the transient phase reduces, the period between the two wave peaks found on each side of the constrained point at $t = 0$ increases slightly with corresponding higher peaks and a deeper trough. The phenomena shown are present because of the narrow band width of the response spectrum, $S_{\eta}(\omega_c)$. The deep trough $t_0 = 0$ sec. corresponds well with the conditional sagging bending moment amidships.

![Figure 3.12: The MLRW (right) and constrained response as functions of time, $x = 0$, (left). The results are generated for $T_z = 8.0$ sec.](image)
3.7 Evaluation of the MLRW/CRRW Model

Figure 3.13: The MLRW (right) and constrained response as functions of time, \( x = 0 \), (left). The results are generated for \( T_z = 10.0 \) sec.

Figure 3.14: The MLRW (right) and constrained response as functions of time, \( x = 0 \), (left). The results are generated for \( T_z = 12.0 \) sec.

Figure 3.15 shows the MLRW plotted for selected space coordinates \( x \), with \( T_z = 10.0 \) sec., otherwise with the same parameters applied. It is characteristic that the wavelength between the two peaks around the conditional location in space roughly equals the hull length. The constrained sagging moment occurs when the vessel is basically supported by two wave crests. Asymmetry in the MLRW profile as a function of space \( x \) reflects the shape of the vessel considered.
The present analyses are made for a constrained sagging response. The simulations have also been performed for conditional hogging responses. These results are not shown here, but are almost a mirrored image of the conditional sagging responses.

### 3.7.3 Effect of Applied Wave Spectrum

Analyses have been performed by application of the JONSWAP spectrum with $T_z = 10.0$ sec. and $H_s = 10.0$ m. The same trends are observed, the transient phase as seen for the MLW in Figure 3.8 becomes longer, which is a consequence of the more narrow banded wave spectrum applied. It is interesting to observe that even by use of different wave spectra (PM and JONSWAP), the shapes of the MLRW from roughly $-125$ m to $125$ m (see Figure 3.15 and 3.17 (right)) are practically identical. The same phenomenon is observed for the results plotted as a function of time.

Figure 3.16: The MLRW (right) and constrained response as functions of time, $x = 0$, (left). The results are generated for $T_z = 10.0$ sec.
3.8 Concluding Remarks

An introduction to conditioning of ship responses has been given and discussed. The so-called “New Wave” by Tromans et al. (1991) has been introduced. It is found that the “New Wave” profile around a peak or trough reduces to the autocorrelation function scaled by the crest \( a \). Friis-Hansen & Nielsen (1995) derived an extension to the “New Wave” model, where the approach is extended to include the conditioning on the random instantaneous frequency as well. The Most Likely Wave (MLW) reduces to the “New Wave” profile by inserting the mean frequency.

Adegeest et al. (1998) presented the Most Likely Extreme Response (MLER) model, which derives the underlying wave profile that derives a conditional linear response. The derived most likely extreme response wave profile may subsequently be used in a non-linear time domain calculation.

The Most Likely Response Wave (MLRW) model derived in the present study is an extension to the models by Friis-Hansen & Nielsen (1995) and Adegeest et al. (1998). The MLRW is established conditional on a given linear response amplitude and the instantaneous response frequency. The present approach derives the MLRW directly. The random background wave is easily included, thus allowing an entire non-linear extreme value distribution to be generated. The deterministic MLRW profile reduces to the MLER wave by inserting the mean response frequency. By introducing the stochastic background wave of the MLRW profile, the Conditional Random Response Wave (CRRW) is established.

The models presented have been evaluated for selected sea states, different wave spectra and operational profiles.

Figure 3.17: The MLRW as function of space \( x \), \( t = 0 \). The results are generated for \( T_z = 10.0 \) sec.
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Chapter 4

Most Likely Wave Analyses

4.1 Introduction

Application of conditional time series is interesting to study as the used CPU time may be reduced significantly compared to the traditional very time-consuming brute force analyses. The simulations in this chapter are focused on estimation of the extreme value distribution $F_{RNL}(r \mid h_s, t_z, v, \beta)$ by application of the Most Likely Wave (MLW) analysis, where $r$ is the response of interest, $h_s$ is the significant wave height, $t_z$ is the zero-upcrossing period, $v$ is the vessel’s speed and $\beta$ is the sailing direction relative to the long-crested ocean waves. A rational procedure is presented with a discussion of advantages and disadvantages of the MLW approach.

Some of the ideas by Taylor et al. (1995) are used in the present analyses. They showed how the Conditional Random Wave (CRW), i.e. the MLW profile with the random wave background included could be used to estimate the extreme dynamic response of a Jack-Up in a severe sea state. The result of their work was a technique for estimating the probability distribution of the extreme response in a stationary stochastic seaway using short constrained simulations. For the analyses of bottom supported structures, it was found that the peak response determined from the deterministic MLW itself would be significantly biased as compared to medians (50 % level) determined from extreme value distributions in a three hour period. This indicates that the random background waves for dynamically responding structures can be important, and that the specific wave amplitude not necessarily is the only parameter to be considered as extreme responses are sought.

The main interest in the present study is the vertical bending moment (VBM) amidships, which is the most important design parameter for the hull girder strength. The responses are studied for different MLW amplitudes given a sea state, selected ship’s velocities and headings and finally by changing the longitudinal location of the constrained wave peak relative to the ship. Furthermore, it is interesting to study bias factors to be applied on
peak responses due to the MLW as compared to medians (50 % levels) determined from extreme value distributions representing a scenario of for example three hours.

In the first section, the location of the vessel relative to the conditional wave is studied. In the following section, a numerical study on ship responses due to the deterministic MLW is performed. The study is focused on application of different operational profiles, different sea states and wave spectra. These analyses provide valuable information on the physical shape of the MLW profiles. Later on, the analyses are extended to embed the deterministic MLW in a stochastic seaway, i.e. the CRW profile. The main objective is to gain a better understanding of ship responses due to the CRW and to derive a method for generating an extreme value distribution for a selected response, using short constrained time domain simulations. It is of interest to investigate whether bias factors between the MLW and CRW approach can be established and used efficiently.

Time domain simulations have been performed using the non-linear time domain sea keeping code ShipStar, Xia et al. (1998). The theory is briefly explained in Appendix A. The derived distributions by application of the MLW and CRW approaches are compared to results from brute force simulations, with good agreements obtained for the selected operational profile and sea state.

Simulations are performed using both the PM and JONSWAP wave spectra. The results from application of the JONSWAP spectrum are mostly found in Appendix E and summarised in Sections 4.5.2 and 4.5.3.

Examples of chosen MLW profiles are shown in Figure 4.1 for crest amplitudes \( a = 12, 15 \) and \( 18 \) m. The more narrow-banded JONSWAP spectrum provides a more energy concentrated wave system with a longer transient phase (larger memory effect) before the constrained peak occurs. The instantaneous frequency is given as \( \tilde{\omega} = m_0/m_1 \), where \( m_0 \) and \( m_1 \) are the spectral moments of order zero and one, respectively.

In Section 3.4.1, the changes in the shape of the MLW profile for selected zero-upcrossing periods were studied. Critical sagging bending moments amidships are often generated as the vessel moves in waves of the same length as the vessel itself. Simulation of a deterministic MLW, where the wavelength between the conditional crest and the peak immediately before equals the hull girder length, corresponds in the current vessel to \( T_z = 11.63 \) sec. for the JONSWAP spectrum and \( T_z = 11.35 \) sec. for the PM wave spectrum.

The analyses for the present study by application of MLW profiles are tuned to obtain wavelength and hull girder length of approximately the same size. If another zero-upcrossing length had been selected, the instantaneous wave frequency should be selected differently. By use of a smaller zero-upcrossing period and corresponding smaller instantaneous conditional wave frequency compared to the mean wave frequency, it is possible to obtain a MLW profile that results in a wavelength which is equal to the hull girder length. Simulations by application of MLW with zero-upcrossing periods \( T_z \neq 11.35 \) sec. for the PM spectrum and \( T_z \neq 11.63 \) sec. for the JONSWAP spectrum have similar been made with resulting smaller peak responses. These are therefore not included within the present study.
4.2 Time Domain Simulation Using a Constrained Signal

Figure 4.1: Applied MLW profiles with the PM (left) and JONSWAP (right) wave spectra, crest amplitudes equal to 12, 15 and 18 m. It is interesting to observe the larger transient phase for the JONSWAP spectrum.

The Panmax Container Ship

The simulations are performed for a Panmax container ship with the main dimensions given in Table 4.1. Further information on the vessel is provided in Appendix B.

Table 4.1: Main dimensions of the Magleby Maersk.

<table>
<thead>
<tr>
<th>Main dimensions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, ( L_{pp} )</td>
<td>276.38 m</td>
</tr>
<tr>
<td>Displacement, ( \Delta )</td>
<td>( \sim 63350 ) tons</td>
</tr>
<tr>
<td>Breadth, ( B_{mld} )</td>
<td>32.20 m</td>
</tr>
<tr>
<td>Block coefficient, ( C_b )</td>
<td>0.62 [-]</td>
</tr>
<tr>
<td>Depth, ( D )</td>
<td>23.50 m</td>
</tr>
<tr>
<td>Service speed, ( v_{design} )</td>
<td>24.8 kn</td>
</tr>
<tr>
<td>Draught, ( T_{mld} )</td>
<td>11.20 m</td>
</tr>
</tbody>
</table>

4.2 Time Domain Simulation Using a Constrained Signal

The MLW profiles in Figure 4.1 are plotted as functions of time \( t \). Ship structures are generally large structures, and it is normal to see vessels with hull lengths above 300 m. It is therefore obvious that the location of the constrained peak relative to the vessel will influence the calculated bending moments amidships.

Figure 4.2 shows a conditional wave surface plotted as a function of both time and space, where the conditional peak is found at \( t_0 = 0 \) and \( x_0 = 0 \). The surface represents the wave
sequences which the vessel will encounter with zero speed in head sea. With increasing forward speed, the incoming wave system will appear more compact and slightly less S-shaped for wave heading directed towards beam sea.

Depending on the wave spectrum chosen, some differences are found for the generated deterministic wave field. For the PM spectrum, the deterministic surface from Figure 4.2 dies out faster, compared to the field generated on the basis of the JONSWAP spectrum, see Appendix E, Figure E.1. For the JONSWAP spectrum, it is additionally found that the wave ridge stays relatively high, which means that by moving the hull one ship’s length in the longitudinal direction, the vessel will encounter a wave which is almost as high as the conditioned peak, but with a slightly changed wavelength.

Figure 4.2: Most likely wave profiles plotted as a function of time and space with the PM spectrum, $H_s = a = 15$ m and $T_z = 11.35$ sec.

Time domain simulations for long and slender floating structures like a large container ship by application of MLW profiles become more complicated in comparison to analyses of for example a fixed bottom-supported offshore structure, as the critical wave should be found within a physically larger area. Imagine a vertical circular cylinder of a given diameter, which from a structural point of view would normally be small compared to a traditional trading vessel. The largest response of interest to the offshore structure may quickly be found by moving the deterministic MLW profile relative to the structure when the most extreme response is sought. The structure would presumably be considered small compared to the wavelength, whereas the occurrence of the highest wave crests would often become the most critical.
4.3 Parameter Study, MLW Responses

By application of the MLW profiles in time domain analyses, the analyses should be made with the hull placed somewhere parallel to the space axis, as illustrated in Figure 4.3, after which it can be moved forward in time.

Figure 4.3: Vessel “movement” (for some of the cases \( v = 0 \) m/s) over a constrained surface.

4.3 Parameter Study, MLW Responses

The main purpose of this parameter study is to show how a given vessel responds to the MLW profile. The effect on the vertical bending moment amidships has been examined by changing the wave spectrum, the forward speed, the wave heading relative to the vessel and finally the location of the MLW crest relative to the vessel.

The linear results are obtained using a linear transfer function. The responses are calculated according to Eq. (3.54). Generation of non-linear responses is more time-consuming as ShipStar has to be executed for each simulation. Both linear and non-linear analyses are performed upon the assumption of a rigid hull girder.

4.3.1 Effects of used Wave Spectrum

As shown previously in Chapter 3, differences are found within the MLW profiles generated from the two wave spectra. Figure 4.4 shows the vertical bending moment amidships as a
function of time, given that the vessel encounters the MLW crest at FP. Results are shown for both the linear and the non-linear case. Operational parameters correspond to zero speed and head sea with the JONSWAP spectrum to the right and the PM spectrum to the left. Sagging responses are represented by positive values in the time series. Note furthermore, that the ship’s flare shape for example induces considerable non-linearity and asymmetry between maximum and minimum vertical bending moments.

It is interesting to observe the differences in the response series using the two spectra. For the selected spectra, ratios of the non-linear to the linear peak responses are found to be between 1.31 and 1.51 for the selected amplitudes. The non-linearities are most pronounced by application of the JONSWAP wave spectrum. In Table 4.2 the peak responses are summarised for the two wave spectra.

Table 4.2: Sagging peak responses - by changing the MLW amplitudes, $S_{PM}$ equals the PM spectrum and $S_{JON}$ the JONSWAP spectrum.

<table>
<thead>
<tr>
<th>VBM (MNm)</th>
<th>Linear</th>
<th>Non-linear</th>
<th>$S_{PM}$</th>
<th>$S_{JON}$</th>
<th>Non-lin./Lin.</th>
<th>Non-lin./Lin.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = 12.0$ m</td>
<td>4151</td>
<td>4757</td>
<td>5428</td>
<td>6728</td>
<td>1.31</td>
<td>1.41</td>
</tr>
<tr>
<td>$a = 15.0$ m</td>
<td>5189</td>
<td>5946</td>
<td>7041</td>
<td>8954</td>
<td>1.36</td>
<td>1.51</td>
</tr>
<tr>
<td>$a = 18.0$ m</td>
<td>6227</td>
<td>7136</td>
<td>8951</td>
<td>10290</td>
<td>1.44</td>
<td>1.44</td>
</tr>
</tbody>
</table>

This small example presents some basic and interesting observations. It seems as if the transient phase, before the MLW peak occurs, has an important impact on the resulting responses obtained for both the hogging and the sagging conditions. The random background wave, which has been eliminated for the current case, therefore becomes important. Use of the PM wave spectrum has been applied for setting up guidelines for application of the MLW profile as a critical design wave for extreme response value estimation.
Figure 4.4: The results to the left are generated by application of the PM spectrum. The uppermost plot shows the MLW profiles for selected amplitudes. The plot in the middle shows linear responses whereas the non-linear simulation is shown at the bottom. The right plot shows the same, but by application of the JONSWAP spectrum. The constrained MLW crest is generated at FP, $t_0 = 150$ sec.

4.3.2 Selection of the Critical MLW Profile

Section 4.3.1, the simulations were performed in such a way that the vessel encounters the MLW at the forward perpendicular. Subsequently, it was possible to collect the largest sagging and hogging bending moments. Essentially, the vessel has experienced a very large wave, but not necessarily located in the most critical place relative to the hull girder. Figure 4.5 shows snapshots of the vessel at the same time instant, namely $t_0 = 150$ sec., but with the MLW crest moved relative to the hull girder. On the uppermost subplots, the simulation is performed so that the MLW crest occurs at FP. The following plots below show the same vessel, but with the MLW crest moved towards the stern in steps of $L_{pp}/4$. The results in
the right column illustrate the corresponding time series given the location of the wave on the left side. First, it is observed that the peak responses move backwards as the constrained peak is moved towards the stern. This phenomenon corresponds well with what is expected and can be verified visually by Figures 4.2 and 4.3. The vessel encounters the MLW peak at different locations along the hull girder with a corresponding critical hogging or sagging response. These are not found at the same time instant and often away from the conditional time instant $t_0$. The calculations are performed using the PM spectrum for zero speed and in head sea.

![Figure 4.5](image)

Figure 4.5: Linear responses, PM spectrum with $H_s = a = 15.0$ m, $T_z = 11.35$ sec., zero speed and head sea. The constrained peak is moved step by step from FP (the uppermost plot) to AP (lowest plot), all at $t_0 = 150$ sec. The circles represent maximum sagging and hogging moments.

Figure 4.5 shows that the size of the sagging or hogging peak responses changes depending on the location of the constrained peak relative to the hull. For the current operational profile and sea state, it is observed that the largest sagging vertical bending moment amidships
obtained from the linear simulations is found when the MLW crest is located at FP. For the corresponding response to the right, the first larger sagging peak is observed at \( t \simeq 150 \) sec. This is followed by an even larger one, which turns out to be the maximum sagging response. This peak is found at \( t \simeq 163 \) sec., which means that the wave has passed the vessel and is approximately found at AP (one “wave” period later). Here the hull girder becomes more or less simply supported by two wave crests.

The most critical hogging situation (plot in the middle) is found as the MLW is located amidships, where the ends of the vessel are basically hanging freely. This corresponding response series on the right side illustrates furthermore that the largest response is found at \( t \simeq 150 \) sec.

The linear and non-linear hogging and sagging peaks of the simulation from Figure 4.5 are found in Table 4.3. Besides, results obtained from a non-linear analysis are listed. Simulations have additionally been made by application of the JONSWAP wave spectrum. The same pattern is found, though with slightly higher peak responses. Generally for the linear case, the variation in the responses is found between 5–10% depending on the location of the MLW relative to the hull girder. Results are summarised in Appendix E, Table E.1.

It is often assumed that the wave pattern which generates an extreme linear response will also likely generate the corresponding non-linear response. For the presented simulations, the largest linear and non-linear hogging moments by application of the PM spectrum are obtained from the same input wave pattern.

Table 4.3: Sagging and hogging peak responses given the location of the constrained peak relative to the hull girder. The calculations have been performed using the PM spectrum with \( H_s = a = 15.0 \) m, \( T_z = 11.35 \) sec., zero speed and head sea. JONSWAP results are summarised in Appendix E, Table E.1.

<table>
<thead>
<tr>
<th>VBM</th>
<th>VBM</th>
<th>PM</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hog</td>
<td>Sag</td>
<td>Hog</td>
</tr>
<tr>
<td>AP</td>
<td>-4636</td>
<td>5113</td>
<td>-3951</td>
</tr>
<tr>
<td>L/4</td>
<td>-4966</td>
<td>5025</td>
<td>-4110</td>
</tr>
<tr>
<td>L/2</td>
<td>-5133</td>
<td>4735</td>
<td>-4204</td>
</tr>
<tr>
<td>3L/4</td>
<td>-5114</td>
<td>4940</td>
<td>-4049</td>
</tr>
<tr>
<td>FP</td>
<td>-4801</td>
<td><strong>5189</strong></td>
<td>-3884</td>
</tr>
</tbody>
</table>

For the selected extreme sea state, the waves generated are fairly steep, and in Figure 4.5 several green water observations are presented, which may alter the overall view. It is a question whether these waves will occur in real life at all. The ratios between the wave
height $H$ and the acceleration of gravity $g$ times the wave period $T$ squared, $H/(gT^2)$ are found to be lower than 0.02, which indicates that the waves will be non-breaking for deep water conditions.

As illustrated in the previous paragraphs, it was found that the location of the MLW peak would influence the extreme hogging and sagging bending moments. For some of the selected cases it was not obvious that the maximum peak response would be found within the selected range. Therefore, it was decided to stretch the range further, with the consequence that the vessel would not physically encounter the constrained peak, but pass by in a near region. Figure 4.6 illustrates the maximum sagging and hogging peaks as functions of the location of the constrained peak relative to the vessel. The simulations have been performed with the PM spectrum, $H_s = a = 15.0$ m, $T_z = 11.35$ sec., zero speed and head sea. The hull is fixed in the position from AP ($x = 0$ m) to FP ($x = 276.38$ m), the stern points therefore towards the positive space coordinate as illustrated in Figure 4.3. From linear calculations it is found that the largest hogging and sagging responses occur if the constrained peak is located from $L/4$ to $L/2$ hull lengths in front of the vessel. This means that the vessel passes by the MLW without encountering it physically. As the constrained peak is moved further away from the vessel, the responses will naturally be smaller, which is a consequence of the wave field flattening out. Local maxima and minima are illustrated by red dots. The phenomena observed are a consequence of a local better fit between the hull and the wavelength from a structural point of view.

The non-linear simulations show a changed picture. By application of the PM spectrum, the extreme non-linear responses are obtained if the constrained peak is located approximately one half to two ship’s lengths aft of the vessel. Figures 4.2 and 4.3 provide a good visual view of the situation. For the non-linear simulations the extreme responses are generated if the vessel passes in a band on the negative space coordinates, whereas the linear extreme responses are found as the vessel passes the constrained peak on the other side. It is seen that the variations of the peak responses within one hull length aft and in front of the vessel change with approximately $\pm 20\%$.

The shape of the critical wave that generates linear and non-linear peak responses has been briefly studied. Figure 4.7 shows realisations of the critical wave profile which are plotted relative to the instantaneous position of the vessel at the time step where the peak responses occur. The peak responses of interest on the right subplot are illustrated by a circle. The two uppermost plots show the linear results, whereas the two lowest represent the non-linear simulations.
The critical waves that generate a sagging vertical bending moment amidships are found on subplots no. 1 and 3 to the left in Figure 4.7. It is seen that the vessel is basically hanging on two wave crests. For the linear case, it is seen that the hull is almost unsupported amidships, and that the bow is more or less fully submerged in water. The situation is almost identical for the non-linear case, here the stern is found to be fully submerged, after which the vessel is purely supported amidships. It is interesting to observe that the largest wave peak along the hull that the vessel encounters physically equals $\sim 10.0\text{ m}$ for the linear case and $\sim 12.0\text{ m}$ for the non-linear case. This is furthermore a significantly smaller wave compared to the constrained peak of 15.0 m. The deepest troughs equal $\sim -11.0\text{ m}$ for both cases. Visual observations indicate that the shape of the waves is far away from the original MLW itself.

The critical waves that generate the peak hogging vertical bending moment are found on subplots no. 2 and 4 to the left. The two situations are almost identical; the hull girder is supported amidships, which results in a large hogging bending moment. The largest wave crests equal $\sim 12.0\text{ m}$ for both cases. The deepest troughs equal $\sim -9.0\text{ m}$ for both linear and non-linear analyses.
The next two sections are extensions of the above analyses. The effect on vertical bending moment amidships is studied for selected forward velocities, vessel headings and locations of the constrained peak relative to the vessel.

### 4.3.3 MLW Responses, Changing the Velocity

The effects on the vertical bending moment amidships by changing the vessel’s velocity and the location of the MLW relative to the hull are seen in Figure 4.8. The results are, as expected, increasing global maximum responses as a function of the vessel’s velocity.
Figure 4.8: Hogging and sagging peak responses as functions of the constrained peak relative to the vessel. The hull is fixed in the position from AP (x = 0.0 m) to FP (x = 276.38 m). The results are given for \( v = 0 \), 5 and 10 m/s and head sea. The filled marks represent maximum sagging and hogging moments.

4.3.4 MLW Responses, Changing the Heading Angle, \( \beta \)

The effect on the vertical bending moment amidships by changing the vessel’s heading relative to the wave fronts has been studied. The simulations are made for \( \beta = 0 \) to 180 degrees. For the analysed sea state, head sea conditions is the most critical. The results are as expected. For quartering, beam and bow quartering seas the responses are smaller as the wavelength decreases. Following sea conditions become additionally critical; here the vessel encounters waves of the same wavelength as the vessel, which results in large sagging responses.
4.3.5 Discussion of Responses due to the Deterministic MLW Profile

The present studies on responses of ship structures due to the MLW profile have shown that the physical size of the structure considered is important.

The selected zero-upcrossing period of 11.35 sec. for the PM spectrum generates a MLW profile, where the hull length and the wavelength almost become identical, given that the vessel encounters the wave at FP. These simulations result in corresponding critical sagging and hogging moments.

