Damage Tolerance of Debonded Sandwich Structures

Christian Berggreen
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Preface

This thesis is submitted as a partial fulfilment of the requirements for the Danish Ph.D degree. The work has been performed in the Section of Maritime Engineering, Department of Mechanical Engineering, the Technical University of Denmark, during the period from September 2000 to September 2004, including 10 months of absence. The project was supervised by Associate Professor Bo Cerup Simonsen and Professor Preben Terndrup Pedersen.

The study was financially supported by the Technical University of Denmark. Furthermore, the study was partly supported by the joint Nordic/Anglo defense project, Inspection and Repair of Sandwich Structures in Naval Ships (saNDI). The support is gratefully acknowledged.

Special thanks to my supervisor, Associate Professor Bo Cerup Simonsen, for an inspiring guidance through this project and for numerous inspiring and educational discussions. His help, support and guidance is highly appreciated. Also thanks to Professor Preben Terndrup Pedersen and Professor Jørgen Juncher Jensen for creating a friendly and inspiring working environment. Finally, thanks to my friends and family for their support and to my colleagues at the Section of Maritime Engineering, especially my officemates Rikard Törnqvist and Jesper Urban for many valuable discussions and to Hugo Heinicke for preparing the many high quality drawings in this thesis.

In a four months period, from April 2003 to July 2003 the studies were carried out at RISØ National Laboratory, Department of Materials Research. I would like to thank the staff at the department for an inspiring stay, including a special thank to Senior Engineer Kaj Kvistgaard Borum and Senior Scientist Povl Brøndsted. Their support throughout my study is highly appreciated.

During nearly the entire part of my Ph.D study my work has been closely linked to the saNDI-project, mentioned above. The value of this connection in terms of priceless input and discussions with leading sandwich experts cannot be underestimated. My sincere thanks go to the partners in the project. In that connection I would especially like to thank Senior Principal Engineer Brian Hayman from Det Norske Veritas for his highly appreciated help, support and encouragement throughout my study.
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Executive Summary

The overall objective of this thesis has been to establish methodologies for the prediction of both crack propagation and residual strength of debond damaged fibre reinforced sandwich structures with foam cores. It is evident that in order to achieve highly optimised structures, which are able to operate in a stochastic loading environment, damage tolerance evaluation based on residual strength prediction is needed. Is a given damage critical for the structural integrity needing immanent repair, or is the damage negligible, where repair can be postponed to the next inspection? These questions are generally interesting for all types of structures, but they are especially relevant for sandwich structures which by nature are highly optimised structures with a high number of possible damage scenarios and consequent failure mechanisms. Presently, no or only limited tools are available to carry out these damage tolerance predictions. Thus the secondary purpose of this thesis has been to develop usable damage tolerance prediction tools for coming production control and in-service inspection/repair manuals for the Nordic/Anglo navies in joint defence funded research project THALES JP3.23 Inspection and Repair of Sandwich Structures in Naval Ships (saNDI).

A major challenge in estimation of residual strength of especially debonded sandwich structures is modelling and prediction of crack propagation and initiation, as these mechanisms are governing for the overall failure load of the structure. Thus a comprehensive and thorough overview of state-of-the-art bimaterial fracture mechanics is given with special aim at methods (mode-mixity methods) capable of predicting the loading of the crack tip front of a debond. Three different mode-mixity methods from the literature are presented and evaluated with the aim of application on face-core interfaces in foam core sandwich structures. However, it is indicated in this thesis that none of the presented methods are suitable for use in an automatic crack propagation routine in a commercial finite element code. Therefore, a new mode-mixity method (the Crack Surface Displacement Extrapolation method) is presented in this thesis which is robust and reliable when applied to interfaces with a high stiffness ratio which is the case for most foam core sandwich structures.

On the basis of a thorough theoretical interface fracture mechanics background and the new fracture mechanical mode-mixity method, two different numerical models are presented in this thesis - a two-dimensional model able to predict crack propagation in sandwich beams and a three-dimensional model able to predict the ultimate failure load of a sandwich panel with a circular debond. Both models, which are able to simulate geometrical non-linearity are based on the finite element method using the commercial finite element program ANSYS.
A comprehensive verification and validation against other numerical and experimental results is carried out. Four different full-scale test series have been carried out on sandwich beams and panels in connection with this thesis. The loading chosen for these test series is comparable with real life loading scenarios for sandwich ships and consists of:

- **Face tearing** (deck-superstructure connection). The failure mode is a growing interface crack.

- **Pure compression** (bottom or deck panel). The failure mode is local buckling of the debonded face and propagation of the interface crack.

- **In-plane bending or non-uniform compression** (side panel). The failure mode is similar to the pure compression case.

- **Lateral pressure** (bottom panel). The failure mode is propagation of the interface crack, kinking, and through-thickness shear failure of the core.

The face tearing tests, which are simulating face tearing in a deck superstructure corner connection in a ship exposed to global hull sagging and hogging, have been carried out by the author at RISØ National Laboratory in a new test rig specially designed for this purpose. By use of experimentally obtained fracture toughness results as input to the two-dimensional propagation model and advanced air-coupled ultrasonic scans, it is shown that as long as no fibre bridging is present, which is the case for the beam specimens with H80 core, good agreement is obtained. In the case of H130 and H200 cores, the numerical model yields conservative results as soon as fibre bridging occur. Additionally, by use of the face tearing fracture problem in a deck-superstructure connection, the propagation model is compared with an independent damage mechanics model using the Bonora damage model. The comparison reveals a very good agreement.

The pure and non-uniform compression test series have both been carried out in new test rigs at The Technical University of Denmark, and the residual strength of panels with various debond sizes has been investigated. Furthermore, the three-dimensional model is validated against the experimental results from both in-plane test series.

For the pure compression panel case it is shown from both the experimental results and the comparison of the experimental and numerical obtained average residual strength factors that the residual strength decreases with increasing debond diameter. Numerical and experimental results show considerable strength reductions with average residual strength factors around 20-25% for debond diameters around Ø200-300 mm in a 560 mm wide specimen. For smaller debond diameters, the experimental results show average residual strength factors around 37%, whereas the numerical results yield a non-conservative 51%. Furthermore some discrepancies are seen for smaller debonds which are most likely due to large influence from imperfections on the debond buckling for high face thickness - debond diameter ratios. However, it can be concluded that all numerical predictions of the residual strength are in good agreement for large debond diameters.
For the non-uniform compression case it is first of all seen that both experimental and numerical investigations show that small debonds below at least Ø100 mm are not critical. As in the pure compression case both experimental and numerical results show considerable strength reduction with average residual strength factors around 32-55% for debond diameters larger than Ø200 mm. For debond diameters from approximately Ø150 mm and up the numerical model yields increasingly conservative results compared to the experimental values. The increasing conservatism may be explained by problems in the numerical model for predicting the correct buckling mode under influence of production introduced imperfections. However, for practical engineering purposes, the results in both pure and non-uniform compression are acceptable.

Finally, with regards to the lateral pressure loaded panels, they have been tested and investigated experimentally in cooperation with the VTT Technical Research Center of Finland by use of their existing test rig and advanced air-coupled ultrasonic scans performed at Risø National Laboratory. It is shown in this thesis that central debonds are non-critical whereas the edge and corner debonds yield residual strength factors of 25% and 35% respectively for the tested panels.

The full-scale tests presented in this thesis serve together with the theoretical methodologies as input to new production control and in-service inspection/repair manuals developed for the Nordic/Anglo navies in the saNDI-project, mentioned above.
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Synopsis


En stor udfordring ved estimering af residualstyrke af specielt debond-skadede sandwichkonstruktioner er modellering og estimering af revnepropagering og initiering, da disse mekanismer er styrende for den overordnede belastning af strukturen. Derfor er der i denne afhandling givet en omfattende præsentation af state-of-the-art bi-materiale brudmekanik, specielt rettet mod metoder, der er i stand til at bestemme belastningen af revnespidsen langs fronten af en debond. Tre forskellige mode-mixity metoder fra litteraturen er præsenteret og sammenlignet med det specielle formål at anvende dem på skind/kærne-samlinger i skum-kærne sandwich-konstruktioner. Det bliver endvidere vist i denne afhandling at ingen af de præsenterede metoder er direkte anvendelige i automatiske propageringsrutiner i et kommersielt finite element program. Derfor bliver en ny mode-mixity metode (the Crack Surface Displacement Extrapolation method) præsenteret, som er tilstrækkelig robust og anvendelig, når den anvendes på samlinger med høj stivhedsforskell, sådan som det er tilfældet i sandwich-konstruktioner.

På basis af den omfattende teoretiske bi-materiale brudmekanik baggrund og den nye brudmekaniske mode-mixity metode, præsenteres to forskellige numeriske modeller i denne afhandling - en to-dimensionel model i stand til at estimere revnepropagering i sandwich-bjælker og en tre-dimensionel model i stand til at estimere den ultimative svigtlast af et panel med
en cirkulær debond. Begge modeller, der kan simulere ikke-lineær geometriske effekter, er baseret på finite element metoden ved anvendelse af det kommercielle finite element program, ANSYS.

En omfattende verifikation og validering mod andre numeriske og eksperimentelle resultater er udført i denne afhandling. Fire forskellige fuld-skala test-serier er udført på sandwich-bjælker og paneler. Belastningen i disse test er valgt, så de er sammenlignelige med belastnings-scenarioer ofte set i sandwich-fartøjer i drift og består af:

- **Skind-aftrækning** (dæk-overbygnings-samling). Svigt-mekanismen er en voksende revne i samlingen.
- **Ikke-uniform kompression** (side panel). Svigt-mekanismen ligner den fra ren kompression.
- **Lateralt tryk** (bundpanel). Svigt-mekanismen er propagering af revnen i samlingen, kinking ud af samlingen og forskydningssvigt gennem tykkelsen af kærnen.


Forsøgene med ren kompression og ikke-uniform kompression er begge udført i nye test-rigge på Danmarks Tekniske Universitet, og residualstyrken af paneler med forskellige debond-størrelser er blevet undersøgt. Endvidere er den tre-dimensionale model blevet valideret ved anvendelse af de eksperimentelle resultater fra begge forsøgsserier.

For tilfældet med ren kompression er det vist, for både eksperimentelle resultater og fra sammenligningen af eksperimentelle og numerisk opnåede residualstyrke-faktorer, at residualstyrken falder ved en voksende debond diameter. Numeriske og eksperimentelle resultater viser betydelige styrkereduktioner med residualstyrke-faktorer fra 20-25% for debond-diametre mellem Ø200-300 mm i et 560 mm bredt emne. For mindre debond-diametre viser de eksperimentelle resultater residualstyrke-faktorer omkring 37% mod de numeriske resultaters 51%. Der ses endvidere nogle fluktuationer i resultaterne for små debonds, som højt
sandsynlig skyldes stor indflydelse fra imperfekutioner på den lokale debond-buling for høje $t_f/D$-forhold. Det kan dog konkluderes, at alle numeriske estimationer af residual-styrken enten er i god overenstemmelse med eksperimentelle resultater for store debonds.

For tilfældet med ikke-uniform kompressionsbelastede paneler ses det umiddelbart fra både eksperimentelle og numeriske undersøgelser, at små centrale debonds, mindre end ca. Ø100 mm, ikke er kritiske. I lighed med resultaterne fra panelerne med ren kompression viser både eksperimentelle og numeriske resultater for større debonds en betydelig styrkereduktion med residualstyrke-faktorer mellem 32-55%. For debond-diametre fra ca. Ø150 mm og op udviser den numeriske model stigende konservative resultater sammenlignet med de eksperimentelle resultater. Den voksende konservatismen kan muligvis forklares ved problemer i den numeriske model med at bestemme den korrekte lokale buleform under indflydelse af produktionsintroducerede imperfektioner. Det kan dog konkluderes for både paneler med ren- og ikke-uniform kompression, at for praktiske ingeniørmssige beregninger, er de opnåede resultater acceptable.

Den sidste testserie med lateralt belastede paneler er blevet undersøgt eksperimentelt i samarbejde med VTT Technical Research Center of Finland ved anvendelse af en eksisterende test-rig og igen avanceret luft-koblet ultralydsscanninger udført på Forskningscenter RISØ. Resultaterne viser, at centrale debonds ikke er kritiske for residualstyrken, hvorimod debonds lokaliseret ved kanten og hjørnet af panelet resulterede i residualstyrke-faktorer på respektiv ca. 25% og 35%.

Resultaterne fra alle fuld-skala forsøg sammen med de teoretiske og numeriske metoder præsenteret i denne afhandling fungerer endvidere som input til kommende produktionskontrol- og in-service inspektion/reparations-manualer til de Nordiske/Engelske flåder i saNDI-projektet beskrevet ovenfor.
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Symbols

Roman Symbols

\( A \) Panel length or crack surface area
\( A_i \) Support length
\( A_P \) Pressure area height
\( A_R \) Support height
\( A_{SG} \) Vertical strain gauge position
\( A_{ij} \) Non-dimensional bimaterial tensor
\( a \) Crack length
\( a_i \) Polynomial coefficients
\( B \) Panel breadth
\( B_i \) Support breadth
\( B_P \) Pressure area breadth
\( B_R \) Support breadth
\( B_{SG} \) Horizontal strain gauge position
\( C \) Compliance
\( c \) Lower beam support length
\( D \) Debond diameter
\( D_{cr} \) Critical damage variable
\( D_i(Z) \) Complex material dependent function
\( E \) Total energy
\( E_0 \) Initial stiffness
\( E_1 \) Face in-plane Young’s modulus
\( E_2 \) Face in-plane Young’s modulus
\( E_3 \) Face out-of-plane Young’s modulus
\( E_c \) Core Young’s modulus
\( E_{ff} \) Effective stiffness
Symbols

- $e$  
  Upper beam support length

- $F_c$  
  Propagation load

- $F(x, y)$  
  Stress function

- $F_{Hj}$  
  Complex nodal force (VCCT)

- $F_{ij}$  
  Nodal forces

- $F_{ijk}$  
  Displacement field parameters

- $f_{ij}(Z_i)$  
  Complex material dependent holomorphic function

- $g_{ij}(Z_1, Z_2)$  
  Complex material dependent function

- $G$  
  Griffith-energy (energy release rate)

- $G_I$  
  Traditional mode I Griffith-energy

- $G_{II}$  
  Traditional mode II Griffith-energy

- $G_1$  
  Modified mode 1 Griffith-energy

- $G_2$  
  Modified mode 2 Griffith-energy

- $G_{core}$  
  Critical core mode I Griffith-energy

- $G_{12}$  
  Face in-plane G-modulus

- $G_{13}$  
  Face out-of-plane G-modulus

- $G_{23}$  
  Face out-of-plane G-modulus

- $G_c$  
  Core G-modulus

- $H$  
  Curved panel height or bimaterial parameter

- $H_{11}$  
  Non-dimensional bimaterial parameter

- $H_{22}$  
  Non-dimensional bimaterial parameter

- $h$  
  Characteristic length

- $h_1$  
  Upper crack flank thickness (semi-analytical method)

- $h_2$  
  Lower crack flank thickness (semi-analytical method)

- $I$  
  Moment of inertia (semi-analytical method)

- $J$  
  J-integral

- $K_I$  
  Homogeneous mode I stress intensity factor

- $K_{II}$  
  Homogeneous mode II stress intensity factor

- $K_1$  
  Bimaterial mode 1 stress intensity factor

- $K_2$  
  Bimaterial mode 2 stress intensity factor

- $k$  
  Fracture toughness distribution parameter

- $L$  
  Active beam length

- $L_E$  
  Edge debond location

- $L_{C1}$  
  Corner debond location

- $L_{C2}$  
  Corner debond location

- $L_{oa}$  
  Overall beam length

- $L_R$  
  Reinforcement length
Symbols

$L_{s}$ Length of idealised superstructure

$L$ Non-dimensional bimaterial matrix

$M_1$ Path independent integral (VCE)

$M_i$ Moment components (semi-analytical method)

$M_{ijk}$ Displacement field parameters

$m$ Non-dimensional bimaterial parameter

$n$ Non-dimensional bimaterial parameter or multiplication factor

$P_i$ Force components (semi-analytical method)

$p$ Pressure

$p(t)$ Pressure as a function of time

$Q_{ij}$ Displacement field parameters

$R$ Non-dimensional bimaterial parameter

$R_l$ Residual strength factor

$r$ Radius

$r_K$ Modulus of the complex stress intensity factor

$S$ Face lamina shear strength (stress)

$S_{e}$ Face lamina shear strength (strain)

$S^c$ Core shear strength (stress)

$S^{LAM}_{e}$ Face laminate shear strength (strain)

$S_{ij}$ Compliances (plane stress)

$S'_{ij}$ Transformed compliances (plane strain)

$[S]$ Stiffness matrix

$T$ Non-singular stress parallel to the crack surface

$T_i$ Traction

$T_{eigen}$ Eigenperiod

$t$ Time

$t_{time}$ Termination time

$t_c$ Core thickness

$t_f$ Face thickness

$t_{f,inner}$ Inner face thickness

$t_{f,outer}$ Outer face thickness

$U$ Non-dimensional parameter (semi-analytical method) or available elastic energy

$u_i$ Displacement tensor

$[u_n]$ Nodal displacement vector

$V$ Non-dimensional parameter (semi-analytical method)

$W$ Strain energy density

$W_d$ Work done by external forces
Symbols

\( W_s \)  
Energy required to create new surfaces

\( X_c \)  
Face lamina compression strength in fibre direction (stress)

\( X_t \)  
Face lamina tension strength in fibre direction (stress)

\( X_{cc} \)  
Face lamina compression strength in fibre direction (strain)

\( X_{ct} \)  
Face lamina tension strength in fibre direction (strain)

\( X_{LAM}^{cc} \)  
Face laminate compression strength (strain)

\( X_{LAM}^{ct} \)  
Face laminate tension strength (strain)

\( X_c \)  
Core compression strength (stress)

\( X_t \)  
Core tension strength (stress)

\( Y_c \)  
Face lamina compression strength in matrix direction (stress)

\( Y_t \)  
Face lamina tension strength in matrix direction (stress)

\( Y_{cc} \)  
Face lamina compression strength in matrix direction (strain)

\( Y_{ct} \)  
Face lamina tension strength in matrix direction (strain)

\( Y_{LAM}^{cc} \)  
Face laminate compression strength (strain)

\( Y_{LAM}^{ct} \)  
Face laminate tension strength (strain)

\( x \)  
Field variable

\( y \)  
Field variable

\( Z_d \)  
Distance to neutral axis

\( Z_t \)  
Complex material dependent field variable

Greek Symbols

\( \alpha \)  
1. Dundur’s parameter or damage exponent

\( \beta \)  
2. Dundur’s parameter

\( \beta_{ij} \)  
Displacement field parameters

\( \Gamma \)  
Fracture toughness or integration path surrounding the crack tip

\( \Gamma (\Psi) \)  
Fracture toughness distribution as a function of mode-mixity

\( \gamma \)  
Non-dimensional parameter (semi-analytical method)

\( \gamma_m \)  
Surface free energy

\( \gamma_{ijk} \)  
Displacement field parameters

\( \Delta \)  
Integration length (VCCT) or neutral axis parameter

\( \Delta l \)  
Incremental distance (VCE)

\( \Delta L \)  
Beam end movement

\( \delta \)  
Lift displacement

\( \delta_H \)  
Complex crack flank displacement parameter

\( \delta_{ij} \)  
Kronecker’s delta function

\( \delta_l \)  
Crack flank shearing displacement
Symbols

\( \delta \)
Crack flank opening displacement

\( \varepsilon \)
Oscillatory index

\( \varepsilon_f \)
Uniaxial ductile fracture strain

\( \varepsilon_{ij} \)
Strain tensor

\( \zeta \)
Modulus of the Beta-function

\( \eta \)
Crack flank thickness ratio (semi-analytical method)

\( \theta \)
Modulus

\( \kappa \)
Hull beam curvature

\( \lambda \)
Non-dimensional bimaterial parameter

\( \mu \)
Non-dimensional bimaterial parameter

\( \nu_{12} \)
Face in-plane Poisson’s ratio

\( \nu_{13} \)
Face out-of-plane Poisson’s ratio

\( \nu_{23} \)
Face out-of-plane Poisson’s ratio

\( \nu_c \)
Core Poisson’s ratio

\( \Pi \)
Propagation initiation index or potential energy

\( \rho \)
Non-dimensional bimaterial parameter

\( \Sigma \)
Non-dimensional parameter (semi-analytical method)

\( \tilde{\sigma} \)
Effective stress

\( \sigma_{eq} \)
Equivalent stress

\( \sigma_H \)
Complex stress parameter or hydrostatic pressure

\( \sigma_{ij} \)
Stress tensor

\( \sigma_{ij}^I \)
Mode I stress function

\( \sigma_{ij}^II \)
Mode II stress function

\( \Upsilon \)
Structural reliability index

\( \Phi_i \)
Mode dependent functions

\( \Phi_2' \)
Modified mode dependent function

\( \chi \)
Argument of the Beta-function

\( \Psi \)
Homogeneous mode-mixity

\( \Psi_G \)
Griffith-energy based bimaterial mode-mixity

\( \Psi_K \)
Stress intensity factor based bimaterial mode-mixity

\( \Omega \)
Kink angle

\( \omega \)
Tabulated non-dimensional parameter (semi-analytical method)

\( \omega \)
Eigenvector
Abbreviations

\begin{align*}
CFRP & \quad \text{Carbon Fibre Reinforced Plastic} \\
CSD & \quad \text{Crack Surface Displacement method} \\
CSDE & \quad \text{Crack Surface Displacement Extrapolation method} \\
FRP & \quad \text{Fibre Reinforced Plastic} \\
GFRP & \quad \text{Glass Fibre Reinforced Plastic} \\
VCCT & \quad \text{(Modified) Virtual Crack Closure Technique} \\
VCE & \quad \text{Virtual Crack Extension method}
\end{align*}
Chapter 1

Introduction

1.1 Overview and Background

In the last 30 years fibre composite materials have seen a growing popularity in a wide spectrum of different industries. Areas of application have first of all been aircraft and spacecraft, but with a decreasing fibre material price of the most commonly used fibre types, composite materials have eventually been applied on a larger scale in ships, cars, trains, wind generator blades, off-shore installations, etc. Common to most of these weight critical applications is the need for reducing the weight of the structure to increase the strength-to-weight and stiffness-to-weight ratios and thus obtain better performance and/or an increased loading capacity. With regard to these strength- and stiffness-to-weight ratios, composite and especially sandwich materials possess a superior performance. Other advantageous properties are thermal and acoustic insulation, fatigue, corrosion and easy manufacturing of aero- and hydrodynamically superior shapes.