Figure 4.9: Sagging and hogging peak responses as functions of the location of the constrained peak relative to the vessel. The hull is fixed in the position from AP (x = 0.0 m) to FP (x = 276.38 m). The results are given for \( v = 0 \) m/s and \( \beta = 0, 45, 135, 180 \) degrees. The filled marks represent maximum sagging and hogging moments.
responses. The obtained responses are sensitive to the selected zero-upcrossing period. Additional simulations for other zero-upcrossing periods have been made. The overall results were much more blurred, and it has not been possible to specify any general trends. These observations are a main drawback of application of most likely wave profiles, as the applied wave profiles do not explicitly take the shape of the vessel into account.

It has been found that the largest responses are generally obtained if the vessel passes the MLW without encountering the maximum conditional wave physically. The slightly larger responses outside the wave crest are obtained because of better correspondence between the hull length and the wavelength and that transient effects are better captured.

For further analyses of application of the MLW profile it seems most correct to locate the most likely wave crest at FP, given a critical sagging moment is desired. Critical hogging responses are considered most likely as the MLW encounters the vessel amidships.

4.4 MLW Responses, Short-Term Response Statistics

An interesting question will be whether the deterministic MLW profile itself can be applied as a useful predictor of the most probable largest response, given a sea state with a corresponding operational profile. The calculations performed using the MLW profile are quickly made, and by applying the correct position of the constrained peak relative to the hull, an extreme response candidate is obtained.

The MLW could be used to obtain short-term response statistics for a Floating, Production, Storage and Offloading (FPSO) vessel. These vessels have an operational profile that reflects long periods of zero speed and head sea, as they are moored.

Some of the ideas by Taylor et al. (1995) showed how the deterministic MLW embedded in a stochastic seaway, i.e. the CRW could be used to estimate the extreme dynamic response of a Jack-Up in a severe sea state have been in the present study.

By use of short wave sequences with the random background wave included, the distribution of the largest response associated with a crest of a given height can be determined. It is known that the distribution of large wave crests within a random sea state fits the tail of a Rayleigh distribution well, hence the marginal distribution of the non-linear VBM peak responses may be derived.

The following sections illustrate how the peak value distributions $F_{R_{NL}}(r \mid h_s, t_z, v, \beta)$ are derived. The obtained extreme value distribution is compared to the response due to the deterministic MLW and to a traditional (and very time-consuming) brute force time domain simulation. On the basis of the analyses made, bias factors are presented for estimation of the median (50 % level) of the most probable largest response in a given sea state of a specified duration and compared to the peak response of the MLW.
4.4.1 The Constrained Signals

Figure 4.10 show examples of a traditional stochastic wave signal by application of the PM and JONSWAP wave spectra. Moreover, the two constrained signals are plotted. The conditional peak occurs at \( t_0 = 100 \) sec. with a crest amplitude of 10.0 m. Very close to the conditional peak, the two signals are significantly different. The effect of applying the JONSWAP wave spectrum is more pronounced and extends further in time before the two signals merge again. This is reflected by the broad-banded nature of the wave field generated.

![Figure 4.10: Applied original and conditional random wave profiles using different wave spectra. The used spectra are PM with \( H_s = a = 10.0 \) m., \( T_z = 11.35 \) sec. and JONSWAP with \( H_s = a = 10.0 \) m, \( T_z = 11.63 \) sec. at \( t_0 = 100 \) sec.](image)

Figure 4.11 shows the constrained wave surface plotted as a function of time and space; the effect of the random background wave is observed by comparison to Figure 4.2.

The constrained crest of \( a = 10.0 \) m is found at \( t_0 = 0 \) and \( x_0 = 0 \). The surface is generated for the PM spectrum and is basically the surface the vessel will encounter for zero speed in head sea. Depending on the wave spectrum chosen, differences are found for the generated wave fields because of the changed spectral density. It is furthermore found difficult to observe differences between the conditional wave field from Figure 4.2 and a purely stochastic wave field except from the crest found in the middle of the plot.
4.5 Extreme Value Distributions

The response distributions \( F_{RNL}(r \mid a, h_s, t_z, v, \beta) \) and \( F_{RNL}(r \mid h_s, t_z, v, \beta) \) have been derived by application of short constrained simulations. \( F_{RNL}(r \mid a, h_s, t_z, v, \beta) \) represents a peak distribution where only one CRW amplitude \( a \) has been considered. The CRW amplitude has been chosen to equal the significant wave height \( H_s \). A wave amplitude of \( a = H_s \) corresponds roughly to the most probable largest wave amplitude during a period of approximately three hours.

\( F_{RNL}(r \mid h_s, t_z, v, \beta) \) covers simulations of a selected range of CRW with different crest amplitudes. From the peak distribution obtained, order statistics can be applied to derive for example the most probable largest response in a sea state of three hours.

Figure 4.12 shows examples of CRW, i.e. the MLW with the random background wave included and the corresponding non-linear response. The CRW profiles are simulated by application of the PM spectrum with \( H_s = a = 10 \) m and \( T_z = 11.35 \) sec. The two upper figures show the CRWs, whereas the corresponding non-linear responses are given on the lower plots. The figures to the left represent the situation where the vessel encounters the CRW at FP, whereas the figures to the right represent the case where the CRW peak is generated amidships, (\( \otimes \)). The black curves show 15 individual simulations (both wave and VBM responses), and the red curve represents the response due to the MLW profile.
The green curve represents the mean of the individual CRW simulations, which has been generated from 250 simulations. These short wave sequences represent roughly the most probable largest linear wave amplitude within a three hours’ return period in a completely random occurrence. The wave signals generated are seen to merge around the selected conditional time instant after which the wave profile returns to its original stochastic nature.

It is interesting to observe that the response due to the MLW profile and the average response of the 250 individual response simulations due to the CRWs are almost identical, which could indicate that the response generated from the MLW is a qualified candidate for prediction of the most likely response. A small phase shift between the red and green curves seems to develop towards the end of the time window showed. If the number of simulations is increased to $N = 500$, the tendency becomes less pronounced and will presumably vanish totally for $N \to \infty$.

Figure 4.12: Upper plots: Random wave simulations constrained to a crest of 10.0 m at $t_0 = 100$ sec. The figures to the left represent the situation where the vessel encounters the CRW profile at FP. To the right, the vessel encounters the CRW profile amidships, ($\circ$). The results of 15 constrained individual simulations are illustrated (black curves), the red curve represents the response due to the MLW profile itself, whereas the green line represents the mean of the 250 simulations due to the CRW profiles. Lower plots: The corresponding response amidships to the simulated conditional wave profiles.

It is found that the random response signals almost merge as the vessel passes the conditional wave crest at $t_0 = 100$ sec. By focusing on the largest responses, it is found that the periods
between individual response simulations are seen to be almost identical, and only the peak and trough values change.

### 4.5.1 Selection of Peak Responses

Significant non-linearities are introduced in the current sea state as seen in Figure 4.12. From a visual observation it is difficult to tell the difference of the peak responses given the location of the conditional wave relatively to the vessel - except from the phase shift. Slightly larger sagging peaks are introduced as the vessel encounters the CRW at FP compared to the case where the CRW is generated amidships. The opposite is true for the hogging responses, here the largest hogging responses are obtained as the CRW is generated amidships.

Simulations have also been made for severer sea states. For these sea states it was visually observed that the peak and trough responses showed considerable less variation than observed in Figure 4.12. It is believed that the effect of moving the CRW peak relative to the hull girder disappears due to the large non-linearities introduced. For less severe sea states differences in the location of the CRW peak relative to the hull girder become more pronounced for both the linear and non-linear, cases which indicates that the location of the CRW crest relative to the hull girder should be taken into account.

From the above simulations it seems most reasonable that sagging peak responses are selected as the vessel encounters the CRW at FP. The peak responses are selected to the same time instant, which for the present case corresponds to $t \sim 112.5$ sec. (same place as the peak response due to the MLW).

The hogging trough responses to be fitted are selected as the vessel encounters the CRW amidships. The responses are selected to the same time instant, which for the present case corresponds to $t \sim 100$ sec. (same place as the peak response due to the MLW)

### 4.5.2 Response Distribution, Part One

This subsection focuses on the peak response distribution $F_{R_{NL}}(r \mid a, h_s, t_z, v, \beta)$, where $r$ is the peak response, $a$ represents the CRW amplitude, $h_s$ is the significant wave height and $t_z$ the zero-upcrossing period, $v$ is the forward velocity and $\beta$ the heading angle.

On the basis of calculated non-linear responses due to the CRW, peak distributions conditional on a wave crest have been generated for the sagging and hogging conditions. The peak responses have been fitted to the asymptotic extreme value distribution from Eq. (2.40). Usually, the Gumbel distribution ($k=0$) is applied. The coefficients $\kappa$, $\alpha$ and $\beta$ are found by least square fitting. Other distributions as for example the Weibull distribution have been tested as well, but the best overall fit is obtained by the present distribution.
Figure 4.13 shows the fitted and cumulative peak distribution $F_{RNL}(r \mid a, h_s, t_z, v, \beta)$ for the non-linear responses due to the CRW wave with $H_s = a = 10.0$ m and $T_z = 11.35$ sec., all for the PM spectrum. The responses due to the deterministic MLW for the four cases are additionally shown by the arrows on the plot. The upper two plots illustrate the peak distribution for the hogging and sagging peaks given that the vessel encounters the CRW at FP. The sagging distribution is found to the left and hogging to the right. The lower two plots present the fitted distributions where the vessel encounters the CRW amidships.

Figure 4.13: Fitted and cumulative peak distribution $F_{RNL}(r \mid a, h_s, t_z, v, \beta)$ due to CRW profiles with $H_s = a = 10.0$ m and $T_z = 11.35$ sec, all for the PM spectrum, zero speed and head sea. The distributions have been fitted on basis of $N = 200$ (hogging) and $N = 300$ (sagging) simulations.

The distributions shown are obtained for zero speed and head sea. The hogging response distributions have been fitted on the basis of $N = 200$ simulations. A few more, namely $N = 300$ simulations, were used to obtain reasonable fitting of the sagging responses.
Good fits to the generalised extreme value distributions for the peaks responses are generally made, with the fitted distribution illustrated by the dashed curves. The 50% levels of the fitted peak distributions from Figure 4.13 have been compared. The results are as expected, the largest 50% levels for the sagging responses are obtained as the vessel encounters the CRW at FP, and the largest hogging response is obtained as the CRW profile is generated amidships.

Results have been generated for $H_s = 5.0, 10.0$ and 15.0 m with the PM and JONSWAP spectra. Table 4.4 lists responses corresponding to a 50% level of $F_{RNL}(r | a, h_s, t_z, v, \beta)$ for hogging and sagging by application of the PM spectrum for selected values of $H_s$. Moreover, the peak responses due to the MLW profile itself are listed. It is found that the peak responses due to the MLW are always biased to the lower side of the found 50% levels. Results of application of the JONSWAP spectrum are found in Appendix E, Table E.2, with the same trends observed again. However, higher response levels due to the longer transient period of the most likely waves are found.

Table 4.4: $H_s = a$ for all the cases. Peak responses due the MLW profile and responses corresponding to a 50% level of $F_{RNL}(r | a, h_s, t_z, v, \beta)$ for hogging and sagging. The PM spectrum is applied with $H_s = 5, 10$ and 15 m, $T_z = 11.35$ sec.

<table>
<thead>
<tr>
<th></th>
<th>PM</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linear response [MNm]</strong></td>
<td>Sag</td>
<td>Hog</td>
<td>Sag</td>
<td>Hog</td>
<td>Sag</td>
</tr>
<tr>
<td>$H_s$ [m]</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRW$_{FP}$</td>
<td>1740</td>
<td>1721</td>
<td>3518</td>
<td>3440</td>
<td>5295</td>
</tr>
<tr>
<td>MLW$_{FP}$</td>
<td>1730</td>
<td>1600</td>
<td>3457</td>
<td>3201</td>
<td>5189</td>
</tr>
<tr>
<td>CRW$_{\odot}$</td>
<td>1551</td>
<td>1755</td>
<td>3118</td>
<td>3495</td>
<td>4625</td>
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<tr>
<td>MLW$_{\odot}$</td>
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<td>1712</td>
<td>3155</td>
<td>3422</td>
<td>4735</td>
</tr>
<tr>
<td><strong>Non-linear response [MNm]</strong></td>
<td>Sag</td>
<td>Hog</td>
<td>Sag</td>
<td>Hog</td>
<td>Sag</td>
</tr>
<tr>
<td>$H_s$ [m]</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRW$_{FP}$</td>
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<td>4736</td>
<td>3210</td>
<td>7658</td>
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<td>4424</td>
<td>2841</td>
<td>7041</td>
</tr>
<tr>
<td>CRW$_{\odot}$</td>
<td>1978</td>
<td>1761</td>
<td>4550</td>
<td>3338</td>
<td>7475</td>
</tr>
<tr>
<td>MLW$_{\odot}$</td>
<td>1808</td>
<td>1662</td>
<td>4126</td>
<td>3151</td>
<td>6818</td>
</tr>
</tbody>
</table>

Figure 4.14 shows ratios of linear and non-linear hogging and sagging responses as given in Table 4.4. These are plotted as a function of $H_s$. The left figure is based on the responses due to the MLW profile; here the peak responses are simply compared. On the right side, the 50% levels of $F_{RNL}(r | a, h_s, t_z, v, \beta)$ are compared. For the selected cases an almost linear trend in the plotted ratios is found. Moreover, it is observed that there seems to be a linear offset between the ratios obtained on the basis of the MLW and the CRW profiles.
Chapter 4. Most Likely Wave Analyses

Figure 4.14: Ratios of the linear and the non-linear calculated vertical bending moment as function of $H_s$, all for the PM spectrum, $H_s = a$.

Figure 4.15 shows the ratio between the 50% level obtained from the fitted response distributions and the peak responses due to the MLW. The ratios are shown for both the linear and non-linear cases (sagging on the left side and hogging to the right). For hogging, an almost constant offset of $\sim 6\%$ for the non-linear case and $\sim 2\%$ for the linear case is found.

For the sagging responses a linear trend is observed as well. Bias factors between the two approaches are given as function of $H_s$ and obtained in an interval of 1.06 to 1.09.

The analyses have furthermore verified that the largest sagging responses are obtained as the vessel encounters the MLW at FP, and that the largest hogging responses are obtained with the MLW located amidships.
Results by Application of the JONSWAP Spectrum

Results have also been generated by application of the JONSWAP spectrum, which are found in Appendix E, Tables E.3 and E.4. Generally larger responses are obtained by application of the JONSWAP spectrum due to the larger transient period before the MLW peak occurs. The ratio between the 50% level obtained from the fitted response distributions and the peak responses due to the MLW due not change significantly as compared to the results by application of the PM wave spectrum.

4.5.3 Response Distribution, Part Two

The above section was focused on the estimation of the distribution $F_{RNL}(r \mid a, h_s, t_z, v, \beta)$, where $a$ was selected as the most probable largest crest within a period of roughly three hours. For design purposes, the distribution $F_{RNL}(r \mid h_s, t_v, v, \beta)$ would appear more correct as it takes the complete range of the large crest into account. It is known, Lindgren (1970), that the distribution of large wave crests in a random sea state fits the tail of a Rayleigh distribution well, hence the marginal distribution of the VBM peaks is derived:

$$f_{RNL}(r \mid h_s, t_z, v, \beta) = \int_{a_{\min}}^{a_{\max}} f_{RNL}(r \mid a, h_s, t_z, v, \beta) f(a \mid h_s, t_z) da$$ (4.1)

where $r$ is the response, $f_{RNL}(r \mid a, h_s, t_z, v, \beta)$ is the fitted distributions of the non-linear responses obtained from the simulations. $f(a \mid h_s, t_z)$ represents the Rayleigh distribution of the wave crests.

For evaluation of Eq. (4.1), an appropriate interval of $a$ is needed. A local trough could be present as conditioning on the curvature is skipped within the present model. A local trough would in theory most likely occur for lowest conditional amplitudes. Taylor et al. (1995) use a lower limit of $a_{\min}$ equal to $0.7H_s$, which has also been used in the present calculations. An appropriate upper limit for $a_{\max}$ has been selected from 1.7 to 2.0 $H_s$ (lowest for the largest significant wave heights). Eq. (4.1) may hereafter be rewritten to

$$f_{RNL}(r \mid h_s, t_z, v, \beta) = \int_{a_{\min}}^{a_{\max}} f_{RNL}(r \mid a, h_s, t_z, v, \beta) f(a \mid h_s, t_z) da$$ (4.2)

The introduced interval of $a$ results in a lower valid response level from where the extreme value distribution may be used. This is discussed later in the present section.

Figure 4.16 shows the results of many constrained simulations. The probability density functions $f_{RNL}(r \mid a, h_s, t_z, v, \beta)$ to be used in Eq. (4.2) are plotted in Figure 4.16. The figure shows the individual fitted extreme value distributions for the non-linear sagging and hogging responses due to CRW profiles conditional on crest heights of $a = 5$ to 20 m, at a step of 1 metre. For each CRW amplitude, the hogging distributions are fitted from $N = 200$ simulations, whereas $N = 300$ has been used for the sagging responses.
The points on the plot correspond to the probability of exceedance of response values from 5% to 95%. The extreme response probability density functions for the selected range of crest heights show the breadth of the extreme response at each crest, which reflects the effect of the random background wave of the CRW. The constant probability lines, which represents the fractiles selected are based on least square curve fitting. These curves are required for the convolution with the Rayleigh distribution for the crest height to provide the unconditional short-term extreme response distribution from Eq. (4.2). From Figure 4.16, it is furthermore seen how well the curves are fitted for the non-linear simulations.

Figure 4.16: Fitting to non-linear response distributions $f_{R_{NL}}(r \mid a, h_s, t_z, v, \beta)$ for a crest of a given amplitude $a$, all for the PM spectrum with $H_s = 10.0$ m and $T_z = 11.35$ sec, zero speed and head sea. The figure to the left represents the fitted hogging peaks, and sagging is shown to the right.

In Figure 4.16, a lower cutoff response level has been introduced. The cutoff response level indicates a lower valid boundary from where the probability density function may be derived, according to Eq. (4.2). The cutoff response level has generally been selected as the 95% fractile for the lowest CRW crest. If the response level in Figure 4.16 is moved further down, i.e. towards smaller response values, it is observed that the probability mass due to crest heights of $a = 4.0$ metres will not be included, as it has not been generated.

An upper cutoff response level could also be applied and should roughly be selected as the 5% fractile. For the present example, the contribution to the distribution $f_{R_{NL}}(r \mid h_s, t_z, v, \beta)$ from the crest elevation $a = 15$ to 20 metres is small due to the low probability of occurrences. The loss of probability mass is therefore limited. However, large waves do occur. Haver & Andersen (2000) report on substantial damage of the jacket platform Draupner when a giant wave ($H_{max} = 25.63$ m) of a crest height $\zeta_{crest} = 18.50$ m hit the platform structure at a water depth of 70 m on January 1$^{st}$, 1995. The significant wave height, $H_s$, was measured to be $11.92$ m, the maximum wave rises here to $H_{max} = 2.15H_s$ with a crest of $\zeta_{crest} = 1.55H_s$.

The contributions from the individual constrained peaks are illustrated by the stacked column plot in Figure 4.17. The figure shows the tail of the probability density function from
4.5 Extreme Value Distributions

Eq. (4.2). For the current case, it is observed that the major contributors at the selected response levels are found by means of CRW amplitudes of 6 to 12 metres, whereas the rest hardly influence the final result for the sea state concerned.

Figure 4.17: Tail of the probability density function \( f_{R_{NL}}(r | h_s, t_z, v, \beta) \). The contribution of each of the CRW crests is additionally illustrated as a bar under the solid line. The figure to the left represents the hogging responses, and sagging is shown to the right.

The extreme value distribution \( F_{R_{NL}}(r | h_s, t_z, v, \beta) \) of the independent simulated non-linear peak response may subsequently be found to be

\[
F_{R_{NL}}(r | h_s, t_z, v, \beta) = \int_{r_{\min}}^{r} f_{R_{NL}}(r | h_s, t_z, v, \beta) \, dr = 1 - \int_{r_{\min}}^{r} f_{R_{NL}}(r | h_s, t_z, v, \beta) \, dr \quad (4.3)
\]

where \( \delta = \int_{r_{\min}}^{\infty} f_{R_{NL}}(r | h_s, t_z, v, \beta) \, dr \).

Order statistics, where the probability distribution of the maximum peak among \( N \) peaks could be found according the Eq. (2.54) and Eq. (2.55).

The extreme value distribution may also be shown by upcrossing rates \( \nu_\gamma(r) \) through a given response level \( r \). Upcrossing rates for the linear response are found by their spectral moments as given in Eq. (2.63). The upcrossing rate for the non-linear responses is obtained on the assumption of a narrow-banded spectral density process as

\[
\nu_\gamma(r) = \nu_0 \left[ 1 - F_{R_{NL}}(r | h_s, t_z, v, \beta) \right] \quad (4.4)
\]

where \( F_{R_{NL}}(r | h_s, t_z, v, \beta) \) is found from Eq. (4.3). \( \nu_0 \) represents the zero upcrossing rate from Eq. (2.51).

Figure 4.18 shows the upcrossing rates of the vertical bending moment amidships. The non-linear hogging and sagging responses due to the deterministic MLW profile are shown as the
Chapter 4. Most Likely Wave Analyses

filled black dots. The amplitude of the MLW was chosen to correspond roughly to the most probable largest crest amplitude within a period of \( \sim 3 \) hours, i.e. \( a = H_s \).

From the results presented in Figure 4.18 it is seen that the peak response due to the deterministic MLW with an amplitude of \( a = H_s \) is found to be bias with a factor of 1.2 as compared to results of the CRW approach from a fully derived extreme value distribution. This upcrossing rate represents the response level that corresponds to the median (50%) level of the cumulative distribution of the largest peak in a period of approximately three hours. Moreover Figure 4.18 shows the results of 600 x 1 hours’ brute force simulations. The upcrossing rates are calculated for response levels of 0 to \( \sim 8000 \) MNm with a step of 200 MNm. A fine correspondence is found between the upcrossing rates obtained by results of brute force simulations and the generated upcrossing rates by application of the CRW approach for the current sea state and operational profile. A slightly better fit to results of brute force simulations is obtained for the hogging responses compared to sagging.

![Upcrossing rates](image)

**Figure 4.18:** Linear and non-linear upcrossing rates. The upcrossing rates are obtained by application of the PM spectrum with \( H_s = 10 \) m and \( T_z = 11.35 \) sec. The simulations are performed for zero speed and head sea. The response due to the deterministic MLW is also found. The results are compared to results obtained from a brute force simulation.

For dynamically responding structures like a container ship, the deterministic MLW therefore seems not directly useful for estimation of extreme responses. The same observation was made by Taylor et al. (1995), when they studied Jack-Up dynamics.

A large number of simulations have been performed by application of the CRW approach. For each of the peak distributions obtained, extreme value distributions have been derived
4.5 Extreme Value Distributions

according to Eq. (2.54). Hereafter 50% levels of the extreme response in a period of approximately three hours’ have been selected. The results are listed in Table 4.5. The responses are obtained by applying the PM spectrum with $H_s = 5.0$, 10.0 and 15.0 m and $T_z = 11.35$ sec. The sagging distributions are obtained where the vessel encounters the CRW at FP, whereas the hogging responses are found by locating the CRW peak amidships. All simulations are made for head sea and zero speed.

Table 4.5: Medians (50% levels) of $F_{R_{max}}(r | h_s, t_z, v, \beta)$ for extreme responses in a period of three hours. The PM spectrum is applied for selected sea states, zero speed and head sea.