The key aspect in the design, which does not only apply to composite weight-critical structures but in general, is to be able to take advantage of the building material and utilise it to its limits. This, in turn, leads to requirements for theoretical tools for accurate prediction of the loads and the structural response. A part of this thesis is focused on structural optimization and on the structural response of curved sandwich panels.

With the increasing ability to optimise the structures to the performance limit of the building materials and with the willingness to do so in practice, the reserve margin for structural degradation and damage tolerance becomes significantly smaller. In Figure 1.1 the reliability index, \( \chi \), versus the ageing of the structure is shown for a typical structural lifetime of a structure optimised to the material performance limit. For this particular example it may be observed that the reliability index is reduced as the ageing of the structure increases. However, the structural integrity is regained because of repair every time the reliability index reaches the accepted minimum value.
In Figure 1.1 the effect of the same sudden damage to the structure is indicated for two different times during the structural lifetime. The damage to the structure means that the structural integrity is suddenly reduced and the reliability index is therefore dropping. As indicated in Figure 1.1, the damage is seen to be non-critical to the first damage case but critical to the second damage case, as the reliability instantly drops below the accepted value.

The above-mentioned example emphasises the importance of being able to evaluate the criticality of a given damage in connection with the redundancy of the structure. Furthermore, it is evident that in order to achieve highly optimised structures, which are able to operate in a stochastic loading environment, damage tolerance evaluation is needed. Furthermore, the damage tolerance approach does not only apply to the design and optimisation of composite structures, but is also highly relevant to composite structures already in service and exposed to minor or major damages. Is a given damage critical to the structural integrity, or is the damage negligible? These questions are especially relevant to sandwich structures\(^1\), which by nature are highly optimised structures with a high number of possible damage scenarios and consequent failure mechanisms.

Among the most critical damages to sandwich structures is debonding of face and core layer (loss of connection between them). This kind of damage can be highly critical to the sandwich

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\(^1\)The sandwich concept will be introduced in detail in the next chapter
1.1 Overview and Background

structure as the basic sandwich principle is compromised when the connection between the face and the core layer is lost.

Several investigations of this damage type have been carried out in the past, beginning with Zenkert (1990), who investigated the strength of foam core sandwich beams with various debonds and interface propagated shear cracks using the finite element method and experimental testing. Somers et al. (1991) followed with an extensive analysis of the debonded beam problem using a continuous fracture mechanical analytical model, where the sandwich beam is divided into four individual parts. Other investigations followed, which all enlightened the criticality of interfacial damages. Among those are Kim and Dharan (1992), who investigated debonded honeycomb sandwich beams by a foundation model and linear fracture mechanics, Hwu and Hu (1992) and Frostig and Sokolinsky (2000), who carried out analytical higher-order analysis of the local debonded face layer buckling mechanism. The latter paper compares the analytical results with an extensive test series carried out in Avery and Sankar (2000) on honeycomb core sandwich beams with debonded carbon reinforced face sheets.

In recent years Professor L. A. Carlsson from Florida Atlantic University and his coworkers have made several fracture mechanically based analytical and experimental studies on the debond problem in sandwich beams, specially aimed at the determination of interfacial fracture toughness and introducing a number of specially designed test specimens. Among these publications are: Carlsson et al. (1991), Carlsson and Prasad (1993), Prasad and Carlsson (1994a) and Prasad and Carlsson (1994b), in which foam core sandwiches with isotropic facings are investigated. Li and Carlsson (1999), Li (2000), Li and Carlsson (2001), Viana and Carlsson (2002b) and Viana and Carlsson (2002a) present and investigate the tilted sandwich specimen, which will be described in more detail later in this thesis.

In addition to the sandwich application publications by Carlsson and coworkers, a wide spectrum of general bimaterial fracture mechanical publications is available. Most of them have their roots in the area of debonding of thin films on elastic substrates, which has many similarities to the debonding of large-scale sandwich structures, even though the length scale is somewhat different. The most important publications are probably the ones by Professor J. W. Hutchinson and Professor Z. Suo, who were the first to establish a firm foundation for the analytical description of the bimaterial interface fracture mechanics, Hutchinson and Suo (1992), Suo (1990). The first gives an extensive overview of the bimaterial fracture mechanics, and the latter gives a comprehensive and in-depth analytical description of the displacement field and fracture mechanics for anisotropic bimaterials. The two last papers also form the theoretical foundation for the fracture mechanical models presented later in this thesis.

With regard to three-dimensional debonds, only very limited investigations of debonded sandwich panels can be found in the literature. However, for three-dimensional delaminations in laminates a fair number of publications may be found. Among those are: Nilsson et al. (1997), Tay et al. (1999), Nilsson et al. (2001a), Nilsson et al. (2001b) and Asp and Nilsson (2002), additionally Xue and Qu (1999) presented a purely general analytical investigation of elliptical cracks in anisotropic bimaterials. The models from these delamination
investigations can in some cases be applied to debonds in sandwich interfaces, neglecting for instance the kinking\(^2\) behaviour.

An isolated example of a both numerical and experimental investigation of circular debonds in a laterally loaded sandwich panel is given in Falk (1994). Linear finite element solutions are used and nodal crack flank data from node pairs close to the crack tip is used to achieve a prediction of the Griffith-energy and mode-mixity distribution along the crack front and a comparison with mode II fracture toughness data is carried out. Additionally, the numerical results are compared with experimental data from seven tested full-scale panels, and good agreement is found between numerical and experimental results. However, as Falk himself points out in his paper, more thorough numerical and experimental investigations are needed to establish a firm foundation for the prediction of residual strength of debonded sandwich panels.

### 1.2 Objectives and Scope of the Work

The objective of this thesis is primarily to establish methodologies for the prediction of both crack propagation and residual strength of debonded fibre reinforced sandwich structures with foam cores. Furthermore, on the basis of a thorough theoretical interface fracture mechanics background and validation against experimental results, the models produced in this thesis will be used to predict residual strength of sandwich panels exposed to various damages, which have resulted in a debond between face and core.

Two different models will be presented in this thesis - a two-dimensional model able to predict crack propagation in sandwich beams and a three-dimensional model able to predict the ultimate failure load of a sandwich panel with a circular debond. Both models, which are able to simulate geometrical non-linearity contrary to the model described in Falk (1994), are based on the finite element method using the commercial finite element program ANSYS, where parametrical subroutines have been produced. The two models are furthermore based on a new fracture mechanical mode-mixity method (the Crack Surface Displacement Extrapolation method), which will also be presented in this thesis.

In addition to the theoretical models mentioned above, four different full-scale test series have also been carried out on sandwich beams and panels in connection with this thesis. The loading chosen for these test series is comparable with real life loading scenarios for sandwich ships and consists of:

- Face tearing (deck-superstructure connection)
- Pure compression (bottom or deck panel)

\(^2\)Kinking out of a sandwich interface and into the core material will be treated later in this thesis
1.2 Objectives and Scope of the Work

- In-plane bending or non-uniform compression (side panel)
- Lateral pressure (bottom panel)

The full-scale tests will both serve as validation of the theoretical models produced in this thesis and together with the theoretical methodologies serve as input to new production control and in-service inspection/repair manuals, produced in connection with the joint Nordic/Anglo defence funded research project THALES JP3.23 Inspection and Repair of Sandwich Structures in Naval Ships (saNDI).

The thesis is composed as follows:

In Chapter 2 an overview and background will be given to the field of sandwich structures, and the concept of damage tolerance will be introduced in connection with the description of debonds and various other damages often seen on in-service sandwich vessels.

In Chapter 3 the non-linear response of curved sandwich panels will be investigated by use of various modelling approaches and finite element codes. Furthermore, weight reduction, by taking the initial membrane effect into account, will be investigated, both in a representative sandwich vessel and in an idealised section where the effect of the surrounding structure will be investigated.

In Chapter 4 a comprehensive study of linear interface fracture mechanics is carried out. The general near tip displacement field will be deducted in detail and used to present the three most popular mode-mixity methods in the literature. Additionally, a new mode-mixity method, specially designed for application to sandwich interfaces, will be presented in both two and three dimensions and compared with the most suited mode-mixity method from the literature using two test cases in 2-D and 3-D. Finally, fracture toughness determination for sandwich interfaces will be described followed by experimentally determined fracture toughness distributions measured at RISØ National Laboratory.

In Chapter 5 the two-dimensional beam propagation model and the three-dimensional initiation model will be described in detail, together with advantages, disadvantages and assumptions for both models.

In Chapter 6 the two-dimensional beam propagation model will be applied to a real life debond problem often seen in sandwich vessels - face tearing of the deck-superstructure connection. Furthermore, the two-dimensional model will be validated against an experimental full-scale beam test series, simulating the face tearing of the deck-superstructure connection.

Chapter 7 is dedicated to validation of the three-dimensional panel model and a residual strength investigation of sandwich panels with circular debonds. Three experimental test series will be presented and compared with the theoretical results. Finally, the results from all three test series will be compared in a residual strength perspective and a proposed manual implementation approach is presented.

Chapter 8 contains conclusions and recommendations for future work.
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Chapter 2

Composite Sandwich Structures and Structural Degradation of Carrying Capacity

2.1 Composite Sandwich Structures and Their Application

2.1.1 The Sandwich Concept and Materials

A tale says that John Montagu (1718-1792), also known as the 4th Earl of Sandwich and British First Lord of the Admiralty during the American Revolution, ate meat between two slides of white bread while playing long billiard games\(^1\) - and thus gave name to the *edible* sandwich.

The *structural* sandwich concept is described very precisely by ASTM:

> A structural sandwich is a special form of laminated composite comprising of a combination of different materials that are bonded to each other so as to utilize the properties of each separate component to the structural advantage of the whole assembly.

\(^1\)Some tales also report it was while playing cards
Chapter 2. Composite Sandwich Structures and Structural Degradation of Carrying Capacity

Figure 2.1: Sketch of the similarity between an ordinary I-beam and a sandwich (a). (b) The normal (top) and shear stress (bottom) distribution in a sandwich.

In order to utilise the material properties to a structural advantage, the normal sandwich configuration consists of two stiff and thin layers (the faces) separated by a soft and light material (the core). The three layers are in most cases glued together, and thus forming two additional glue layers in the sandwich.

The material choice and location of the different materials in the sandwich can be compared to an ordinary I-beam, which can be regarded as optimised with regard to the cross-sectional geometry, see Figure 2.1a. The big advantage of the sandwich compared to the I-beam is that the optimization can be expanded to panel level, resulting in a highly optimised lightweight structure, whereas the geometrical cross-sectional optimisation in the I-beam can be carried out at beam level only. However, in both the I-beam and the sandwich, nearly all normal stresses will be carried by the flanges or faces and shear stresses by the body plate or core respectively, see Figure 2.1b. Additionally to these tasks, the core in the sandwich should separate the faces, keeping the bending stiffness constant. Furthermore, the core should be sufficiently stiff to avoid local buckling (wrinkling) of the faces.

Almost any structural material which is available in the form of thin sheets may be used to form the faces of a sandwich panel, and today a vast number of possibilities are available, making it possible to tailor the sandwich for the actual demands. The face materials can be divided into two groups: Metallic and non-metallic materials. The metallic faces include aluminium, stainless steel, titanium etc., whereas the non-metallic faces are dominated by the fibre reinforced composites (short FRP), like glass (Glass Fibre Reinforced Plastic - short GFRP), carbon (Carbon Fibre Reinforced Plastic - short CFRP) and aramid (Kevlar) fibres, with various resin systems, polyester, epoxy, vinylester etc. Other types of non-metallic face materials have also been used, like plywood, veneer and even cement. But common to all materials is that the primary demands on the face materials are:
2.1 Composite Sandwich Structures and Their Application

- High stiffness giving high flexural rigidity
- High tensile and compressive strength
- Impact resistance
- Surface finish
- Environmental resistance (chemical, UV, heat etc.)
- Wear resistance

The core is in many cases the most important part of the sandwich, but unfortunately least knowledge is gained of the materials normally used in the core. In Figure 2.2 the four main types of core materials are presented, the corrugated, the honeycomb, the balsa and the cellular foam cores. The corrugated cores are normally used in heavy industries like shipbuilding but have however also found their way into the packaging industry. The honeycomb cores are to a great extent used in the aeronautical industry as they possess the highest performance compared to the weight. The honeycomb cores are made of for example aluminium, aramid (NOMEX®) or resin impregnated paper, which is the cheapest version and seldom used for structural purposes. Honeycomb is also produced in a large number of different geometries, but the hexagonally shaped type, shown in Figure 2.2b, is today the most popular. Unfortunately, structural honeycombs are also very expensive and less tolerant of impact loads, which limits their application to relatively protected structures. The balsa and especially the structural cellular foams possess a good compromise between performance and price and compared to the honeycomb cores, they are more tolerant of localised loads. The cellular foams are therefore the favoured core type in the maritime structures, which are typically operating in harsher environments compared to the aeronautical structures. There are several foam core types on the market, but the most popular foam core material is the structural polyvinyl chloride (PVC) foam. The PVC cores are available in a wide range of densities and material properties and may be used in both a ductile (linear foam structure) and a brittle version (cross-linked foam structure). However, the linear ductile version is slowly being replaced by the styrene acrylonitrile (SAN) foam core type, which is more tolerant of high temperatures and in general a better performing material for structural use compared to the linear PVC foam. Other core materials are the cheap polyurethane (PUR), which is blown in between the faces in a liquid form to subsequently densify, the polystyrene (PS), the polyisocyanurate (PIR), the polyether imide (PEI) and the polymethacryl imide (PMI), which is more expensive compared to the PVC core type and enjoys an increased popularity in the aeronautical industry as an alternative to the honeycomb cores types.
The most important demands on the core materials are:

- Low density
- Sufficient stiffness to prevent decrease in thickness under lateral loading (a limited decrease in thickness leads to rapid decrease in flexural rigidity)
- Sufficient shear stiffness to ensure unwanted out-of-plane shear deformations
- Sufficient stiffness to prevent local buckling of the faces (wrinkling)
- Sufficient shear strength to prevent global core shear failure under lateral loading
- Sufficient thermal insulation
2.1.2 Applications

Limited pioneer use of structural sandwich was seen from about 1820, but large production applications were not seen until the invention of structural adhesives in the 1930s, which allowed the application of bonded sandwich composites. The de Havilland Mosquito, Figure 2.4, built 1940-1950, was the first mass-produced aeronautical structure to take advantage of the sandwich concept. The sandwich consisted of plywood faces and balsa core, and the choice of the sandwich concept was a product of earlier experience with two predecessors, the Comet racer aircraft and the Albatros commercial passenger aircraft, which were built in relatively small numbers. However, the experiences gained proved that a much lighter and especially superior aerodynamically shaped streamlined aircraft could be achieved compared to similar metal-made aircraft. Furthermore, the Mosquito proved to possess superior performance in terms of speed and agility, making it ideal for precision bombing runs in Nazi-occupied cities and other missions otherwise considered as impossible at that time. It also included design principles first seen in mass production military aircraft at the present time with fibre composites. It was in many ways ahead of its time.

Figure 2.4: The first mass produced (1940-1950) sandwich structure, the multi role aircraft de Havilland Mosquito.

Beginning in the late 1980s the application of composites increased rapidly in both civilian and military aviation, and in both cases the sandwich concept has been exploited. As regards to civilian aviation earlier mass-produced airliners like the Airbus A300 only included about 1% composite parts, while newer aircraft like the Airbus A340 and A380, Figure 2.5a, include 13% and 25% respectively, where the main applications for composites are control surfaces and secondary structures.

Military aviation applications are always somewhat ahead of the civilian counterparts. But the same increasing use of fibre composites has been seen here. The General Dynamics F-16, developed in the late 1970s, uses about 3% composite parts while the newer fighter-bomber Lockheed/Boeing F-22, Figure 2.5b, developed in the early 1990s, uses approximately 23%.

The latest-state-of-the-art application in the aviation industry is the Joint Strike Fighter F-35, Figure 2.5c, which was laid out as a competition between American Boeing and Lockheed Martin. Lockheed Martin proved to have the superior design when the two planes flew in
1999 and received the order for the next century fighter aircraft. The plane has extraordinary performance and is in the STOVL-version able to start and land vertically. The Lockheed Martin F-35 will in 2008, when the first aircraft will be operational, be equipped with the largest amount of composite parts to date, about 45%, which includes a continuous wing/body structure, with a complete CFRP composite centre fuselage and wings additional to the standard composite applications like control surfaces and secondary structures. Orders count at present about 2800 planes, for the United States, the United Kingdom, Italy, Holland, Australia, Canada, Norway and Denmark. The production of the Joint Strike Fighter will have a tremendous impact on the complete industry, including the spin-off effect to research and development within damage tolerance.

![Mass-produced composite aircraft. The Airbus A380 (a), the Lockheed/Boeing F-22 and the Lockheed Martin F-35 (Joint Strike Fighter).](image)

In the maritime industry composite materials and sandwich structures have been utilised since the middle of the last century. In the beginning mainly in smaller vessels like pleasure boats, but also in more high-performance applications like power boats.

The step towards larger composite vessels was just as in the aeronautical industry taken by the military. In the beginning mainly single skin concepts were used, like the British HUNT and SANDOWN mine hunter classes, but in the early 1970s the Royal Swedish Navy received the HMS VIKSTEN, which is still in service and built with GFRP faces and PVC foam core. The sandwich concept with fibre composite and PVC foam material choice has proved to be both very efficient, with respect to performance and weight, and economical in service, especially with regard to mine counter-measure vessels, which were earlier were wooden vessels demanding a high level of maintenance.

In the last 15 years a number of newer fibre composite sandwich vessels have followed suit, dominated by vessels from the Scandinavian navies. Among those are the Royal Danish Navy’s Standard Flex 300 (FLYVEFISKEN-class), Figure 2.6a, which is a multi role patrol vessel. The Standard Flex concept built on a number of container modules, which can be replaced in a very short time and thus transform the vessel from for example a fully armed combat vessel to a peaceful pollution counter-measure vessel or a mine hunter. In the mine hunter role the Standard Flex 300 is backed up by two unmanned mine hunter drones, the
Standard Flex 100 (MSF-class), Figure 2.6b, and thus forms an extremely efficient mine hunter concept. The Standard Flex 100 can furthermore be used as a Standard Flex module transport vessel or a diver vessel. Both vessels are built with GFRP faces\(^2\) and PVC foam cores.

The evolution in the Scandinavian navies has culminated in the Royal Swedish Navy’s YS-2000 (VISBY-class), Figure 2.6c, which is the second largest composite vessel ever built, of a length of 73 m and a displacement of only 600 tons fully equipped. The VISBY-class is a combined corvette/mine hunter and a fully CFRP/PVC foam stealth vessel with a service speed well above 35 knots.

\[\text{Figure 2.6: Composite sandwich vessels. The Royal Danish Navy’s Standard Flex 300 (FLYVEFISKEN-class) (a) and the mine hunting drone Standard Flex 100 (MSF-class) (b). The Royal Swedish Navy’s stealth YS-2000 (VISBY-class)(c).}\]

The civilian applications of large composite and sandwich vessels have mostly been oriented towards either high-performing competition oceangoing sailing boats or very luxurious yachts. An example of the latter is the British-built Mirabella V, Figure 2.7, which to date is the largest sandwich vessel ever built, beating the VISBY-class by only 3 m. The Mirabella V is a combined CFRP/Kevlar face and PVC foam core vessel and also includes an 88 m high mast completely built of CFRP.

The above examples are only from the aeronautical and maritime industries, but composites and sandwich enjoy a widespread popularity in a number of other applications like the aerospace industry, which probably includes the most exotic and advanced applications. In the energy industry, wind generator blades are built entirely of composites. At the moment only GFRP layups are used, but as the demand for high-effect output constantly increases resulting in longer blades, the application of CFRP and more widespread use of sandwich in the main load carrying part of the wing profile will be unavoidable.

In the train and automotive industry composites have earlier only been used to a limited extent in mass production. However, in the last decade composites have also here found their way into applications such as bonnets, crash beams and secondary parts in automotive

\(^2\)Varying between woven and non-crimp (multi-axial) laminates in different series.
production and as panels and front sections of trains. Especially in the train industry the application of composites is immensely popular, and the first complete train sets build entirely of composites and sandwich have been planned and designed.

![Image of a vessel](image.png)

Figure 2.7: The British Mirabella V.

### 2.2 Production and In-Service Damages

In the aeronautical, train and automotive industries, the goal is to produce structures entirely built of composites, as they possess the potential of significantly reducing the weight of the structure and thus increasing the performance of the structure. This goal is still somewhat in the future, as high demands on the damage tolerance of the structure result in high safety factors, which put high penalties on the structure and thus reduce the performance. In the maritime industry the goal of structures completely built in composite and sandwich has already been reached, but experience from the navies, using sandwich vessels with FRP faces and PVC foam cores, has shown that a number of common and critical damages cause both a decrease in performance of the structure and are also a source of repeated repair actions.
2.2 Production and In-Service Damages

Figure 2.8: Typical damages. (a) Impact with sharp object, (b) impact with blunt object, (c) slamming introduced damage and (d) debond due to a production flaw.

Figure 2.9: (a) Typical slamming impact damage, where plugs of core material have been torn out. (b) Various impact types including crushing of the underlying core material (bottom). Photo courtesy of Royal Institute of Technology, Sweden.
In Figure 2.8 the typical damages are presented. It is common to these damages that they are all a result of a production defect or a damage directly linked to overloading of the structure, for example impact damage with an object or slamming events.

The slamming event, Figures 2.8c and 2.9a, mostly causes core shear failure near the supports of the panel, which will propagate up to the face core interface during the pressure loading. The following suction loading on the panel by the slamming event will continue propagating the crack at the interface and eventually kink the crack back into the core leaving a damage pattern, which resembles core pieces being torn out of the core by either of the faces. The effect of slamming events on sandwich panels have among others been investigated by Hayman et al. (1991) and Hayman et al. (1992).

Damages directly related to impact events with sharp or blunt objects, Figures 2.8a and b and 2.9b, are often seen on sandwich structures, and they are examples of typical in-service damages. The extent of damage to the structure is directly related to the energy transmitted to the structure at the moment of impact, and the impact event can introduce various damage types to the face, for example face shear fracture resulting in delaminations, where the individual plies in the face laminate are separated, through-the-thickness compression failure of the face laminate and in-plane compression or tension failure of the face laminate near the impact location. The impact damage events and the resulting residual carrying capacity of the sandwich panels have among others been investigated in great detail both analytically and experimentally by Bull (2004).

Damages to the core related to the impact event could be core shear fracture near the impact zone and/or crushing of the core under the impact zone, Figure 2.9b, which is investigated thoroughly by Shipsha (2001). The core crushing results in a permanent dent and for some core types accompanied with a cavity, leaving the face and core separated in a zone extending in a radial direction from the impact location. The phenomenon of separation of face and core in an isolated area is from now on designated a debond.

Another damage scenario which results in a debond damage is production flaws, Figure 2.8d. If the glue forming the interface between face and core is missing in parts of the panel, because of poor production quality a debond is initially present in the panel. The production debonds are typically more difficult to identify, as they do not necessarily leave any visible damage to the face sheet or permanent dent in the surface. However, they have been suspected to have been responsible for severe and numerous large-scale debonds in Swedish mine sweepers.

In many cases the emergence of debonds is accompanied with outgassing from the core, which is furthermore accelerated when the panel is exposed to direct sunshine. This kind of damage is also often designated sun blisters, Figure 2.10. It is believed that when the combination of polyester resin and non-heat stabilised PVC cores is used, a chemical reaction is happening which produces de-gassing from the core. The effect of degassing is in all cases a gas pressure inside the debond accompanied with a debond opening, which will act as a geometrical imperfection. Debond propagation can furthermore be driven by both the pressure inside the sun blister and the in-service loads.
Continued service of the vessel can for both the in-service and the production introduced debond result in a propagation of the debonding, where the debond spreads in a pattern governed by the external loading on the debonded panel. Figures 2.11, 2.12 and 2.13, show three examples of debond spreading as a result of continued in-service loading. Common to all three cases is that a relatively small damage in terms of either an impact or slamming damage, which has introduced a debond between face and core, has propagated to a fairly large part of the vessel. This underlines the criticality of debonds in a damage tolerance context, as the general sandwich concept is compromised, when the interaction between the two face layers is removed and the debonded face and the remaining sandwich are deforming independently. Debond propagation can therefore result in complete loss of load carrying capacity of the sandwich structure. This can happen in various ways:

- **BUCKLING**: Local buckling of the face layer over the debonding, which may lead to global failure of the panel
- **KINKING**: The debond kinks at some length along the crack front into the core and triggers trough-the-thickness shear failure
- **SEPARATION**: Unstable and very rapid crack growth in the interface, which leads to complete separation of face and core

However, one of the key aspects in damage tolerance of structures is that a damage introduced into a structure does not necessarily mean that the structure will collapse immediately. The damage might grow over some time, increasing the criticality of the damage. This might lead to a fatal failure of the structure, as indicated above, but the propagation might also end up settling, because of stress redistribution in the surrounding structure or because of another event, for example fibre bridging. Three major questions now emerge:

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Figure 2.10: Example of a debond in a CFRP/PVC panel, which has blistered. (a) Schematic representation, (b) photo courtesy of Royal Institute of Technology, Sweden.
Figure 2.11: Example of a debond after removal of the face layer. This debond is believed to have occurred as a slamming damage on the keel. The core is H130 and the skin is 8-10 mm. In some areas a high amount of fibre bridging occurs.

Figure 2.12: Example of a debond propagation under continued in-service loading. (a) Impact keel location with an underwater steel wire. (b) The extent of propagation when the outer face has been removed.

Figure 2.13: Example of blistered debonds which could be a result of slamming on the bottom structure. The resin and subsequently act as load transferring bridges between face and core.
1. What is the critical damage size?

2. When should the structure be repaired?

3. Is the damage critical to the structure at all?

With these questions stated, it is natural to define a quantity which is a measure of the remaining carrying capacity of the structure at a given type and size of damage, compared to the intact structure. This quantity will be designated the *residual strength factor*.

For the debond damage, which might be the result of either a slamming, impact or production event/defect, it is also clear that the residual strength of the structure is highly governed by both the debond crack front loading and the subsequent propagation of the debond. So in order to answer the questions posed above and to predict the residual strength factor of the structure, it is necessary to be able to estimate the crack front loading and to simulate the interface crack propagation using for example fracture mechanics.

In the remaining part of this thesis the debond damage event will be investigated. As mentioned above the debond may be a product of any of the damages described earlier. Therefore, the main scope of this thesis is not to investigate the actual debond producing damage event itself, but rather the consequences of the subsequent debond with regard to the structural integrity.

A number of assumptions have been adopted in order to treat the debond damage in a general way by use of Linear Elastic Fracture Mechanics (LEFM):

- The debonded face is assumed to be intact and homogeneous, with the same apparent orthotropic material properties as the remaining intact face layer
- The underlying core is also assumed to be intact in all cases, and it is treated as an isotropic solid, thus neglecting the cellular microstructure of the PVC foam materials
- The current debond geometry in the panel case is idealized by a circular debond, with the diameter $D$, as indicated in Figure 2.14
- The interfaces between the faces and the core will be treated as plane interfaces between two solids, and the debond will be represented by an area where there is no continuous adhesion between the two solids
- The size of the microstructure is assumed to be much smaller compared to any other dimensions
- The failure process zone is assumed to be much smaller compared to the K-dominated zone$^4$
Figure 2.14: Idealisation of a natural debond to circular-shaped debond geometry.

In Figure 2.15 different through-the-thickness crack locations for models with varying detail levels are presented. With regard to the last assumption mentioned above, it is often observed for low density cores with low fracture toughness that the crack propagation is taking place just below the interface as in Figure 2.15b and d. Li and Carlsson (2001) made an extensive analysis of the different crack locations and detail levels in Figure 2.15 using the TSD specimen\textsuperscript{5}. They report negligible influence of the glue layer, but do observe a phase shift in mode-mixity between the pure interface and the sub-interface models, levels a+c and b+d respectively in Figure 2.15. Li and Carlsson (2001) assume linear fracture mechanics to be valid and the same assumption is made here, as most of the core materials exhibit only minor crack tip plasticity.

It should be noted that the negligible difference between especially the level a and c models is only valid, if no non-linear effects are present. It has been observed that for heavier density cores the crack tends to propagate directly in the interface or kink back and forth between the face/glue and glue/core interface, with considerably fibre bridging as the result. In these cases linear fracture mechanics will first of all be very doubtful and secondly the fracture toughness will vary considerable between an interface crack propagating between the face and core or glue layer and between the glue layer and the core.

However, despite the reported phase shift and the fact that the crack is actually propagating just below the interface for lighter density cores, and to minimise the number of unknowns in the analysis, for example the propagation depth in the subinterface models, the crack is

\textsuperscript{4}The K-dominated zone will be explained in chapter 4, but the K-dominated zone is normally comparable to $h/50$, where $h$ is the characteristic length of the crack geometry.

\textsuperscript{5}The TSD specimen is mentioned in chapter 4
assumed to propagate directly in the interface as in Figure 2.15a. Furthermore, the fracture toughness values, which are governing for the initiation of the crack front propagation and which will be presented later in the thesis, are all determined using the same type of interface model, thus excluding errors introduced by using two models with different detail levels.

Figure 2.15: Different through-the-thickness crack locations for models with varying detail levels. (a) Interface, (b) subinterface, (c) interface/core and (d) subinterface/core.
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Chapter 3

Non-Linear Behaviour and Weight Reduction in Curved Sandwich Panels

This chapter is focused on the theoretical tools and their ability to predict the structural response of a curved sandwich panel. Furthermore, it will be presented how the theoretical tools can be used to exploit the geometrical non-linearity and gain significant weight reductions. The work presented in this chapter was carried out in the joint Nordic research project, NORDSANDWICH.

The first part of this chapter was presented at the 6th International Conference on Sandwich Structures, Ft.Lauderdale, Florida, USA, 31 March to 2 April 2003 and is found in Berggreen et al. (2003a).

The second part of this chapter was presented at the 8th International Symposium on Practical Design of Ships and Other Floating Structures (PRADS), Shanghai, China, 16-21 September 2001, and is found in Berggreen and Simonsen (2001).

3.1 Introduction to Curved Sandwich Panels

The understanding of geometrically non-linear behaviour due to large lateral deflections may in some cases be essential in order to produce an optimal design. For initially flat plates it has long been known that even at relatively low levels of load, the effect of the geometrical non-linearity may be significant. The non-linearity arises because the membrane stresses in the plate gradually develop as the deflections become large. This effect has been documented experimentally by Bau-Madsen et al. (1993), Karjalainen and Jolma (2001), Hayman et al. (2002), Hayman et al. (2003a) and Hayman et al. (2003b) for flat sandwich plates. Furthermore, Riber (1997) presented a thorough theoretical analysis of flat sandwich plates and showed that it is possible to derive a closed-form expression for the non-linear, membrane
effect of a sandwich plate, similar to the one DNV (1999) uses for single skin panels. Lately, Det Norske Veritas has introduced a non-linear dimensioning tool for sandwich plates in the High Speed Class Rules, which can be seen in DNV (2003).

The same type of membrane effect will also occur in a laterally loaded panel which is initially curved outwards. For such geometry the membrane loads are activated immediately without requirements for a finite deflection from the initial geometry. Bozhevolnaya and Frostig (1997), Skvortsov et al. (2000), Berggreen (2000) and Berggreen and Simonsen (2001) have presented analyses of this type of geometry. The two types of membrane effects are schematically illustrated in Figure 3.1.

The ship designer has two classes of theoretical tools available for the structural analysis: the analytical rules of the classification societies and the so-called direct calculations. The analytical rules are quite straightforward to use and they do not require any computer-aided modelling. A comparison between the latter theoretical tools with regard to optimisation of curved sandwich panels is seen in Berggreen (2000) and Berggreen and Simonsen (2001) and in the next sections.

In order to use direct calculations in practical design it is important to know how various modelling techniques influence the results. Sandwich panels can be modelled in a number of different ways and each technique has its advantages and disadvantages. It is also important to clarify to which degree of detail the model should be refined, with special attention to the boundary conditions. These aspects are investigated in the following.

Figure 3.1: Schematic presentation of the membrane effect for flat and curved panels.
3.2 Non-Linear Behaviour of Curved Sandwich Panels

The purpose of this section is to investigate the most common modelling techniques used in practical design today by use of three different commercial FE codes, and to compare the numerical calculation results to two experimentally tested curved sandwich panel specimens. Furthermore the objective is to investigate the effect of the level of modelling detail on the general curved panel response.

The experimental testing was carried out at the VTT Technical Research Center of Finland.

3.2.1 Description of Specimens

A single curved sandwich panel under uniform lateral load is analyzed. The panel boundary conditions are simply supported and free to move in the panel plane direction, as described above. The dimensions of the panel are presented in Figure 3.2 and Table 3.1.

The sandwich faces consist of Devold AMT DBL 700-C12 stitched triaxial reinforcement with three 205 g/m² layers. The face fibre directions are [0/45/-45], with the 0°-layer in the curved edge direction. Ampreg Prime 20 epoxy resin is used and the specimens are manufactured with resin infusion. The material properties of the individual laminas¹ and the calculated laminate failure strains, by application of the maximum strain criterion, are seen in Table 3.2. The stiffness parameters of the face laminates can be found by use of the classical lamination theory.

The core is made of 10 mm thick cross-linked Divinycell H160 PVC foam. The material parameters given by the manufacturer are seen in Table 3.2.

¹Tested at the VTT Technical Research Center of Finland
Table 3.1: *Panel geometry parameter values.*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel length</td>
<td>$A$</td>
<td>1050 mm</td>
</tr>
<tr>
<td>Panel breadth</td>
<td>$B$</td>
<td>690 mm</td>
</tr>
<tr>
<td>Panel height</td>
<td>$H$</td>
<td>45 mm</td>
</tr>
<tr>
<td>Support length</td>
<td>$A_l$</td>
<td>1000 mm</td>
</tr>
<tr>
<td>Support breadth</td>
<td>$B_l$</td>
<td>625 mm</td>
</tr>
<tr>
<td>Face thickness</td>
<td>$t_f$</td>
<td>0.7 mm</td>
</tr>
<tr>
<td>Core thickness</td>
<td>$t_c$</td>
<td>10 mm</td>
</tr>
</tbody>
</table>

Table 3.2: *Face lamina (CFRP UNI-axial) and core (Divinycell H160) mechanical parameter values from DIAB (2000).*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face lamina Young’s modulus</td>
<td>$E_1$</td>
<td>117000 MPa</td>
</tr>
<tr>
<td>Face lamina Young’s modulus</td>
<td>$E_2$</td>
<td>7800 MPa</td>
</tr>
<tr>
<td>Face lamina G-modulus</td>
<td>$G_{12}$</td>
<td>4400 MPa</td>
</tr>
<tr>
<td>Face lamina Poisson’s ratio</td>
<td>$\nu_{12}$</td>
<td>0.34</td>
</tr>
<tr>
<td>Face lamina tension strength</td>
<td>$X_{et}$</td>
<td>1.7 %</td>
</tr>
<tr>
<td>Face lamina compression strength</td>
<td>$X_{ec}$</td>
<td>-0.9 %</td>
</tr>
<tr>
<td>Face lamina tension strength</td>
<td>$Y_{et}$</td>
<td>0.9 %</td>
</tr>
<tr>
<td>Face lamina compression strength</td>
<td>$Y_{ec}$</td>
<td>-1.3 %</td>
</tr>
<tr>
<td>Face laminate tension strength</td>
<td>$S_{et}^{LAM}$</td>
<td>1.0 %</td>
</tr>
<tr>
<td>Face laminate compression strength</td>
<td>$X_{et}^{LAM}$</td>
<td>-0.85 %</td>
</tr>
<tr>
<td>Face laminate tension strength</td>
<td>$Y_{et}^{LAM}$</td>
<td>0.90 %</td>
</tr>
<tr>
<td>Face laminate compression strength</td>
<td>$Y_{ec}^{LAM}$</td>
<td>-1.30 %</td>
</tr>
<tr>
<td>Face laminate shear strength</td>
<td>$S_{et}^{LAM}$</td>
<td>1.70 %</td>
</tr>
<tr>
<td>Core Young’s modulus</td>
<td>$E_c$</td>
<td>170 MPa</td>
</tr>
<tr>
<td>Core G-modulus</td>
<td>$G_c$</td>
<td>73 MPa</td>
</tr>
<tr>
<td>Core Poisson’s ratio</td>
<td>$\nu_c$</td>
<td>0.164</td>
</tr>
<tr>
<td>Core tension strength</td>
<td>$X_t^c$</td>
<td>4.7 MPa</td>
</tr>
<tr>
<td>Core compression strength</td>
<td>$X_c^c$</td>
<td>-3.4 MPa</td>
</tr>
<tr>
<td>Core shear strength</td>
<td>$S_c^c$</td>
<td>2.6 MPa</td>
</tr>
</tbody>
</table>
3.2.2 Experimental Setup

Figure 3.3: Test arrangement as described by Karjalainen and Jolma (2001).

The test setup was composed of a rigid steel loading frame which was loaded with a 400 kN universal testing machine, similar to the test rig described in connection with the laterally loaded debond damaged panels in the last chapter. The panel was pressed via its boundaries against a water-filled cushion on the outer curved surface to create uniformly distributed loading. The test arrangement is seen in Figure 3.3, and more details about this loading arrangement are given in Karjalainen and Jolma (2001).

The boundary conditions are close to simply supported with regard to both moment (M=0) and in-plane loads (N=0) and act in the radial direction with respect to the panel surface. The boundary conditions are implemented by resting the panel on 15 mm wide and 8 mm
thick 70 Sh polyurethane (PU) strips. These introduce the support reaction in a wider area than traditional steel rollers while also allowing rotation with minor resistance.

One panel was instrumented with five strain gauges. Two gauges were located on each face at the panel centre. One pair consisted of a gauge in the curved edge direction (x) and another in the panel straight edge direction (y). In addition a single shear strain gauge was located near the panel corner on the support side skin. The panel centre displacement was monitored with an inductive displacement transducer. The total load was measured by the test frame load cell. The water cushion pressure was monitored with a pressure transducer to determine the load footprint area with respect to the total load magnitude. The panels were loaded up to failure in displacement control.

3.2.3 Description of Finite Element Models

In total five different finite element models have been built to analyse the curved panel problem. Three commercial FE codes (ANSYS 6.1, COSMOS/M 2.7 and LS-DYNA 950) were used to model the different models, and various modelling methods have been used to investigate the influence of element type, boundary conditions, solver type and modelling detail level. The five models can be briefly described as follows:

- **COSMOS [SS]**\(^2\): Linear shell model (SHELL4L) with "semi-out-of-plane sandwich behaviour". Implicit code and linear elastic core material properties. 1/4-model, 2668 shell elements with 5185 nodes.

- **ANSYS [SS]**: Parabolic shell model (SHELL91) with out-of-plane sandwich behaviour. Implicit code and linear elastic core material properties. A full model is used, 1632 shell elements with 5061 nodes.

- **ANSYS [CS/S]**\(^3\): Parabolic shell/solid model (SHELL91+SOLID95). Implicit code and linear elastic core material properties. 1/4-model, 1800 shell elements and 3600 solid elements with 17963 nodes. Four elements are used through the thickness of the core.

- **LS-DYNA [CS/S,1/4]**: Linear shell/solid model. Explicit code and elastic-plastic core material properties. 1/4-model, 28800 shell elements and 91776 solid elements with 139905 nodes. Six elements through the core thickness.

- **LS-DYNA [CS/S,1/2]**: Linear shell/solid model. Explicit code and elastic-plastic core material properties. 1/2-model, 57600 shell elements and 183552 solid elements with 220487 nodes. Six elements through the core thickness.

The COSMOS models have been produced by the VTT Technical Research Center of Finland.

\(^{2}\text{Sandwich Shell}\)

\(^{3}\text{Composite Shell/Solid}\)
3.2 Non-Linear Behaviour of Curved Sandwich Panels

LS-DYNA Models

Besides the element types resulting in a higher mesh density, the LS-DYNA [CS/S]-models differ in the way the PU layer at the support frame used in the test is included in the model. The PU layer is modelled with solid elements and contact definitions are used to model the contact between the lower face of the curved panel and the PU layer of the support frame. No friction is assumed between the two materials. Another notable difference between the LS-DYNA [CS/S]-models and the other models used is the material characterisation, which is explained below.

Model Details

Boundary conditions:

- **COSMOS [SS]**: The centre plane of the panel is restrained from movement in the radial direction at the centre line of the support frame. Symmetry conditions are introduced in the middle of the straight and the curved plane, and in the curved plane the symmetry conditions are introduced by application of a cylindrical coordinate system.

- **ANSYS [SS]**: The same as for the COSMOS [SS] except that the full model is used, and the panel centre point is restrained from in-plane movement.

- **ANSYS [CS/S]**: The same as for the COSMOS [SS]-model except that BC’s at the support frame are introduced at center point of the lower face layer.

- **LS-DYNA [CS/S, 1/4]**: The same as for the ANSYS [CS/S]-model, except that no cylindrical symmetry condition can be used along the curved edge. Instead a normal Cartesian symmetry condition is used.

- **LS-DYNA [CS/S, 1/2]**: Only one symmetry condition along the straight edge is used.

Loading:

- **COSMOS and ANSYS models**: The load is introduced as a uniform surface pressure on the specified area indicated in Figure 3.2.

- **LS-DYNA [CS/S, 1/4 & 1/2]**: The same as for the COSMOS and ANSYS. Furthermore, the loading is added using the trigonometric relationship between surface pressure and time shown in Eq. (3.1), Urban (2003):

  \[ p(t) = 1 - \cos\left(\frac{\pi t}{2t_{\text{time}}}\right) \quad t_{\text{time}} = \frac{2n + 1}{2} T_{\text{eigen}} \]

  (3.1)
In Eq. (3.1) \( t_{\text{time}} \) is the termination time, \( T_{\text{eigen}} \) is the eigenperiod of the curved panel and \( n \) designates a multiplication parameter. By use of the expression in Eq. (3.1) for the termination time, a quasi-static response is achieved. \( n \) is chosen to be 5 in order to minimise the kinetic energy of the system. Normally, a quasi-static response is assumed when the kinetic energy is below approximately 10% of the total energy.

**Material Models**

- **COSMOS and ANSYS models**: The faces are modelled as orthotropic linear elastic using the modified ply properties as shown in Table 3.2. The maximum strain failure criterion is used to predict face failure. The core is assumed to be isotropic and linear elastic with the same Young’s modulus in both tension and compression. The stress values in the core are evaluated using the maximum stress criterion.

- **LS-DYNA [CS/S, 1/4 & 1/2]**: With regard to the faces, the same orthotropic description has been applied as in the ANSYS and COSMOS models, also the maximum strain failure criterion has been used to predict failure, except that LS-DYNA cannot distinguish between tension and compression failure in the lamina matrix direction, when the max. strain criterion is used, so a numerical average value of 1.1% has been applied.

In order to try to model the failure of the core more accurately, a simple damage mechanics approach are adopted. The core is still modelled as isotropic, completely linear elastic and with brittle failure in tension, but in compression the yielding-like plateau stress of the PVC foam is included. Physically, the individual core cells in the PVC foam are crushed and compacted. This response is modelled by setting Poisson’s ratio equal to zero in this area. The plateau stress area extends to a strain of approx. 50%. Continued loading leads to completely crushed and compacted cells, and therefore almost solid PVC properties. No direct failure of the material is modelled in compression.

In shear the material behaviour almost equals the behaviour in compression, except that the material fails at a prescribed shear strain, which has been set to 20%.

When a core element fails in either tension or shear, or the maximum strain failure criterion is reached for a face element, the element is completely removed from the model.
3.2.4 Results from Tests and Finite Element Calculations

The collected results for the panel centre point deflection are illustrated in Figure 3.5. The last data points after the failure are removed from experimental results because the displacement transducer was struck by the failing panel, and it moved on its attachment. Three experimental results are included. Experiment 1.1 is solely panel response without panel failure. The loading had to be aborted, because the panel corners lifted away from the support frame and were about to touch the loading arrangement. More water had to be added to the water cushion, with the result that the loading area decreased slightly. This effect might explain why higher maximum deflections are seen in experiments 1.2 and 2, see Figure 3.5.

The FE simulations can be divided into three groups with respect to the general panel response: One class (A) with the LS-DYNA [CS/S, 1/2]-model and one class (B) with the LS-DYNA [CS/S, 1/4]-model and one last class (C) with the remaining models. The class A benefits from the fact that the modelled support frame and contact conditions make it possible for the panel corners to lift away from the support. The class B also allows the panel corners to lift, but it suffers from inaccurate symmetry conditions at the curved edge. The class C models do not allow the panel corners to lift, so these models exhibit a slightly stiffer response. In Figure 3.4 the deformation of the PU strips at the support frame is shown.