<table>
<thead>
<tr>
<th>$H_s$ [m]</th>
<th>Sag</th>
<th>Hog</th>
<th>Sag</th>
<th>Hog</th>
<th>Sag</th>
<th>Hog</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRW&lt;sub&gt;FP&lt;/sub&gt;</td>
<td>1985</td>
<td>1865</td>
<td>3830</td>
<td>3740</td>
<td>5615</td>
<td>5605</td>
</tr>
<tr>
<td>CRW&lt;sub&gt;⊙&lt;/sub&gt;</td>
<td>1817</td>
<td>1890</td>
<td>3652</td>
<td>3740</td>
<td>5452</td>
<td>5518</td>
</tr>
</tbody>
</table>

From Table 4.5 a difference of $\sim 2 - 5\%$ in the calculated vertical bending moment is still found by moving the CRW peak from a midships position to FP. Ratios of the non-linear and linear responses from Table 4.5 have been compared and provide to some extent results similar to those seen in Figure 4.14. The ratios are given in Table 4.6, where CRW<sub>FP</sub> represents sagging and CRW<sub>⊙</sub> hogging. The analyses have been made by application of the JONSWAP spectrum as well.

Table 4.6: Ratios of the linear and the non-linear calculated vertical bending moment as a function of $H_s$, by application of the PM spectrum.

<table>
<thead>
<tr>
<th>$H_s$</th>
<th>5.0</th>
<th>10.0</th>
<th>15.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sagging, CRW&lt;sub&gt;FP&lt;/sub&gt;</td>
<td>1.17</td>
<td>1.39</td>
<td>1.52</td>
</tr>
<tr>
<td>Hogging, CRW&lt;sub&gt;⊙&lt;/sub&gt;</td>
<td>0.99</td>
<td>0.95</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Figure 4.19 shows ratios of medians (50% levels) of $F_{R_{max}}(r | h_s, t_z, v, \beta)$ for extreme responses
in a period of three hours to peak responses due to the deterministic MLW for selected values of $H_s$. The MLW amplitude is given as $a = H_s$ which almost correspond to the most probable largest wave amplitude in period of three hours for the given zero-upcrossing period. The figure to the left presents the ratio between linear and non-linear sagging response, whereas the figure to the right presents the similar results for the hogging responses.

From the non-linear results it is seen that the MLW model is biased with factors from 1.15 to 1.22 for sagging and 1.13 to 1.16 for hogging as related to results from the CRW approach. Some variations are found for the selected sea states and additional analyses will presumably show that further variations will occur as forward speed and different headings are introduced.

It seems therefore difficult to apply the deterministic MLW profile as predictor for extreme responses in a long-term response analysis as bias factors should be established for each sea state and operational profile.

Results by Application of the JONSWAP Spectrum

Results have also been generated by application of the JONSWAP spectrum, which are found in Appendix E, Tables E.5, E.6, and E.7. The results by application of the JONSWAP spectrum is to some extend similar to results by application of the PM spectrum. It is characteristic that the bias factors to be applied between the MLW and CRW approaches are smaller. This is mainly an identification of the importance of the transient period of the wave before the conditional response occurs and that the MLW is not established to capture the important memory effects. The MLW model is discussed further in relation to the MLRW approach in Chapter 5.
4.6 Concluding Remarks

The simulations have shown that from a fairly small number of simulations using the CRW, i.e. the MLW with the random background wave included, it is possible to generate useable extreme value response characteristics. The derived extreme value distributions for zero speed and head sea conditions have been compared to brute force simulations with good agreements obtained for the selected sea state.

Bias factors between medians (50 % levels) of extreme responses in a period of three hours established from the CRW approach to peak responses due to the deterministic MLW for selected values of $a = H_s$ have been established. The bias factors were found to be in the range of 1.07 to 1.15 for the linear model and 1.12 to 1.22 for the non-linear model by application of the PM wave spectrum. Slightly lower ratios have been obtained by application of the JONSWAP wave spectrum. The results have indicated that the random background waves for dynamically responding structures can be important, and that the specific wave amplitude not necessarily is the only parameter to be considered as extreme responses are sought.

The simulations performed have indicated that the location of the MLW relative to the vessel is important. Given that the vessel has encountered the MLW crest physically, the largest linear and non-linear sagging responses are obtained if the vessel meets the MLW at the forward part of the vessel and most often at FP. The largest hogging responses are found if the MLW is located amidships.

If a larger number of vessels were examined in different sea states and for selected operational profiles, it would presumably be possible to formulate guidelines for application of a factor related to the peak response due to the deterministic MLW, after which a design response would be available. The method has one drawback as it is sensitive to the selected zero up-crossing period and that the MLW is not designed to capture the important memory effects. Thus, a significant effort should be spent on the correct location of the MLW peak relative to the hull girder and to unconditioning the results for the zero-upcrossing period.

The next chapter overcomes this problem well by the introduction of most likely response waves, which takes into account the structure considered.
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Chapter 5

Most Likely Response Wave Analyses

5.1 Introduction

Fast and accurate methods for estimating the non-linear extreme value ship response statistics using 2-D or 3-D time domain codes are of interest for the industry. Such procedures will be helpful for more reliable estimation of a vessel's actual safety level and thus guide the industry how to design safer and more economic ships. This study presents a new approach termed the Most Likely Response Waves (MLRW), by use of which it becomes possible to estimate the entire non-linear extreme response value distribution for a selected operational profile.

The analyses in Chapter 4 focused on application of the Most Likely Wave (MLW) profile as a critical wave episode for estimation of the non-linear extreme peak responses. The analyses indicated that the peak responses due to the MLW were significantly biased as compared to results from brute force simulations. From the MLW analyses it was furthermore found that the correct location of the MLW peak relative to the hull girder was important. Application of conditional MLW profiles was therefore considered inadequate for estimation of peak response distributions as information on the vessel considered is neglected and as the random background wave of the Conditional Random Wave (CRW) influenced significantly on the peak responses obtained.

By application of MLRW this problem is taken into account as the wave profile is established with information on the linear transfer function. Now, given that the MLRW established from the linear model is a good predictor of the non-linear extreme events, a very powerful approach for estimation of extreme value statistics become available, Adegeest et al. (1998).

The idea of using the linear model as a predictor for the non-linear response is inspired by the so-called Model Correction Factor Method, Ditlevsen & Arnbjerg-Nielsen (1991). The idea of that procedure is to use a simple idealised model that is fast and captures part of
the real non-linear model. At specific points (here: selected up-crossing probability levels) of the stochastic variables the linear model is "model corrected" such that it becomes equal to the non-linear response. The factor that must be applied on the linear response to make it equal to the non-linear response is called the model correction factor. Clearly the model correction factor will in general be a function of all the stochastic variables in the model. Ditlevsen & Arnbjerg-Nielsen (1991) illustrated that for an appropriately chosen simplified model zero or first order approximation at carefully selected points in general will result in satisfactory results. Friis-Hansen (1994) applied the model correction factor method in the evaluation of the ultimate hull bending capacity. In that study a zero order approximation gave very good agreement with the full non-linear model.

In this study the zero order model is equivalent to the response predicted by the MLRW whereas the higher order model is represented by the Conditional Random Response Wave (CRRW). The bias factor that is presented in this study is the ratio between the higher order response model (CRRW) and and the zero order response model (MLRW). Hence, the bias factor describes the "goodness" of the zero order model correction factor for identifying the non-linear response from a linear response.

Furthermore, since a linear response model can capture only a smoothly varying subset of non-linear transients it is expected that the non-linear response calculated from the MLRW will underestimate (for sagging moment - overestimate for the hogging moment) the "true" non-linear response at the analysed up-crossing probability level. Averaging results from the application of the CRRW model will give the "true" non-linear response level.

During the first part of the present chapter a parameter study by application of MLRW is performed. Secondly, establishment of short-term response statistics by application of MLRWs and CRRWs is given and compared to results of brute force simulation. Hereafter the results obtained are evaluated and bias factors for the model correction factor method is provided. The approaches presented are discussed on assessment of long-term response statistics in Chapter 7.

The numerical results are obtained for a Panmax container ship by means of the non-linear time domain sea keeping code ShipStar, Xia et al. (1998). The analyses performed are made for a rigid hull girder whereas a flexible hull girder is considered in Chapter 6.

### 5.2 Parameter Study, Most Likely Response Waves

The analyses performed in this section show simulations of the linear constrained response and the corresponding MLRW profile. To the left in Figure 5.1 results generated by application of the PM spectrum are shown. The JONSWAP spectrum has been applied throughout the simulations performed on the right side. For both cases zero speed and head sea with $H_s = 10.0$ m and $T_z = 11.35$ sec. are applied.
In Figure 5.1 there are 2 x 2 plots, of which the uppermost plots illustrate the MLRWs which generate a constrained linear response of 5000 MNm. The lowest plots show the corresponding non-linear response due to the MLRW. For the two cases presented, it is characteristic that the vessel encounters a deep trough amidships at $t_0 = 100$ sec. Thus, a large sagging response is introduced as seen from the corresponding response at $t_0 = 100$ sec. The effect of applied wave spectrum is observed on the MLRW. By application of the JONSWAP spectrum, a longer transient growth of the wave profile is seen with a corresponding larger non-linear response of 7302 MNm compared to the non-linear response of 6890 MNm, which is obtained from the PM spectrum.

![MLRW with the PM (left) and the JONSWAP spectrum (right) with $H_s = 10$ m and $T_z = 11.35$ sec. The linear constrained VBM is 5000 MNm, whereas the non-linear peak response equals 6890/7302 MNm. The upper figures illustrate the wave as a function of time and recorded amidships.](image)

It is interesting to compare non-linear peak responses by application of MLWs in Figure 4.4 with the non-linear peak responses in Figure 5.1. It is observed that large differences in wave amplitude are present given that roughly the same non-linear response level is achieved. This observation illustrates how memory effects are better captured by application of the MLRW.

Figure 5.2 shows a small “cartoon” of the instantaneous position of the vessel relative to MLRW as it approaches the conditional time instant $t_0$. The figure are made for a conditional sagging response of 5000 MNm for $v = 0, 5$ and $10$ m/s by application of the PM spectrum. The snapshots are taken to five individual time steps from $t = 73$ sec. to $t = 100$ sec., where the conditional linear response occurs. The individual five time instants correspond to the peak responses in the time series of the vertical bending moment, (see for example the lower left plot in Figure 5.1, $v = 0$ m/s).
It is characteristic that the transient phase of the wave encountered tend to look identical for the three velocities selected. Wave amplitudes become slightly lower and it is important to observe that bottom slamming do not occur at the current response level. As forward speed is introduced, the total time where the vessel is exposed to waves become naturally reduced. This is a general observation for head sea, very similar snapshots can be established by application of different zero-upcrossing wave periods and significant wave heights. The wave amplitudes observed become naturally smaller as a lower response level is considered. For hogging, the trend is the same where almost identical wave patterns before the conditional hogging moment occurs can be sampled.

Figure 5.2: MLRW relative to the instantaneous position of the vessel. The snapshots are taken to five individual time steps from $t = 50$ sec. to $t = 100$ sec. The conditional linear response occurs at $t_0 = 100$ sec.

To the left in Figure 5.3, the instantaneous position of the vessel relative to the MLRW is
shown. The simulations are made for selected forward speeds in head sea with the same sea state. The snapshots are taken at the conditional time instant \( t_0 \) of the prescribed linear sagging bending moment. The constrained sagging moment occurs when the vessel is basically supported on two wave crests. Symmetry in the MLRW profile is almost found around the midships section.

It is interesting to observe that the MLRW becomes smaller for increasing forward velocities given that the conditional linear response level is kept constant. This furthermore indicates that the MLRW is “designed” using the relevant linear transfer function and that the wave captures memory effects and effects of forward speed.

![Critical waves along the hull for constrained sagging and hogging responses obtained on the basis of the MLRW approach, by applying the PM spectrum with \( H_s = 10.0 \text{ m} \) and \( T_z = 11.35 \text{ sec.} \) for selected velocities in head sea.](image)

In the right column of Figure 5.3 the snapshots are taken at the time instant \( t_0 \) of prescribed linear hogging bending moment. The constrained hogging moment occurs when the vessel is supported by the wave amidships and it is found to be of similar shape as compared to the MLW profile from previously.

It is found that the MLRW crest moves slowly towards FP and decreases slightly in amplitude for increasing velocities. The vessel is located almost in the same position relative to the mean water level for the three velocities selected.

Figure 5.4 shows the MLRW relative to the hull girder for constrained sagging responses of 5000 MNm (left) and hogging responses of -5000 MNm (right) by application of the PM spectrum with \( H_s = 10.0 \text{ m} \) for different values of \( T_z \) in head sea and zero speed. \( T_z \) has
been selected in the range 10.0 to 14.0 sec., which is a representative range for the selected significant wave heights.

For the three sea states selected, similarities are found for the plotted MLRW. The wave profiles change slightly in length with corresponding small changes of the wave amplitudes. Anyhow, the MLRW approach does not give rise to the problem of locating the critical wave correctly along the hull girder as what was the case for the MLW approach.

Figure 5.4: Critical waves along the hull for constrained sagging or hogging responses obtained from the MLRW by applying the PM spectrum with $H_s = 10.0$ m for selected values of $T_z$ in head sea and with zero speed.

Figure 5.5 shows the MLRW relative to the hull girder for constrained responses of 5000 MNm, for two wave headings and zero speed. Sagging is illustrated to the left and hogging to the right. The overall trends are the same. The MLRW that generates a sagging response increases slightly in length for $\beta = 135$ degrees with corresponding deeper troughs.

MLRWs simulated for a selected range of constrained sagging and hogging responses in the same sea conditions have additionally been studied. The wave height naturally increases as a function of the constrained response level. This procedure has of course its limitations as the wave steepness sooner or later will exceed the valid range for applying linear wave theory.
5.3 CRRW, Short-Term Response Statistics

The present section focuses on establishment of short-term response statistics by application of the Most Likely Response Wave (MLRW) and the Conditional Random Response Wave (CRRW) profiles. Results of the two models are compared to results of brute force simulations and later related to the Model Correction Factor Method.

The basis for deriving non-linear short-term response statistics using the CRRW approach is outlined by the following steps.

1. Select a stationary sea state with \((h_s, \ell_z)\) and the operational profile \((v, \beta)\) to be examined.

2. Calculate the linear response amplitude operator of the selected response to obtain amplitude and phase information.

3. Perform the constrained linear response calculations (Eqs. (3.61) to (3.65)) and derive the mean constrained coefficients \(\overline{V}_{c,n}\) and \(\overline{W}_{c,n}\).
4. Use the linear CRRW in a non-linear analysis. It is thus assumed that the linear response is a good identifier for the location of the maximum non-linear response using the random constrained coefficients $V_{c,n}$ and $W_{c,n}$.

5. Select an appropriate response range and perform roughly 100 constrained simulations for each of them.

6. Fit the extreme value distribution to the non-linear peak responses for each of the conditional linear response levels. The probability distribution $F_{R_{NL}}(r_{NL} | h_s, t_z, v, \beta)$ for the non-linear peak response is found by unconditioning with respect to the linear response, thus

$$F_{R_{NL}}(r_{NL} | h_s, t_z, v, \beta) = \int_0^\infty F_{R_{NL}}(r_{NL} | R_L = r_L, h_s, t_z, v, \beta) f_{R_L}(r_L | h_s, t_z) \, dr_L \quad (5.1)$$

where $r_{NL}$ is the non-linear response given the linear response level $r_L$. $F_{R_{NL}}(r_{NL} | R_L = r_L, h_s, t_z, v, \beta)$ is obtained by the simulation in step 5 for the individually selected response amplitudes. $f_{R_L}(r_L | h_s, t_z)$ is the Rayleigh distribution with variance $m_0$, obtained from the linear response spectrum as given in Eq. (2.24).

The extreme value distribution $F_{R_{NL}}(r_{NL} | h_s, t_z, v, \beta)$ is obtained by unconditioning the selected range of constrained response simulations. The unconditioning process is done on the assumption that the linear response peaks can be suitable modelled by a Rayleigh distribution.

Figure 5.6 represents a simple Bayesian network where the relation between the linear and non-linear responses through the sea environment and the vessel is illustrated.

The amplitudes of a stationary zero-mean Gaussian distributed response process are described by the Rice distribution, Rice (1945), as given in Eq. (2.45). In the case of a narrow-banded response spectrum, $\epsilon \to 0$, the amplitudes become Rayleigh distributed, as given in Eq. (2.36). Also for extreme amplitudes $u >> 1$, the distribution tends towards a Rayleigh distribution. The Rayleigh distribution is used for the unconditioning of the extreme response, as the band width of the linear response is small and large responses are
sought. Note that the non-linear peak responses are to be sampled in the vicinity of the conditional time instant, \( t_0 \). Sampling of peak responses are discussed further in relation with Figure 5.8.

The main advantages of application of the MLRW or CRRW approach are:

- The approach is based on a full wave spectrum (the entire frequency range can be applied).
- The approach is based on a frequency dependent linear transfer function and this contains linear memory effects.
- As the random background wave is included in the CRRW, transient effects not captured by the linear model like those from whipping are accounted for.
- The approach allows a detailed non-linear response distribution to be made.

The method requires only a limited number of non-linear calculations in each stationary sea state and thus introduces the possibility for generating long-term extreme value distributions on the basis of detailed non-linear time domain simulations.

Several questions may now be asked:

- What is the effect of the random background wave from the CRRW?
- Will the deterministic MLRW profile be a sufficient estimator for the mean of the most likely non-linear extreme response or will the predictor be biased again?
- Will the effect of forward speed be captured well by application of the MLRW?

It is furthermore important to verify that the steepness of the MLRW generated stays within a valid range. This may simply be checked by observing the ratio of the wavelength and the height from the waves generated. However, to steep waves have not been a problem for the current analyses.

Effects of whipping and springing events on the hull girder will moreover depend on the random background wave in the MLRW. This will be discussed in Chapter 6.

### 5.3.1 CRRW Responses

Figure 5.7 shows simulations of application of the CRRW model, with analyses made for the Panmax container ship as described previously. The main dimensions are given in Table 4.1 with further information on the vessel provided in Appendix B.
Example of Numerical Results

The main principles of generation of non-linear responses by application of CRRWs are given in Figure 5.7. For the present example, the vessel operates with zero speed in a head sea characterised by a PM wave spectrum with $h_s = 10.0$ m and $t_z = 11.35$ sec.

![Images of numerical results showing linear and non-linear responses.

Figure 5.7: Upper figures: 10 conditional linear responses with a response peak of 5000 MNm (sagging, right) and -5000 MNm (hogging, left) at $t_0 = 100$ sec. Middle figures: The corresponding CRRWs. Lower figures: Non-linear response derived using these waves. The mean is given by the red curves.

The figures to the left show conditional hogging simulations, whereas the figures to the right represent sagging conditions. The uppermost plot shows ten conditionally independent linear constrained responses (black curves).

The plot in the middle illustrates the generated MLRW and CRRWs. The wave process which generates the conditional sagging response of 5000 MNm at $t_0 = 100$ sec. is seen to develop from a purely stochastic signal, after which it builds up to a set of large nearly
deterministic peaks and one trough. The red curve represents the mean of the sample curves and is similar to the MLRW, if \( N \) is large. Similar observations can be made for the constrained hogging analyses, which seem almost mirrored around the horizontal \( x \)-axis. The constrained hogging vertical hogging moment equals -5000 MNm. The lowest plot in Figure 5.7 shows the non-linear responses due to the derived CRRW for both hogging and sagging. Some variation around the mean is found, but for the present sea state and operational parameters this is considered small. It is interesting to note that the CRRW only show a very limited variation in the interval \( \pm 20 \) sec. around the time point of conditioning.

Figure 5.8 shows a zoom of the non-linear response at the constrained time instant \( t_0 \). Note that the non-linear peak responses in theory should be sampled at \( t = t_0 \) as the conditional variance is zero here. From simulations it is found that the largest peak responses occur in a small time window of approximately \( \pm 2.0 \) sec. By selecting the peak responses here a small error will be introduced when the unconditioning is performed, but as the conditional variance remains close to zero some seconds before and after \( t_0 \) the error introduced becomes negligible.

![Non-linear hogging VBM amidships](image1)
![Non-linear sagging VBM amidships](image2)

Figure 5.8: A zoom of the non-linear responses from the lowest graph in Figure 5.7.

It is furthermore interesting to observe how the non-linear responses show significantly less variation in the vicinity of the conditional time instant \( t_0 \) compared to the non-linear responses due to the MLW, as previously seen in Figure 4.12.

Generally, it may be concluded that the non-linear responses are controlled very well at the vicinity of the conditional time instant \( t_0 \). The simulations from Figure 5.7 represents a general picture of non-linear responses sampled by application of CRRWs as \( h, t_z, v, \beta \) and the conditional response level are changed.

### 5.3.2 Calculation Procedures

The MLRW model described in Section 3.6.1 may be used to calculate non-linear extreme responses. The calculations may be performed by application of the MLRW or the CRRW
with the random background wave included. The results of both techniques have been compared to a brute force simulation of 600 x 1 hour in duration. The non-linear brute force simulations represent statistically independent time series, which have been generated for head sea conditions and a selected forward speed with $H_s = 10.0 \text{ m}$ and $T_z = 11.35 \text{ sec}$. The two methods are further discussed for use within a long-term response assessment in Chapter 7. The focus throughout the present study is kept on the vertical bending moment amidships.

Figure 5.9 shows the results of a large number of constrained hogging (to the left) and sagging (to the right) simulations. The probability density functions of the non-linear peak response are plotted as functions of the linear constrained response. The fractile levels show the variability in the extreme non-linear responses at each linear response level, which reflects the importance of the random background wave of the MLRW.

![Figure 5.9: Results of several constrained response calculations. The non-linear hogging (to the left) and sagging (to the right) responses given the CRRW are plotted as functions of linear constrained responses. The 5, 25, 50, 75 and 95 % fractiles are given for the individually fitted probability density functions.](image)

The variation of the non-linear responses increases as a function of the linear response levels for both hogging and sagging. The variations are found to be smaller compared to the previous observations in Figure 4.16, where the MLW profiles were applied. The constant probability lines represented through the fractile levels selected are based on least square curve fitting. A lower cutoff response level representing the lowest level for which upcrossing rates are calculated is introduced figure 5.9. The cutoff response level indicates a lower valid boundary for the derived probability distribution. Lower values cannot be used due to the assumption of a maximum linear peak at $t = t_0$. Eq. (5.1) may therefore be rewritten as

$$F_{RNL}(r_{NL} \mid h_s, t_z, v, \beta) = 1 - \delta + \int_{r_{min}}^{r} F_{RNL}(r_{NL} \mid R_L = r_L, h_s, t_z, v, \beta) f_{RL}(r_L \mid h_s, t_z) \, dr_L$$  \hspace{1cm} (5.2)
where \( \delta = \int_{r_{\text{min}}}^{\infty} F_{R_{NL}}(r_{NL} \mid R_L = r_L, h_s, t_z, v, \beta) f_{R_L}(r_L \mid h_s, t_z) \, dr_L \). The cutoff response level has generally been selected as the 95\% fractile of the probability density distribution generated from the lowest constrained linear response.

Considering the sagging responses, it is seen that by moving the cutoff response level towards a lower non-linear response level, probability mass due to linear conditional crest responses less than 2500 MNm will be omitted.

The asymptotic extreme value distribution, as given in Eq. (2.40) have been fitted to the non-linear peak responses of the individual conditional probability density functions. The coefficients to be fitted are obtained by least square fitting. Usually, the Gumbel distribution \((k = 0)\) is obtained.

Figure 5.10 shows an example of the peak distributions for the non-linear hogging (to the left) and sagging (to the right) induced responses. The non-linear peak distributions are calculated at the same time step as the generated linear conditional response. The results are obtained for zero speed and head sea.

![Figure 5.10: The cumulative distribution functions of the non-linear hogging (to the left) and sagging (to the right) peaks. Full lines: Direct calculations, dashed lines: Extreme value distribution fit.](image)

Figure 5.11 illustrates upcrossing rates for linear and non-linear hogging and sagging responses in a sea state of \( H_s = 10.0 \) m and \( T_z = 11.35 \) sec. The results are given for zero speed and head sea conditions. The linear results are given as the solid line and represent a good reference level for the hogging and sagging results. The sagging responses are characterised by positive values.

The upcrossing rates \( \nu_0(r) \) for the linear response in Figure 5.11 are found from their spectral moments and given as

\[
\nu_0(r) = \nu_0 \exp \left[ -\frac{r^2}{2m_0} \right]
\] (5.3)
Chapter 5. Most Likely Response Wave Analyses

where $\nu_0$ represents the zero-upcrossing rate and is given as

$$\nu_0 = \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}} \quad (5.4)$$

with the spectral moments $m_n$ from Eq. (2.24). For the non-linear response, the upcrossing rate is calculated on assumption of a narrow-banded spectral density as

$$\nu_\eta(r) = \nu_0 \left[ 1 - F_{R_{NL}}(r \mid h_s, t_z, v, \beta) \right] \quad (5.5)$$

where $F_{R_{NL}}(r \mid h_s, t_z, v, \beta)$ is obtained from Eq. (5.2).

The results of the MLRW and CRRW analyses have been compared to results of brute force simulations through upcrossing rates. From 600 hours of simulation, upcrossing rates are calculated for response levels of 0 to $\sim 8000$ MNm with a step of 200 MNm. Non-linear effects are clearly observed for both the hogging and the sagging responses. It is seen that the results from the brute force simulations (the rightmost curve) agree well with the results obtained by application of the MLRW profiles and slightly better using the CRRW approach, where the random background wave is included. For practically the entire probability level of exceedance the CRRW approach fits almost perfectly to results of the brute force simulation.

![Upcrossing rates](image)

Figure 5.11: Upcrossing rates on the basis of the MLRW approach and brute force simulations for zero speed and head sea. The results are generated from the PM spectrum with $H_s = 10.0$ m and $T_z = 11.35$ sec.

Results from application of MLRW profiles are generated very fast. The present results are based on less than 15 sagging and hogging constrained non-linear time domain simulations, each of a duration of approximately 100 sec.
The constrained sagging simulations in the CRRW method are generated for response levels of 2500 to 6000 MNm, at steps of 250 MNm. For each of the constrained bending moments, 250 individual simulations have been performed, after which the non-linear peaks have been fitted to the generalised extreme value distribution from Eq. (2.40). For the constrained hogging responses, the simulations have been generated for response levels of -5500 to -2500 MNm, at steps of 250 MNm. Here, 100 simulations were performed for each linear response level to obtain consistent data fitting.

The upcrossing rate generated by the MLRWs fits well to the results from brute force simulations and only small differences are present for the present example. It should be noted that the linear model determines the MLRWs, which means that these waves for example are not constrained to reflect non-linear contributions like slamming and green water on deck. Similarly, it should be mentioned that added mass and damping in the non-linear simulations are determined using the instantaneous wave elevation along the hull girder, whereas the MLRW is derived from added mass and damping terms calculated to the design draught as usually seen in linear strip theory. For the present case, hardly any differences between results obtained by application of the MLRW and CRRW model are seen.

From the above, it is seen that the application of MLRW is a promising procedure for further use in relation to the estimation of a long-term extreme value distribution. The evaluation of the MLRWs is further extended by studying the effect of forward speed and different heading angles.

5.3.3 Effect of Vessel’s Speed

Figures 5.12 and 5.13 show the effect of forward speed. Upcrossing rates are illustrated of non-linear peak responses generated from the MLRW and CRRW profiles.
Chapter 5. Most Likely Response Wave Analyses

Figure 5.12: Upcrossing rates on the basis of MLRW simulations and brute force simulations for $v = 5$ m/s and $\beta = 180$ deg. The results are generated from the PM spectrum with $H_s = 10.0$ m and $T_z = 11.35$ sec.

The simulations are compared to brute force simulations for $v = 5$ and $10$ m/s with a fine agreement obtained for both hogging and sagging. The upcrossing rates for the CRRW approach fit better to the results of brute force simulations than those of the MLRW.

Figure 5.13: Upcrossing rates on the basis of MLRW simulations and brute force simulations for $v = 10$ m/s and $\beta = 180$ deg. The results are generated from the PM spectrum with $H_s = 10.0$ m and $T_z = 11.35$ sec.
The effect of the random background wave on the MLRW profile is observed and most pronounced for the lowest upcrossing rates and for increasing speed, and found most pronounced for sagging. Bias factors for both hogging and sagging are given and discussed in Section 5.4.1. Anyhow, it is characteristic that transient effects are captured less well by the MLRW model, which is an issue for further studies.

From the results given in Figures 5.11 to 5.13 it is characteristic that upcrossing rates by application of brute force simulations roughly is limited to an upcrossing level of $10^{-6}$ with corresponding very few crossings and associated statistical uncertainties introduced.

By application of the MLRW or the CRRW approaches it become easier to establish reliable information on the upcrossing rates for the highest response levels as unconditioning simply is made by the Rayleigh distribution.

Non-linear effects are captured as illustrated in Table 5.1 where sag/hog ratios are compared given an upcrossing level of $\nu_{\eta}(r | h_s, t_z, v, \beta) \approx 10^{-5}$.

Table 5.1: Comparison of upcrossing rates at $\nu_{\eta}(r | h_s, t_z, v, \beta) \approx 10^{-5}$ for selected extreme value prediction methods. The results are obtained by application of the PM wave spectrum with $H_s = 10.0 \, \text{m}$, $T_z = 11.35 \, \text{sec.}$, head sea, $v = 0$, 5 and 10 m/s.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Hogging, [MNm]</th>
<th>Sagging, [MNm]</th>
<th>Sag/Hog</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v = 0 , \text{m/s}$</td>
<td>4257</td>
<td>4257</td>
<td>1.00</td>
</tr>
<tr>
<td>$v = 5 , \text{m/s}$</td>
<td>4475</td>
<td>4475</td>
<td>1.00</td>
</tr>
<tr>
<td>$v = 10 , \text{m/s}$</td>
<td>5137</td>
<td>5137</td>
<td>1.00</td>
</tr>
<tr>
<td>MLRW</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$v = 0 , \text{m/s}$</td>
<td>3720</td>
<td>5735</td>
<td>1.54</td>
</tr>
<tr>
<td>$v = 5 , \text{m/s}$</td>
<td>3480</td>
<td>6028</td>
<td>1.73</td>
</tr>
<tr>
<td>$v = 10 , \text{m/s}$</td>
<td>3205</td>
<td>6290</td>
<td>1.96</td>
</tr>
<tr>
<td>CRRW</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v = 0 , \text{m/s}$</td>
<td>3618</td>
<td>5860</td>
<td>1.62</td>
</tr>
<tr>
<td>$v = 5 , \text{m/s}$</td>
<td>3570</td>
<td>6265</td>
<td>1.75</td>
</tr>
<tr>
<td>$v = 10 , \text{m/s}$</td>
<td>3360</td>
<td>7158</td>
<td>2.13</td>
</tr>
<tr>
<td>Brute force</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$v = 0 , \text{m/s}$</td>
<td>3830</td>
<td>5811</td>
<td>1.52</td>
</tr>
<tr>
<td>$v = 5 , \text{m/s}$</td>
<td>3577</td>
<td>6520</td>
<td>1.82</td>
</tr>
<tr>
<td>$v = 10 , \text{m/s}$</td>
<td>3468</td>
<td>7254</td>
<td>2.09</td>
</tr>
</tbody>
</table>
5.3.4 Effect of Bow Quartering Seas

Figure 5.14 shows the effect of bow quartering seas. The trend as observed for head sea illustrated in Figure 5.11 is obtained again. The MLRW and CRRW approaches agree well for hogging and sagging responses. Even for the very low upcrossing rates, the two approaches agree well. Results have additionally been generated for $\beta = 0, 45$ and $90$ degrees with the same good agreement. For beam sea, the response level naturally becomes significantly smaller. For a long crested sea the wave induced vertical bending moment is theoretically equal to zero.

![Figure 5.14](image)

Figure 5.14: Upcrossing rates on the basis of MLRW simulations for $v = 0$ m/s and $\beta = 135$ deg. The results are generated from the PM spectrum with $H_s = 10$ m and $T_z = 11.35$ sec.

5.4 Model Correction Factor Method

Direct calculations or brute force simulations are very time consuming and are not considered practical feasible for day to day engineering. The model correction factor method by Ditlevsen & Arnbjerg-Nielsen (1991) becomes therefore very handy. The model has previously been applied on marine structures with success by Friis-Hansen (1994) who studied hull girder strength.

The model is formulated on the assumption that a simplified model, which for the present case is represented by the MLRW model, is correlated by a factor $\psi$ to establish the advanced model. The simplified model has to be realistic enough to capture non-linear effects. The model is outlined as follows. The vertical bending moment $M_{\text{Ideal}}(Z)$ of the simplified model
has to be corrected with a model correction factor or bias factor $\psi$. By application of the MLRW, the variables of $\psi$ is a function of the upcrossing level, the forward speed, and the heading angle. The bias factors calculated may be considered independent of $H_s$ and $T_z$ due to the very large similarities within the MLRW calculated. The model follows hereby as:

$$M_{\text{Real}}(Z) = \psi(Z) M_{\text{Ideal}}(Z)$$  \hspace{1cm} (5.6)

where $M_{\text{Real}}(Z)$ in theory is the ultimate vertical bending moment from the time-consuming advanced model.

A complete evaluation of the model correction factors $\psi(Z)$ would be very time consuming as brute force simulations should be applied for all combinations of forward speed, heading. The CRRW model is considered representative for the advanced model after which significantly simulation time is gained. It is furthermore possible to obtain information on upcrossing rates for the highest response levels where it practically would be impossible to obtain results from the brute force analysis.

By application of the model correction factor method significantly simulation time is gained on a long-term response assessment as discussed later.

### 5.4.1 Bias Factors for the MLRW Model

The MLRW tend to look very similar for the selected zero-upcrossing periods as seen previously. It is therefore assumed that the bias factors in Figure 5.15 apply for the entire scatter diagram.

Bias factors, $\psi(Z)$ are given in Figure 5.15 where the ratio of $\frac{\text{CRRW}_{\text{NL}}}{\text{MLRW}_{\text{NL}}}$ is plotted as a function of the upcrossing rates for the selected vessel velocities and summarised in Table 5.2.

Figure 5.15 shows that the MLRW is a fairly good identifier of the non-linear peak responses. Only small variations are present and most pronounced for the lowest upcrossing levels. By application of the MLRW approach the bias factors to be applied are found in an interval from $0.97 \rightarrow 1.16$ with the largest uncertainties for $v = 10 \text{ m/s}$. 
Chapter 5. Most Likely Response Wave Analyses

Figure 5.15: Ratio of \( \frac{CRRW_{NL}}{MLRW_{NL}} \) as a function of the upcrossing rate for different velocities and head sea.

Table 5.2: Bias of model correction factor for selected forward speeds.

<table>
<thead>
<tr>
<th>( v ), [m/s]</th>
<th>Hogging</th>
<th>Sagging</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.010 → 1.046</td>
<td>1.015 → 1.040</td>
</tr>
<tr>
<td>5.0</td>
<td>0.998 → 1.061</td>
<td>1.030 → 1.072</td>
</tr>
<tr>
<td>10.0</td>
<td>0.972 → 1.063</td>
<td>1.072 → 1.160</td>
</tr>
</tbody>
</table>

5.4.2 Correction Factors to the Linear Model

Figure 5.16 shows the model correction factors to the linear model for selected forward speeds and head sea conditions. The ratios of the non-linear response to the linear response are plotted as function of the linear response on application of the brute force, MLRW and CRRW approach.

Sagging: The ratios are plotted for linear response amplitudes of 1000 to 7000 MNm. Non-linearities are clearly observed for the simulation approaches selected and found most pronounced for the CRRW and brute force approaches, which almost are identical. The introduced non-linearities are almost of similar size for \( v = 0 \) and 5 m/s, and obtained slightly lower for \( v = 10 \) m/s. This is a general observation for the three approaches examined and
## 5.4 Model Correction Factor Method

according to the expected as the non-linear contribution will exceed an upper limit for both hogging and sagging. It is important to remember that the probability of exceedance for a given response level changes as function of the energy in the response spectrum. Results by application of the MLRW approach predict generally the lowest ratios.

![Graphs](image)

**Figure 5.16:** Ratios of non-linear responses to linear response plotted as function of the linear response for selected forward velocities and head sea. The results are shown for the brute force, MLRW and CRRW approaches and generated for a short-term sea state of $H_s = 10.0$ m and $T_z = 11.35$ sec.

**Hogging:** The ratios are provided for a linear response range of 1000 to 6000 MNm. For the three approaches considered, a general decrease is obtained within the ratio of the non-linear response to the linear response as function of the linear response. The ratios are furthermore found most pronounced for forward velocity of $v = 10$ m/s. By application of the MLRW and CRRW approach almost similar results are obtained. These are furthermore found slightly more optimistic on the non-linear to the linear ratio compared to results of the brute force simulations.
5.5 Discussion on the MLW and MLRW Approaches

Two approaches for application of conditional processes to estimate the short-term response statistics have been studied. For the MLRW model and even better for the CRRW model, good agreement to results of brute force simulations has been found for the selected sea states and operational profiles. For the MLW or CRW models significantly effect should be applied to obtain usable results.

By application of the MLW approach, it was found that the zero-upcrossing wave period and the location of the MLW crest relative to the hull girder affected the overall results. For the same operational profile, bias factors of $1.15 \rightarrow 1.23$ (see Figure 4.19) were found on the response due to the MLW as compared to the most probable largest response in a sea state of approximately three hours. The bias factors were furthermore found to depend on the actual sea state.

The MLRW/CRRW approach has the advantage that the critical waves are derived with the effect of forward speed and wave heading and the size of the vessel is included in the analyses through the transfer function. This is a significant advantage compared to the MLW approach. The CPU time is thus reduced significantly compared to both the MLW approach and the brute force analysis.

Response statistics calculated by application of the MLRW profiles have been found to agree well with the CRRW approach for zero forward speed. As the forward speed is increased, the effect of the random background wave become more and more important.

Presently, it is difficult to indicate any general trends as regards the effect of forward speed, as only one vessel has been examined. Adegeest et al. (1998) obtained acceptable agreement between simulation and model tests by application of the MLER wave profile with forward speed included. Here non-linearities were introduced with a sag to hog ratio of $\sim 1.93$. The experiments were made for the container ship Snowdrift within a sea state of $H_s = 4.8 \text{ m}$ and $T_z = 8 \text{ sec}$. The selected sea state with relatively smaller vessel motions compared to the sea state considered in the present study may explain some of the differences between the response due to the MLRW profile and the upcrossing rate obtained by application of the CRRW approach. Difference in the bow flare design may also account for some of the differences found.

5.6 Concluding Remarks

The main objective of this chapter was to present a fast and reliable method for estimation of non-linear extreme value ship response statistics.

Calculations of the MLRW profiles relative to the instantaneous position of the vessel considered showed that very similar wave profiles at $t_0$ were generated give different combinations
of $T_z$ and $H_s$. The MLRW that generates a sagging-induced vertical bending moment amidships is characterised by a wave length of almost similar length as the hull girder. The wave amplitudes change according to the linear conditional response level and the forward speed selected. MLRW that generates a hogging-induced bending moment is characterised by a large wave located amidships, where the main variations are found in the amplitude of the MLRW crest as the conditional response level is changed.

The present study derives a new approach by application of Most Likely Response Waves (MLRW) or the Conditional Random Response Waves (CRRW) to estimate the entire non-linear extreme value distribution for a selected operational profile in a given sea state. Comparison between results of the CRRW model and results of brute force analyses for the scenarios selected show a perfect match, and it is considered that the CRRW model captures most of the important memory effects.

The results of the MLRW approach have been compared with the CRRW method, and a brute force simulation. Good agreement between the MLRW approach and brute force simulation has been found. Bias factors between results of the MLRW and CRRW models are derived for $v = 0, 5$ and $10$ m/s and considered representative for the entire scatter diagram due to the very similar MLRW profiles derived. The bias factors are obtained between $1.015 \rightarrow 1.160$ for sagging and $0.972 \rightarrow 1.063$ for hogging. The largest variations introduced are obtained for increasing forward speed. The MLRW profile captures most of the important memory effects at zero forward speed, but with increasing difficulties as forward speed is introduced. The effects of the random background wave of the CRRW profile is therefore important given an exact solution is desired.

It is therefore recommended that the CRRW approach is used to establish short-term response statistics. The model correction factor method and derived bias factors may hereafter be used as long-term response statistics are required. The latter is discussed in Chapter 7 with other approaches.
Chapter 6

Hydroelastic Responses

6.1 Introduction

The hull girder is exposed to a low-frequency wave-induced bending moment, which contributes the most to the overall load, and a high-frequency part due to slamming events or green water on deck. A slamming event and the generated impact occur as the bottom or bow flare of the vessel hits the water surface after a series of large heave and pitch motions. In the case where the bottom of the vessel emerges from the water, bottom slamming occurs. Similarly, bow flare slamming (also known as momentum slamming) may occur as the vessel makes a combined heave and pitch motion down into the wave. Slamming events result in shock-like impact forces in the bow regions and should be studied and considered in the design of a vessel. For vessels with a flat bottom and low dead rise angle, the duration of the impulse is very short, \( \sim 0.01 - 0.1 \) sec., Ochi & Motter (1973). For container ships, the transient whipping responses are introduced by bow flare slamming. The duration of the impulse is slightly longer, \( \sim 1.0 \) sec., as the wave is exposed to a longer area in the longitudinal direction, Jensen (2001).

The majority of the class societies have included the effects of bow flare slamming and forward speed in their rules by introducing a factor to be multiplied onto the rule for estimation of a “corrected” wave-induced bending moment. The factor normally depend on the vessel considered. The applied empirical formulas are generally based upon data from various types of vessels and common engineering judgement. However, the empirical formulations do not always generate accurate results, particularly for novel ship designs requiring a direct time domain simulation to be performed.

Several studies have addressed the effect of slamming-induced whipping responses. Jensen & Mansour (2002) assumed in their work on closed-form expressions that the wave- and whipping-induced sagging maxima occur at the same time. A fast approach for estimation of the long-term response statistics with whipping responses included was presented. Good
agreement with class rule results was obtained. Baarholm & Jensen (2002) studied the effect of whipping on the long-term vertical bending moment. Their analyses were based on independence of the wave- and whipping-induced response, which simplifies the establishment of the short-term response distribution. It was shown that whipping may increase the vertical bending moment significantly and that the correlation between the wave- and the whipping-induced response can be significant.

However, estimation of short-term response statistics may be performed very effectively by application of the MLRW or CRRW profiles without introducing the assumption of independence or a full correlation between the wave- and whipping-induced response. One would generally expect that a large wave-induced response is correlated with a large whipping-induced response. The results of the brute force simulations made show that this may not always be the case and discussed later in Section 6.6. By application of the MLRW or CRRW this dilemma is avoided to some extent as the non-linear peak response can be selected directly followed by unconditioning with respect to the linear response.

Hydroelastic responses are studied by application of the MLRW and CRRW profiles. The calculations have been performed for the Panmax container ship as presented in Appendix B and are based on the non-linear time domain strip theory as described in Appendix A, Xia et al. (1998). A model for estimation of short-term response statistics is presented. The current approach takes into account the effect of whipping given the selected response levels. Results of the model presented are compared to results of brute force simulations and show very good agreement.

The first part of this chapter discusses results of the brute force simulations and hydroelastic responses. The slamming problem is reviewed in general by application of MLRW. Hereafter short-term response distribution are established. Bias factors are estimated as previously discussed in Section 5.4.1. Finally, the results of the various models are compared and discussed.

### 6.2 Brute Force Simulation

Brute force simulations are performed for an elastic hull girder in order to obtain validation data for comparison with results of the MLRW and CRRW approaches. The simulations provide further knowledge of hydroelastic responses and how peak response distributions for the traditional analyses may be established. The latter is reviewed in Section 6.5.

The brute force analyses are performed for \( v = 0, 5 \) and 10 m/s and head sea. Here 300 x 1 hours’ of simulations were performed for each of the forward velocities selected.
6.2 Brute Force Simulation

6.2.1 The Wave- and Whipping-Induced Responses

Figure 6.1 shows an example of a short sampled response signal of the vertical bending moment amidships by application of an elastic hull girder for a Panmax container ship operating at zero speed in irregular head waves with $H_s = 10.0$ m and $T_z = 11.35$ sec. The total response signal is shown to the left, where it can be observed that a low- and high-frequency part is present.

To the right, the total response signal is filtered by truncating the generated power spectrum (see Figure 6.2) into two parts, after which an inverse discrete Fourier transform has been performed to obtain the filtered signal. The high-frequency whipping-induced part becomes the difference between the total vertical bending moment and the low-frequency wave-induced part. Furthermore, it is found that both the negative and positive whipping-induced maxima occur for the wave-induced maxima. It is also characteristic that a large wave-induced response is not necessarily followed by a large whipping-induced response.

![Figure 6.1: Left: Vertical bending moment amidships for an elastic hull girder. Right: Filtering of the signal into a low- and high-frequency part.](image)

Figure 6.2 shows calculated spectral density Fourier transforms of the vertical bending moment amidships of the signal presented above. The analyses are based on 60 minutes of simulation. If it is of interest to establish a peak distribution from the results of brute force simulations, the wave- and whipping-induced parts may be considered independent as assumed by Baarholm & Jensen (2002). The establishment of a short-term response distribution simplifies hereby.
Figure 6.2: Predicted power spectrum of the vertical bending moment amidships for a Pan-max vessel moving in irregular head waves with $H_s = 10.0$ m and $T_z = 11.35$ sec., $v = 0$ m/s.

Figure 6.3 shows results of brute force simulations. The results for $v = 0$ m/s are plotted at the top, $v = 5$ m/s in the middle, and finally the results for $v = 10$ m/s on lowest plot. Hoggings responses are found to the left and sagging to the right. The results are shown through upcrossing rates and are derived on the assumption of a rigid and a flexible hull girder. These are referred to as BF$_{\text{Rigid}}$ and BF$_{\text{Flexible}}$.

The upcrossing rates BF$_{\text{Rigid}}$ may either be established from the low-frequency wave-induced part by filtering of the total response signal or by performing the brute force simulation for a rigid hull girder as seen in Chapter 5. The results provided in Figure 6.3 are derived by filtering the total response signal into two parts. Small variations are found as the upcrossing rates are compared to results from Chapter 5. It should furthermore be mentioned that the brute force simulations in Chapter 5 are established from 600 hours of simulation whereas only 300 hours of simulation has been generated for the present example. For both the rigid and flexible hull girder approach the upcrossing rates are given as $\nu_\eta(r) = \nu_0[1 - F_{RNL}(r)]$, where $F_{RNL}(r)$ represents the discrete cumulative peak distribution. This approach is considered necessary to apply as the presence of the high-frequency whipping-induced responses will introduce too many upcrossings for the individual response levels selected.

Therefore, for the low-frequency part (rigid hull), the largest peak response between each up- and downcrossing has been selected for further use. The peak responses of the original response signal, i.e. including both wave- and whipping-induced responses, are collected as the largest peak response within each up- and downcrossing of the low-frequency wave-induced response.
Figure 6.3: Brute force simulations of the combined wave- and whipping-induced maxima. Results are shown for \( v = 0, 5 \) and \( 10 \) m/s. Only 10\% of the applied data for estimation of the upcrossing rates is shown here.