![Figure 3.4: Deformation of the PU strips in the LS-DYNA models.](image)

Generally, it is observed that all models show a transition from an initially typical curved panel response (see Figure 3.1) to a flat panel response: The membrane effects and consequently the panel stiffness are gradually decreasing in the beginning and then increasing again, as the membrane effects return to the panel as the deflections rise, and the panel moves on to a flat-panel-like response.

An important conclusion to be drawn from Figure 3.5, which has also been shown earlier, is that a linear FE calculation is completely insufficient in analyses of this kind of curved sandwich panels, which is in agreement with Berggreen (2000) and Berggreen and Simonsen (2001). The linear calculation only captures the initial membrane effect of the curved panel and fails to simulate the following decrease in membrane effect, which results in a too stiff panel response.
Figure 3.5: Panel centre point deflection.

Figure 3.6: Panel midpoint strains in the curved direction.
3.2 Non-Linear Behaviour of Curved Sandwich Panels

Figure 3.7: Panel midpoint strains in the straight direction.

Figure 3.8: Panel strain results for a high-strain location near panel corner.
Chapter 3. Non-Linear Behaviour and Weight Reduction in Curved Sandwich Panels

Figure 3.9: Failure in test specimen on the pressure side.

Figure 3.10: Panel corner lift-off as observed during testing and modelled by the LS-DYNA models.
3.2 Non-Linear Behaviour of Curved Sandwich Panels

Results for panel centre skin strains are illustrated in Figure 3.6 for the curved direction and in Figure 3.7 for the straight direction. In Figures 3.6 and 3.7, the post-failure results of the LS-DYNA models are removed for clarity. In the experiments it was observed that the panel fails near the centre of the panel on the pressure side skin. The fracture propagated instantaneously across the skin causing a considerable increase in panel centre deflection and a decrease in the carried load, see Figure 3.9. This observed failure mechanism is explained well by the results from the strain gauges at the panel centre. In Figure 3.6 it is seen that the laminate compressive strain in the curved direction nearly reaches the laminate maximum compressive strength value (-0.85%) in the 0°-direction.

The FE model results shown in Figures 3.6 and 3.7 all qualitatively follow the general trend, but none of them exactly match the strain values from the test. This may have several reasons. The strongest and probably the most important reason is already discussed above under the general response: the corners are lifting away from the support, see Figure 3.10. On the pressure side of the panel centre it is clearly seen that the class C models do not capture the high compressive strains, contrary to the class A, which includes the panel corner lifting. The consequence of neglecting the cylindrical symmetry conditions (the class B model) can also be seen in both Figures 3.6 and 3.7, where this model fails to simulate the centre strains.

In Figure 3.8 the in-plane shear strain response of the corner location has been plotted. Generally, all models capture this shear strain response quite well and the model differences do not influence this parameter in the same way as in the case of the normal strains at the panel centre.

In order to compare failure loads and mechanisms of all models used, these values and mechanisms have been listed in Table 3.3. The first observation, which can be made from the simulations, is that all class C models fail in the core close to the boundary condition along the straight direction of the panel, regardless of the reference area, element type and interactive/separate mode criterion applied. Thus, the conclusion is (as could be expected) that the boundary conditions, and the fact that the corners are restricted from lifting, have a great influence in these models.

From the results in Table 3.3 it is also seen that the class B model (LS-DYNA [CS/S, 1/4]) fails very near to the test value. Unfortunately, this result is impaired by the fact that the strain response is highly distorted near the panel centre, as discussed above.

The best result is that of the class A model (LS-DYNA [CS/S, 1/2]), which predicts correctly both the failure mechanism and a failure load relatively close to that of the experiments. But because face failure is controlled at the lamina level, the difference of 7-9 kN between the class A model and the test results could be explained by the fact that the LS-DYNA model does not distinguish between tensile and compressive matrix failure, which triggers a slightly different progressive lamina failure. This is emphasised by the strain results from the panel centre in the curved direction, see Figure 3.6, where the class A model fails at a lower laminate strain than the corresponding failure strain given in Table 3.2.
Table 3.3: Failure predicted by different models and observed in experiments.

<table>
<thead>
<tr>
<th>Model</th>
<th>Class</th>
<th>Failure load</th>
<th>Mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment 1.1</td>
<td>-</td>
<td>Loading aborted at 41 kN</td>
<td>No detectable failure</td>
</tr>
<tr>
<td>Experiment 1.2</td>
<td>-</td>
<td>47 kN</td>
<td>Face failure (pressure side)</td>
</tr>
<tr>
<td>Experiment 2</td>
<td>-</td>
<td>45 kN</td>
<td>Face failure (pressure side)</td>
</tr>
<tr>
<td>COSMOS [SS]</td>
<td>C</td>
<td>38 kN (complete panel)</td>
<td>Core failure (along straight BC)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50 kN (loaded panel area)</td>
<td></td>
</tr>
<tr>
<td>ANSYS [SS]</td>
<td>C</td>
<td>32 kN (complete panel)</td>
<td>Core failure (along straight BC)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50 kN (loaded panel area)</td>
<td></td>
</tr>
<tr>
<td>ANSYS [CS/S]</td>
<td>C</td>
<td>19 kN (complete panel)</td>
<td>Core failure (along straight BC)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>51 kN (loaded panel area)</td>
<td></td>
</tr>
<tr>
<td>LS-DYNA [CS/S,1/4]</td>
<td>B</td>
<td>46 kN (First ply failure)</td>
<td>Face failure (pressure side)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>48 kN (Last ply failure)</td>
<td></td>
</tr>
<tr>
<td>LS-DYNA [CS/S,1/2]</td>
<td>A</td>
<td>34 kN (First ply failure)</td>
<td>Face failure (pressure side)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38 kN (Last ply failure)</td>
<td></td>
</tr>
</tbody>
</table>

### 3.3 Weight Reductions in Curved Sandwich Panels

The purpose of the second part of this chapter is to investigate whether curved sandwich panel designs utilizing membrane effects can be designed to weigh less than designs not utilizing these membrane effects. Previous studies have indicated that significant weight reductions may be obtained, but the studies have been based on highly idealised boundary conditions and none of the studies have actually quantified the weight savings. In this section a representative bottom panel in a sandwich vessel is considered. The considered panel is 2000 mm wide and 2700 mm high, has a camber of 240 mm and is subjected to uniform lateral pressure, of an amplitude varying between 37 kPa and 180 kPa.

In the first part of the paper the panel and the surrounding ship structure are modelled in the commercial finite element program **ANSYS**. In order to determine the effect of panel curvature a model with a plane panel with the same dimensions is also considered.

In order to utilise the membrane effect of the shell and obtain the weight reduction, it is essential, however, that the structure around the panel can hold the compressive membrane forces. In the second part of this section, different surrounding structures are investigated to determine under which conditions the advantages of the panel curvature can be obtained. First of all, it turns out that the real structure behaves very differently from the idealised, fully clamped boundary conditions considered in other studies. Secondly, it is shown how much structure is needed around a panel edge to obtain the desired membrane effect.
3.3 Weight Reductions in Curved Sandwich Panels

3.3.1 The Vessel, Materials and Design Parameters

The analysis is based on one specific case: a bottom panel in a representative medium-sized vessel. The vessel’s length is 25m, the breadth is 7.0 m, the draught is 1.8 m, the displacement is 125 tons and the speed is 12 knots. A specific representative bottom panel is considered in the analysis. The panel is located between 16.0 and 18.0 m from the aft end of the vessel. The width of the panel is 2000 mm and the average height (the chord length) is 2700 mm. The panel is approximately cylindrical with a camber of 240 mm, i.e. a radius of curvature of 3917mm. The geometry of the considered panel is presented in Figure 3.11.

The faces of the sandwich panel consist of quadro-axial GFRP mats, with polyester resin. The layup configuration is [0/45/90/-45]. As normally used in the face laminates in navy applications, a chopped stand mat (CSM) is on top and bottom of each face laminate. The mechanical properties of each type of lamina in the layup sequence are found in Table 3.4. The core consists of Divinycell H series PVC foam with densities ranging from 45 to 250 kg/m³. The mechanical properties for this material are found in DIAB (2000).

Table 3.4: Mechanical properties for the face laminas, GFRP UNI-directional and GFRP CSM.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UD-directional</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E_1$</td>
<td>25800 MPa</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E_2$</td>
<td>8000 MPa</td>
</tr>
<tr>
<td>G-modulus</td>
<td>$G_{12}$</td>
<td>4900 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu_{12}$</td>
<td>0.26</td>
</tr>
<tr>
<td>Tension strength</td>
<td>$X_t$</td>
<td>720 MPa</td>
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<tr>
<td>Tension strength</td>
<td>$X_c$</td>
<td>-351 MPa</td>
</tr>
<tr>
<td>Compression strength</td>
<td>$Y_t$</td>
<td>70 MPa</td>
</tr>
<tr>
<td>Compression strength</td>
<td>$Y_c$</td>
<td>-122 MPa</td>
</tr>
<tr>
<td>Shear strength</td>
<td>$S$</td>
<td>63 MPa</td>
</tr>
<tr>
<td><strong>CSM</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E_1$</td>
<td>6000 MPa</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E_2$</td>
<td>6000 MPa</td>
</tr>
<tr>
<td>G-modulus</td>
<td>$G_{12}$</td>
<td>2300 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu_{12}$</td>
<td>0.35</td>
</tr>
<tr>
<td>Tension strength</td>
<td>$X_t$</td>
<td>80 MPa</td>
</tr>
<tr>
<td>Tension strength</td>
<td>$X_c$</td>
<td>-100 MPa</td>
</tr>
<tr>
<td>Compression strength</td>
<td>$Y_t$</td>
<td>80 MPa</td>
</tr>
<tr>
<td>Compression strength</td>
<td>$Y_c$</td>
<td>-100 MPa</td>
</tr>
<tr>
<td>Shear strength</td>
<td>$S$</td>
<td>20 MPa</td>
</tr>
</tbody>
</table>
Figure 3.11: Finite element presentation of the section.
3.3.2 Panel Design by Use of Analytical and Numerical Calculations

The purpose of this section is to investigate how much weight can be saved by taking advantage of the panel curvature. Furthermore, the analysis will show how much weight is saved by going from a linear analytical rule calculation (based on a flat plate) to a direct calculation for this particular configuration.

The designs are developed in agreement with the Rules of Det Norske Veritas (1991, 1996, 1999). The vessel is designed to Class Light Craft (LC), Type: Patrol, Service Restriction: R0, see DNV (1999). The design pressure is 36.4 kPa and acts from the keel to the middle deck, see Figure 3.11.

When the panel is designed by use of the analytical formulas given in the Rules, the optimum weight is found to be 80.8 kg, assuming fully clamped boundaries according to normal standard.

In order to design the panel by use of direct calculations, the finite element model shown in Figure 3.11 is used. The model includes the ship structure around the panel, from keel to deck and between the frames 13.5 m and 19.5 m from the aft section. The free boundaries of the shell plating are fully fixed and the nodes in the centre plane are treated with a symmetry condition. The model contains 5116 elements and 15220 nodes. A non-linear 8-noded layered composite shell element (SHELL91) has been used. The element has four integration points in the plane and three through the thickness for each layer. The element incorporates furthermore a sandwich option, which takes into account the out-of-plane shear deformation, see Figure 3.12.

![Deformed shape](image)

**Figure 3.12:** The effect of out-of-plane shear deformation - the sandwich option.

The weight of the optimised curved panel becomes 72.8 kg, when the rules for direct calculation are applied, see configuration 3 in Table 3.5. The final design is determined by the required minimum face laminate thickness, the stress level in the core and the maximum
panel deflection. This design is 9.8% lighter than the basis design, configuration 1 in Table 3.5. The saving in weight is caused particularly by going from a linear plate theory to direct non-linear calculations and by going from a flat plate to a curved panel. In order to extract the effect of panel curvature configuration 2 was considered. Configuration 2 was designed on the basis of direct calculation by a modified finite element model with a plane panel. It is seen that in this case the minimum weight becomes 89.7 kg, which corresponds to a weight increase of 11%. The reason why this panel becomes heavier than in configuration 1 is that this design was based on the assumption of fully clamped boundaries. By comparison of configurations 2 and 3 it is seen that the weight saved by use of the panel curvature is 18.8% of the total panel weight. Thus, the reduction in weight is quite significant. The extra load-carrying capacity is obtained by taking advantage of membrane loads. In order to reach general conclusions regarding the effect of panel curvature, it is therefore necessary to investigate the in-plane stiffness and strength required for the structure around the panel. This is the objective of the following section.

### Table 3.5: Weight optimisation results of the chosen panel, see Figure 3.11.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$t_{f,\text{inner}}$</th>
<th>$t_{f,\text{outer}}$</th>
<th>$t_c$</th>
<th>Core density</th>
<th>Weight</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Theory (CLASS)</td>
<td>3.64</td>
<td>3.64</td>
<td>60</td>
<td>80</td>
<td>80.8</td>
<td>0</td>
</tr>
<tr>
<td>Direct calc., flat panel</td>
<td>3.64</td>
<td>3.64</td>
<td>80</td>
<td>80</td>
<td>89.7</td>
<td>-11.0</td>
</tr>
<tr>
<td>Direct calc., curved panel</td>
<td>3.64</td>
<td>3.64</td>
<td>26</td>
<td>130</td>
<td>72.8</td>
<td>+9.8</td>
</tr>
<tr>
<td>Direct calc., curved panel (allow small $t_{f,\text{inner}}$)</td>
<td>2.46</td>
<td>3.64</td>
<td>37</td>
<td>100</td>
<td>59.5</td>
<td>+26.4</td>
</tr>
</tbody>
</table>

#### 3.3.3 Effect of Boundary In-Plane Stiffness on Minimum Panel Weight

In order to investigate the effect of boundary stiffness on the weight of the curved panel, the model shown in Figure 3.13 is considered. The model is an idealization of the real geometry using a perfectly cylindrical centre part surrounded by two plane parts extending in the tangential direction to the keel and the weather deck. The original angle between the middle deck and the curved panel is maintained and the geometry of the remaining internal structure is adjusted accordingly. The basic idea of this model is that strips may be removed from the upper boundary of the model, thus reducing the in-plane stiffness of the upper boundary. The plates are restrained from rotation around the q- and r-lines, and the section is furthermore clamped in the plane at frame 0, 6 and the centre plane and a symmetry condition is introduced at frame 3, see Figure 3.13. The finite element model of the idealised section is presented in Figure 3.14.
3.3 Weight Reductions in Curved Sandwich Panels

Figure 3.13: Geometry of idealised section with removable plate strips.

Figure 3.14: FE presentation of the idealised geometry from frame 0 to 3, with two panel strips removed. The shell elements are here shown with thickness.
The results are shown in the two graphs in Figure 3.15 and in Tables 3.6 and 3.7 for two different pressures. For both configurations the internal structure is made sufficiently stiff so that it does not deflect significantly. Figure 3.15 shows the weight of the optimised panel as a function of the strip size, i.e. the amount of structure above the panel, see Figure 3.13. The weight is made non-dimensional by the weight of the panel if the boundaries are fully fixed. From the Figure 3.15a it is seen that the weight ratio is gradually decreasing from 1.5 to 1.37 as the strip size is increased from 0 to 2000 mm. Thus, if the upper boundary is a free edge (for example at the weather deck) the weight is 13% higher than if the panel was embedded with a large amount of structure around it. On the other hand, if the in-plane deformations of the boundary could be fully held, 50% weight could be saved.

For the panel with a very high load, Figure 3.15b, the numbers are different but the tendencies are the same. Note furthermore that in this case the weight of the panel has converged for an amount of surrounding structure corresponding to a strip size of 1500 mm, and the weight of the panel is then approximately the same as for the clamped case. The core thickness has not been chosen to be higher than 100 mm, as it is assumed to be practically inconvenient.

In both cases it is very clear, that a large amount of surrounding structure is needed to hold the membrane forces, if the membrane stiffness of the curved panel is going to be exploited.

Figure 3.15: The effect of the surrounding structure on the optimum panel weight. The panel weight is made non-dimensional with the minimum weight of the fully clamped panel. This is shown as a function of the width of the plate strip above the panel. (a) $p=60$ kPa, (b) $p=180$ kPa.
Table 3.6: Weight optimised panel data for increasing strip size, \( p = 60 \text{kPa} \).

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( t_{f,\text{inner}} ) [mm]</th>
<th>( t_{f,\text{outer}} ) [mm]</th>
<th>( t_c ) [mm]</th>
<th>Core density [kg/m(^3)]</th>
<th>Weight [kg]</th>
<th>Savings [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel clamped</td>
<td>3.64</td>
<td>3.64</td>
<td>20</td>
<td>130</td>
<td>67.5</td>
<td>0</td>
</tr>
<tr>
<td>Strip size 0 mm</td>
<td>3.64</td>
<td>3.64</td>
<td>67</td>
<td>130</td>
<td>101.2</td>
<td>-49.9</td>
</tr>
<tr>
<td>Strip size 500 mm</td>
<td>3.64</td>
<td>3.64</td>
<td>63</td>
<td>130</td>
<td>98.4</td>
<td>-45.6</td>
</tr>
<tr>
<td>Strip size 1000 mm</td>
<td>3.64</td>
<td>3.64</td>
<td>58</td>
<td>130</td>
<td>94.8</td>
<td>-40.3</td>
</tr>
<tr>
<td>Strip size 1500 mm</td>
<td>3.64</td>
<td>3.64</td>
<td>56</td>
<td>130</td>
<td>93.3</td>
<td>-38.2</td>
</tr>
<tr>
<td>Strip size 2000 mm</td>
<td>3.64</td>
<td>3.64</td>
<td>55</td>
<td>130</td>
<td>92.6</td>
<td>-37.1</td>
</tr>
<tr>
<td>Full section with top deck</td>
<td>3.64</td>
<td>3.64</td>
<td>53</td>
<td>130</td>
<td>91.2</td>
<td>-35.0</td>
</tr>
</tbody>
</table>

Table 3.7: Weight optimised panel data for increasing strip size, \( p = 180 \text{kPa} \).

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( t_{f,\text{inner}} ) [mm]</th>
<th>( t_{f,\text{outer}} ) [mm]</th>
<th>( t_c ) [mm]</th>
<th>Core density [kg/m(^3)]</th>
<th>Weight [kg]</th>
<th>Savings [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel clamped</td>
<td>3.64</td>
<td>3.64</td>
<td>100</td>
<td>250</td>
<td>191.0</td>
<td>0</td>
</tr>
<tr>
<td>Strip size 0 mm</td>
<td>4.88</td>
<td>4.88</td>
<td>100</td>
<td>250</td>
<td>215.5</td>
<td>-12.8</td>
</tr>
<tr>
<td>Strip size 500 mm</td>
<td>4.48</td>
<td>4.48</td>
<td>100</td>
<td>250</td>
<td>207.6</td>
<td>-8.6</td>
</tr>
<tr>
<td>Strip size 1000 mm</td>
<td>4.28</td>
<td>4.28</td>
<td>100</td>
<td>250</td>
<td>203.7</td>
<td>-6.6</td>
</tr>
<tr>
<td>Strip size 1500 mm</td>
<td>4.08</td>
<td>4.08</td>
<td>100</td>
<td>250</td>
<td>199.7</td>
<td>-4.5</td>
</tr>
<tr>
<td>Strip size 2000 mm</td>
<td>4.08</td>
<td>4.08</td>
<td>100</td>
<td>250</td>
<td>199.7</td>
<td>-4.5</td>
</tr>
<tr>
<td>Full section with top deck</td>
<td>4.08</td>
<td>4.08</td>
<td>100</td>
<td>250</td>
<td>199.7</td>
<td>-4.5</td>
</tr>
</tbody>
</table>
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Chapter 4

Fracture Mechanics in Sandwich Structures

4.1 Basic Homogeneous Fracture Mechanics

According to the First Law of Thermodynamics, when a system goes from a non-equilibrium state to equilibrium, there will be a decrease in potential energy. In 1920 Griffith applied this theorem to the formation of a crack in Griffith (1920) and wrote:(from Anderson (1995))

*It may be supposed, for the present purpose, that the crack is formed by the sudden annihilation of the tractions acting in its surface. At the instant following this operation, the strains, and therefore the potential energy under consideration, have their original values; but in general, the new state is not one of equilibrium. If it is not a state of equilibrium, then, by the theorem of minimum potential energy, the potential energy is reduced by the attainment of equilibrium; if it is a state of equilibrium the energy does not change.*

Thus, a crack may form (or an existing crack may grow) only if such a process causes the total energy to decrease or remain constant. The critical conditions for fracture can be defined as the point where crack growth occurs under equilibrium conditions, which means no net change in the total energy.
This consideration can be illustrated with a crack of length $2a$ in an infinite plate (this means that the plate width and length are $>> 2a$) under plane stress conditions, see Figure 4.1. If the crack is going to increase in size, then a sufficient amount of potential energy must be present in the plate to overcome the surface energy of the material. If the above-mentioned consideration is applied, the following energy balance can be formed for an incremental increase in the crack area, $dA$:

$$
\frac{dE}{dA} = \frac{d\Pi}{dA} + \frac{dW_s}{dA} = 0 \iff \frac{d\Pi}{dA} = \frac{dW_s}{dA}
$$

where $E$ is the total energy, $\Pi$ the potential energy supplied by internal strain energy and external forces and $W_s$ is the energy required to create new surfaces. This energy balance is also called the \textit{Griffith energy balance}.
4.1 Basic Homogeneous Fracture Mechanics

Irwin defined in 1956 the Griffith-energy $G$, also called the energy release rate\(^1\), as a measure of the energy available for an increment of crack extension (Irwin (1956)):

$$G = -\frac{d\Pi}{dA}$$  \hspace{1cm} (4.2)

and furthermore by use of the Griffith energy balance, the critical Griffith-energy can be defined as

$$G_c = \frac{dW_s}{dA} = \Gamma$$  \hspace{1cm} (4.3)

This parameter is also called the fracture toughness of the material.

Irwin was also among the first to derive the singular stress field close to a sharp crack tip in plane stress or strain for a homogeneous, isotropic elastic solid. This stress field can be defined in the following polar form proposed by Hutchinson and Suo (1992):

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} \sigma_{ij}^I(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} \sigma_{ij}^{II}(\theta) + T(r, \theta) \delta_{i1} \delta_{1j}$$  \hspace{1cm} (4.4)

where $\delta_{ij}$ is Kronecker’s delta, $T$ the non-singular stresses parallel to the crack surfaces. Note the $\frac{1}{\sqrt{r}}$ singularity. The two functions $\sigma_{ij}^I(\theta)$ and $\sigma_{ij}^{II}(\theta)$ define the shape of the stress field.