The results of the brute force simulations are as expected. For the lowest sagging-induced response levels, only small differences between results of BF\textsubscript{Rigid} and BF\textsubscript{Flexible} are obtained, and is an identification of that the effect of whipping is limited. As the response level is increased, the effect of whipping becomes more and more pronounced as seen from the increasing separation between the filtered and original data. For hogging the overall trend is the same. The lowest response levels are not affected, whereas an increasing contribution is observed for increasing response levels.
6.3 Parameter Study, Hydroelastic Responses

Hydroelastic responses by application of conditional waves have been studied. The analyses are focused on head sea operations for selected forward velocities applying the PM spectrum with $H_s = 10.0$ m and $T_z = 11.35$ sec. From the analyses using the MLW and MLRW models from Chapters 4 and 5, it was found that the MLRW approach was superior in capturing the dominating effect of the non-linear response. Hence, the MLRW approach will also be applied in the study of the inclusion of the hydroelastic responses. A small review of the application of the MLW approach is included in Appendix E, Section E.2.

Firstly, the slamming problem is introduced through snapshots of the instantaneous position of the wave relative to the hull girder where either a conditional hogging- or sagging-induced response is generated at $t = 100$ sec. Secondly, several small response series on assumption of a flexible hull girder are studied to gain further knowledge on hydroelastic responses due to the deterministic MLRW.

6.3.1 The Slamming Problem - MLRW

Figure 6.4 shows snapshots of the vessel relative to the wave. To the left, five snapshots illustrate different scenarios of the wave and hull girder before a conditional linear sagging moment is established at $t_0 = 100$ sec. To the right, similar snapshots are shown. These result in a conditional hogging moment. The waves are plotted to a turning point, where the arrows show the future motion of the wave profile. From the illustrated snapshots, it is observed that bottom slamming do not occur for the given operation profile and response level. This is furthermore found to be a general trend by application of the MLRW for the response levels, operational profiles and sea states examined.

**Conditional sagging response:** At $t = 95$ sec. the wave is at a turning point relative to the vessel. The surface of the wave profile is on it’s way up and the vessel is about to perform a combined heave and pitch motion down into the wave, after which a bow flare slamming event will occur at approximately $t \approx 98$ sec. The present observation points towards that a whipping-induced response very likely will be present in addition with the wave-induced response at the conditional time instant $t_0 = 100$ sec. The whipping-induced response from bow flare slamming will therefore only be a few seconds old and will presumably still be found to dominate the total response. A new whipping-induced response will occur as the next wave crest passes the amidship section whereafter the whipping-induced response will be increased again.

**Conditional hogging response:** The situation is to some extend similar. Bow flare slamming are likely to occur at $t \approx 93$ sec. as the vessel will perform a combined pitch and heave motion down into the wave. The whipping-induced response at the conditional time instant, $t_0$, will therefore be represented by a smaller peak in the tail of transient whipping response.
At $t_0 = 100$ sec. the wave profile will be located in a new turning position whereafter bow flare slamming will be introduced again at approximately $t \approx 102$ sec.

This small example illustrates nicely the nature of the slamming problem by application of MLRW and to some extend where limitations may occur. Figures 6.5 to 6.8 illustrate responses due to the MLRW.

Figure 6.4: Waves relative to the hull girder. Sagging to the left and hogging to the right.

It is furthermore important to observe that by application of the MLRW which has been generated from the PM spectrum only two peaks are encountered before the conditional time instant. Memory effects are therefore to be captured quickly. This may introduce limitations for the MLRW approach as the wave profile is established on the assumption of a rigid hull girder.

Torhaug et al. (1998) studied critical wave episodes and how long the critical wave episode had to be to avoid degrading the results when the non-linear analysis is started with the exact initial condition estimated from a full irregular linear analysis. Their results indicated
that by applying three critical wave episodes in a non-linear simulation only slightly affected the maximum peak response as compared to an infinitively long critical wave episode.

### 6.3.2 MLRW, Hydroelastic Responses

Hydroelastic responses have been studied by application of MLRW profiles, i.e. the most likely response wave without the random background wave included. The MLRWs are simulated from a linear transfer function assuming a rigid hull girder.

The main purpose of the present section is to verify whether the MLRW can be applied as a predictor of whipping-induced responses, and to study the relation between the non-linear peak response due to the conditional linear peak response.

Figure 6.5 shows non-linear responses of the vertical bending moment amidships for an elastic hull girder. The simulations are performed conditional on a linear sagging response level at $t_0 = 100$ sec. for selected forward velocities and head sea. The effect of whipping is clearly observed for most of the total response signals as the faster transient oscillating signal and found to be the most pronounced for the largest linear conditional response levels with $v = 0$ and 5 m/s. The most likely response waves are not plotted for all the selected cases. However the snapshots found in Figure 6.4 represents a general picture of the waves encountered.

It is important to remember that the MLRWs decrease in size with increasing forward speeds, given the same linear conditional response level. This furthermore explains the observation of slightly smaller total peak responses with increasing forward speeds.

It is seen that the whipping-induced responses are increased as a function of the conditional response levels for the same velocity and that the local whipping-induced peak responses occur very close to the same time instants. It follows further that the peak response of the total response tends to move backwards, which means that a local whipping trough or peak may be present at $t = t_0$.

Further analyses of conditional linear response levels smaller than 3000 MNm have also been studied. Here the effect of whipping at the vicinity of the conditional time instant was very limited and basically negligible. These results are therefore not shown here.

Figure 6.6 shows filtered response signals from Figure 6.5. The low-frequency part corresponds to the wave-induced response and the high-frequency part to the whipping-induced response. The wave-induced response tends to decrease with increasing forward speeds, given the same response level as seen previously in Section 5.2.

The whipping-induced responses are observed to increase with increasing linear conditional response levels. It is found that the first larger whipping peak is introduced as the vessel encounters the first large wave crest of the MLRW profile. By comparison of the snapshots from Figure 6.4 (left column) and the third plot in the second column in Figure 6.6, it is
observed that the first whipping-induced response occurs at $t \approx 98$ sec. which correspond to the expected. The whipping-induced response slowly decreases whereafter a new bow flare slamming impulse adds energy to the system at $t \approx 105$ sec. (off course smaller this time).

Figure 6.5: Hydroelastic responses by application of MLRW profiles. The simulations are generated for head sea, selected velocities and conditional linear sagging response levels. The Pierson Moskowitz spectrum is applied with $H_s = 10.0$ m and $T_z = 11.35$ sec. CLR: Conditional Linear Response.

As the whipping-induced response is generated very close to the conditional time instant,
variations in the CRRW profile around the MLRW profile will be small as illustrated in Figure 5.7 (right column, middle plot). It is therefore expected that the MLRW will predict the non-linear peak response well on assumption of a flexible hull girder as compared to results of CRRW or brute force analyses.

Figure 6.6: The wave- and whipping-induced sagging responses after filtering.

Figure 6.7 shows similar results of the non-linear vertical bending moment amidships for an elastic hull girder. The simulations are now performed by application of MLRWs conditional on linear hogging response levels at $t_0 = 100$ sec. for selected forward velocities.
The effect of whipping is observed on most of the response series and found to be the most pronounced for the largest linear conditional response levels. For the selected cases, the largest transient whipping responses are observed away from the linear conditional hogging response, and it is observed that only small whipping-induced responses are present in the near vicinity of the conditional time instant \( t_0 \).

Figure 6.7: Hydroelastic responses by application of MLRW profiles. The simulations are performed for head sea conditions, selected velocities and linear hogging response levels, by application of the Pierson Moskowitz spectrum with \( H_s = 10.0 \text{ m} \) and \( T_z = 11.35 \text{ sec} \).
Figure 6.8 shows the results of filtering the response signals from Figure 6.7. The non-linear wave-induced response tends to decrease with increasing forward speeds as seen previously in Section 5.2, and reflects the nature of the smaller MLRWs generated. The whipping-induced responses develop basically as described above, with the largest whipping responses located away from the largest non-linear hogging response. The results are as expected and correspond well with the observations made in Figure 6.4.

By observing the third plot in the second column in Figure 6.8 it is seen that the first larger
whipping-induced peak occurs at \( t \simeq 93 \text{ sec.} \) (just after the slamming event). Hereafter, the transient whipping-induced response decreases slowly in size whereafter a new slamming event is generated at \( t \simeq 103 \text{ sec.} \).

The whipping response at the vicinity of \( t_0 \) represents therefore a response at the tail of the transient whipping vibration and is therefore generated away from the conditional time instant. Variations in the CRRW profile around the MLRW will be larger here as illustrated in Figure 5.7 (left column, middle plot), it is therefore expected that the MLRW will predict the non-linear hogging peak response less well on assumption of a flexible hull girder as compared to results of CRRW or brute force analyses.

By application of the CRRW the instantaneous position of the hull girder will be different because of the wave episodes previously experienced. It is therefore much more likely that bottom slamming also will be present in the response signal.

### 6.3.3 Selection of Peak Responses, MLRW / CRRW Approach

One of the main ideas by application of the MLRW or CRRW profiles is the to derive fast and accurate models for estimation of short-term response statistics. It is of special interest if the deterministic MLRW profile can be used as a good predictor as seen previously by application of a rigid hull girder. The analyses from Chapter 5 showed good agreement between results of the brute force simulation and the MLRW approach.

By application of MLRW, the non-linear peak response may be selected directly and unconditioned through the Rayleigh distribution. One may have to collect the largest peak response a few sample points (\( \pm 1.0 \text{ sec.} \)) away from the conditional time instant \( t_0 \) without introducing any errors. The peak distribution is hereby obtained directly, wherefore the assumption of independence between the wave- and whipping-induced response is irrelevant to introduce. The above accounts for both conditional hogging- and sagging-induced peak responses.

### 6.3.4 Effect of Changed Linear Constrained Response and Forward Speed

The effect of non-linear responses given a flexible hull girder has been studied by application of MLRW profiles. Analyses are performed for linear conditional sagging and hogging responses of 3000 to 6000 MNm and forward speeds of 0, 5 and 10 m/s. All the analyses are performed for head sea conditions, applying the Pierson Moskowitz wave spectrum with \( H_s = 10.0 \text{ m} \) and \( T_z = 11.35 \text{ sec.} \).

Figure 6.9 shows the effect of non-linearities on the vertical bending moment amidships by assuming a flexible hull girder. For both hogging and sagging, the ratio \( x_{totalNL}/x_{evaNL} \)
is plotted as functions of the conditional linear response $x_{wa_L}$. The non-linear wave- and whipping-induced sagging and hogging responses are sampled from the near vicinity of the conditional time instant $t_0$.

**Sagging**: For the selected velocities, the effect of whipping is clearly observed. The contribution of whipping seems almost linearly increasing for the response interval chosen. The largest whipping contributions are found when forward speed is introduced. It is observed that the largest non-linear wave- plus whipping-induced bending moment is found to be approximately 1.40 times the non-linear wave-induced response for the severest cases.

For conditional response intervals lower than 3000 MNm, the effect of whipping is small. Comparison to results of the brute force simulations, where whipping-induced responses also were found for the lowest response interval, may indicate that the random background wave has to be included in order to obtain reliable results.

**Hogging**: For hogging, similar results are obtained. The effect of whipping is increasing for increasing linear response levels. A linear trend is almost found, and the effect tends to disappear for the lowest response levels.

![Figure 6.9: Effect of elasticity on hogging and sagging conditions for selected response levels and forward velocities by use of the MLRW wave profile.](image)

Figure 6.9: Effect of elasticity on hogging and sagging conditions for selected response levels and forward velocities by use of the MLRW wave profile.
6.4 Hydroelastic Responses, Short-Term Statistics

Short-term response statistics of combined wave- and whipping-induced responses by application of the MLRW and CRRW profiles have been studied and compared to results of brute force simulations. The exact peak distributions and corresponding upcrossing rates are established like the MLRW approach from Chapter 5.

6.4.1 Hydroelastic Response Analyses of Application of the CRRW

This section presents analyses of hydroelastic responses using the CRRW profile. The upper plot in Figure 6.10 shows non-linear results of constrained simulations by application of the CRRW for an elastic hull girder. Typical examples of most likely response waves are shown in Figure 5.7. The present example shows 12 individual response simulations of the vertical bending moment amidships (black curves), and all the results are generated from a linear conditional rigid body sagging response of 4000 MNm at \( t_0 = 100 \) sec., \( v = 5 \) m/s and head sea. The non-linear results are subsequently derived on the assumption of an elastic hull girder. The effect of whipping is observed and found to be the most pronounced in the surrounding area of the constrained time instant \( t_0 \). The red curve represents the response due to the MLRW profile. All the response series are found to merge around \( t_0 = 100 \) sec., and it is found that the random background wave only slightly affects the total response for the current case. The lower plot shows the filtered signals, where the low- and high-frequency time series represent the wave- and whipping-induced responses. As previously indicated from the analyses by application of the MLRW approach, the effect of whipping is controlled well.

![Figure 6.10: Wave- and whipping-induced responses for \( v = 5 \) m/s. A linear constrained sagging vertical bending moment amidships of 4000 MNm at \( t_0 = 100 \) sec. has been applied. The red curve represents the response curve obtained from the application of the MLRW.](image)
Figure 6.10 shows furthermore a typical example of the response series generated for the sea states and operational profiles selected.

The upper plot in Figure 6.11 shows results of constrained simulations, and the corresponding filtered signals are shown below. The non-linear simulations are generated for head sea with $v = 5 \text{ m/s}$ by application of CRRWs that generate a linear rigid body hogging moment of -4000 MNm at $t_0 = 100 \text{ sec}$. Typical examples of most likely response waves that generate conditional hogging moments are given in Figure 5.7.

Figure 6.11 represents a slightly more blurred picture. The responses are found in a thicker band and not as merged as seen from the previous analyses. Filtering of the response signal into a low- and high-frequency part indicates that the wave-induced response performs as previously seen in Section 5.3.1, and that main variations are to be found in the whipping-induced response.

The whipping-induced responses found in the vicinity of $t_0$ are presumably generated as the vessel encounters the wave peak that builds up to the CRRW a few seconds later. The whipping-induced responses observed at $t_0$ become a peak or trough at the tail of the transient whipping vibrations as discussed previously. As larger variations of the CRRW profile naturally are present at the time instant where the whipping-induced responses are generated, it is more likely that significantly different peak responses are present at the vicinity of the conditional time instant $t_0$. The whipping-induced signal may furthermore be increased if bottom slamming additionally occurs. For the current model, the same observations have been found for basically all head sea conditions with $v = 0, 5$ and 10 m/s by application of conditional hogging response levels of -6000 to -2000 MNm.

![Wave- and whipping-induced responses for $v = 5 \text{ m/s}$. A linear constrained hogging vertical bending moment amidships of -4000 MNm at $t_0 = 100 \text{ sec}$ has been applied. The red curve is response from the MLRW.](image-url)
6.4.2 Short-Term Response Statistics - Hydroelastic Responses

The basic idea of estimating short-term response statistics by application of CRRWs on the assumption of an elastic hull girder is outlined below. The approach is to some extent similar to the method by application of a rigid hull girder as given in Section 5.1. The method is based on the assumption that the linear model is a good predictor of the non-linear extreme events.

Establishment of Non-linear Response Distribution

1. Select a stationary sea state with selected $H_s, T_z$. Furthermore select the operational profile to be examined.

2. Calculate the linear transfer functions for the selected response to obtain amplitude and phase information, assuming a rigid hull girder. This is not strictly necessary but recommended.

3. Perform the constrained linear response calculations, Eq. (3.61), and derive the coefficients $V_{c,n}$ and $W_{c,n}$.

4. Derive the CRRW that generates the linear extreme conditional response, $a$. The most likely response wave can be generated directly with or without the random background wave, depending on the initial conditions.

5. Use the linear CRRW in a non-linear analysis, assuming an elastic hull girder. It is thus assumed that the linear response is a good identifier for the location of the non-linear response, also by use of an elastic hull girder.

6. Select an appropriate response range and perform $\sim 100$ constrained simulations for each of them.

7. Select the non-linear peak responses from a small time window around the conditional time instant $t_0$ for each of the applied linear conditional response levels.

8. The probability distribution $F_{R_{NL}}(r \mid h_s, t_z, v, \beta)$ of the non-linear response is found by unconditioning with respect to the linear response. The lower cutoff response level as introduced in Eq. (5.2) should additionally be applied. The extreme value distribution fits well to the non-linear peak responses for each of the conditional response levels.

This approach allows short- or long-term response statistics to be made for both hogging and sagging conditions. Comparison with brute force analyses is made in Section 6.4.3, indicating good agreement for both hogging and sagging. The approach by application of CRRW is considered very powerful as it takes into account the fraction of whipping-induced responses as a function of the response.
By introducing the model correction factor method on the upcrossing rates generated by application of MLRW profiles, an even faster approach is available. The model is discussed in Section 6.4.4.

6.4.3 Short-Term Response Distributions of Wave- and Whipping-Induced Responses

Figure 6.12 shows peak response distributions of combined wave- and whipping-induced responses for both hogging and sagging. The results are derived for head sea conditions and forward speeds of $v = 0, 5$ and $10 \, \text{m/s}$. Both the MLRW, CRRW and brute force simulations are made for a sea state of $H_s = 10.0 \, \text{m}$ and $T_z = 11.35 \, \text{sec}$. Sagging-induced responses are shown within the right column, and hogging to the left. Results of the traditional linear analysis are also included.

The results of the brute force simulations are identical to the previous results in Figure 6.3. The upcrossing rates are given for original signals, BF\textsubscript{Flexible} and established as previously mentioned. The non-linear effect on the hogging- and sagging-induced responses is clearly observed as the results are compared to the linear results.

The plotted red circles represent results obtained by application of the MLRW profile. The results generated from the MLRW profiles are found to be fluctuating around the results established from the CRRW approach. This indicates that the random background wave of the MLRW profile is important, especially for increasing forward speeds. Peak responses due to the MLRW on assumption of a rigid hull girder are also plotted, whereupon the effect of whipping for the selected upcrossing levels is illustrated.

For both hogging and sagging, the CRRW approach is found to predict the results of brute force simulations well. Even the tail of the upcrossing rates is well-predicted.

Sagging-induced responses by application of the MLRW model are predicted well. The model captures most of the transient whipping effects, and it is seen that only small bias factors should be applied to correct the MLRW approach. Bias factors are shown in Figure 6.13. The hogging-induced responses by application of the MLRW model is less well predicted. The model as discussed previously in Section 6.3.2 has difficulties on predicting the whipping-induced responses whereas the peak responses become biased with a factor up to 1.25 as shown in Figure 6.13.
Figure 6.12: Upcrossing rates established by application of the MLRW approach for an elastic hull girder in a sea state of $H_s = 10.0 \text{ m}$ and $T_z = 11.35 \text{ sec}$. The results are generated for $v = 0, 5$ and $10 \text{ m/s}$, and are compared to results of brute force simulations. The legend illustrated applies to all of the individual subplots.

Wave- and whipping-induced responses obtained by the selected calculation methods are summarised in Table 6.1. The results are sampled for a probability level corresponding to $\nu_{\eta}(r | h_s, t_z, v, \beta) = 10^{-5}$. 
Table 6.1: Results of hydroelastic analyses for head sea and selected forward speeds. The simulations are performed for $H_s = 10.0$ m and $T_z = 11.35$ sec. The responses collected correspond to an upcrossing rate of $\nu(q|h_s, t_z, v, \beta) = 10^{-5}$.

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<td>5811</td>
</tr>
<tr>
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<td>3577</td>
<td>4428</td>
<td>6520</td>
</tr>
<tr>
<td>10</td>
<td>3468</td>
<td>4410</td>
<td>7254</td>
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<tr>
<td>CRRW</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>0</td>
<td>3618</td>
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</tr>
<tr>
<td>5</td>
<td>3570</td>
<td>4337</td>
<td>6265</td>
</tr>
<tr>
<td>10</td>
<td>3360</td>
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<td>6028</td>
</tr>
<tr>
<td>10</td>
<td>3205</td>
<td>3837</td>
<td>6290</td>
</tr>
</tbody>
</table>

6.4.4 Model Correction Factor Method - Hydroelastic Responses

Bias factors as derived previously in Section 5.4.1 are similarly obtained by application of an flexible hull girder. The bias factors, $\psi(Z)$, are given in Figure 6.13, where the ratio of $\frac{CRRW_{NL}}{MLRW_{NL}}$ is plotted as a function of the upcrossing rates for the vessel velocities selected. As seen from the previous example, only small variations between the MLRW and CRRW approach are present. The bias factors are though found to be slightly more fluctuating as compared to results by application of a rigid hull girder.

Table 6.2: Model correction factors for selected forward speeds.

<table>
<thead>
<tr>
<th>v, [m/s]</th>
<th>Hogging</th>
<th>Sagging</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.045 → 1.105</td>
<td>0.955 → 1.027</td>
</tr>
<tr>
<td>5.0</td>
<td>1.040 → 1.282</td>
<td>1.005 → 1.122</td>
</tr>
<tr>
<td>10.0</td>
<td>1.025 → 1.255</td>
<td>0.972 → 1.110</td>
</tr>
</tbody>
</table>

By application of the MLRW approach the model correction factors to be applied are found
in an interval from 0.95 → 1.25 with the most deviation for $v = 10$ m/s. The ratios are summarised in Table 6.2.

![Graph showing ratio of $\frac{\text{CRBW}_{NL}}{\text{MLRW}_{NL}}$ as a function of upcrossing rate for different velocities.]

Figure 6.13: Ratio of $\frac{\text{CRBW}_{NL}}{\text{MLRW}_{NL}}$ as a function of the upcrossing rate for different velocities.

### 6.5 Short-Term Response Distribution - Approximative Solution

Analyses of whipping responses are traditionally made by considering time series of a given length, after which the derived response statistics may be used to estimate short- or long-term response statistics. The peak distribution may be modelled by different approaches. One way would be to fit the largest peak or trough of the non-linear response signal between each of the zero-crossings. This would be one solution as the exact peak response distribution is hereby obtained. It would presumably be difficult to obtain acceptable fitting, especially at the tail of the response distribution where the effect of whipping will be the most pronounced, for which reason approximations can be applied to establish a peak distribution.

#### 6.5.1 Establishment of Peak Distributions - Brute Force Simulations

On the assumption that the wave- and whipping-induced peak responses occur approximately simultaneously and that the two peaks are independent, the probability distribution
of combined wave- and whipping-induced maxima smaller than $r$ is given as

$$F_{RNL}(r) = \int_0^r F_{X_{NL}}^{wa}(r-x) f_{X_{NL}}^{wh}(x) dx$$  \hspace{1cm} (6.1)$$

where $F_{X_{NL}}^{wa}$ is the wave-induced bending moment, which for example may be described by the Weibull or the generalised extreme value distribution. Leadbetter et al. (1996) found that whipping responses are well described by an exponential distribution

$$f_{X_{NL}}^{wh}(x) = \lambda \exp(-\lambda x)$$  \hspace{1cm} (6.2)$$

where $\lambda$ is a scale parameter. Alternatively, it is possible to fit the distribution $F_{X_{NL}}^{wa+wh}$ directly to the sum of wave- and whipping-induced maxima. Since not all of the wave-induced peak responses are associated with a slamming event, the distribution has to be modified to account for this:

$$F_{RNL}(r) = F_{X_{NL}}^{wh}(r)(1 - N_{SL}) + F_{RNL}(r)N_{SL}$$  \hspace{1cm} (6.3)$$

or

$$F_{RNL}(r) = F_{X_{NL}}^{wh}(r)(1 - N_{SL}) + F_{RNL}^{wa+wh}(r)N_{SL}$$  \hspace{1cm} (6.4)$$

$N_{SL}$ is the fraction of wave peaks with an associated slamming event. The two last examples are approximations as it is assumed that the whipping-induced responses occur for the entire response range.

### 6.5.2 Selection of Peak Responses

Sampling of wave- and whipping-induced peak responses is done by applying low- and high-frequency filters to the simulated time series. Figure 6.14 shows the vertical bending moment by application of an elastic hull girder. Additionally shown are the filtered wave- and whipping-induced responses. From the small time window selected, it is characteristic that a large wave-induced response is not necessarily followed by a large whipping-induced response.