\hspace{1cm}

![Figure 4.2: Near tip homogeneous crack field definitions.](image)

\(^1\)The term Griffith-energy will be used in this thesis
In Figure 4.3 the different crack modes are defined. Three modes are used: The opening mode (mode I), the in-plane shear mode (mode II) and the out-of-plane shear mode (mode III). When 2-D is assumed only the two first modes are used. If this definition is applied to the stress field, then a pure mode I field will be symmetric with respect to the crack line with \( \sigma_{22}^I = 1 \) and \( \sigma_{12}^I = 0 \) for \( \theta = 0 \) and a pure mode II field will be anti-symmetric with \( \sigma_{12}^{II} = 1 \) and \( \sigma_{22}^{II} = 0 \) also for \( \theta = 0 \).

\( K_I \) and \( K_{II} \), called the stress intensity factors, are then defined as the stress amplitudes according to

\[
\sigma_{22} = \frac{K_I}{\sqrt{2\pi r}} \quad \sigma_{12} = \frac{K_{II}}{\sqrt{2\pi r}} \quad \text{for} \quad \theta = 0 \quad (4.5)
\]

Furthermore, the present mode-mixity for a certain stress field can be defined as

\[
\Psi = \arctan \left( \frac{K_{II}}{K_I} \right) \quad (4.6)
\]

Similar to the stress field, a displacement field can be derived and furthermore refined to describe the relative crack flank opening, \( \delta_2 \), and shearing, \( \delta_1 \), displacements as indicated for a bimaterial crack in Figure 4.5\(^2\):

\(^2\)Note that in the homogeneous case the 1-2 coordinate system is used whereas the x-y system is used for the interface crack tip.
4.2 General Bimaterial Fracture Mechanics

\[ \delta_2 = \frac{8K_I}{E} \sqrt{\frac{r}{2\pi}} \quad \delta_1 = \frac{8K_{II}}{E} \sqrt{\frac{r}{2\pi}} \]  

(4.7)

where \( E \) is given as Young’s modulus for plane stress and for plane strain:

\[ \overline{E} = \frac{E}{1 - \nu^2} \]  

(4.8)

and where \( \nu \) is Poisson’s ratio.

Irwin also defined a relationship between the stress intensity factors and the Griffith-energy for straight ahead quasi-static crack advance:

\[ G = \frac{K_{I}^2 + K_{II}^2}{E} \]  

(4.9)

In short the above theory is known as homogeneous isotropic linear elastic fracture mechanics (LEFM). This theory applies to brittle materials where the failure process zone in terms of crack tip plasticity is only very limited and much smaller compared to the K-dominated zone. This applies for most PVC foam materials with the exception of very heavy foams, where increasing crack tip plasticity is observed, thus resulting in a more doubtful LEFM assumption.

4.2 General Bimaterial Fracture Mechanics

In section 4.1, the basic crack mechanics has briefly been outlined for homogeneous isotropic materials, and the basic parameters such as Griffith-energy and mode-mixity have been introduced.

In this section the theory will be expanded to include inhomogeneous materials, more specifically bimaterials with a straight interface, where a crack is advancing.

Before a more thorough derivation of the theory behind interface fracture mechanics is carried out, one important observation will be made.

When a crack is advancing in a homogeneous material, it mostly happens in mode I. Even though mixed mode crack propagation can happen in homogeneous materials, the crack
will immediately try to kink into a path where pure mode I exists. When an interface crack is advancing between two dissimilar materials, it will mostly happen in a mixed mode condition. The detailed explanation for this fact will be given later in this section, but the main consequence is that not only is the present Griffith-energy level needed for an interface crack in a structural simulation, but also the mode-mixity. Normally, the mode-mixity is not linked directly to the opening and shearing displacements of the crack flanks or the normal and shear stresses in front of the crack tip, but a distortion exists, so special mode-mixity methods have to be employed in order to extract the mode-mixity.

Another consequence of this difference between homogeneous and interface fracture mechanics is that the dependence of the critical Griffith-energy on the mode-mixity is more important in interface than in homogeneous fracture mechanics, because mostly only the mode I critical Griffith-energy is needed in the homogeneous case, while the complete distribution is needed for the interface case. A typical distribution of the critical Griffith-energy as a function of mode-mixity is shown in Figure 4.4 for a bimaterial.

In order to extract Griffith-energy and mode-mixity automatically from any complex cracktip loading in a bimaterial in a FE calculation, a robust numerical mode-mixity method is needed. Several methods have been described in the literature, these methods are presented in this chapter. However, first the background theory will be explained and derived.

A crack in a sandwich construction often propagates just beneath the face/core interface. Because the crack is propagating so close to the interface, it is modelled as an interface crack between two dissimilar orthotropic (face) and isotropic (core) materials. The crack geometry is seen in Figure 4.5. The problem is treated as planar, in plane strain, and linear elastic fracture mechanics (LEFM) is assumed to be valid.
The displacement and the stress field close to the crack tip can be described by the Lekhnitskii-Eshelby-Stroh (LES) formulation, and the derivation of the displacement field can be seen in Appendix A for both complex and real numbered expressions.

The displacement field in Eq. (A.1) can be specialised to describe only the opening $\delta_y$ and shearing $\delta_x$ relative displacements of the crack flanks, see Figure 4.5, and likewise the stresses as normal $\sigma_{yy}$ and the shear $\sigma_{xy}$ stresses in front of the crack tip with $\theta = 0$:

$$\sqrt{\frac{H_{11}}{H_{22}}} \delta_y + i \delta_x = \frac{2H_{11}(K_1 + iK_2)|x|^{1/2 + i\varepsilon}}{\sqrt{2\pi(1 + 2i\varepsilon)} \cosh \pi \varepsilon}$$  \hspace{1cm} (4.10)$$

$$\sqrt{\frac{H_{22}}{H_{11}}} \sigma_{yy} + i \sigma_{xy} = \frac{K x^\varepsilon}{\sqrt{2\pi x}}$$  \hspace{1cm} (4.11)$$

The oscillatory index $\varepsilon$ is defined in Eq. (A.15) and the material depended parameters $H_{11}$ and $H_{22}$ in Eq. (A.13).

These expressions are similar to those defined for homogeneous materials in the previous Section, Eqs. (4.7) and (4.5), but it should be noted in this case that the stress intensity factors $K_1$ and $K_2$ no more act as individual stress amplitudes for respectively mode I and II. This is due to the mix-up of the traditional stress intensity definition in bimaterial fracture mechanics. It is also seen that the term $x^\varepsilon$ is responsible for this mix-up:

$$K x^\varepsilon = [K_1 \cos (\varepsilon \ln x) - K_2 \sin (\varepsilon \ln x)] + i [K_2 \cos (\varepsilon \ln x) + K_1 \sin (\varepsilon \ln x)]$$  \hspace{1cm} (4.12)$$

This mix-up is also the reason for the different choice of mode designation with Arabic numerals, in order to distinguish between homogeneous and inhomogeneous stress intensity factors.
The direct consequence of this mode mix-up is that for example an opening displacement of the crack flanks does not necessarily mean that $K_2 = 0$, but that it is rather a mixed-mode condition where both $K_1$ and $K_2$ are different from zero.

In both the relative crack flank displacements and the stresses in front of the crack tip, the part $x^{\varepsilon}$ is also responsible for a violent oscillation in the stress values for $x \to 0$ (towards the crack tip). Fortunately, this oscillation can be filtered out in a mode-mixity method, as described below, or the oscillations can be neglected, because they are only noticeable very close to the crack tip.

By application of the definition of mode-mixity suggested by Hutchinson and Suo (1992):

$$
\Psi_K = \arctan \left[ \frac{\Im(K h^{\varepsilon})}{\Re(K h^{\varepsilon})} \right]
$$

(4.13)

where $h$ is the characteristic length of the problem, which is explained in more detail below.

The modified Irwin expression linking $K_1$ and $K_2$ to the Griffith-energy:

$$
G = \frac{H_{11} |K|^2}{4 \cosh^2(\pi \varepsilon)}
$$

(4.14)

The stated displacement field stated above in Eq. (4.10) can be specialised, so that mode-mixity and Griffith-energy can be expressed in terms of the relative crack flank displacements.

$$
\Psi_K = \arctan \left( \frac{H_{22}}{H_{11}} \frac{\delta_x}{\delta_y} \right) - \varepsilon \ln \left( \frac{|x|}{h} \right) + \arctan (2\varepsilon)
$$

(4.15)

$$
G = \frac{\pi (1 + 4\varepsilon^2)}{8 H_{11} |x|} \left( \frac{H_{11}}{H_{22}} \delta_y^2 + \delta_z^2 \right)
$$

(4.16)

The characteristic length has the effect that the phase of the mode-mixity is shifted when the length is chosen arbitrarily. Generally, the characteristic length has no physical meaning, but is normally chosen, so that the minimum encountered in the distribution of the critical Griffith-energy is located at $\Psi_K = 0$. In sandwich debonding problems the characteristic length is normally chosen as the face thickness, which will approximately place the minimum of the critical Griffith-energy distribution at $\Psi_K = 0$. 

The mode-mixity can also be described by the Griffith-energies linked to the mode I and II deformations. In this case, the roman numbers refer to the mode dependent Griffith-energies before the oscillations have been filtered out.

The total Griffith-energy can be expressed as a limit value in terms of the stresses in front of the crack tip and the crack flank displacements behind the crack tip (Beuth (1996)):

\[
G = G_I + G_{II} \\
= \lim_{\Delta \to 0} \frac{1}{2\Delta} \int_0^\Delta \left[ \sigma_{yy} (x) \delta_y (\Delta - x) + \sigma_{xy} (x) \delta_x (\Delta - x) \right] dx
\]

The individual oscillatory mode depending Griffith-energies are given as

\[
G_I = \lim_{\Delta \to 0} \frac{1}{4\Delta} \left[ \Phi_1 + \Phi_2 \right]
\]

\[
G_{II} = \lim_{\Delta \to 0} \frac{1}{4\Delta} \left[ \Phi_1 - \Phi_2 \right]
\]

where \( \Phi_1 \) and \( \Phi_2 \) are

\[
\Phi_1 = \Re \left[ \int_0^\Delta \sigma_H (x) \delta_H (\Delta - x) dx \right] \\
= \frac{H_{11}}{\pi \cosh \pi \varepsilon} K K \Delta
\]

\[
\Phi_2 = \Re \left[ \int_0^\Delta \sigma_H (x) \delta_H (\Delta - x) dx \right] \\
= \frac{2\Delta H_{11}}{\pi \cosh \pi \varepsilon} \Re \left[ \frac{Kh}{2 + 4i\varepsilon} \left( \frac{\Delta}{h} \right)^{\frac{2i\varepsilon}{1 + \frac{3}{2} + i\varepsilon}} B (\frac{1}{2} + i\varepsilon, \frac{3}{2} + i\varepsilon) \right]
\]

by use of Eqs. (4.10) and (4.11) with \( \sigma_H \) and \( \delta_H \) defined as

\[
\sigma_H = \sqrt{\frac{H_{22}}{H_{11}}} \sigma_{yy} + i\sigma_{xy} \\
\delta_H = \sqrt{\frac{H_{11}}{H_{22}}} \delta_y + i\delta_x
\]
B is the Beta-function and $\Delta$ the integration length, measured from the crack tip.

In Eq. (4.19) only $\Phi_2$ oscillates because of the part $(\frac{\Delta}{h})^{2\epsilon}$, which produces oscillations between -1 and 1. By simply extracting this part from $\Phi_2$,

$$\Phi'_2 = \Re \left[ \left( \frac{\Delta}{h} \right)^{-2\epsilon} \cdot \int_0^\Delta \sigma_H(x) \delta_H(\Delta - x) \, dx \right]$$

(4.21)

it is possible to define two non-oscillating mode dependent Griffith-energies:

$$G_1 = \lim_{\Delta \to 0} \frac{1}{4\Delta} \left[ \Phi_1 + \Phi'_2 \right]$$

$$G_2 = \lim_{\Delta \to 0} \frac{1}{4\Delta} \left[ \Phi_1 - \Phi'_2 \right]$$

(4.22)

which are now non-oscillating for $\Delta \to 0$.

$G_1$ and $G_2$ are then used for mode-mixity designation:

$$\Psi_G = \arctan \left( \sqrt{\frac{G_2}{G_1}} \right)$$

(4.23)

This mode-mixity designation is not the same as the one proposed by Suo and Hutchinson (1990), $\Psi_K$, but a transformation can be carried out and is included in the chapter dealing with the virtual crack closure technique.
4.3 The J-Integral

The J-integral was first proposed by Rice (1968) and is equal to the Griffith-energy in linear elastic fracture mechanics.

In its simplest form, the J-integral can be defined as a path independent line integral that measures the strength of the singular stresses and strains near a crack tip. Eq. (4.24) is an expression for J in its 2-D form. It assumes that the crack lies in a local crack front Cartesian x-y plane, with the x-axis parallel to the crack and the interface, see Figure 4.6.

\[
J_{2D} = \int_{\Gamma} \left( W dy - T_i \frac{\partial u_i}{\partial x} ds \right)
\]

(4.24)

where \(\Gamma\) is a path surrounding the crack tip going from the lower to the upper crack flank. \(W\) is the strain energy density defined as

\[
W = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij}
\]

(4.25)

and \(T_i\) are the tractions defined as

\[
T_i = \sigma_{ij} n_j
\]

(4.26)
where \( n_j \) is the normal to the path.

The J-integral can also be defined in 3-D, Machida (1998), and it depends on a surface integral over the area \( A \) surrounded by the path \( \Gamma \), see Figure 4.7.

\[
J_{3D} = \int_\Gamma \left( W\,dy - T_i \frac{\partial u_i}{\partial x} \,ds \right) - \int_A \left( \sigma_{iz} \frac{\partial u_i}{\partial x} \right)_{,z} \,dA
\]  

(4.27)

Figure 4.7: Definition of path for 3-D J-integral.

In order to implement the 2-D J-integral theory in a commercial finite element program, like for example ANSYS, a numerical routine has been produced. This routine uses the path facilities typically available in commercial finite element programs to determine the value of the J-integral.

The normal vectors and the strain energy density can in ANSYS, as well as in most programs be listed along the path automatically, so that the first part of the J-integral can be determined by integration with respect to the path length, \( s \).

The tractions \( T_i \) can be listed in the same way along the crack path, but in order to obtain a numerical estimation of the derivatives of the displacements with respect to the x-direction, the following approach is used, see Figure 4.8.
Figure 4.8: Shifting of 2-D J-integral path.

By translating the path a small distance $\Delta x/2$ back along the x-axis and calculating the x- and y-displacements along this new path, and hereafter shifting the path a distance $\Delta x$ forward and calculating the same displacements, a numerical estimate of the derivatives is obtained:

$$\frac{\partial u_i}{\partial x} \approx \frac{u^{(2)}_i - u^{(1)}_i}{\Delta x} = \frac{u_i\rvert_{\Gamma + \Delta x/2} - u_i\rvert_{\Gamma - \Delta x/2}}{\Delta x}$$

(4.28)

The integration over the path length for the second part of the J-integral can now be carried out, and the total value of the 2-D J-integral calculated.
4.4 Theory of Mode-Mixity Methods

In order to extract the Griffith-energy and the mode-mixity from a bimaterial interface in a finite element solution, to predict whether or not crack propagation is happening, several methods have been described in the literature. The three most commonly used methods are:

- The Virtual Crack Extension method (short VCE), first described by Matos et al. (1989) for isotropic bimaterials and later by Charalambides and Zhang (1996) for orthotropic bimaterials.
- The Virtual Crack Closure Technique (short VCCT), first presented in the classical form by Rybicki and Kanninen (1977) and further developed in a modified form for orthotropic bimaterials by Beuth (1996).
- The Crack Surface Displacement method (short CSD), described in many places. Among those are: Smelser (1979), Matos et al. (1989) and Charalambides and Zhang (1996).

A new mode-mixity method has been developed by the author and RISØ National Laboratory for both 2-D and 3-D crack geometries. The new mode-mixity method is designated the Crack Surface Displacement Extrapolation method (short CSDE). It aims specifically at application to typical sandwich bimaterials in an automatic crack propagation simulation.

The above-mentioned four mode-mixity methods have been implemented in the commercial finite element code ANSYS and investigated with special attention to typical sandwich bimaterials used in the maritime industry.

In the following subsections the above-mentioned mode-mixity methods will be described and investigated. Advantages and disadvantages of the methods will be outlined with special emphasis on practical implementation and use in structural calculations using a commercial finite element code.

4.4.1 The Virtual Crack Extension Method (VCE)

Theory

The virtual crack extension method is an energy method, where the difference in Griffith-energy from separately imposing arbitrary mode I and II displacement fields is used to calculate the mode I and II stress intensity factors.
The calculation of the Griffith-energy, using the compliance of an element ring surrounding the crack tip, was presented by Parks (1974) and Hellen (1975), who investigated homogeneous materials by the finite element method. Nevertheless, the procedure can also be used for calculating the Griffith-energy for bimaterials. The expression can be derived by differentiating the potential energy, given in terms of the master stiffness matrix and the nodal displacements, with respect to the crack length. Deforming an element ring surrounding the crack tip by moving an inner rigidly held element core parallel to the crack surfaces, and thus achieving an incremental change in the crack length, will only change parts of the stiffness matrix originating from the element ring, so that only the stiffness matrix of the element ring has to be considered. The Griffith-energy, assuming that no loads are prescribed at the crack flanges, can therefore be given as:

\[
G = J (u_n) = -\frac{1}{2} [u_n]^T \frac{\partial [S]}{\partial a} [u_n]
\]

where \([u_n]\) is the displacements in the nodes of the element ring indicated in Figure 4.9. The term \(\frac{\partial [S]}{\partial a}\) is the stiffness matrix of the element ring differentiated with respect to the crack length.

Figure 4.9: Element ring surrounding the crack tip.
The fraction is calculated by specifying all elements outside the chosen element ring as rigid, and then moving the inner rigid core a small incremental distance $\Delta l$ parallel to the crack surface, and using the deformed elements new stiffness matrix for the numerical approximation, as indicated in Eq. (4.29) and mentioned above.

In commercial codes the stiffness matrix is generally not accessible, but the element ring can be represented by a superelement. However, when parabolic elements are used to form the superelement, they are treated as linear elements in the process. This linearisation should have a very small influence, because the inner element core is only moved a very small distance $\Delta l$ and thus results in very small deformations in the element ring.

In order to extract the mode-mixity from the finite element solution, additional actions have to be taken. First of all assume that a displacement field, designated field $A$ with the associated $J$-integral $J_A$, and another displacement field, designated field $B$ with the associated $J$-integral $J_B$, are added, then a resulting field $C$ with the associated $J_C$ will emerge. $J_C$ will then be given as, Parks (1974) and Matos et al. (1989):

$$J_C = J_A + J_B + M_1$$  \hspace{1cm} (4.30)

where $M_1$ is a path independent integral:

$$M_1 = \int_{\Gamma} \left[ \sigma_{ij}^{A} \varepsilon_{ij}^{B} dy - \left( n_i \sigma_{ij}^{B} \frac{\partial u_{ij}^{A}}{\partial x} + n_i \sigma_{ij}^{A} \frac{\partial u_{ij}^{B}}{\partial x} \right) ds \right]$$ \hspace{1cm} (4.31)

which can also be represented by the stress intensity factors using Eqs. (4.30) and (4.14) to link $J_A$, $J_B$ and $J_C$ to their respective stress intensity factors, $(K_{1A}, K_{2A})$, $(K_{1B}, K_{2B})$ and $(K_{1A} + K_{1B}, K_{2A} + K_{2B})$:

$$M_1 = \frac{2}{H} (K_{1A} K_{1B} + K_{2A} K_{2B})$$  \hspace{1cm} (4.32)

where

$$H = \frac{4 \cosh^2 \pi \varepsilon}{H_{11}}$$  \hspace{1cm} (4.33)

Now let the displacement field $A$ be the original field and the displacement field $B$ be a pure mode I displacement field. A pure mode I displacement field, $\Delta u_x^I$ and $\Delta u_y^I$, can be
found analytically for each node in the chosen element ring, by the displacement field given in Eq. (A.1) and making an arbitrary choice of $\Delta K_1$ and setting $\Delta K_2 = 0$. The mode I displacement field is seen in Appendix A.

The J-integral (and the Griffith-energy) which results from imposing the pure mode I displacement field $J (u_n + \Delta u_1^I)$ is calculated numerically by means of the already found differentiated stiffness matrix for the element ring $\frac{\partial [S]}{\partial a}$. The change in J-integral value and Griffith-energy is thus found to be

$$\Delta^I J = J (u_n + \Delta u_1^I) - J (u_n)$$

and if it is utilised that $\Delta^I J$ is equal to $J_B + M_1$ in Eq. (4.30) and Eq. (4.32) is applied, the following relation between the change in J-integral and the mode I stress intensity factor $K_1$ and the arbitrarily chosen mode I stress intensity factor $\Delta K_1$ is found:

$$\Delta^I J = \frac{1}{H} \left( \Delta K_1^2 + 2 K_1 \Delta K_1 \right)$$

By isolating $K_1$ in Eq. (4.35) and using the numerically found $\Delta^I J$ from Eq. (4.34), the mode I stress intensity factor for the mixed-mode condition may finally be calculated as

$$K_1 = \frac{2 \cosh^2 \pi \varepsilon}{H_{11}} \frac{\Delta^I J}{\Delta K_1} - \frac{1}{2} \Delta K_1$$

The mode II stress intensity factor is calculated in the same way as mode I, by just choosing $\Delta K_2$ to be arbitrary and setting $\Delta K_1 = 0$. The mode II displacement field is also included in Appendix A.

The mode II stress intensity factor for the mixed-mode problem may be calculated as

$$K_2 = \frac{2 \cosh^2 \pi \varepsilon}{H_{11}} \frac{\Delta^II J}{\Delta K_2} - \frac{1}{2} \Delta K_2$$

By application of the mode-mixity definition of Hutchinson and Suo, the mode-mixity is found by Eq. (4.13).

\textsuperscript{3}Note that the above mentioned "pure" mode I displacement field is \textit{physically} a mixed mode case, because of the the mix-up of the traditional stress intensity definitions for an interface crack, see Eq. (4.12)
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Implementation

The VCE method has been implemented by a superelement representation in the commercial FE program ANSYS to access the stiffness matrix for the element ring. The ANSYS program is not able to work with complex numbers, so a comprehensive derivation of real-numbered displacement field expressions has been carried out. This is found in Appendix A. Similar expressions have been reported in Charalambides and Zhang (1996), but unfortunately these expressions are erroneous, so that the complete set of real-numbered displacement field expressions has been rederived by the author.