The peak responses of interest will naturally be the largest peak of the total response signal, i.e. the unfiltered signal, between each zero-crossing. These are sampled in the vector $X_{total}$. Similarly, the wave-induced peak response can be selected between each of the zero-crossings. These are sampled in the vector $X_{wa}$ and are almost found at the same time instant as peak responses of the unfiltered signal. The related whipping-induced peak responses, $X_{wh}$, are selected at the same time instants as where the peak responses of the total response signal occurred.

The selected peaks are therefore related through the model

$$X_{total} = X_{wa} + X_{wh} - E$$  \hspace{1cm} (6.5)$$
where $E$ represents the small error introduced as the peaks of interest are sampled at different time instants.

6.5.3 Peak Distributions

The wave-induced responses have been fitted to the three-parameter Weibull distribution, Baarholm & Jensen (2002). The whipping-induced maxima have been fitted to the exponential distribution, Leadbetter et al. (1996). The analyses are performed for $v = 0$, 5 and 10 m/s showing acceptable fitting of the maxima sampled for the lowest response levels.

The number of maxima where both wave- and whipping-induced responses occur will depend on the design of the vessel, the sea state and the operational parameters. Normally one would select $N_{SL}$ to be constant. According to NordForsk (1987), an operating vessel voluntarily reduces the speed if it is judged that slamming occurs for more than 3% of the time.

Figure 6.15 shows fitting of the low-frequency wave-induced maxima and the high-frequency whipping-induced maxima, $v = 0$ m/s. The wave- and whipping-induced maxima are collected as described in Section 6.5.2 with sagging responses characterised by positive response values.
Figure 6.15: Distributions of wave- and whipping-induced maxima with zero speed and in head sea conditions.

Figure 6.16 shows the cumulative distribution of the combined wave- and whipping-induced maxima for selected values of $N_{SL}$. $N_{SL} = 0\%$ and $100\%$ indicates the lower and upper boundaries of the approximative peak distribution. The “true” distribution should in theory be found somewhere in between. The results are only given for $v = 0$ m/s with hogging responses to the left and sagging to the right.

The wave- and whipping-induced peak responses are fitted to the three-parameter Weibull distribution and exponential distribution and combined according to Eq. (6.3). The results are shown through upcrossing rates and given as $\nu_{\ell}(r) = \nu_{0}(1 - F_{R_{NL}}(r))$. The fitted upcrossing rates are clearly dominated by the large amount of data available for the lowest response levels, which results in poor fitting of the important peaks at the largest response levels.

Figure 6.16: Brute force simulations of the combined wave- and whipping-induced maxima. Results are shown for $v = 0$ m/s. Only $10\%$ of the applied data for estimation of the upcrossing rates is shown here.
6.6 Correlation between Wave- and Whipping-Induced Maxima

The above outlined approximate solution could in theory also be applied to the MLRW or CRRW approaches. Some work has been applied on this subject but generally with poorer results obtained as compared to more direct approaches as seen illustrated previously in this chapter. To establish peak distributions from the approximative solutions by application of the MLRW or CRRW models would require that the signal had to be filtered and subsequently added together again. Therefore one could ask: Why take unnecessary trouble?

6.6 Correlation between Wave- and Whipping-Induced Maxima

The correlation between the wave- and whipping-induced responses has been studied for the data presented in Figure 6.3. Figure 6.17 shows the correlation between the wave- and whipping-induced maxima for each of the forward speeds considered. It is found that whipping-induced responses are present for the entire response range for both hogging and sagging, given the velocities selected. It is also observed that the largest whipping-induced responses occur for the largest wave-induced responses.

The correlation $\rho(X_{wa}, X_{wh})$ between the wave-induced peak response, $x_{wa}$, and the associated whipping response, $x_{wh}$, has been studied and give as follows

$$\rho(X_{wa}, X_{wh}) = \frac{\text{Cov}[X_{wa}, X_{wh}]}{D[X_{wa}]D[X_{wh}]} = \frac{E[X_{wa}X_{wh}] - E[X_{wa}]E[X_{wh}]}{D[X_{wa}]D[X_{wh}]}$$  \hspace{1cm} (6.6)$$

where Cov[•] is the covariance, and D[•] the standard derivation.

Table 6.3 shows the correlation coefficients $\rho(X_{wa}, X_{wh})$ between the wave- and corresponding whipping-induced peak responses for selected forward speeds. It follows that hogging and sagging is similarly correlated. Baarholm & Jensen (2002) found a slightly higher correlation between the wave- and whipping-induced maxima for the S175 container ship.

Table 6.3: Correlation coefficients between the wave- and the corresponding whipping-induced peak for selected forward speeds.

<table>
<thead>
<tr>
<th>$v$, [m/s]</th>
<th>Hogging</th>
<th>Sagging</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.32</td>
<td>0.35</td>
</tr>
<tr>
<td>5.0</td>
<td>0.37</td>
<td>0.38</td>
</tr>
<tr>
<td>10.0</td>
<td>0.26</td>
<td>0.32</td>
</tr>
</tbody>
</table>

The correlation $\rho(X_{total}, (X_{wa} + X_{wh}))$ between the total peak response, $x_{total}$ and the wave-plus the whipping-induced response, $x_{wh}$, has been calculated and found very close to 1.0 for both hogging and sagging within the present velocity range.
Figure 6.17: Correlation between wave- and whipping-induced maxima. Only 10% of the sampled data is shown in the present figures.

The correlations $\rho(X_{wa}, E)$ and $\rho(X_{wh}, E)$ have also been studied. For the selected forward velocities, the correlations for both hogging and sagging were found between 0.3 and 0.4.

Figure 6.18: Cumulative distribution functions of the error $E$
6.7 Concluding Remarks

Hydroelastic responses have been studied for a Panmax container ship in a full load condition. The analyses are performed for head sea and selected forward speeds in a severe sea state of $H_s = 10.0 \text{ m}$ and $T_z = 11.35 \text{ sec}$.

A model is presented for application of the MLRW or CRRW wave profiles for prediction of the combined wave- and whipping-induced responses. The effect of whipping, given a wave-induced sagging response, is predicted well by application of the MLRW and CRRW approaches. The latter provides the best fit compared to brute force simulations, this also holds as the forward speed is increased. The MLRW model captures transient effects that influences the sagging moment well. From the present simulations, bias factors of $0.95 \rightarrow 1.13$ should be applied to correct the model properly. As mentioned previously, the MLRW approach introduces a fast and accurate approach for estimation of extreme value distributions. Only five to ten non-linear time domain simulations of approximately 100 sec. are required to establish a reliable fit to the true upcrossing rates.

The CRRW model presented for predicting the effect of whipping, given a wave-induced hogging response, corresponds well with brute force simulations, also when forward speed is included. The transient effects from slamming are captured well. The MLRW model predicts less well the transient whipping-induced responses. Results of the MLRW model are biased with factors of $1.05 \rightarrow 1.25$ for the selected operational parameters and sea states. The main reason for this is to be found within the time step where the slamming-induced force is generated.

The MLRW and CRRW approaches should be verified through further analyses of various types of ships to obtain further validation data, the this study carry persuasive indications generally it is believed that the present model of most likely response waves represents a very powerful tool for estimation of response statistics.

It is therefore recommended that the CRRW approach is used to establish short-term response statistics as the model practically captures all the non-linear effects. The model correction factor method and derived bias factors are recommended as long-term response statistics are sought.
Chapter 7

Long-Term Response Statistics by Application of CRRW

7.1 Introduction

The present chapter briefly discusses long-term response statistics by application of the MLRW and CRRW models. A few examples are given with recommendations for further work on this topic. Non-linear time domain simulations would normally be considered practically impossible for day to day engineering, but with the present models by application of the MLRW or CRRW it become possible to establish results during an acceptable period.

7.2 Long-term contribution factor and CRRW Approach

Videiro & Moan (1999), Sagli (2000) and Baarholm & Moan (2001) presented the long-term contribution factor approach and demonstrated that a long-term extreme load analysis for marine structures can be estimated in an efficient manner by considering only a few short-term sea states instead of determining responses for all sea states within the scatter diagram.

The initial step of the long-term contribution factor approach is to calculate the coefficients of contribution, $C_R(h_s, t_z)$, for the long-term response of wave heights and ship responses. The analyses are performed within the frequency domain and are therefore quickly made.

$$C_R(h_s, t_z) = \frac{Q_R(R > r | h_s, t_z, v, \beta)f_{H_s,T_z}(h_s, t_z)W_P(h_s, t_z, v, \beta)}{Q_{LT}(r)} \quad (7.1)$$

where $Q_R(R > r | h_s, t_z, v, \beta)$ is the short-term response distribution, $f_{H_s,T_z}(h_s, t_z)$ is the long-term joint probability distribution of the significant wave height, $H_s$, and mean peak
Chapter 7. Long-Term Response Statistics by Application of CRRW

period, $T_z$, $W_P(h_s, t_z, v, \beta)$ is a weight function which expresses the relative rate of peak responses within each sea state, and finally $Q_{LT}(r)$ which represents probability that the long-term value $R$ will be larger than $r$.

The derived contribution factor lines will split the scatter diagram into two parts. One area that contributes to most to the long-term response distribution, and an area that hardly influences the overall result. Focusing on the areas that contributes the most, the probability $Q_{LT}(r)$ that the long-term value $R$ will be larger than $r$ may thus be approximated as

$$Q_{LT}(r) = \sum_{i=1}^{N_i|C_R(h_s, t_z) \succ a} Q_R(R > r | h_{s,i}, t_{z,i}, v, \beta) f_{H_s, T_z}(h_{s,i}, t_{z,i}) W_P(h_s, t_z, v, \beta) \Delta h_{s,i} \Delta t_{z,i}$$

(7.2)

where $N_i | C_R(h_s, t_z) \succ a$ represents a number of discrete that contributes in the area selected.

Therefore, the contribution factor approach specifies the areas in the scatter diagram that contributes the most to the long-term response distribution. Several possibilities by application of the MLRW and CRRW models become now interesting. It was previously found that the MLRW model provided practically identical wave profiles for all combinations of $H_s$ and $T_z$ given the same operational profile and response level. It was further found that the transient phase of the wave profile before the conditional time instant $t_0$ tends to look similar for the operational profile selected.

One approach would be to apply the CRRW model directly in addition with the contribution factor approach. The previous analyses indicated that the very fine agreements to results of brute force simulations were obtained. Therefore, depending on the long-term operational profile reliable results would relatively easy be obtained.

Alternatively the model correction factor method could by applied. Firstly bias factors to the model should be obtained. The CRRW approach has to be applied for each of the combinations of $v$ and $\beta$ selected. The MLRW model and established bias factors can subsequently be applied to the entire scatter diagram or more limited in the areas where the most energy present as given though the contribution factor line approach. The latter is only considered possible as the analyses in this report points toward that the MLRW seems to be independent of $H_s$ and $T_z$ given the same operational profile and response level.

Both of the two approaches are considered as powerful tools on establishing long-term response distributions and most be considered very competitive to the rules and regulations from the classification societies.

7.3 MLRW, Long-Term Response Statistics

Floating Production Storage and Offloading (FPSO) vessel has an operational profile characterised by zero speed and head sea conditions throughout the entire life at sea. The vessel moves very rarely from its moored position because of the subsea system attached.
Figure 7.1 shows upcrossing rates for selected non-linear sagging response levels and combinations of $T_z$ and $H_s$ given an operational profile. The contour plots have been limited to observed combinations of the significant wave height and the zero-upcrossing period for the North Atlantic Ocean as represented in the scatter diagram from Table 2.1. The non-linear sagging response levels are generated by application of the MLRW approach with corresponding bias factors applied for zero speed and head sea. It is characteristic that the largest contributions to the individual response levels are naturally obtained from areas with the highest significant wave height within the available range.

The upcrossing rates $\nu_{\gamma}$ given $v$ and $\beta$ are obtained as

$$\nu_{\gamma}(r \mid v, \beta) = \int_{H_s} \int_{T_z} f_{H_s,T_z}(h_s,t_z) n_0(h_s,t_z) \left[ 1 - F_{R_{NL}}(r \mid h_s,t_z,v,\beta) \right] dt_z dh_s$$

(7.3)

where the peak distribution is derived fast by application of the MLRW and corresponding model correction factors for the upcrossing levels considered. It is assumed that the sea states are statistically independent with the probability density function $f_{H_s,T_z}(h_s,t_z)$. An example of weighted upcrossing rates is presented in Figure 7.4.

![Figure 7.1: Upcrossing rates for selected sagging response levels using the complete scatter diagram from Table 2.1. The non-linear results are obtained by application of the MLRW approach for zero speed and head sea.](image)

The upcrossing rates $\nu_{voyage}$ given $v$ and $\beta$ are obtained as

$$\nu_{voyage}(r \mid v, \beta) = \int_{H_s} \int_{T_z} \frac{1}{t_{voyage}} \left[ 1 - F_{R_{max}}(r \mid h_s,t_z,v,\beta) \right] f_{H_s,T_z}(h_s,t_z) dt_z dh_s$$

(7.4)
Here the upcrossing rates \( \nu_{\text{voyage}} \) from Eq. (7.4) or Eq. (7.5), the distribution for the maximum wave-induced bending moment amidships for any voyage of duration \( T \) is available through Eq. (2.73).

If the effect of speed and heading had to be included as well the upcrossing rate \( \nu_{\text{voyage}} \) would be obtained as

\[
\nu_{\text{voyage}}(r) = \int_{H_s} \int_{T_z} \int_{V} \int_{B} \frac{1}{t_{\text{voyage}}} \left[ 1 - F_{R_{\text{max}}}(r \mid h_s, t_z, v, \beta) \right] \times f_{H_s, T_z}(h_s, t_z) f_{V, B}(v, \beta \mid h_s, t_z) d\beta dv dt_z dh_s \tag{7.5}
\]

Figure 7.2 shows the areas in the applied scatter diagram which contribute the most to the individual sagging response levels. The red areas represent a crest on the contour plot. These areas are located for zero-upcrossing periods which generated waves of a length equal to the hull length.

Figure 7.2: Areas within a scatter diagram contributing to a given sagging response level. The results are based on zero speed and head sea. The red areas represent a peak.
Figure 7.3 shows the same principle, but made for hogging responses. The overall picture is the same. It is found, however, that the lowest response levels obtain energy from a slightly larger area.

From the two figures, it is found that energy is obtained from roughly two areas given that the response level is kept constant. One area which relatively hardly influences the results at all (blue areas), and an area of significantly larger importance (remaining part). Focus should therefore be applied within these areas.

![Figure 7.3: Areas within a scatter diagram contributing to a given hogging response level. The results are based on zero speed and head sea.](image)

Figure 7.4 shows upcrossing rates for the vertical bending hogging and sagging moment amidships, which are obtained from Eq. (7.3). The probability density function $f_{H_s,T_z}(h_s, t_z)$ is based on the scatter diagram from Table 2.1. The linear responses represent both hogging and sagging. The non-linear sagging upcrossing rates are found to the right and hogging to the left of the linear upcrossing rates. The current analyses are made very quickly, where the most time consuming part basically represents the calculation of the linear transfer functions.

For a upcrossing rate of $\nu_{g}(r) = 1.0 \cdot 10^{-9}$, the ratio $\text{VBM}_{NL}/\text{VBM}_{L}$ between the non-linear and linear responses equals 1.30 for sagging and 0.71 for hogging. The sag to hog ratio for the current upcrossing level equals therefore 1.84. Introduction of forward speed and heading is similarly easy made. It requires of course that model correction factors are established for the selected headings.
Figure 7.4: The upcrossing rates $\nu_{h}(r|v, \beta)$ are generated for a vessel which only operates at zero speed and in head sea like an FPSO. The non-linear results are given for both hogging and sagging.

### 7.4 Concluding Remarks

Long-term response statistics by application of the MLRW and CRRW models have briefly been studied. The analyses are performed for a rigid hull girder and for a simple operational profile. However, the can be established very quickly and most be considered very competitive to the rules and regulations from the classification societies.

Further work on this topic become interesting, as it is important to verify if the bias factors applied may be established from only one analysis by application of the CRRW approach for each of the operational profiles. It is furthermore of interest to perform the analyses for a flexible hull girder.
Chapter 8

Conclusions and Suggestions for Further Work

8.1 Conclusions

Application of conditional waves as critical wave episodes to predict extreme loads on marine structures has been studied. The introduction of short constrained simulations points towards faster estimation of short- and long-term response distributions, as the severer events are significantly faster identified.

Application of short conditional processes to the design of marine structures represents a powerful tool for the design of a vessel, and it must be considered a competitive candidate to the rules and regulations from the classification societies.

In the present thesis, focus is kept on application of the Most Likely Wave (MLW) and a new approach called the Most Likely Response Wave (MLRW). The vertical bending moment amidships has mainly been studied, which is the most important structural design parameter within ship construction.

Most Likely Wave Analyses

By application of the MLW with the random background wave included, i.e. the Conditional Random Wave (CRW), it is possible to generate short-term response statistics which compare well with brute force simulations. Results and operations are made for a Panmax container ship operating with zero speed and in head sea.

Application of MLW profiles has shown that the location of the MLW crest relative to the hull girder and the applied zero-upcrossing period relative to the hull girder are important
parameters when the short-term response statistics are sought. Given that the vessel has encountered the MLW crest physically, the largest linear and non-linear sagging responses are obtained if the vessel meets the MLW at the forward part of the vessel and most often at FP. The largest hogging responses are found if the MLW is located amidships.

Bias factors between medians (50 % levels) of extreme responses in a period of three hours established from the CRW approach to peak responses due to the deterministic MLW for selected values of $a = H_s$ have been established. The bias factors were found to be in the range of 1.07 to 1.15 for the linear model and 1.12 to 1.22 for the non-linear model by application of the PM wave spectrum. Slightly lower ratios were obtained by application of the JONSWAP wave spectrum. The results have indicated that the random background waves for dynamically responding structures can be important, and that the specific wave amplitude not necessarily is the only parameter to be considered as extreme responses are sought.

Application of the MLW as a critical wave episode, however, has a drawback as it does not include any information on the size and shape of the marine structure considered.

**Most Likely Response Wave Analyses - Rigid Hull Girder**

The application of Most Likely Response Waves (MLRW) to estimation of the entire non-linear extreme value distribution for a selected operational profile in a given sea state has been studied.

The MLRW approach is based on conditioning on the linear response process. The underlying wave process which causes the conditional response is subsequently derived with amplitude and phase information from the linear transfer function. A non-linear time domain analysis may then be performed with the derived wave profile as input. It is thus assumed that the linear model is a good predictor of the non-linear responses.

Simulations of the MLRW profiles relative to the instantaneous position of the vessel considered showed that very similar wave profiles were generated. MLRW that generates a sagging-induced vertical bending moment amidships is characterised by a wave length of almost similar length as the hull girder. The wave amplitudes change according to the linear conditional response level and the forward speed selected. MLRW that generates a hogging-induced bending moment tend to look much like the MLW profiles. Main variations are found in the amplitude of the MLRW crest.

The results of the MLRW approach have been compared with those of the MLER method, Adegeest et al. (1998), and a brute force simulation. Good agreement between the new MLRW approach and brute force simulation has been found. The effects of the random background wave included in the MLRW, i.e. the Conditional Random Response Wave (CRRW) are observed, showing increasing importance with increasing vessel velocities, which indicates that the effect is not to be neglected.
8.1 Conclusions

The present study derives a new approach by application of MLRW to estimate the entire non-linear extreme value distribution for a selected operational profile in a given sea state. The estimates are based on the model correction factor method where bias factors are obtained by application of the CRRW approach. Bias factors are obtained for \( v = 0, 5 \) and \( 10 \) m/s and considered representative for the entire scatter diagram due to the very similar MLRW profiles derived.

The results of the MLRW approach have been compared with the CRRW method, and a brute force simulation. Good agreement between the MLRW approach and brute force simulation has been found. Bias factors of \( 1.015 \rightarrow 1.160 \) are obtained for sagging and \( 0.972 \rightarrow 1.063 \) for hogging. The largest variations introduced are obtained for increasing forward speed. The MLRW profile captures most of the important memory effects at zero forward speed, but with increasing difficulties as forward speed is introduced. The effects of the random background wave of the CRRW profile is therefore important given an exact solution is desired.

Presently, it is considered difficult to specify general trends as regards the effect of forward speed as only one vessel has been examined. Adegeest et al. (1998) obtained good agreement between simulation and model tests with forward speed included; here non-linearities were introduced with a sag to hog ratio of \( \sim 1.93 \). The experiments were made for a container ship in a sea state of \( H_s = 4.8 \) m and \( T_z = 8.0 \) sec. The selected sea state by Adegeest et al. (1998) with relatively smaller vessel motions compared to the sea state of \( H_s = 10.0 \) m and \( T_z = 11.35 \) sec. considered in the present study may explain the difference in the response due to the MLER wave profile and the MLRW profile.

It is therefore recommended that the CRRW approach is used to establish short-term response statistics as the model practically captures all the non-linear effects. The model correction factor method and derived bias factors are recommended as long-term response statistics are sought.

Most Likely Response Wave Analyses - Elastic Hull Girder

Two models are presented for application of the MLRW or CRRW wave profiles for prediction of the combined wave- and whipping-induced responses. The effect of whipping, given a wave-induced sagging response, is predicted well by application of the MLRW and CRRW approaches. The latter provides the best fit to brute force simulations, also as the forward speed is increased. The MLRW model is found to capture transient effects well. From the present simulations, bias factors of \( 0.95 \rightarrow 1.13 \) should be applied to correct the model properly. As mentioned previously, the MLRW approach introduces a fast and accurate approach for estimation of extreme value distributions. Only five to ten non-linear time domain simulations of approximately 100 sec. are required to derive a reliable fit to the true upcrossing rates.

The CRRW model presented for predicting the effect of whipping, given a wave-induced hogging response, corresponds well with brute force simulations, also when forward speed is
Chapter 8. Conclusions and Suggestions for Further Work

The transient effects from slamming are captured well. The MLRW model predicts less well the transient whipping-induced responses. Results of the MLRW model are biased with factors of $1.05 \rightarrow 1.25$ for the selected operational parameters and sea states.

The MLRW and CRRW approaches should be verified through further analyses of various types of ships to obtain further validation data, but generally it is believed that the present model of most likely response waves represents a very powerful tool for estimation of response statistics.

Long-Term Response Statistics by Application of the MLRW and CRRW Models

Long-term response statistics by application of the MLRW and CRRW models have briefly been studied. The present approaches allow the long-term response statistics to be established fast and most be considered very competitive to the rules and regulations from the classification societies.

The MLRW and CRRW models have been discussed in relation to the contour line approach. It is recommended that the CRRW model is applied directly with the contour line approach as the analyses have indicated very fine agreements to results of brute force simulations.

Alternatively, the model correction factor method could be applied. Here the CRRW approach has to be applied for each of the combinations of $v$ and $\beta$ selected to obtain bias factors. The MLRW model and established bias factors can subsequently be applied to the entire scatter diagram. The latter is only considered possible as the analyses in this report points toward that the MLRW seems to be independent of $H_s$ and $T_z$ given the same operational profile and response level.

8.2 Suggestions for Further Work

Validation of the MLRW and CRRW Approaches

The MLRW approach presented has been validated by the non-linear time domain code ShipStar, Xia et al. (1998). Further analyses with other types of vessels and comparison with model or full scale measurements would therefore be very interesting. One of the main objective should be to gain further knowledge of the mode correction or bias factors to be applied.

The present approach points additionally towards the application of three-dimensional time-domain theories like SWAN, Vada & Helmers (1992) or for example LAMP, Lin et al. (1996). The simulation costs are still relatively heavy compared with an alternative approach on application of for example a time domain strip theory code. Application of the MLRW wave
8.2 Suggestions for Further Work

profile for the first simulations to be made would therefore be recommended. Extended analyses on application of the CRRW would be relevant to obtain further information on the response statistics.