Below the advantages and disadvantages of the VCE method are outlined, gained through implementation in a commercial finite element program and used for the structural calculations.

Advantages:

- The VCE method is based on a J-integral calculation and therefore in theory a path independent method.
- The mode mixity is measured far away from near tip uncertain displacement and stress fields in the finite element solution.
- No high-density near tip mesh is needed.

Disadvantages:

- The VCE method is very CPU expensive when used in a commercial finite element code, because a superelement representation is needed to achieve the stiffness matrix for the element ring.
- The superelement representation results in exporting and importing large stiffness matrices out of and into the program.
- The VCE method also includes large matrix multiplication, which again increases the CPU time.
- The VCE method is also very sensitive to the structural crack return rotations, which is explained in section 4.5.

The VCE method is not suited for an automatic crack propagation calculation in a commercial finite element code, mainly because of the large consumption of CPU time, when the matrix multiplications are done in the postprocessor. On the other hand if a specially constructed finite element code is used and the VCE method is implemented in this code, and the matrix multiplications are done in integration with the other matrix operations carried out in a finite element code, the VCE method is very elegant and powerful, because there is no need of refined near tip mesh. This is valid for the case of 2-D geometries. In 3-D, the method can prove to be more complicated to use.
4.4 Theory of Mode-Mixity Methods

4.4.2 The Virtual Crack Closure Technique (VCCT)

Theory

By application of the non-oscillatory representation in Eq. (4.22), it is possible to extract the mode mixity in a finite element solution from nodes close to the crack tip, because the oscillations are filtered out analytically.

Complex node forces, \( F_{Hj} \), and complex node displacements, \( \delta_{Hj} \), are formed from the normal and shear nodal forces ahead of the crack tip and the relative crack flank displacements behind the crack tip:

\[
F_{Hj} = \sqrt{\frac{H_{22}}{H_{11}}} F_{yj} + i F_{xyj}
\]

\[
\delta_{Hj} = \sqrt{\frac{H_{11}}{H_{22}}} \delta_{yj} + i \delta_{xj}
\]

For node pairs according to Figure 4.10, where the last node pair \((j)\) will always be the outermost crack flank nodes and the crack tip node, and by use of Eqs. (4.19), (4.21) and (4.22), the mode 1 and mode 2 Griffith-energies can be calculated for a chosen value \( \Delta \):
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\[ G_1 = \frac{1}{4\Delta} \left[ \Re \left( \sum_{j \text{nodes}} F_{H_j} \delta_{H_j} \right) \right] + \Re \left( \frac{\Delta}{h} \right)^{-2\iota \varepsilon} \left[ \sum_{j \text{nodes}} F_{H_j} \delta_{H_j} \right] \]

\[ G_2 = \frac{1}{4\Delta} \left[ \Re \left( \sum_{j \text{nodes}} F_{H_j} \delta_{H_j} \right) \right] - \Re \left( \frac{\Delta}{h} \right)^{-2\iota \varepsilon} \left[ \sum_{j \text{nodes}} F_{H_j} \delta_{H_j} \right] \]

(4.39)

If the near tip mesh is sufficiently refined the Griffith-energies should be independent of \( \Delta \), because the oscillating part of the mode dependent Griffith-energies has been filtered out. Therefore a small \( \Delta \) can theoretically be used.

This forms the modified VCCT method first presented by Beuth (1996) contrary to the classical VCCT method presented by Rybicki and Kanninen (1977), where the oscillation is not filtered out analytically, and consequently a non-constant mode-mixity is achieved for a varying \( \Delta \), as shown by Beuth (1996).

The mode-mixity is found by use of Eq. (4.23). However, because the modified VCCT method is to be compared to the CSDE method in a following section, it is necessary to translate the mode 1 and 2 Griffith-energy based mode-mixity into the mode-mixity definition proposed by Hutchinson and Suo, which is based on stress intensity factors.

This may be done by defining the complex stress intensity factor in modulus and argument representation:

\[ K h^{i\varepsilon} = (K_1 + iK_2) h^{i\varepsilon} = r_K e^{i\Psi_K} \]  

(4.40)

and by using Eq. (4.19) and the ratio between the mode 2 and 1 Griffith-energies, the modulus and argument of the complex stress intensity factor can be found, as shown in detail by Narayan and Beuth (1998):

\[ r_K^2 = K K = 2\Re \left( \sum_{j \text{nodes}} F_{H_j} \delta_{H_j} \right) \cos^2 \frac{\pi \varepsilon}{H_{11} \Delta} \]

\[ \Psi_K = \frac{1}{2} \left[ \arccos \left( \frac{\frac{\pi}{4\zeta \cosh (\pi \varepsilon) \cos \arctan 2\varepsilon} \left( 1 + \frac{G_2}{G_1} \right) \arctan 2\varepsilon - \chi \right) \right] \]

where \( \zeta \) and \( \chi \) are the modulus and the argument of the Beta-function:

\[ \frac{1}{2} B \left( \frac{1}{2} + i \varepsilon, \frac{3}{2} + i \varepsilon \right) = \zeta e^{i\chi} \]  

(4.42)
4.4 Theory of Mode-Mixity Methods

Implementation

The modified VCCT method has also been implemented in the commercial FE program ANSYS.

Below the advantages and disadvantages of the modified VCCT method are outlined, gained through implementation in a commercial finite element program and used for the structural calculations.

Advantages:

- It is a very elegant and simple method to use in practice. In theory the two node pairs closest to the crack tip can be used, because the $\Delta$ dependence has been removed from the expressions.
- The CPU time for calculation of the Griffith-energy and the mode-mixity is relatively low.
- Only moderate density near tip mesh is needed.

Disadvantages:

- Inaccurate near tip stresses are used to calculate the crack tip node force. The errors on the first couple of elements are carried on when more node pairs are used in the integration, because the integration has to be carried out starting from the crack tip.
- Some material and load cases result in non-converged mode mixity values, because the errors on the first elements around the crack tip are so large that a very long integration length is needed to suppress the errors of the first elements. This might also in some cases bring the integration length out of the near tip dominated zone (some places also called the K-zone) and thus result in permanent non-convergence.
- If non-structural crack tip rotations are present (will be explained in section 4.5), the structural return rotations can be inaccurate.
- The choice of root in the arc cos term in Eq. (4.41) is for some bimaterials not obvious and depends on the sign of the shear deformation in front of the crack tip. Unfortunately, the sign has to be chosen from case to case, which is not suited for an automatic calculation.

The modified VCCT method may be usable for an automatic crack propagation calculation in some controlled cases, where only small errors are present at the first near tip elements and the shear stress direction does not change sign. But generally the method is not robust for structural use.
4.4.3 The Crack Surface Displacement and Extrapolation Methods (CSD and CSDE)

Theory

If the two relations for the mode-mixity and the Griffith-energy, Eqs. (4.15) and (4.16), are applied these quantities can be found in a finite element solution solely from the relative nodal displacements of the crack flanks. Consequently, both the mode-mixity and the Griffith-energy will be affected by the oscillations, but they will be small away from the near crack tip zone and therefore without practical importance. In Figure 4.11 the Griffith-energy has been plotted for crack flange node pairs with increasing distance from the crack tip. The same graphs can be plotted for the mode-mixity.

Figure 4.11: The Griffith-energy plotted for crack flange node pairs with increasing distance from the crack tip. Additionally, the drawback of the CSD method has been indicated.

In theory both the Griffith-energy and the mode-mixity should be constant in a zone purely dominated by the crack tip displacement field (the K-zone). Unfortunately, this zone is very close to the crack tip and needs a very high mesh density to be picked up in the finite element calculation. Furthermore, this zone is limited by two barriers: an inner barrier
where numerical errors from the first elements corrupt the results, because the elements are not able to pick up the correct displacement field close to the crack tip, and an outer barrier where the external displacement field is slowly starting to dominate. In practice it is not possible to use a mesh density high enough to capture this zone, so an alternative method is needed.

The CSD (Crack Surface Displacement) method uses an externally determined Griffith-energy, for example by means of a J-integral calculation, and the mode-mixity is used, from the node pair where the Griffith-energy, calculated by use of Eq. (4.16), equals the externally determined value. This method does not strictly give a physical meaning and might lead to erroneous results if a node is chosen inside the numerical error zone as indicated in Figure 4.11.

Because of the inconsistencies in the CSD method a modified method based on the CSD method has then been derived.

This method, which is presented schematically in Figure 4.12, uses solely the results from the relative crack flank displacements and needs no externally determined Griffith-energy. It is observed in numerous investigations that the transition from the external displacement field to the internal crack dominated field is more or less linear, until the barrier to the numerical error zone is crossed. By use of this information, the linear transition zone is simply linearly
extrapolated into the crack tip. This can be done for both the Griffith-energy and the mode-mixity calculated by relative nodal crack flank displacements, but normally it is sufficient to do this numerical extrapolation on the Griffith-energy and then reuse the linear borders on the mode-mixity curve. The new mode-mixity method is designated the *Crack Surface Displacement Extrapolation* method (short CSDE).

Applied in connection with sandwich problems, where the face-core-interface is dominated by a high stiffness difference and large distortions of the crack tip elements, the method has proved to be very robust compared to the mode-mixity methods in the literature.

A numerical routine has been produced to identify the linear transition zone. The routine steps forwards and backwards up the Griffith-energy curve until the borders of the linear transition zone have been found. The linear extrapolation of the transition area performed in the CSDE method is also seen in Figure 4.12.

**Implementation**

Like the VCE and the modified VCCT method, the CSDE method has been implemented in the commercial FE program **ANSYS**.

Below the advantages and disadvantages of the CSDE method are outlined. They are gained through implementation and used for the structural calculations.

**Advantages:**

- It is a very simple method, and there is no altering of or tampering with the finite element results, for example by means of analytical fields.
- The CSDE method is stable in most cases.
- It does not use inaccurately predicted stresses/forces near the crack tip. The Griffith-energy and the mode-mixity do not depend on these stresses/forces and are "over shot" by the method.
- The CSDE method avoids the numerical error zone in bimaterials by "shooting over" results from this zone.
- It is relatively easy to expand to 3-D.

**Disadvantages:**

- In some isolated cases the linear extrapolation of the transition zone is no good fit.
If local crack tip rotations are present, the structural return rotations can be inaccurate (this is the same as in the VCE and VCCT method, and will be explained in section 4.5).

Since the numerical error zone is avoided, and the results are therefore relatively stable even for bimaterials with high stiffness difference, as in sandwich bimaterials, the CSDE method is robust and usable for an automatic crack propagation calculation in most cases for both 2-D and 3-D applications.

### 4.4.4 Semi-Analytical Mode-Mixity Method for Isotropic Bimaterials

To carry out a benchmark test of some of the mode mixity methods presented above, an analytical or semi-analytical method is needed. Such a method has been derived for isotropic bi-materials by Suo and Hutchinson (1990) mainly for applications like thin films on a substrate, but the method is general and can also be applied to other problems like a sandwich beam. The main characteristics of this method are described below.

In Figure 4.13 the cross-section of an infinite bilayer with a crack in the interface is seen. The layers are taken to be homogeneous, isotropic and linear elastic. The uncracked interface is perfectly bonded with continuous displacements and tractions. The three edges of the bilayer are taken to be loaded uniformly with moments and forces per unit width.

![Bimaterial geometry used in the semi-analytical method.](image)

The bilayer may be regarded as a composite beam far ahead from the crack tip. The neutral axis lies a distance $h_1 \Delta$ above the bottom of the beam. $\Delta$ is given as

$$\Delta = \frac{1 + 2\Sigma \eta + \Sigma \eta^2}{2\eta(1 + \Sigma \eta)}$$  \hspace{1cm} (4.43)
where

\[ \Sigma = \frac{1 + \alpha}{1 - \alpha} \quad \eta = \frac{h_1}{h_2} \]  

(4.44)

\( h_1 \) and \( h_2 \) are the thicknesses of the individual bilayers. \( \alpha \) and \( \beta \) are the 1st and 2st Dundur parameters respectively, and for isotropic bimaterials they are given as

\[ \alpha = \frac{G^{(1)}(\kappa_2 + 1) - G^{(2)}(\kappa_1 + 1)}{G^{(1)}(\kappa_2 + 1) + G^{(2)}(\kappa_1 + 1)} \]  

(4.45)

\[ \beta = \frac{G^{(1)}(\kappa_2 - 1) - G^{(2)}(\kappa_1 - 1)}{G^{(1)}(\kappa_2 + 1) + G^{(2)}(\kappa_1 + 1)} \]  

(4.46)

where \( G^{(1)} \) and \( G^{(2)} \) are the shear modulus of material 1 and 2 respectively. \( \kappa_i = 3 - 4\nu_i \) for plane strain and \( \kappa_i = (3 - \nu_i)/(1 + \nu_i) \) for plane stress, with \( \nu_i \) as Poisson’s ratio.

Eq. (A.14) equals Eq. (4.46) for isotropic bimaterials, and furthermore the following couplings exist between \( \alpha \) and \( \beta \):

\[ \beta = \begin{cases} \frac{4}{3} & \text{for } \nu_1 = \nu_2 = \frac{1}{3} \text{ and plane strain} \\ \frac{2}{3} & \text{for } \nu_1 = \nu_2 = \frac{1}{3} \text{ and plane stress} \end{cases} \]  

(4.47)

With the applied loads the composite beam can be considered to be in a state of pure stretch combined with pure bending. The only non-zero stress is \( \sigma_x \), and the corresponding linear strain distribution through the thickness of the beam can then be calculated by use of classical beam theory:

\[ \varepsilon_x = -\frac{1}{E_i} \left( \frac{P_3}{h_1 A} + \frac{M_3}{h_1^2 I_y} \right) \]  

(4.48)

where \( \overline{E_i} = E_i / (1 - \nu_i^2) \) in plane strain and \( \overline{E_i} = E_i \) in plane stress. \( y \) is measured from the neutral axis according to Figure 4.13. The dimensionless cross-section, \( A \), and the moment of inertia, \( I \), are found to be
4.4 Theory of Mode-Mixity Methods

\[ A = \frac{1}{\eta} + \sum I = \sum \left[ \left( \Delta - \frac{1}{\eta} \right)^2 - \left( \Delta - \frac{1}{\eta} \right) + \frac{1}{3} \right] + \frac{\Delta}{\eta} \left( \Delta - \frac{1}{\eta} \right) + \frac{1}{3\eta^3} \] (4.49)

The Griffith-energy and the complex stress intensity factor \( K_1 + iK_2 \) can now be calculated in closed form. (For details about the derivation, see Suo and Hutchinson (1990)).

\[ G = \frac{1}{E_1} \left( \frac{P^2}{h_1} + 12 \frac{M_3^2}{h_1^3} \right) + \frac{1}{E_2} \left( \frac{P^2}{h_2} + 12 \frac{M_2^2}{h_2^3} - \frac{P^2}{Ah_1} - \frac{M_3^2}{Ih_1^3} \right) \] (4.50)

\[ K = h^{-i\varepsilon} \sqrt{\frac{1 - \alpha}{1 - \beta^2}} \left( \frac{P}{\sqrt{2hU}} - ie^{i\gamma} \frac{M}{\sqrt{2h^3V}} \right) e^{i\omega} \] (4.51)

where \( P \) and \( M \) are linear combinations of the edge loads:

\[ P = P_1 - C_1 P_3 - \frac{C_2 M_3}{h_1} \quad M = M_1 - C_3 M_3 \] (4.52)

and \( C_1, C_2 \) and \( C_3 \) are geometrical factors given as

\[ C_1 = \frac{\Sigma A}{A} \quad C_2 = \frac{\Sigma I}{I} \left( \frac{1}{\eta} + \frac{1}{2} - \Delta \right) \quad C_3 = \frac{\Sigma 12I}{12I} \] (4.53)

\( U, V \) and \( \gamma \) are found to be

\[ \frac{1}{U} = 1 + \Sigma \eta \left( 4 + 6\eta + 3\eta^2 \right) \quad \frac{1}{V} = 12 \left( 1 + \Sigma \eta^3 \right) \quad \sin \gamma = \frac{6\Sigma \eta^2 (1 + \eta)}{\sqrt{UV}} \] (4.54)

The remaining parameter, \( \omega \), is an angle and a function of Dundur’s parameters, \( \alpha, \beta \) and the relative height, \( \eta \). Since this angle is only a function of the relative height and material parameters, it can be calculated numerically and tabulated for all crack cases of the type shown in Figure 4.13. \( \omega \) is tabulated in Suo and Hutchinson (1990).

It is important to notice that this method is derived under the assumption that classical beam theory is valid. This means that the method is only valid if sufficiently thin and slender crack flanks are used.
4.5 Structural Return Rotations

In all the mode-mixity methods described above, only deformations directly linked to the opening and shearing of the bimaterial interface crack are considered.

Generally, in a larger structural calculation by use of for example the finite element method, these deformations are only part of the deformations of the entire structure. The translations and rotations of the nodes near the crack tip are coupled with translations and rotations of the complete structure. Only the deformations due to the opening and shearing of the crack tip area are wanted in the mode-mixity methods, because all methods compare the numerically found deformations with the analytically determined displacement fields for a bimaterial interface crack.

The global or structural translations are normally easy to filter out of the numerical results, because the translation of the crack tip node is known.

The global or structural rotations are another matter, as in this case the local and structural rotations are not easy to distinguish. Nevertheless, normally local rotations of the crack tip area should be small, and the structural rotations are easy to filter out, by just using a fixed node pair away from the crack tip area as a measure of the structural rotations of the crack tip area. This procedure is called structural return rotations.

When on the other hand local rotations are large in a geometrical non-linear analysis, results will become inaccurate, both with regard to linear fracture mechanics and the structural return rotations. The effect may, however, be limited, if a fixed node pair for structural rotation determination far away from the crack tip zone is applied. Normally, in practical structural calculations the geometrical non-linearities has an effect on the response of the global structure, whereas the response of the local crack geometry will mostly be restricted by the fracture toughness of the interface crack, thus making LEFM a good estimate even though a non-linear calculation is carried out.

Structural return rotations are needed in all mode-mixity methods when used in practical structural calculations. The effect of structural return rotations, both combined and not combined with local rotations of the crack tip, is seen in Figures 4.14 and 4.15.

In Figure 4.14 the initial crack geometry in the bottom is translated upwards, and at the same time the structure is globally rotated, indicated by the x'-y'-coordinate system, but without any local rotation of the crack tip zone.

In Figure 4.15 the same global translations and rotations are observed, but in this case the global rotation is mixed with a local rotation of the crack tip zone, which makes it not obvious to filter out the global rotations, compared to the case of no local crack tip rotations, unless measures are taken as described above.
4.5 Structural Return Rotations

Figure 4.14: Structural return rotations without local crack tip rotations.

Figure 4.15: Structural return-rotations with local crack tip rotations.
4.6 Comparison of Mode-Mixity Methods

In order to test which mode-mixity method is most suited for fracture problems in typical sandwich materials, the two best performing methods, i.e. the modified VCCT method and the CSDE method presented in the previous chapter, have been tested against each other in two test cases. Furthermore, the two mode-mixity methods are benchmarked against the semi-analytical method by Suo and Hutchinson (1990) in the first test case, where isotropic bimaterials are used.

The VCE method is not included in this comparison, primarily because of the huge CPU consumption when it is used in a commercial finite element code where the matrix operation has to be done in the postprocessor, which makes it unsuited for larger structural calculations, whose model sizes in many cases are considerably larger.

Considerably more tests have been made, but these two test cases have been selected to represent both a comparison with a semi-analytical method and how the individual methods perform on orthotropic/isotropic sandwich-like bimaterials encountered in sandwich vessels.

4.6.1 Prenotched Tension Four-Point Bending Delamination

Figure 4.16: Definition of the prenotched tension four-point bending delamination specimen.
4.6 Comparison of Mode-Mixity Methods

The prenotched tension four-point bending delamination specimen was first proposed by Charalambides (1989) and is shown in Figure 4.16.

When this specimen is used, both the VCCT and CSDE methods can be compared to the semi-analytical method proposed by Suo and Hutchinson (1990) for isotropic bimaterials. This is test no. 1.

In test no. 2 an isotropic bimaterial is chosen with a stiffness difference over the interface comparable to that of a typical sandwich GFRP/PVC bimaterial interface. In this case no analytical method can be used as a benchmark, because the semi-analytical method by Suo and Hutchinson (1990) only supports moderate bimaterials, with \( \alpha \) up to 0.8. In this case, \( \alpha \) is near to 1.0 because of the large stiffness difference between a typical GFRP face and a typical density PVC core, used in the maritime industry.

The parameters used for the specimens in both test 1 and 2 are presented in Table 4.1. It should be noted that for specimens 1A, 1B and 1C, \( \nu_1 \) and \( \nu_2 \) are chosen to be 1/3 in order to exploit the coupling between \( \alpha \) and \( \beta \) in Eq. (4.47). Furthermore, only four-point bending and pure tension load cases have been chosen.

Table 4.1: Series 1 and 2 specimen parameters, \( L = 1000\text{mm} \), \( l = 200\text{mm} \) and \( a = 400\text{mm} \). The load cases refer to FPB=Four-point bending \( (Q_0 = 0) \), T=Pure tension \( (P_0 = 0) \).

<table>
<thead>
<tr>
<th>Test case</th>
<th>Load case</th>
<th>( \alpha )</th>
<th>( \eta )</th>
<th>( E_1/E_2 )</th>
<th>( \nu_1 = \nu_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>FPB</td>
<td>0.8</td>
<td>0.5</td>
<td>9</td>
<td>0.3</td>
</tr>
<tr>
<td>1B</td>
<td>FPB</td>
<td>0.8</td>
<td>0.1</td>
<td>9</td>
<td>0.3</td>
</tr>
<tr>
<td>1C</td>
<td>FPB</td>
<td>0.8</td>
<td>( \approx 0 )</td>
<td>9</td>
<td>0.3</td>
</tr>
<tr>
<td>2A</td>
<td>FPB</td>
<td>( \approx 1 )</td>
<td>0.5</td>
<td>100</td>
<td>0.3</td>
</tr>
<tr>
<td>2B</td>
<td>T</td>
<td>( \approx 1 )</td>
<td>1</td>
<td>100</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Test 1 can briefly be described as

- Three sub-cases (A,B and C)
- Isotropic bimaterials \( (E_1/E_2 = 9) \), with moderate difference in stiffness
- Long crack flanks in subcases 1 and 2 compared to the layer thickness to comply with the classical beam theory
- Can be compared to the semi-analytical model of Suo and Hutchinson (1990)
- Excellent results from both methods
In Table 4.2 the results from the three subcases in test 1 are given. When the VCCT and CSDE methods are directly compared, the differences between the calculated Griffith-energies and the mode-mixities lie from 1.46% – 1.72% and 0.01° – 0.05° respectively, which is negligible.

When the VCCT method is compared to the semi-analytical method, the differences are 0.28% – 2.93% for the Griffith-energy and 0.37° – 0.89° for the mode-mixity, which is also a relatively good result. Finally, when the CSDE method and the semi-analytical method are compared the differences lie between 1.44% – 1.97% for the Griffith-energy and 0.41° – 0.84° for the mode-mixity. Again good agreement.

It should also be noted that in the case of $\eta \approx 0$ the results from the analytical method are uncertain, because the lower crack flank is not long compared to the thickness, as assumed in the theory for the semi-analytical method. The differences in the Griffith-energy of 1.44% for the CSDE method and 2.93% for the VCCT method should be considered with this inconsistency in mind.

Test 2 can briefly be described as

- Two subcases - one four-point bending (A) and one pure tension (B)
- Isotropic ”sandwich-like” bimaterials ($E_1/E_2 = 100$), with high stiffness difference
- Typical GFRP/PVC isotropic parameters
- For values of $\eta < 0.5$, the use of contact elements is required to avoid element overlap of the crack flanks near the crack tip
- Good agreement for bending
- The VCCT method is unstable for the tension case

The results from the first of the two test cases in test 2 are also given in Table 4.2. The loading of this test case is pure four-point bending ($Q_0 = 0$), and the top layer is twice the thickness of the bottom layer. The differences in Griffith-energy and mode-mixity between the VCCT and CSDE methods for this case are 1.74% and 0.00°. Acceptable agreement is achieved for the Griffith-energy and excellent agreement is achieved in this case for the mode-mixity.

The loading in the second case is taken as pure tension ($P_0 = 0$). In this case the differences are 2.90% for the Griffith-energy and 3.34° for the mode-mixity, see Table 4.2. The difference in the Griffith-energy is acceptable, but this is not the case for the mode-mixity, when compared to the accuracy achieved in the other cases in both test 1 and 2. The Griffith-energy and the mode-mixity results have also been plotted in Figures 4.17 and 4.18 respectively as functions of the distance from the crack tip. It is seen that the extrapolation carried out
in the CSDE method works well in both cases, but the VCCT method never or only slowly converges against a constant value for the mode-mixity. The difference of 3.34° is achieved by using the average result of the VCCT method, with the integration length ranging from two node pairs up to 15 node pairs, see Figure 4.10.

As mentioned earlier this non-convergence of the VCCT method is most likely caused by large numerical errors on the first elements next to the crack tip. These errors are carried on and only slowly suppressed by the following node pairs. It also makes sense that these large numerical errors must be more likely to happen, when a large stiffness difference over the interface is present as the elements on the flexible side of the interface close to the crack tip exhibit large distortions. These errors are difficult to quantify and they also depend on the loading situation, as can be seen in these two test cases where it seems that the pure tension case is more critical than the four-point bending load case. Furthermore, mesh convergence analysis has been carried out, but with very little effect because the mesh density is already extremely high, as it is seen from Figures 4.19 and Figure 4.20.

Note in Figure 4.17 how the VCCT method behaves numerically when the integration length becomes small. It is clearly observed that oscillations emerge in the results. The result is shown for specimen 2B, but the same behaviour is seen in all specimens investigated. These oscillations are most likely addressed to numerical errors in the finite element solution, and they emerge when the analytically determined oscillations are used to extract the oscillations from the numerical solution. If the numerical errors are not completely equal, then the oscillations are not totally filtered out of the numerical solution. This effect must furthermore be most dominant for small integration lengths, where the results are only determined on the basis of the first erroneous crack tip elements.

As mentioned earlier, only the results from the two extreme load cases, four-point bending and pure tension load cases, have been presented here. For test 1 similar results are seen for the pure tension load case and for mixed loading as for the pure four-point bending load case. For test 2 the mixed loading cases show results which gradually converge towards the two extreme cases presented here.

Table 4.2: Differences between the VCCT, CSDE and semi-analytical methods (S-A).

<table>
<thead>
<tr>
<th>Test case</th>
<th>VCCT vs. CSDE</th>
<th>VCCT vs. S-A</th>
<th>CSDE vs. S-A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>∆G %</td>
<td>∆ΨK</td>
<td>∆G %</td>
</tr>
<tr>
<td>1A</td>
<td>1.72</td>
<td>0.05°</td>
<td>0.28</td>
</tr>
<tr>
<td>1B</td>
<td>1.46</td>
<td>0.01°</td>
<td>0.25</td>
</tr>
<tr>
<td>1C</td>
<td>1.47</td>
<td>0.04°</td>
<td>2.93</td>
</tr>
<tr>
<td>2A</td>
<td>1.74</td>
<td>0.00°</td>
<td>-</td>
</tr>
<tr>
<td>2B</td>
<td>2.90</td>
<td>3.34°</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 4.17: Griffith-energy, calculated for specimen 2B, by use of the VCCT and CSDE methods, as a function of distance from the crack tip.

Figure 4.18: Mode-mixity, calculated for specimen 2B, by use of the VCCT and CSDE methods, as a function of distance from the crack tip.
Figure 4.19: Finite element model of the prenotched tension four-point bending delamination specimen 1B, showing the deformed geometry with a scale factor of 5.

Figure 4.20: Near tip mesh used in the prenotched tension four-point bending delamination specimen 1B (similar for all specimens). In specimen 1B the smallest element length is approx. 0.0033$h_1$. 

The 2-D finite element model and the near tip mesh density are seen in Figures 4.19 and 4.20. The model consists solely of 8-noded plane strain elements (PLANE82), equally spaced in the near tip zone with 30 element rings. No singular elements have been used as crack tip elements. Linear calculations have been carried out. The boundary conditions are modelled according to Figure 4.16, and symmetry is exploited at the midspan.

### 4.6.2 Face Tearing Debonding

![Face tearing debonding specimen](image)

**Figure 4.21: Face tearing debonding specimen.**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$P_0$ [N/mm]</th>
<th>$a$ [mm]</th>
<th>$L$ [mm]</th>
<th>$t_f$ [mm]</th>
<th>$t_c$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.5</td>
<td>50</td>
<td>1000</td>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>B</td>
<td>0.5</td>
<td>100</td>
<td>1000</td>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>C</td>
<td>0.5</td>
<td>200</td>
<td>1000</td>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>D</td>
<td>0.5</td>
<td>400</td>
<td>1000</td>
<td>5</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 4.3: Parameters for the face tearing debonding specimens.

---

4 Including a transition zone between circular and quadratic element ring shapes, see Figure 4.20
5 Beuth (1996) showed that singular quarter-point elements are not needed in connection with the modified VCCT-method
4.6 Comparison of Mode-Mixity Methods

Table 4.4: Mechanical properties for a typical GFRP laminate.

<table>
<thead>
<tr>
<th>Typical GFRP laminate</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1 \ [MPa]$</td>
<td>14000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{12} \ [MPa]$</td>
<td>2500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_2 \ [MPa]$</td>
<td>8000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{13} \ [MPa]$</td>
<td>5000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_{13}$</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_3 \ [MPa]$</td>
<td>8000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{23} \ [MPa]$</td>
<td>5000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_{123}$</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Mechanical properties for the Divinycell H160 PVC foam material.

<table>
<thead>
<tr>
<th>Divinycell H160</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E \ [MPa]$</td>
<td>230</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G \ [MPa]$</td>
<td>73</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The face tearing specimen, Figure 4.21, has been chosen to show how the VCCT and CSDE methods agree on a real sandwich beam with typical sandwich materials like GFRP faces and PVC foam core. The mechanical properties of the typical face and core materials are found in Tables 4.4 and 4.5.

The results cannot in this case be benchmarked against an analytical method, because an orthotropic material for layer 1 and an isotropic material is used for layer 2. No analytical method is yet available for orthotropic bimaterials.

This test can briefly be described as

- Four specimens of increasing crack length, see Table 4.3
- Simulation of a face tearing mechanism
- Orthotropic-isotropic bimaterials
- Typical GFRP/PVC material parameters
- Again good agreement between the methods

Table 4.6 shows that the differences between the VCCT and CSDE methods are very small for increasing crack length. For the Griffith-energy the differences lie between 0.83% and 0.88% and for the mode-mixity between 0.03° and 0.04°. The agreement between the VCCT and CSDE methods is for this test very good, which is very encouraging for the application to similar sandwich structures.
The finite element model used is in this case quite similar to the one used in the previous section, apart from the boundary condition, which is in this case according to Figure 4.21.

Figure 4.22: Finite element model of the face tearing debonding specimen showing the deformed geometry for $a = 400\text{mm}$.

Table 4.6: Face tearing debonding specimen results.

<table>
<thead>
<tr>
<th>Case</th>
<th>$G_{V\text{CCT}} [J/m^2]$</th>
<th>$G_{\text{CSDE}} [J/m^2]$</th>
<th>$\Delta G %$</th>
<th>$\Psi_{K,V\text{CCT}}$</th>
<th>$\Psi_{K,\text{CSDE}}$</th>
<th>$\Delta \Psi_K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3.260</td>
<td>3.233</td>
<td>0.83</td>
<td>$-16.78^\circ$</td>
<td>$-16.82^\circ$</td>
<td>0.04$^\circ$</td>
</tr>
<tr>
<td>B</td>
<td>10.513</td>
<td>10.424</td>
<td>0.85</td>
<td>$-17.93^\circ$</td>
<td>$-17.97^\circ$</td>
<td>0.04$^\circ$</td>
</tr>
<tr>
<td>C</td>
<td>37.503</td>
<td>37.182</td>
<td>0.86</td>
<td>$-18.07^\circ$</td>
<td>$-18.10^\circ$</td>
<td>0.03$^\circ$</td>
</tr>
<tr>
<td>D</td>
<td>141.915</td>
<td>140.682</td>
<td>0.88</td>
<td>$-17.05^\circ$</td>
<td>$-17.09^\circ$</td>
<td>0.04$^\circ$</td>
</tr>
</tbody>
</table>
4.6.3 Conclusion on the Mode-Mixity Method Investigation

Four mode-mixity methods have been investigated for structural use and three of these methods have been implemented in the commercial finite element code, ANSYS. The methods include a new mode-mixity method (CSDE) proposed by the author based on a modification of the traditional CSD-method.

The VCE method proved to be unsuited for practical use in a commercial finite element code, where the matrix multiplication demanded by the VCE-method is done in the postprocessor. Nevertheless, the method is very elegant and will be very powerful in a specialised finite element code, where it can be integrated into the source code.

The CSDE and VCCT methods have been compared for bimaterials, with moderate and large stiffness difference and benchmarked against a semi-analytical mode-mixity method by Suo and Hutchinson (1990). Excellent agreement between the VCCT, CSDE and semi-analytical methods is observed in almost all cases. Only in one case of a bimaterial with very high stiffness difference and pure tension loading, the VCCT method proved unstable because the errors were relatively high on the first element near the crack tip. Still, errors were less than 5%.

Both the VCCT and CSDE methods can be used in most cases in practical structural finite element calculations where information about crack propagation in a bimaterial interface is needed. However, when the mode-mixity method is used in an automatic calculation procedure, where manual monitoring of the results during the calculation is not possible, the VCCT method proves to be unsuited, because it is unstable for certain load cases for high stiffness difference in the bimaterial, and because of the arccos term in Eq. (4.41), where the correct root has to be chosen manually from case to case.

The CSDE method has proved to be very stable and robust in all test cases and has therefore been chosen as the most optimal for structural use in an automatic crack propagation model in a commercial finite element code.
4.7 Mode-Mixity Determination in 3-D

Figure 4.23: Crack front definitions, (a) in a straight crack front geometry and (b) in curved crack front.

In Figure 4.23 the crack front is shown schematically for a straight and a curved crack front geometry. For both crack front geometries an additional crack tip movement is included which is out-of-plane shearing of the crack tip, see Figure 4.3.

First it should be noted that the 3-D method is only aimed at predicting crack initiation, i.e. the evolving crack geometry is not predicted.

In the previous sections the CSDE method was presented, and it was chosen to be the most suited mode-mixity method for structural use. The CSDE method has up to now been presented in 2D, but in this section a 3-D version of the CSDE method will be presented and verified.

The accuracy of the CSDE method is in general dependent on how far the extrapolation zone is extended from the crack tip. If the CSDE method is to achieve high linearity in the extrapolation, it is necessary that the high mesh density zone is located so close to the crack tip that the structural outer displacement field has only a small influence on the Griffith-energy and the mode-mixity, determined from the relative crack flank displacements.

In 2-D, the effect on the CPU time consumption of a high mesh density close to the crack is only limited. But as the 2-D model is extended into 3-D, the influence of the high mesh density is increased considerably. Thus, the mesh density close to the crack tip has to be chosen to be much lower than in 2-D. The natural consequence of this choice is that non-linearity in the extrapolation zone to a certain extent is encountered.

To overcome these non-linear effects, the following actions can be taken:
4.7 Mode-Mixity Determination in 3-D

- The linear extrapolation can be carried out, accepting a certain error on both Griffith-energy and mode-mixity.

- The extrapolation procedure can be extended, so that the extrapolation is done by non-linear curve fitting of the zone outside the numerical error zone close to the crack tip.

- A path independent J-integral can be used along the crack front to determine the Griffith-energy. The mode-mixity is determined in the same way as in 2-D accepting a certain non-linearity in the extrapolation.

The first action is the easiest way of overcoming the problems with non-linearity, but unfortunately this approach can give quite erroneously results if the non-linearity in the extrapolation zone is strong. This is most critical for the Griffith-energy, because this parameter directly gives the magnitude of how much the crack tip is loaded. The effect is somewhat less critical for the mode-mixity, because it is only used to compare the Griffith-energy value with the fracture toughness. If the mode-mixity is close to zero, the fracture toughness will only vary little, and the effect of non-linearity in the extrapolation zone will be limited.

The second action can solve these problems, but the procedure demands a considerable implementation effort, and it is doubtful if the procedure is robust enough to be used for practical applications.

The third action demands implementation of a numerical procedure for calculation of a path independent J-integral for each point along the crack front.

This third procedure has been chosen and adopted for the CSDE method as a 3-D variant. However, in order to achieve usable results at a structural level in 3-D with acceptable calculation accuracy, certain simplifications have to be made.

First it has been chosen to neglect the mode III deformations and use the 2-D version of the expressions for the Griffith-energy and the mode-mixity as a function of the crack flank displacements, Eqs. (4.15) and (4.16), as well as the 2-D version of the path independent J-integral. This limitation can be justified, because the mode III contribution will be limited in most debond cases compared to the opening mode I part of the Griffith-energy. Furthermore, the fracture toughness is only determined experimentally for mode-mixity including mode I and II deformations.

Second, as most laminate types used in this thesis are quasi-isotropic naval applications, no further action has been taken to adjust the mechanical properties of the face laminate parallel to the investigated crack plane along the crack front. However, if a more orthotropic face laminate is applied, the face laminate properties have to be adjusted and possibly also compared to separate fracture toughness data for each bimaterial type, depending on the level of orthotrophy.
Nevertheless, both the 2-D J-integral and the 2-D expressions for the Griffith-energy and the mode-mixity have to be verified for use in 3-D. In order to verify the 3-D version of the CSDE method, two numerical verification examples have been chosen:

**CASE 1:** Face tearing of a debonded sandwich beam, modelled in 2-D and 3-D

**CASE 2:** Sun blister debond in a sandwich panel, modelled in 2-D axisymmetry and 3-D

The first case is chosen to investigate, by application of the already verified 2-D version of the CSDE method, if the 3-D version is able to capture the shift from plane stress on the sides of the beam to the plane strain state at the middle plane of the beam, when Griffith-energy and mode-mixity are evaluated at longitudinal cuts made through the beam width.

The second case is chosen in order to investigate if the 3-D CSDE- method can predict the Griffith-energy and the mode-mixity along a circular crack front, when plane strain state is assumed. The 3-D result is compared with a 2-D model using axisymmetry and the 2-D version of the CSDE method.

### 4.7.1 CASE 1: Face Tearing of a Debonded Sandwich Beam

Figure 4.24 shows the face tearing beam specimen, and the geometrical and mechanical properties are found in Table 4.7. The materials used are isotropic for both face and core. The face properties are similar to a quasi-isotropic quadro-axial Kevlar laminate and the core is chosen to be an isotropic Divinycell H100 PVC foam.

The specimen is similar to the one used in connection with the verification and the comparison of the 2-D VCCT and CSDE methods, except that in this case the specimen is loaded with a vertical point unit displacement load at the upper face at the beam midspan. The beam ends are clamped, and the lower face is restricted from translations in all directions over a horizontal distance of \(2c = 80\) mm at the beam midspan, see Figure 4.24.

A 2-D and 3-D model have been produced. The two models are seen in Figures 4.26 and 4.27 respectively. Both are 1/2-models, with a symmetry condition imposed at the beam midspan.

The 2-D model consists of isoparametric 8-noded plane elements, except in a region extending three elements from the crack tip, where the element type is linear 4-noded elements, see Figure 4.25. The 4-noded elements are chosen because large deformations are expected very near to the crack tip, and the linear elements have shown a higher tolerance against these deformations than the parabolic 8-noded elements. The linear element choice has no influence on the Griffith-energy and mode-mixity calculation by the CSDE method, since the method “shoots” over the results from these elements. Outside the inner region with linear elements the mesh has a high density of parabolic elements in order to capture the Griffith-energy and the mode-mixity through the relative crack flank displacements. Furthermore, both plane strain and stress models have been used in 2-D.
The 3-D model consists of parabolic 20-noded solid elements, and just like the 2-D model, the inner region consists of linear elements. Compared with the 2-D model, the high density region near the crack tip, including both the linear element region and the high density parabolic element mesh region, is much larger in the 3-D model, as the number of elements in the region is the same in both models. The mesh density is therefore much larger in the 2-D model compared to the 3-D model. Through the width of the beam five evaluation “cuts” have been prepared. The Griffith-energy and the mode-mixity are evaluated in these positions.

The goal of this investigation is to show that by comparing Griffith-energy and mode-mixity from the relatively precise result from the high near tip mesh density 2-D model to the low
near tip mesh density 3-D model, but at the same time by use of the 3-D version of the CSDE method as described in the last section, comparable results can be achieved.

A direct comparison between the plane strain/stress 2-D models and the 3-D model can be achieved on the beam sides of the 3-D model, where pure plane stress is expected to be present, and at the mid-width of the beam, where near plane strain conditions are expected.

In Figures 4.28 and 4.29 the Griffith-energy and the mode-mixity are shown for the 3-D model, where the Griffith-energy and the mode-mixity are evaluated using both the J-integral and nodal extrapolation in seven positions through the beam breadth, and the 2-D model in plane stress and strain. Two points are on the sides of the beam and the other five are equidistantly placed over the beam width.

First it is seen that the Griffith-energy increases as the stress state changes from pure plane stress to something very near to plane strain. When the results from the 2-D and 3-D models using the 2-D and 3-D versions (J-integral) of the CSDE method, respectively, are compared, it is seen that the agreement between the two models is good for the pure plane strain with a difference of 1.4%. In plane stress at the beam sides the Griffith-energy predictions from the 3-D model using the J-integral based CSDE method yields results which are not far from the precise predictions from 2-D model, but an increase in Griffith-energy can be observed near the beam side. The same variation is seen at both beam sides 9.1%, which can be expected because of symmetry. The Griffith-energy calculated for the 3-D model has also been carried out by use of nodal extrapolation, and these results predict a decreasing Griffith-energy going
4.7 Mode-Mixity Determination in 3-D

Figure 4.26: CASE 1 2-D model.

Figure 4.27: CASE 1 3-D model.
towards the beam ends, but with a considerable higher error compared to the 2-D model. However, it seems that the J-integral predictions are more doubtful near free surfaces, which could be explained by the fact that the mode III deformations are neglected. But as these free edges are not present in most of the main applications of the 3-D model (panels), these deviations are of less concern.

With regard to the mode-mixity a difference of 0.3° and 0.19° are seen at the beam midplane and sides respectively. In the 3-D version of the CSDE method only plane strain is assumed at the nodal extrapolation when using the 2-D expressions, and for that reason an error is deliberately introduced on the beam sides, where something near to pure plane stress conditions are dominating. In Figure 4.29, the results from using plane stress conditions in 3-D can also be seen. Surprisingly, it can be observed that a considerable larger deviation is present. Thus, using plane strain conditions in 3-D, seems to give acceptable results in all cases. However, as mentioned above, this free edge condition is though highly unlikely in debonded sandwich panels, where the 3-D CSDE method will be applied.

The conclusion covering CASE 1 is therefore that for both Griffith-energy and mode-mixity the differences are highly acceptable, assuming that no significant mode III deformations are present.

Figure 4.28: The Griffith-energy calculated using both the J-integral and nodal extrapolation in the seven evaluation positions by use of the 3-D model and compared with the 2-D model in plane stress and strain.
4.7 Mode-Mixity Determination in 3-D

4.7.2 CASE 2: Sun Blister Debond in a Sandwich Panel

A typical damage to sandwich vessels is sun blisters. They typically occur as a consequence of a debond damage. Heat from the sun exposure of the sandwich structure accelerates the processes that produce degassing from the core and into the debond. A gas pressure is thus building up inside the debond, which makes it open and in some cases also grow.

In Figure 4.30 the geometry of the panel specimen is shown. Geometrical and mechanical properties are seen in Table 4.8. The materials used are the same as in CASE 1.

As for CASE 1, 2-D and 3-D models were produced for this test case. The two models are seen in Figures 4.31 and 4.32 respectively. In 3-D a 1/4-model is used, with symmetry conditions imposed. The 2-D axisymmetric model is a 1/2-model, with symmetry conditions imposed on the beam midspan. Furthermore, a beam length is chosen to be comparable to the diagonal of the panel, which is 706 mm.

The loading in both cases consists of a unit uniform opening pressure load on the crack flanks. For the 3-D model the edges are clamped, and for the 2-D axisymmetric model the beam end is also clamped.

With regard to the element types in both cases, the same choices have been made as in CASE 1.
Figure 4.30: Geometry of a sandwich panel with a sun blister defect.