Long-Term Response Statistics by Application of the MLRW Model

It is of significant interest to make further studies on establishment of bias factors for the model correction factor method in relation to long-term response statistics. Given that analyses verify that the bias factors can be considered independent of $H_s$ and $T_z$ for other operational parameters, loading conditions and other types of vessels, it become natural to apply the MLRW approach.

Additional Applications of the MLRW

Currently, the design of a vessel is mainly based on rules developed by the classification societies. These rules and regulations are based on semi-analytical models for response and strength, and are to some extent modified empirically to agree with observations made. It is therefore not straightforward to compare individual designs.

The empirical formulations do not always generate accurate results, particularly for novel ship designs with for example a sandwich hull structure, requiring a direct time domain simulation to be performed. It is not considered reasonable to perform non-linear time domain simulations in the random sea over multiple sea states to identify extreme ship responses to be applied in the design process, as these analyses are too time-consuming.

Introduction of the linear MLRW as design wave makes it possible to identify a relatively short wave pattern that leads directly to an ultimate design value for the response. Using this wave pattern, or design wave, in combination with a finite element analysis will thus effectively lead to consistent estimation of for example the stress distribution in the hull section.
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References


Appendix A

Ship Responses and Load Theory

A.1 Introduction

Ships operating in heavy seas are exposed to non-linear wave- and slamming-induced forces. Full-scale and model measurements have shown that the wave-induced sagging bending moments can be considerably larger than the wave-induced hogging moments, Wang (2000).

This appendix describes briefly the non-linear time domain strip theory by, Xia et al. (1998). The time domain code, which is called ShipStar, takes into account non-linearities in wave- and slamming-induced rigid-body motions and structural responses of ships such as heave, pitch and the vertical bending moment. Moreover, an optional hydrodynamic model for prediction of sectional green water loads is included. The hydrodynamic memory effect due to the free surface wave motion is approximated by solving a higher order ordinary differential equation. The hydrodynamic and restoring forces for the non-linear case are estimated over the instantaneous wetted surface of the hull girder. The momentum slamming force is also accounted for. The extreme sagging wave bending moments are normally determined by taking into account the non-linearities due to momentum slamming and hydrodynamic restoring actions. These non-linearities are sensitive to the flare size and are therefore important to consider in the design of container ships. The fluid force expression is coupled with the structure, which is modelled as a Timoshenko beam. The representation of the hull girder allows a hydroelastic analysis to be made.

ShipStar has systematically been verified by experimental results, Xia et al. (1998) and Wang (2000). The linear solution agrees well with existing strip theory solutions. The predicted non-linear behaviour of rigid-body motions and vertical hogging/sagging bending moment ratios of ships corresponds well with experimental results.


A.2 Non-Linear Time Domain Strip Theory

A.2.1 The Hydrodynamic Force

The non-linear hydrodynamic force $F(x, t)$ acting on the hull girder in the longitudinal position $x$ at a given time step $t$ is given as

\[
\begin{cases}
F(x, t) = \frac{D I}{D t} \\
\sum_{j=0}^{J} (B_j I - A_j \frac{D \pi}{D t})^{(j+1)} = 0
\end{cases}
\tag{A.1}
\]

where $I$ represents both the impulsive and the memory effects in the hydrodynamic momentum, $I^{(j)} = \frac{D^j I}{D t^j}$. $A_j(x, \pi)$ and $B_j(x, \pi)$ are the frequency independent hydrodynamic coefficients, which are derived from the frequency dependent added mass $m(x, \pi, \omega)$ and the damping coefficient $N(x, \pi, \omega)$:

\[
i \omega m - N = \sum_{j=0}^{J} \frac{A_j(-\omega)^{j+1}}{\sum_{j=0}^{J} B_j(-\omega)^{j}}
\tag{A.2}
\]

with $J = 3$ to fit most of the sectional shapes. Examples are given by Wang (2000). For the linear model, $A_j(x, \pi)$ and $B_j(x, \pi)$ are taken as functions of only $x$ whereas the effect of change of wetted body surface is neglected. $\pi(x, t)$ represents the relative motion between the structure and the wave surface and is given as

\[
\pi(x, t) = w(x, t) - \bar{\zeta}(x, t)
\tag{A.3}
\]

where $w(x, t)$ is the vertical motion of the hull and $\bar{\zeta}(x, t)$ is the wave elevation with Smith correction. The total derivative $\frac{D}{D t}$ with respect to time $t$ is given as

\[
\frac{D}{D t} = \frac{\partial}{\partial t} - v \frac{\partial}{\partial x}
\tag{A.4}
\]

where $v$ is the forward speed of the vessel.

By integration of the higher order coupled differential equations from Eq. (A.1) and by introduction of the hydrostatic buoyancy force $f_b$ acting on the instantaneous wave surface and the green water force $f_{gw}$, the total non-linear external fluid force $F_t(x, t)$ acting on a give section may be expressed as

\[
F_t(x, t) = -\overline{m} \frac{D^2 \pi}{D t^2} + v \frac{\partial \overline{m} D \pi}{\partial x} - \frac{\partial \overline{m}}{\partial \pi} \left( \frac{D \pi}{D t} \right)^2 - \frac{D q \overline{m}}{D t} + f_b + f_{gw}
\tag{A.5}
\]

The first term of Eq. (A.5) represents change of the added mass $\overline{m}(x, \pi)$ of the ship section when the oscillating frequency tends towards infinity. The second term represents the wave diffraction force. The third term of $F_t(x, t)$ represents the momentum slamming force, which
A.2 Non-Linear Time Domain Strip Theory

is assumed to be zero as the vessel exits the water. The fourth term \( \frac{Dq_j}{Dt} \) accounts for the memory hydrodynamic effect where \( q_j \) is given by the set of differential equations

\[
\left\{ \begin{array}{l}
\frac{\partial q_j(x,t)}{\partial t} = q_{j-1}(x,t) - B_j q_j(x,t) - (mB_{j-1} + A_{j-1}) \frac{Dq}{Dt} \\
q_0(x,t) = 0 \end{array} \right. \quad j = 1, 2, ..., J \quad (A.6)
\]

The hydrostatic buoyancy force \( f_b \) under the sectional water surface is given as

\[
f_b(x,t) = \rho g S(x,\zeta) \quad (A.7)
\]

where \( \rho \) is the sea water density, \( g \) is the acceleration of gravity and \( S(x,\zeta) \) the instantaneous wetted area of the selected ship section.

The green water force \( f_{gw} \) is briefly introduced here, see also Xia et al. (1998) and Wang (2000). The vertical load directed upwards per unit length from green water on deck in the longitudinal direction \( x \) at time \( t \) is given as

\[
f_{gw}(x,t) = -gm_{gw}(x,t) - \frac{D}{Dt} \left[ m_{gw}(x,t) \frac{Dz_e}{Dt} \right] \quad (A.8)
\]

where \( m_{gw} \) is the instantaneous mass per unit length of green water. The effective relative motion \( z_e \) is given as the nominal relative motion \( Z_n(x,t) = w(x,t) - \zeta(x,t) \). Here \( \zeta(x,t) \) is the undisturbed wave elevation.

\[
z_e(x,t) = \left\{ \begin{array}{l}
(2.1 - c_s)z_n(x,t); \quad h_e(x,t) \leq D_f(x) \\
(3 - 2c_s)z_n(x,t); \quad h_e(x,t) > D_f(x)
\end{array} \right. \quad (A.9)
\]

where \( c_s \) is the Smith correction factor for the instantaneous submerged surface of the vessel, and \( h_e(x,t) \) represents the effective green water height on the deck \( h_e(x,t) \), which is taken to be proportional to the green water mass, \( m_{gw} \).

\[
m_{gw} = \rho B_e(x)h_e(x,t) \quad (A.10)
\]

with \( B_e \) being the effective breadth of the green water and \( h_e(x,t) = -z_e(x,t) - D_f(x) \) where \( D_f(x) \) denotes the freeboard height.

A.2.2 Hydroelastic Equation of Vertical Motion

The fluid force expression is coupled with the structure, which is modelled as a Timoshenko beam. The representation of the hull girder allows a hydroelastic analysis to be made. Due to small rigid body motions and structural deformations a model superposition of the displacement may be allowed:

\[
w(x,t) = \sum_{r=0}^{n} w(x)p_r(t) \quad (A.11)
\]
where $w_r$ is the $r$’th unit dry mode shape with $w_r(0) = 1$ and $p_r$ being the $r$’th generalised principal coordinate of the motion. It follows that $r = 0$ and $1$ represents heave and pitch motions, whereas $r = 2, 3, \ldots n$ represents vertical elastic deformations (distortions) of the hull girder.

The equation of motion of a ship structure in the vertical direction per unit length can be written as, Bishop & Price (1979):

$$\sum_{r=0}^{n} \left[ a_{sr} \ddot{p}_r(t) + b_{sr} \dot{p}_r(t) + c_{sr} p_r(t) \right] = \int_L (F_t - \mu g) w_s dx, \quad s = 0, 1, \ldots n \quad \text{(A.12)}$$

The total non-linear external force from Eq. (A.5) is observed on the righthand side. $\mu(x)$ is the longitudinal mass distribution of the ship structure and $L$ the hull length. The components $a_{sr}$, $b_{sr}$, and $c_{sr}$ on the left side are the generalised structural mass, damping and stiffness matrices.

$$a_{sr} = \delta_{sr} \int_L (\mu w_r w_s + I_y \theta_r \theta_s) dx$$
$$b_{sr} = \delta_{sr} a_{sr} \omega_r \alpha_r / \pi$$
$$c_{sr} = \delta_{sr} a_{sr} \dot{\omega}_r \quad \text{(A.13)}$$

where $\delta_{sr}$ is the Kronecker delta function and $I_y$ is the longitudinal distribution of the inertia of the hull girder. $\theta_r$ is the $r$th mode angular function of the hull girder beam in the $x - z$-plane induced by vertical bending. $\alpha_r$ is the $r$th mode logarithmic damping decrement of the structure in vacuum. Finally, $\dot{\omega}_r$ is the $r$th mode natural mode of the dry hull.

The equations of the fluid-structure interaction can be obtained by inserting Eq. (A.5) into Eq. (A.12). The detailed description is given by Wang (2000).

### A.3 Modifications to the Existing ShipStar Code

#### A.3.1 Implemented Functions in the Existing Code

The most likely response wave approach has been implemented in ShipStar. From the input files it is possible to apply a traditional analysis, the MLW or the MLRW approach. For the two last-mentioned methods, the analyses may be performed for the mean deterministic wave profile or the wave profile where the random background wave is included.

The wave profile generated within ShipStar is obtained as a linear superposition of sinusoidal wave components which generate an irregular wave train. The ocean surface may be seen as statistically stationary in a given area and during a limited period of time. The wave elevation $Z(x, t)$ is given as

$$Z(x, t) = \sum_{n=1}^{N} a_{\xi,n}^e \left[ V_n \cos(k_{e,n}x - \omega_{e,n}t) + W_n \sin(k_{e,n}x - \omega_{e,n}t) \right] \quad \text{(A.14)}$$
A.3 Modifications to the Existing ShipStar Code

where $N$ is large. $V_n$ and $W_n$ are independent standard normal random variables. For the individual wave components, $k_{e,n} = k_n \cos(\beta)$ with $k_n = \frac{\omega_n}{g}$ being the wave number obtained by the dispersion relation for deep water waves. The coefficients $a_{e,n}$ are determined from the wave spectrum.

From the desired wave profile to be applied, the independent standard normal random variables $V_n$ and $W_n$ are changed to a set of constrained variables $V_{c,n}$ and $W_{c,n}$ given through Eq. (3.61). Before performing the simulation, it is necessary to derive the linear transfer function for the selected operational profile. Simulations using ShipStar are hereafter continued on a normal basis.

The formulation of the wave profile from Eq. (A.14) apply naturally to both the traditional analysis and the constrained approaches.

A.3.2 Using ShipStar

It is assumed that the user is familiar with the "ShipStar, User’s Guide and Reference Manual", Wang (2001). This section describes the input files to be made and how to perform constrained response simulations. There are two possibilities for performing a constrained simulation. One can either perform conditioning on the wave process or the response process.

Preparation of Input Files and Executing ShipStar

A number of input files are needed before ShipStar can be executed as well as the file "fname.dat" which lists these files:

<table>
<thead>
<tr>
<th>File</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;name&gt;.out</td>
<td>Description of the ship hull geometry</td>
</tr>
<tr>
<td>a&lt;name&gt;.dat</td>
<td>General information on the calculation</td>
</tr>
<tr>
<td>c&lt;name&gt;.dat</td>
<td>Output file from tico3.exe</td>
</tr>
<tr>
<td>s&lt;name&gt;.dat</td>
<td>Description of the ship structure</td>
</tr>
<tr>
<td>rewa.dat</td>
<td>Information on the waves to be applied</td>
</tr>
<tr>
<td>LinTrans.dat</td>
<td>The linear transfer function</td>
</tr>
</tbody>
</table>

Table A.1: Input files for ShipStar, "fname.dat".

In the version of ShipStar used new input parameters have been added to the "a<name>.dat" compared to previous versions. The input lines are listed in Table A.2.
Table A.2: Additional input for ShipStar, "a<name>.dat".

<table>
<thead>
<tr>
<th>USECOND</th>
<th>NCONP</th>
<th>MLWNC</th>
<th>ALPHAMLMW</th>
<th>UseMorR</th>
<th>UseRAO</th>
<th>ALPHAVBM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2000</td>
<td>51</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>2400000000</td>
</tr>
</tbody>
</table>

- USECOND = 1 Perform a conditional calculation (MLW/CRW or MLRW/CRRW)
  - The time series length is limited to 5000 steps
- USECOND = 0 Perform a normal regular or irregular response simulation
- NCONP : Number of time steps before the conditional time instant
- MLWNC : Longitudinal station number for the conditional peak
- ALPHAMLMW : Amplitude of the MLW or CRW, [m]

UseMorR = 1 The MLW or MLRW profile will be obtained
UseMorR = 0 The CRW or CRRW profile will be obtained
  (For both cases, the variable USECOND should equal 1)

UseRAO = 1 Conditioning is applied to the response process. --> The MLRW or CRRW profile will be obtained.
  Remember to correct the file "LinTrans.dat" if changes are made to NW, U, β or linear/non-linear case
UseRAO = 0 Conditioning is applied to the wave process. --> The MLW or CRW profile will be obtained

ALPHAVBM : The constrained peak value of the VBM in [MNm]. The constrained time and location of the peak response is given though the parameters, NCONP and MLWNC

UseRAO = 1
UseRAO = 0

ALPHAVBM : 2400000000
A response analysis given a conditional wave process only requires changes to the file "a<name>.dat". The variable USECOND should equal 1, UseMorR should equal 0 or 1 and indicates whether the random background wave should be included or not. NCONP represents the number of time steps to the conditional time instant where ALPHAMLW provides information on the conditional wave amplitude. MLWNC specifies the longitudinal station number along the hull girder where the conditional peak should occur. The output station numbers are specified in the variable STA. The variable UseRAO should equal 0.

ShipStar can now be executed from "ShipStar.exe". Output from the wave and the response process are given in "22.dat" and "44.dat". See Wang (2001) for additional information.

Before a conditional response simulation can be performed, the linear transfer function of interest should be estimated. By executing ShipStar in its linear mode and using regular waves, the amplitude and phase information are written in the file "AmpPhase.DAT". It is important that the frequencies applied are similar to the frequencies calculated by ShipStar. The amplitude and phase information should be used as input in the file "LinTrans.dat". This input file contains information on the transfer function of the response and corresponding phase angle for the given frequencies. The information are obtained by running ShipStar using regular waves, amplitude = 1.0 m.
Table A.3: *Input file for ShipStar, "LinTrans.dat".*

<table>
<thead>
<tr>
<th>LWFRAO</th>
<th>: Number of frequencies, LWFRAO &lt;= 300</th>
</tr>
</thead>
<tbody>
<tr>
<td>WFRAO</td>
<td>: Encounter frequencies, should be changed each time the operational parameters are changed.</td>
</tr>
</tbody>
</table>

LWFRAO
WFRAO(1:LWFRAO)
*

100

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<th>0.244</th>
<th>0.264</th>
<th>0.283</th>
<th>0.302</th>
<th>0.321</th>
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<td>0.476</td>
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### Table A.3 continued.

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<tr>
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</tr>
<tr>
<td>Given in MNm. The responses are obtained using a regular wave with an amplitude of 1.0 m at the selected frequencies.</td>
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</tr>
<tr>
<td>LAMP</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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| 17.630 | 26.816 | 38.482 | 52.916 | 70.270 | 90.751 | 114.372 | 141.143 |
| 170.913 | 203.421 | 238.201 | 274.666 | 312.029 | 349.263 | 385.198 | 418.487 |
| 447.585 | 470.892 | 486.814 | 493.848 | 490.795 | 477.028 | 452.223 | 417.544 |
| 374.755 | 326.809 | 276.956 | 227.778 | 180.524 | 135.440 | 94.867 | 69.850 |
| 77.905 | 107.723 | 138.927 | 162.331 | 173.980 | 172.398 | 157.865 | 132.111 |
| 98.030 | 59.517 | 21.506 | 19.636 | 47.411 | 67.078 | 76.214 | 74.677 |
| 63.721 | 45.694 | 23.686 | 1.051 | 19.091 | 34.186 | 42.900 | 44.754 |
| 4.864 | 4.538 | 4.346 | 4.385 |
Table A.3 continued.

<table>
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<th>LPHS</th>
<th>RAOPHS(1:LPHS)</th>
</tr>
</thead>
<tbody>
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<td>Corresponding phase angles</td>
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<tr>
<td>RAOPHS(1:LPHS)</td>
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<tr>
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<td>0.162</td>
</tr>
<tr>
<td>3.279</td>
<td>4.068</td>
<td>4.913</td>
<td>5.748</td>
</tr>
</tbody>
</table>

The conditional response simulation may now be performed. It is important that the parameter UseRAO = 1, and that the station used to obtain the linear transfer function corresponds with the station selected by the variable STA (from "a<name>.dat"). The input variable UseMorR controls the use of MLRW or CRRW.
Appendix B

The Panmax Container Ship

All the analyses in this thesis are made for the Panmax container ship as shown in Figure B.1. In this appendix, the main hull dimensions, the structural data on the midship section, a body plan, a selected weight distribution and the corresponding linear transfer functions are presented. These are given for different operational parameters and obtained by application of ShipStar as described in Appendix A. The linear transfer functions are derived from a rigid hull girder.

Figure B.1: *The Magleby Maersk.*
B.1 The Panmax Vessel

The main dimensions of the Panmax container ship are given in Table B.1.

Table B.1: Main dimensions of the Panmax container ship.

<table>
<thead>
<tr>
<th>Main dimensions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length overall, $L_{oa}$</td>
<td>294.30 m</td>
</tr>
<tr>
<td>Length between perpendiculars, $L_{pp}$</td>
<td>276.38 m</td>
</tr>
<tr>
<td>Breadth mld $B_{mld}$</td>
<td>32.20 m</td>
</tr>
<tr>
<td>Depth mld, $D_{mld}$</td>
<td>21.50 m</td>
</tr>
<tr>
<td>Draught, $T$</td>
<td>11.20 m</td>
</tr>
<tr>
<td>LCG from AP</td>
<td>138.50 m</td>
</tr>
<tr>
<td>Block coefficient, $C_b$</td>
<td>0.627 [-]</td>
</tr>
<tr>
<td>Max service speed, $v$</td>
<td>24.80 knots</td>
</tr>
</tbody>
</table>

The structural dimensions of the midships section are given in Table B.2

Table B.2: Structural dimensions of the Panmax containership.

<table>
<thead>
<tr>
<th>Structural dimensions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of inertia, $I_{yy}$</td>
<td>351.6 m$^4$</td>
</tr>
<tr>
<td>Section modulus, $W_{Deck}$</td>
<td>32.3 m$^3$</td>
</tr>
<tr>
<td>Section modulus, $W_{Keel}$</td>
<td>33.1 m$^3$</td>
</tr>
<tr>
<td>Neural axis, from BL.</td>
<td>10.6 m</td>
</tr>
<tr>
<td>Modulus of elasticity, $E$</td>
<td>2.1 e11 N/m$^2$</td>
</tr>
<tr>
<td>Yield stress, HTS36</td>
<td>360 MN/m$^2$</td>
</tr>
</tbody>
</table>
B.2 Body Plan

Figure B.2 shows the body plan of the Panmax container ship.

Figure B.2: Body plan, the Magleby Maersk.

B.3 Weight Distribution

Figure B.3 shows the weight distribution applied. The current weight distribution equals a zero trim condition with a mean draught $T$ of 11.2 m.

Figure B.3: Applied weight distribution.
B.4 Modelling of the Hull Girder

For the hydrodynamic calculations, the bending rigidity $EI$ is assumed to follow a trapezoidal shape. The hull rigidities applied are given in Table B.3.

<table>
<thead>
<tr>
<th>Hull rigidities</th>
<th>Station 1 (AP)</th>
<th>10</th>
<th>29</th>
<th>40 (FP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EI, [Nm^2]$</td>
<td>3.25 $\cdot 10^{13}$</td>
<td>7.23 $\cdot 10^{13}$</td>
<td>7.23 $\cdot 10^{13}$</td>
<td>3.25 $\cdot 10^{13}$</td>
</tr>
</tbody>
</table>

where $E$ is the modulus of elasticity and $I$ is the bending stiffness amidships.
B.5 Linear Transfer Functions

Linear transfer functions of the vertical bending moment amidships are given for selected forward velocities and heading angles. The transfer functions are given for \( v = 0 \), 5 and 10 m/s, and \( \beta = 180 \) (head sea) and 135 degrees.

![Graphs showing linear transfer functions for different velocities and headings.](image)

Figure B.4: Linear transfer function, \( v = 0 \), 5 and 10 m/s, \( \beta = 180 \) degrees.
Figure B.5: Linear transfer function, $v = 0$, 5 and 10 m/s, $\beta = 135$ degrees.
Appendix C

Estimation of Wave Spectra

C.1 Introduction

The aim of the present study is to provide a rational procedure for predicting a wave spectrum from a recorded strain amidships at the deck of the vessel. For shipmasters who operate large vessels, an accurate method for estimation of the present sea state is of great importance, especially during night where it is extremely difficult to predict the specific sea state visually.

The present analyses are made for the Panmax container ship as presented in Appendix B.

C.2 Full-Scale Measurements

The data used for the present study has been sampled during a sea trail from Rotterdam to Halifax in Canada from 1 December to 7 December 2002, where several periods with harsh weather were experienced. During the sea trail, a strain signal was recorded on the deck on both port and starboard sides amidships. Green water on deck, relative wave motion at FP and accelerations in the wheelhouse and at FP were also recorded.

The first harsh period was selected for further studies, where satellite observations of the surface were additionally recorded for comparison. An example is given in Figure 2.1, with the satellite estimated significant wave heights illustrated by different colour codes. During the first period, an average significant wave height $H_s$ of 6.6 m was recorded. Vessel’s speed and direction relative to the waves were recorded to be $v = 12$ knots and $\beta = 195$ degrees. The relatively low speed was due to the very harsh weather, with wind speeds exceeding 30 m/s for shorter periods.
C.2.1 The Recorded Signals

Two strain signals amidships on port (BB) and starboard sides (SB) were recorded. The measurement devices were located just below the main deck (19.25 m. above the keel). The port side signal was used for the analyses, with an example given in Figure C.1. The sampled data series had some irregularities, which have been repaired. The signal was mainly filtered for some very large unrealistic peak responses recorded.

The left plot in Figure C.1 shows the sampled data as a function of time (sampling frequency 10Hz). The figure to the right shows a Fourier transform of the signal where the two node whipping induced hull girder vibration is observed at $\omega_e \sim 4.0 \text{ rad/s}$, which corresponds well with the observations made in Chapter 6. Hogging events are identified by positive peaks whereas sagging events are given as troughs.

Change in Weather and Response

The standard deviation and the mean value of the recorded signal on the port side have been calculated continuously for individually specified lengths throughout a part of the sampled signal. Figure C.2 shows the variation of the mean value and standard deviation over time. The results are derived for individual sample lengths of 10, 30 and 60 min.

From the observations shown in Figure C.2 and Table C.1, a short term period from 07:00 to 10:00 was selected for further studies.
C.3 Sea State Estimation

From time series of measured ship motions and/or sea loads the sea state can be estimated. Different procedures have been suggested in the literature, e.g. Huss & Olander (1994). The most straightforward method is to derive the wave spectrum $S(\omega)$ from the response...
Appendix C. Estimation of Wave Spectra

The wave spectra predicted in this study are obtained in two steps. First by minimising the overall error $f_{err}$ between the estimated response spectrum $S_{\eta,\zeta}^{est}(\omega_e,n)$ and the measured response spectrum $S_{\eta,\zeta}^{mes}(\omega_e,n)$:

$$f_{err} = \sum_{n=1}^{N} |S_{\eta,\zeta}^{est}(\omega_e,n) - S_{\eta,\zeta}^{mes}(\omega_e,n)|^2$$

(C.2)

with the estimated response spectrum given as

$$S_{\eta,\zeta}^{est}(\omega_e) = \sum_{j=1}^{J} \alpha_j S_\zeta(\omega_e | h_{s,j}, t_{z,j}, \gamma_j) | \Phi_\eta(\omega_e | \eta, \beta) |^2$$

(C.3)

Subsequently, small changes in the derived values of $H_s$, $T_z$ and $\gamma$ were made by minimising the difference between the spectral moments $m_0$ of the calculated and the measured response spectrum. The best fits were obtained by application of four to five individual spectra.

The current analysis are derived on application of a summation of JONSWAP spectra which are a function of the significant wave height, the zero-upcrossing and the peakedness factor $\gamma$. The analyses could additionally have been made on application of the gamma spectrum as presented in section 2.2.3. The method presented above is practical from an engineering point of view as the spectral moments can be generated. A minor drawback of the current approach is present as the obtained values of $H_s$, $T_z$ and $\gamma$ may show some variation within the derived values.

C.3.1 Estimated Significant Wave Height and Zero-Upcrossing Period

Figure C.3 shows an example of estimated wave spectra obtained from a recorded strain signal amidships. The uppermost figure illustrates two wave spectra. One of them is obtained from the direct approach, where the smoothed measured response spectrum has simply been
divided by the response amplitude operator squared. The second wave spectrum has been fitted by application of Eq. (C.3). For the present case four individual wave spectra are used. The figure in the middle presents the applied response amplitude operator for $v = 12$ knots and $\beta = 195$ degrees, where 180 degrees represent head sea. The response amplitude operator has been generated by the strip theory program I-SHIP applying a linear analysis. The lower plot represents the measured response spectrum after smoothing. The illustrated figures are all plotted as functions of the encounter frequency, $\omega_e$.

![Wave Spectrum](image)

**Figure C.3:** Top: Wave spectra, direct and fitted approach. Middle: The applied response amplitude operator. Bottom: Measured and fitted response spectra, all as functions of the encounter frequency, $\omega_e$.

For the current analyses, long-crested waves are applied. Introduction of a directional wave spectrum for the optimisation would presumably generate a slightly better overall fit. Table C.2 illustrates the results obtained by applying the above procedures. The significant wave height has been obtained as $H_s = 4\sqrt{m_0}$ and $T_z = 2\pi \sqrt{\frac{m_0}{m_2}}$, where $g$ is the acceleration of
Appendix C. Estimation of Wave Spectra

gravity and $m_n$ the spectral moment of order $n$. Fine agreement between predicted significant wave height and satellite observations is found for the example concerned.

Table C.2: Estimated and observed sea states, 2 December, 2002 at 07:00 hours.

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Satellite</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_s$</td>
<td>5.97 m</td>
<td>6.6 m</td>
</tr>
<tr>
<td>$T_z$</td>
<td>10.99 s</td>
<td>NA</td>
</tr>
</tbody>
</table>
Appendix D

Most Likely Wave

Friis-Hansen & Nielsen (1995) derived the Most Likely Wave (MLW) profile conditioning on both the wave amplitude, $a$, and the instantaneous wave frequency, $\omega$. The wave process may be written as

$$\zeta(t)_{a,\omega} \equiv \hat{E}[Z(t) \mid Z(0) = a, \dot{Z}(0) = 0, \Omega = \omega]$$  \hspace{1cm} \text{(D.1)}

where $\hat{E}[\cdot]$ is the conditional mean of the process $Z(t)$. Here $\Omega \equiv \dot{Z}(0)/Z(0)$, where $\dot{Z}(t)$ is the Hilbert transform and $\dot{Z}(t)$ being its first derivative. The MLW is established from a vector process $Z(t) = (Z(t), \dot{Z}(t))$ as

$$\hat{E}[Z(t) \mid (Z(0), \dot{Z}(0)) = (a, 0), (\dot{Z}(0), \ddot{Z}(0)) = (0, \omega a)]$$  \hspace{1cm} \text{(D.2)}

The model is obtained by use of Slepian model processes, Ditlevsen (1985). They described the response process by an envelope process, Cramer & Leadbetter (1967), with the envelope $R(t)$ given as

$$R(t) = \sqrt{Z(t)^2 + \dot{Z}(t)^2}$$

where $\dot{Z}(t)$ is the Hilbert transform of $Z(t)$

$$Z(t) = \sum_{n=1}^{N} a_{\zeta,n} [V_n \cos(\omega_n t) + W_n \sin(\omega_n t)]$$  \hspace{1cm} \text{(D.3)}

$$\dot{Z}(t) = \sum_{n=1}^{N} a_{\zeta,n} [V_n \sin(\omega_n t) - W_n \cos(\omega_n t)]$$  \hspace{1cm} \text{(D.4)}

where $a_{\zeta,n}$ are coefficients determined on basis of the wave spectrum. $V_n$ and $W_n$ are independent Gaussian random variables with zero mean.

The initial conditions of the Slepian model process from Eq. (3.15) may be written in terms of a random vector $S = (S_1, S_2)$ and $T = (T_1, T_2)$, Ditlevsen & Lindgren (1988), for $t = 0$ the initial conditions become

$$S_1 = \sum_{n=1}^{N} a_{\zeta,n} V_n - Z(0)$$  \hspace{1cm} \text{(D.5)}
Appendix D. Most Likely Wave

\[ S_2 = \sum_{n=1}^{N} a_{\zeta,n} \omega_n V_n - \dot{Z}(0) \]  
(D.6)

\[ T_1 = \sum_{n=1}^{N} a_{\zeta,n} W_n - \dot{Z}(0) \]  
(D.7)

\[ T_2 = \sum_{n=1}^{N} a_{\zeta,n} \omega_n W_n - \dot{Z}(0) \]  
(D.8)

From Eq. (D.5) to Eq. (D.8) it is seen that the coefficients \( V = (V_n, W_n) \) depend on the choice of \( Y \). For \( S = (0, 0) \) and \( T = (0, 0) \) the wave profile will satisfy the given initial conditions, and as the variance of the residual of the conditional random vector \([V|S]\) is constant for any \( S \), the conditional vector becomes

\[ [V|S = (0,0)] = V - \hat{E}[V|S] + \hat{E}[V|S = (0,0)] = V - \left\{ E[V] + Cov[V, S^T]Cov[S, S^T]^{-1}(S - E[S]) \right\} \]

\[ \left\{ E[V] + Cov[V, S^T]Cov[S, S^T]^{-1}(S - E[S]) \right\}|_{S=(0,0)} = V - Cov[V, S^T]Cov[S, S^T]^{-1}S \]  
(D.9)

since \( E[S] = 0 \). The conditional vector and the conditional random vector \([V|T]\) are derived from the same assumptions.

The \( n \)'th component of the vector \( V \) becomes

\[ [V_n | S = (0,0)] = V_n - \frac{a_{\zeta,n}}{m_0 m_2 - m_1^2} x \]

\[ \sum_{j=1}^{N} a_{\zeta,j} (m_2 - m_1 (\omega_n + \omega_j) + m_0 \omega_n \omega_j) V_j - Z(0)(m_2 - m_1 \omega_n) - \dot{Z}(0)(m_0 \omega_n - m_1) \]  
(D.10)

and for \( W_n \) as

\[ [W_n | T = (0,0)] = W_n - \frac{a_{\zeta,n}}{m_0 m_2 - m_1^2} x \]

\[ \sum_{j=1}^{N} a_{\zeta,j} (m_2 - m_1 (\omega_n + \omega_j) + m_0 \omega_n \omega_j) W_j - \dot{Z}(0)(m_2 - m_1 \omega_n) - \dot{Z}(0)(m_0 \omega_n - m_1) \]  
(D.11)

By inserting the expected values \( Z(0) = a, \dot{Z}(0) = 0, \dot{Z}(0) = 0 \) and \( \dot{Z}(0) = \omega a \) into Eq. (D.10) and (D.11), the constrained coefficients \( V_{c,n} \) and \( W_{c,n} \) which define the mean wave profile become

\[ V_{c,n} = E[V_n | S = (0,0)] = \frac{a_{\zeta,n}}{m_0 m_2 - m_1^2} \left[ a(m_2 - m_1 \omega_n) - \omega a(m_0 \omega_n - m_1) \right] \]  
(D.12)
and
\[ \overline{W}_{c,n} = \mathbb{E}[W_n \mid \mathbf{T} = (0, 0)] = 0 \] (D.13)

Hence, the mean conditional on both the amplitude and the frequency, Friis-Hansen & Nielsen (1995)
\[ \zeta(t)_{a,\omega} \equiv \hat{\zeta}(Z(t) \mid Z(0) = a, \dot{Z}(0) = 0, \Omega = \omega) = \frac{a}{m_0m_2 - m_1^2} \sum_{n=1}^{N} a_{\zeta,n}^2 \left[ (m_2 - m_1\omega_n) + \omega(m_0\omega_n - m_1) \right] \cos(\omega_n t) \] (D.14)

where the coefficient \( a_{\zeta,n} \) is given as
\[ a_{\zeta,n} = \sqrt{S_{\zeta}(\omega_n) \Delta \omega} \] (D.15)

the variance is similarly found as
\[ \mathbb{E} \left[ (Z(t) - \mathbb{E}[Z(t) \mid Z(0) = a, \dot{Z}(0) = 0, \Omega = \omega])^2 \mid Z(0) = a, \dot{Z}(0) = 0, \Omega = \omega \right] = \frac{m_0}{m_0m_2 - m_1^2} \left[ -m_2^2 + m_0m_2(1 - c(t)^2 - r(t)^2) + 2m_0m_1(c(t)\dot{r}(t) + \dot{c}(t)r(t)) - m_0^2(\dot{c}(t)^2 + \dot{r}(t)^2) \right] \] (D.16)

with \( c(t) \) and \( r(t) \) given as
\[ c(t) = \frac{1}{m_0} \int_{0}^{\infty} S_{\zeta} \cos(\omega t) d\omega \] (D.17)
\[ r(t) = \frac{1}{m_0} \int_{0}^{\infty} S_{\zeta} \sin(\omega t) d\omega \] (D.18)

By inserting the mean frequency, \( \bar{\omega} = \frac{m_1}{m_0} \), the solution by Friis-Hansen & Nielsen (1995) reduces to the New Wave profile introduced by Tromans et al. (1991).

Realizations of the most likely wave with the random background wave included are obtained by entering the coefficients \( \mathbf{V} = (V_n, W_n) \) from Eq. D.9 as a zero mean Gaussian distributed random vector before the conditioning is performed, whereas the mean MLW is obtained by entering the coefficients \( \mathbf{V} = (V_n, W_n) \) as a zero vector.
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Appendix E

Additional Results by Application of the MLW

This appendix contains results of time domain simulations by application of the most likely wave. The simulations are performed with the non-linear time domain program ShipStar, Xia et al. (1998). Figure E.1 shows a conditional wave surface plotted as function of both time and space by application of the JONSWAP spectrum.

Figure E.1: MLW as function of time and space with the JONSWAP spectrum, \(H_s = a = 15 \text{ m}, T_z = 11.63 \text{ sec}\). The conditional peak is found at \(t_0 = 0\) and \(x_0 = 0\).
Linear and non-linear hogging and sagging peaks of the simulation illustrated in Figure 4.5 are found in Table E.1 by application of both the PM and JONSWAP spectra.

Table E.1: Largest sagging and hogging peaks given the location of the constrained peak relative to the hull girder. The calculations are performed for different wave spectra, $H_s = a = 15.0$ m., $T_z = 11.35$ sec. (PM) and 11.63 sec. (JONSWAP), zero forward speed and head sea.

<table>
<thead>
<tr>
<th>VBM</th>
<th>PM</th>
<th>JONSWAP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear</td>
<td>Non-linear</td>
</tr>
<tr>
<td></td>
<td>Hog</td>
<td>Sag</td>
</tr>
<tr>
<td>AP</td>
<td>-4636</td>
<td>5113</td>
</tr>
<tr>
<td>L/4</td>
<td>-4966</td>
<td>5025</td>
</tr>
<tr>
<td>L/2</td>
<td>-5133</td>
<td>4735</td>
</tr>
<tr>
<td>3L/4</td>
<td>-5114</td>
<td>4940</td>
</tr>
<tr>
<td>FP</td>
<td>-4801</td>
<td><strong>5189</strong></td>
</tr>
</tbody>
</table>
E.1 Response Distribution, JONSWAP Spectrum

Results by application of the JONSWAP spectrum due to the MLW are given here. In some of the tables presented, the results by application of the PM spectrum are additionally given, which makes comparison easier.

E.1.1 Response Distribution, Part One

Table E.2 lists medians (50% levels) of $F_{\text{RNL}}(r\mid a, h_s, t_z, v, \beta)$ for hogging and sagging by application of the JONSWAP spectrum for selected values of $H_s$.

Table E.2: $H_s = a$ for all the cases. Peak responses due the MLW profile and responses corresponding to a 50% level of $F_{\text{RNL}}(r\mid a, h_s, t_z, v, \beta)$ for hogging and sagging. The JONSWAP spectrum is applied with $H_s = 5, 10$ and $15$ m, $T_z = 11.63$ sec.

<table>
<thead>
<tr>
<th>JONSWAP</th>
<th>Linear response [MNm]</th>
<th>Sag</th>
<th>Hog</th>
<th>Sag</th>
<th>Hog</th>
<th>Sag</th>
<th>Hog</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_s$ [m]</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRW$_\text{FP}$</td>
<td>2017</td>
<td>1972</td>
<td>3962</td>
<td>3952</td>
<td>6039</td>
<td>6000</td>
<td></td>
</tr>
<tr>
<td>MLW$_\text{FP}$</td>
<td>1982</td>
<td>1875</td>
<td>3964</td>
<td>3751</td>
<td>5946</td>
<td>5627</td>
<td></td>
</tr>
<tr>
<td>CRW$_\beta$</td>
<td>1848</td>
<td>2008</td>
<td>3731</td>
<td>3991</td>
<td>5570</td>
<td>6025</td>
<td></td>
</tr>
<tr>
<td>MLW$_\beta$</td>
<td>1873</td>
<td>1966</td>
<td>3746</td>
<td>3921</td>
<td>5619</td>
<td>5897</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>JONSWAP</th>
<th>Non-linear response [MNm]</th>
<th>Sag</th>
<th>Hog</th>
<th>Sag</th>
<th>Hog</th>
<th>Sag</th>
<th>Hog</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_s$ [m]</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CRW$_\text{FP}$</td>
<td>2548</td>
<td>1947</td>
<td>5713</td>
<td>3475</td>
<td>9260</td>
<td>4655</td>
<td></td>
</tr>
<tr>
<td>MLW$_\text{FP}$</td>
<td>2405</td>
<td>1744</td>
<td>5390</td>
<td>3233</td>
<td>8954</td>
<td>4415</td>
<td></td>
</tr>
<tr>
<td>CRW$_\beta$</td>
<td>2421</td>
<td>1974</td>
<td>5592</td>
<td>3571</td>
<td>9205</td>
<td>4668</td>
<td></td>
</tr>
<tr>
<td>MLW$_\beta$</td>
<td>2280</td>
<td>1890</td>
<td>5292</td>
<td>3340</td>
<td>8674</td>
<td>4311</td>
<td></td>
</tr>
</tbody>
</table>
Table E.3: Ratios of linear and non-linear calculated vertical bending moments as a function of $H_s$, using the PM and JONSWAP spectra. The results are obtained for zero speed and in head sea.

<table>
<thead>
<tr>
<th>$H_s$</th>
<th>JONSWAP</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.0</td>
<td>10.0</td>
</tr>
<tr>
<td>VBM$<em>{NL}$/VBM$</em>{L}$</td>
<td>MLW$_{FP}$</td>
<td>1.213</td>
</tr>
<tr>
<td></td>
<td>MLW$_{\odot}$</td>
<td>0.961</td>
</tr>
<tr>
<td>VBM$<em>{NL}$/VBM$</em>{L}$</td>
<td>CRW$_{FP}$</td>
<td>1.263</td>
</tr>
<tr>
<td></td>
<td>CRW$_{\odot}$</td>
<td>0.983</td>
</tr>
</tbody>
</table>

Table E.4 summarises the results for Figure 4.14. Results by application of the JONSWAP spectrum are also given.

Table E.4: Ratios of linear and non-linear sagging or hogging vertical bending moments as a function of $H_s$, by application of the PM and JONSWAP wave spectrum. The ratios illustrate the difference between medians of $F_{R_{NL}}(r | a, h_s, t_z, v, \beta)$ from CRW analyses and the peak responses obtained on the basis of the MLW profile with $H_s = a$.

<table>
<thead>
<tr>
<th>$H_s$</th>
<th>JONSWAP</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.0</td>
<td>10.0</td>
</tr>
<tr>
<td>VBM$<em>{CRW}$/VBM$</em>{MLW}$</td>
<td>Sag</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Non-linear</td>
<td>1.059</td>
</tr>
<tr>
<td></td>
<td>Linear</td>
<td>1.017</td>
</tr>
<tr>
<td>VBM$<em>{CRW}$/VBM$</em>{MLW}$</td>
<td>Hog</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Non-linear</td>
<td>1.044</td>
</tr>
<tr>
<td></td>
<td>Linear</td>
<td>1.021</td>
</tr>
</tbody>
</table>
E.1.2 Response Distribution, Part Two

Table E.5: Medians (50% level) $F_{R_{\text{max}}} (r | a, h_s, t_z, v, \beta)$ for extreme responses in a period of three hours. The JONSWAP spectrum is applied for selected sea states, zero speed and head sea. The sagging distributions are obtained where the vessel encounters the CRW at FP, whereas the hogging responses are found by locating the CRW peak amidships.

<table>
<thead>
<tr>
<th>$H_s$ [m]</th>
<th>Sag</th>
<th>Hog</th>
<th>Sag</th>
<th>Hog</th>
<th>Sag</th>
<th>Hog</th>
</tr>
</thead>
<tbody>
<tr>
<td>JONSWAP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear response [MNm]</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
</tr>
<tr>
<td>CRW&lt;sub&gt;FP&lt;/sub&gt;</td>
<td>2216</td>
<td>2090</td>
<td>4230</td>
<td>4180</td>
<td>6255</td>
<td>6205</td>
</tr>
<tr>
<td>CRW&lt;sub&gt;⊥&lt;/sub&gt;</td>
<td>2094</td>
<td>2110</td>
<td>4090</td>
<td>4120</td>
<td>6040</td>
<td>6178</td>
</tr>
<tr>
<td>Non-linear response [MNm]</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
</tr>
<tr>
<td>CRW&lt;sub&gt;FP&lt;/sub&gt;</td>
<td>2610</td>
<td>2008</td>
<td>5946</td>
<td>3655</td>
<td>9571</td>
<td>4936</td>
</tr>
<tr>
<td>CRW&lt;sub&gt;⊥&lt;/sub&gt;</td>
<td>2565</td>
<td>2015</td>
<td>5915</td>
<td>3652</td>
<td>9489</td>
<td>4930</td>
</tr>
</tbody>
</table>

Ratios of the non-linear and linear responses from Table E.5 (JONSWAP) have been compared. Results by application of the PM spectrum is additionally shown. The ratios are given in Table E.6, where CRW<sub>FP</sub> represents sagging and CRW<sub>⊥</sub> hogging.

Table E.6: Ratios of linear and non-linear calculated vertical bending moment as a function of $H_s$, by use of the JONSWAP and PM spectrum.

<table>
<thead>
<tr>
<th>$H_s$</th>
<th>JONSWAP</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.0</td>
<td>10.0</td>
</tr>
<tr>
<td>$VBM_{NL}$</td>
<td>CRW&lt;sub&gt;FP&lt;/sub&gt;</td>
<td>1.17</td>
</tr>
<tr>
<td>$VBM_{L}$</td>
<td>⊥</td>
<td>0.96</td>
</tr>
</tbody>
</table>
Table E.7: Ratios of medians of $F_{R_{\text{max}}}(r \mid a, h_s, t_z, v, \beta)$ for extreme responses in a period of three hours to peak responses due to the deterministic MLW for selected values of $H_s = a$. Results are given for both the PM and JONSWAP spectra.

<table>
<thead>
<tr>
<th>$H_s$</th>
<th>JONSWAP</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Sag</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-linear</td>
<td>1.0852</td>
<td>1.1032</td>
</tr>
<tr>
<td>Linear</td>
<td>1.1181</td>
<td>1.0671</td>
</tr>
<tr>
<td>Hog</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-linear</td>
<td>1.0661</td>
<td>1.0934</td>
</tr>
<tr>
<td>Linear</td>
<td>1.0732</td>
<td>1.0508</td>
</tr>
</tbody>
</table>
E.2 Most Likely Wave, Hydroelastic Responses

The main purpose of the present section is very briefly to investigate the vertical bending moment amidships by application of MLW profiles for an elastic hull girder.

Figure E.2 shows time series of the vertical bending moment amidships by application of an elastic hull girder. The vessel encounters a most likely wave of a crest height of 10.0 m at the forward perpendicular at $t_0 = 100$ sec. The left plot shows the total unfiltered response where the transient whipping vibration is observed throughout most of the signal. The right plot shows the filtered wave- and whipping-induced responses. The wave-induced response is given as the low-frequency solid line and represents clearly the largest part of the total response. The whipping-induced response is observed as the faster oscillating signal, and it is seen that both whipping-induced peaks and troughs are found in the vicinity of the wave-induced maxima.

Figure E.2: Low- and high-frequency time series by application of the mean MLW. The left plot shows the total response, whereas the filtered signals are given to the right. The calculations are performed for head sea and zero speed, by use of the Pierson Moskowitz spectrum with $H_s = a = 10.0$ m and $T_z = 11.35$ sec.

E.2.1 Effect of MLW Amplitude and Forward Speed

Whipping-induced responses have been studied. Analyses are performed for the MLW profile with amplitudes of 5.0 to 15.0 m and forward velocities from 0 to 10 m/s. Only head sea conditions have been studied.

Figure E.3 shows the effect of whipping on the non-linear wave-induced response for both hogging and sagging. The left plot shows the effect for head sea and selected forward velocities with $H_s = a = 10.0$ m. The peak responses are selected as described in Section 6.5.2. The results illustrate increasing whipping contributions to the sagging bending moment with increasing forward velocities, which seems reasonable. The effect on hogging is found to be slightly decreasing for increasing forward speeds.
The right plot shows the effect on whipping with increasing MLW amplitudes and zero forward speed. The contribution seems constant for both hogging and sagging.

Figure E.3: **Effect of MLW height and forward speed.**

Application of MLW profiles as predictor of whipping-induced responses has only been studied briefly. Whipping responses are observed. However, in view of the conclusions from Chapter 4, where it was found that considerable effort should be be spent on selecting the correct location of the MLW peak relative to the hull girder, no further analyses are performed.
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    Bølgeenergimaskiner.

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