Table 4.8: Geometrical and mechanical properties for CASE 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel length</td>
<td>A</td>
<td>500 mm</td>
</tr>
<tr>
<td>Panel breadth</td>
<td>B</td>
<td>500 mm</td>
</tr>
<tr>
<td>Debond diameter</td>
<td>D</td>
<td>40 mm</td>
</tr>
<tr>
<td>Face thickness</td>
<td>$t_f$</td>
<td>3.6 mm</td>
</tr>
<tr>
<td>Core thickness</td>
<td>$t_c$</td>
<td>50 mm</td>
</tr>
<tr>
<td>Face Young’s modulus</td>
<td>$E_f$</td>
<td>21000 MPa</td>
</tr>
<tr>
<td>Face G-modulus</td>
<td>$G_f$</td>
<td>8154 MPa</td>
</tr>
<tr>
<td>Face Poisson’s ratio</td>
<td>$\nu_f$</td>
<td>0.3</td>
</tr>
<tr>
<td>Core Young’s modulus</td>
<td>$E_c$</td>
<td>105 MPa</td>
</tr>
<tr>
<td>Core G-modulus</td>
<td>$G_c$</td>
<td>40 MPa</td>
</tr>
<tr>
<td>Core Poisson’s ratio</td>
<td>$\nu_c$</td>
<td>0.3125</td>
</tr>
</tbody>
</table>
4.7 Mode-Mixity Determination in 3-D

Figure 4.31: CASE 2 2-D axisymmetric model.

Figure 4.32: CASE 2 3-D model.
Chapter 4. Fracture Mechanics in Sandwich Structures

The goal of this investigation is to examine how well the Griffith-energy and the mode-mixity can be determined along a crack front in 3-D. The debond geometry and loading chosen dictate that approximately the same Griffith-energy and mode-mixity should be calculated along the crack front in the 3-D model. Furthermore, a test case is chosen, which can be modelled in both 3-D and in 2-D axisymmetry, as well as a debond diameter so that the effect of the panel edges is negligible. As in CASE 1, the mesh density can be chosen to be much higher in the 2-D model than in the 3-D model. The effect of this mesh density reduction can also be studied in this analysis.

In Figures 4.33 and 4.34 the Griffith-energy and the mode-mixity are shown in a polar diagram on the basis of calculations with both the 3-D and 2-D axisymmetric models. The Griffith-energy or the mode-mixity can be read in the radial direction and the crack front position in the circumferential direction. The results from the 2-D axisymmetric model have furthermore been plotted with the same value in all crack front positions, and likewise for the 3-D 1/4-model where the results have been copied to the other four quadrants.

When the two models are compared with regard to the Griffith-energy, the overall conclusion is that only small differences are observed. In Figure 4.33 the Griffith-energy has been determined on the basis of the J-integral and nodal displacements of the crack flanks. No significant differences are seen in the 2-D model and the difference is 3.0%. In the 3-D model a more fluctuating result is seen for the Griffith-energy determined from the nodal crack flank displacements, whereas the Griffith-energy determined from the J-integral gives approximately the same value for all crack front positions as expected. The differences all lie inside a window of 2.4% – 8.9%, but the J-integral shows a much more steady and reliable result and will subsequently be used in the 3-D version of the CSDE method.

With regard to the mode-mixity, there is almost complete agreement between the 2-D axisymmetric and the 3-D model for all crack front positions, except in the crack front positions where a symmetry condition is present. It is believed that the symmetry condition introduces some disturbances in the nodal crack flank displacement in these positions. However, the errors are below 2° and therefore fairly small and without practical significance for structural use.

The overall conclusion on CASES 1 and 2 is that a reliable 3-D version of the CSDE method has been presented and verified for use in structural calculations.
4.7 Mode-Mixity Determination in 3-D

Figure 4.33: Polar diagram showing the calculated Griffith-energy in each crack front position by use of the 3-D model and compared with the 2-D axisymmetric model. (Note the axis)

Figure 4.34: Polar diagram showing the calculated mode-mixity in degrees in each crack front position by use of the 3-D model and compared with the 2-D axisymmetric model.
4.8 Kinking

It is often seen in sandwich vessels that an interface debond between face and core propagates. The crack may either continue to propagate in the interface or it may continue at an angle into the core. This phenomenon is called kinking.

Automatic mesh routines in 2-D have been made to account for any kinked crack geometry. Kinking from the interface into the core can be treated in two ways: Semi-analytical or numerical, and both procedures will be presented below.

![Kinked crack geometry.](image)

4.8.1 Semi-Analytical Approach

He and Hutchinson (1989) and He et al. (1991) presented semi-analytical approaches to the kinking problem for isotropic bimaterials. This approach was furthermore expanded to anisotropic bimaterials by Wang et al. (1992) and Wang (1994).

The Griffith-energy at a presumed kinked crack into the core, see Figure 4.35, is maximised with respect to the kinking angle, $\Omega$, through the complex stress intensity factors at the kinked crack tip by use of Eq. (4.9):

\[
K_{\text{tip}} = K_{I}^{\text{tip}} + i K_{II}^{\text{tip}} = c(\Omega, \alpha, \beta) K_{\text{int}} a^{i\varepsilon} + d(\Omega, \alpha, \beta) K_{\text{int}} a^{-i\varepsilon} + b(\Omega, \alpha, \beta) T \sqrt{a} \quad (4.55)
\]

$b$, $c$ and $d$ are complex material functions, $K_{\text{int}}$ is the complex stress intensity factor for the interface crack and $T$ is the non-singular part of $\sigma_{11}$, which acts on the parent crack tip parallel to the crack prior to kinking and has to be determined numerically. The kink angle is then found where the Griffith-energy is maximum.

By this procedure, $K_{\text{tip}}$ and $G_{\text{tip}}$ for the kinked crack and the kink angle, $\Omega$, are estimated for a presumed kinked crack of length $a$ into the core.
4.8.2 Numerical Approach

In the numerical approach, the Griffith-energy of the kinked crack is simply calculated for a number of kinked angles into the core.

The Griffith-energy can easily be found for the kinked crack, because in this case the crack tip lies in an isotropic homogeneous material. The J-integral, or crack flank nodal displacements can be used. Both procedures are normally implemented in modern finite element codes as trivial functions.

The Griffith-energy results from each kinked crack are then approximated to a curve, and the maximum is found at the presumed kink angle.

User routines have been produced, so that ten kinking angles to or from 45° are investigated. The routine continues until a maximum Griffith-energy has been found and the angle is then found by approximation of the last three Griffith-energies with a second degree polynomial. Furthermore, automatic mesh routines have been produced in order to investigate the ten kinking angles, see Figure 4.36.
4.8.3 Kinking Criterion

Hutchinson and Suo (1992) presented the following criteria for kinking out of the interface and into the core:

\[ \frac{G(\Psi_K)}{G_{\text{tip}}} < \frac{\Gamma(\Psi_K)}{G_{\text{core}}} \]

(4.56)

where \( \frac{G(\Psi_K)}{G_{\text{tip}}} \) is the ratio between the Griffith-energy at the crack tip and the maximised Griffith-energy for the kink angle, found by either the analytical or the numerical approach. \( \frac{\Gamma(\Psi_K)}{G_{\text{core}}} \) is the ratio between the interface fracture toughness and the mode I fracture toughness of the core material.

4.8.4 Kinking Conclusions

In order to choose whether the semi-analytical or the numerical procedure should be used, positive and negative factors concerning the two approaches have to be compared:

Semi-analytical approach:

- In the finite element solution the non-singular stresses are difficult to quantify, because the non-singular part of the normal stress parallel to the interface is difficult to filter out and identify, especially in an automatic crack propagation simulation.
- The complex material functions are not available for "sandwich-like" bimaterials and have to be produced numerically for the specific bimaterial, which has proved to be difficult in practice.

Numerical approach:

- Easy to use and control, and the material functions, \( c(\Omega, \alpha, \beta), d(\Omega, \alpha, \beta) \) and \( \bar{d}(\Omega, \alpha, \beta) \), are not needed as in the semi-analytical method.
- A number of subcalculations have to be made (equal to the number of kink angles), which means increased CPU time.
- The accuracy of the kinking angle depends on the number of kink angles which are used in the numerical routine.
4.9 Fracture Toughness

The numerical approach has been chosen and is implemented in the finite element procedure. The procedure is used for each crack position in 2-D models to calculate a possible kink angle. In 3-D the application is more difficult, because a complete crack front will have to be kinked into the core. Furthermore, this approach will result in a massive CPU consumption, because of the subcalculations for each kink angle into the core. For these reasons, kinking will only be investigated in 2-D.

The question whether a crack will continue to propagate in the interface or kink into the core, will be treated in the next chapter.

4.9 Fracture Toughness

4.9.1 General Background

In Eq. (4.3) the fracture toughness was on the basis of the Griffith energy balance, defined as the energy required to make new surfaces. By extending Griffith’s hypothesis, which describes a quasi-static crack propagation, the energy criterion for fracture can be reformulated in the following way (Kinloch (1987)):

$$\frac{\partial (W_d - U)}{\partial a} \geq \gamma_m \frac{\partial A}{\partial a}$$

(4.57)

where $\partial A$ is the increase in surface area associated with an increment of crack growth $\partial a$. $W_d$ and $U$ are the work done by the external forces and the available elastic energy stored in the bulk of the specimen respectively. Finally $\gamma_m$ is the surface free energy.

If the criterion is rewritten for a crack propagating in a 2-D lamina of thickness $b$, the criterion becomes

$$\frac{1}{b} \frac{\partial (W_d - U)}{\partial a} \geq 2\gamma_m$$

(4.58)

This criterion implies that only twice the surface energy is required to cause crack growth. As early as in the papers by Orowan (1948) and Rivlin and Thomas (1953) the criterion was investigated with respect to metals, cross-linked rubbers and polymers, and they found that the energy needed to make the crack propagate was far greater than twice the surface free energy. The reason is mainly that the surface free energy only accounts for the energy needed to rupture the secondary bonds such as the van der Waals forces and does not take into account the primary bonds, such as bonds between interfaces in the material at a
microscopic level. Secondly, localised visco-elastic and/or plastic energy dissipative processes around the crack tip will be present for even the most brittle adhesives, like phenolic resins, but of course the distribution between the different ”energy consumers” at the crack tip will be different between a brittle PVC foam material and a highly visco-elastic material, like a rubber modified structural adhesive.

However, as described in Kinloch (1987), if it is assumed that energy dissipation around the crack tip occurs in a manner independent of both the test geometry and the way in which the forces are applied to the specimen, then the term $2\gamma_m$ may be replaced by $G_c$ designated the fracture toughness. By this definition $G_c$ includes all energy losses in the crack tip area and is therefore the energy required to increase the crack by a unit length on a specimen of unit width. Thus, Eq. (4.58) can be rewritten as

$$\frac{1}{b} \frac{\partial (W_d - U)}{\partial a} \geq G_c$$

(4.59)

Furthermore, if the laminas exhibit bulk linear-elastic behaviour, they will obey Hooke’s law and Eq. (4.59) can be expressed as

$$G_c = \frac{F_c^2}{2b} \frac{\partial C}{\partial a}$$

(4.60)

where $F_c$ is the load at which crack propagation is initiated and $C$ is the compliance of the structure, which can be determined analytically or numerically as a function of the crack length. Eq. (4.60) is the foundation of many calculations of the fracture toughness $G_c$, since by determining the compliance of the structure, $G_c$ can be found by just determining the force at the onset of crack propagation.

In Figure 4.37 and Table 4.9 the fracture toughness for cross-linked Divinycell H-series PVC foam is shown. The results have been produced by Viana and Carlsson (2002a) and represent the latest results reported in the literature.

Table 4.9: $G_{lc}$-values for Divinycell H-series as reported by Viana and Carlsson (2002a).

<table>
<thead>
<tr>
<th>Density [kg/m$^3$]</th>
<th>H30</th>
<th>H80</th>
<th>H100</th>
<th>H200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{lc}$ [J/m$^2$]</td>
<td>112 ± 17</td>
<td>186 ± 39</td>
<td>309 ± 13</td>
<td>625 ± 100</td>
</tr>
</tbody>
</table>
4.9 Fracture Toughness

4.9.2 Interfacial Fracture Toughness

As mentioned earlier in the beginning of the section about bimaterial fracture mechanics, an interface crack is restricted to propagating along or directly in an interface. It is thus forced to propagate at mode-mixities different from mode I, which is the case for cracks in homogeneous materials.

A direct consequence of this propagation behaviour is that the fracture toughness depends on the mode-mixity at the crack tip, because mechanisms like asperity contact of the fracture surfaces and crack tip plasticity will increase for increasing mode II in the mode-mixity. The amount of contact of the crack flanks at some distance behind the crack tip and the mode-mixities for which this occurs have been presented by Rice (1988) and can be deducted from Eq. (4.10). With regard to the crack tip plasticity, the plastic zone increases in size as the mode-mixity increases, with $G$ held fixed as shown by Shih and Asaro (1988). Furthermore as described earlier, a high mode-mixity tends to make the crack kink away from the interface on a microscopic scale, which makes the asperity even larger and therefore increases the contact forces and restricts the propagation of the crack.

It is common for all mechanisms described above that they make the fracture toughness increase when the mode-mixity increases.

In the bimaterial fracture mechanics section earlier in this chapter, it was shown that the mode-mixity definition in Eq. (4.13) depends on the length scale, $h$. As mentioned earlier,
the length scale can be chosen arbitrarily, but it is normally chosen as the face thickness, when the mode-mixity is determined for sandwich interfaces. Changing the length scale from \( l_1 \) to \( l_2 \) will result in the following phase change in the mode-mixity:

\[
\Psi_2 = \Psi_1 + \varepsilon \ln \left( \frac{l_2}{l_1} \right)
\]

(4.61)

but the length scale may be changed several orders of magnitude without noticeable changes in the mode-mixity, considered within the frame of accuracy also seen in the determination of fracture toughness for typical sandwich interfaces, for instance for GFRP/PVC foam core.

As an example it should be noted that by application of the material properties described later in this report the phase change is only about 4°, when the length scale is changed from 3.2 mm to 10 mm. As it will be revealed later, the fracture toughness is nearly constant for the mode-mixities encountered in the applications treated in this report. The phase change will therefore have negligible influence on the results, and the fracture toughness will therefore be treated as independent of the length scale in practical sandwich structure applications.

Furthermore, in order to distinguish between the fracture toughness of homogeneous materials and the mode-mixity dependent fracture toughness encountered in bimaterial interface crack problems, the fracture toughness for the latter will be designated by \( \Gamma (\psi) \).

As described by Hutchinson and Suo (1992) efforts have been made in recent years to measure the interfacial fracture toughness, beginning with Trantina (1972) and Anderson et al. (1974) and carried on by a relatively high number of studies on different bimaterials with the emphasis on developing specimens to be used for mixed-mode fracture toughness determination. Among these are Cao and Evans (1989), Wang and Suo (1990) and Liechti and Chai (1991), who all studied interfaces in epoxy/glass bimaterials, and Thouless (1990), who carried out experiments on brittle wax and glass interfaces. Furthermore, Kinloch (1987) presented mixed-mode fracture specimens designed to measure delamination toughness associated with ply separation in polymer matrix composites.

In the last 15 years special attention has been paid to the development of mixed-mode specimens for fracture toughness determination of debonded sandwich structures. Li (2000) reviewed these specimens and presented the tilted sandwich debond specimen. All specimens are seen in Figure 4.38.
4.9 Fracture Toughness

Figure 4.38: Mixed-mode sandwich specimens. CSB (a), DCB (b), SCS (c), ELSS (d), TPBS (e) and TSD (f).

The presented mixed-mode specimens in Figure 4.38 from the literature:

- The Cracked Sandwich Beam (CSB) specimen, Carlsson et al. (1991)
- The Double Cantilever Beam (DCB) specimen, Prasad and Carlsson (1994a) and Prasad and Carlsson (1994b)
- The Single Cantilever Sandwich (SCS) specimen
- The End-Loaded Sandwich Structure (ELSS) specimen
- The Three Point Bending Sandwich (TPBS) specimen, Cantwell et al. (1997)
- The Tilted Sandwich Debond (TSD) specimen by Li and Carlsson (1999), analysed in Li (2000), Li and Carlsson (2001) and Viana and Carlsson (2002b)

According to Li (2000), most of the above shown specimens perform satisfactorily for some face and core combination, but they are also connected with practical and analytical problems, among others sizing difficulties (CSB and TPBS), crack kinking (DCB) and geometrical non-linearity (SCS, ELSS and TPBS). The tilted sandwich debond specimen has none of these problems, but seems to have a quite narrow range of possible mode-mixities when the tilt angle is changed, see Figure 4.38, however, peeling has been reported to minimal.
In connection with the saNDI-project, RISØ National Laboratory’s task was to measure the interfacial fracture toughness for typical sandwich interfaces used in practical applications. The fracture toughnesses should furthermore be measured for the largest possible mode-mixity range. RISØ National Laboratory produced a new mixed-mode specimen and a new test rig to be used for this purpose. The specimen and the test rig are shown in Figure 4.39. The main foundation of this new mixed-mode specimen is an expanded version of the semi-analytical method, presented in section 4.4.4, to a sandwich specimen. Furthermore, only moments are added to the beam ends by the test rig, and by changing the magnitude of these moments by changing the distance between the wire rollers as indicated on Figure 4.39b, the mode-mixity can actually be varied to span the complete mode-mixity spectrum. The mixed-mode specimen and the modified semi-analytical method will be presented in Østergaard and Sørensen (2004).

Fracture toughness distributions as a function of mode-mixity (with the face thickness as length scale) are presented by the author on the basis of the fracture toughness results produced by RISØ National Laboratory. These distributions are seen in Figures 4.40, 4.41 and 4.42 for three different interface types, and the expressions are found in Table 4.10.

Table 4.10: Sandwich interface types used in fracture toughness tests.

<table>
<thead>
<tr>
<th>Type</th>
<th>Face fibre</th>
<th>Face layup</th>
<th>Face resin</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPE 1</td>
<td>GFRP</td>
<td>CSM + Quadro-axial + CSM</td>
<td>Polyester</td>
<td>Divinycell H80</td>
</tr>
<tr>
<td>TYPE 2</td>
<td>GFRP</td>
<td>CSM + Quadro-axial + CSM</td>
<td>Polyester</td>
<td>Divinycell H130</td>
</tr>
<tr>
<td>TYPE 3</td>
<td>GFRP</td>
<td>CSM + Quadro-axial + CSM</td>
<td>Polyester</td>
<td>Divinycell H200</td>
</tr>
</tbody>
</table>

As shown in Figures 4.40, 4.41 and 4.42, the experimentally determined fracture toughness values are to some extent scattered. Therefore, the values have been ordered in a band, with an assumed upper and lower limit distribution, and in between these borders assuming an average distribution, which will predominantly be used in the structural applications.

For the H80 and H130 specimens fracture toughness data points exists, for which the crack has propagated just below the interface on the core side. Additionally, data points also exists which represent fracture values where fibre bridging is observed to some extent on the fracture surfaces. As it is seen, fibre bridging only occurs for some specimens for mode-mixities over approximately $-30^\circ$. It is clear that considerably higher fracture toughness values are measured for specimens with fibre bridging, which could also be expected, because the bridged fibre restrains the crack from opening and shearing. For the H200 specimens fibre bridging is observed in all specimens.

For all core densities mode-mixities range up to approximately $0^\circ$, with a large nearly constant fracture toughness level from about $-30^\circ$ up to $0^\circ$. Beyond this limit and up into

---

6The distribution expressions are variants of the ones presented by Hutchinson and Suo (1992).
the positive mode-mixity area the crack kinks into the core. Furthermore, it is observed in Figure 4.42 for the H200 specimen that a group of data points is located just on the positive side of 0° mode-mixity with an increased fracture toughness. It is believed these increased fracture toughness values can be addressed to a shift in fracture mechanism, as the crack propagation changes from interface propagation to kinking. As a consequence, these values are neglected in the practical application of the measured fracture toughness distributions.

In the fracture toughness test performed by RISØ National Laboratory the face thickness was several orders of magnitude greater than used anywhere else in the structural specimens, as described later in this thesis. Because the face consists of both quadro-axial mats with different thicknesses between fracture mechanics and structural specimens and chopped strand mats (CSM) of nearly equal thickness, the $\beta$, $\lambda$ and $\rho$ parameters for the bimaterial interface will vary slightly, but throughout the implementation of these test results in structural applications, these small variations will be neglected.

In order to apply the measured fracture toughness values in practice in connection with a finite element calculation using one of the mode-mixity methods described earlier in this chapter, fracture toughness distributions as a function of mode-mixity have to be given. As mentioned earlier both an upper, a lower and an average distribution have been used to estimate the test values. Furthermore, an average distribution for the fracture toughness values connected with fibre bridging will also be given for the H80 and H130 specimens.

The distributions have been chosen to be similar to the fracture toughness expressions presented in Hutchinson and Suo (1992), but some modifications have been implemented in order to make the expressions fit the measured values. These expressions are given in Table 4.11.
Figure 4.39: Mixed-mode sandwich specimen and test rig presented in Østergaard and Sørensen (2004). Schematic presentation of test rig (a) showing a specimen with nearly pure mode II loading (b). Photo courtesy of RISO National Laboratory.
4.9 Fracture Toughness

Figure 4.40: Interfacial fracture toughness for a GFRP/H80 PVC foam core bimaterial.

Figure 4.41: Interfacial fracture toughness for a GFRP/H130 PVC foam core bimaterial.
Figure 4.42: Interfacial fracture toughness for a GFRP/H200 PVC foam core bimaterial.

Table 4.11: Fracture toughness distributions.

<table>
<thead>
<tr>
<th>$\Gamma(\psi)$</th>
<th>TYPE 1</th>
<th>TYPE 2</th>
<th>TYPE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No fibre bridging</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UPPER $(G_{lc}[J/m^2], k)$</td>
<td>$G_{lc}((1 - k)\tan^2\Psi + 1)$ $(350, 0.8)$</td>
<td>$G_{lc}((1 - k)\tan^4\Psi + 1)$ $(550, 0.2)$</td>
<td>-</td>
</tr>
<tr>
<td>AVERAGE $(G_{lc}[J/m^2], k)$</td>
<td>$G_{lc}((1 - k)\tan^2\Psi + 1)$ $(310, 0.95)$</td>
<td>$G_{lc}((1 - k)\tan^4\Psi + 1)$ $(400, 0.2)$</td>
<td>-</td>
</tr>
<tr>
<td>LOWER $(G_{lc}[J/m^2], k)$</td>
<td>$G_{lc}((1 - k)\tan^2\Psi + 1)$ $(270, 1.15)$</td>
<td>$G_{lc}((1 - k)\tan^4\Psi + 1)$ $(250, 0.2)$</td>
<td>-</td>
</tr>
<tr>
<td>Fibre Bridging</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UPPER $(G_{lc}[J/m^2], k)$</td>
<td>-</td>
<td>-</td>
<td>$G_{lc}(k\tan^8\Psi + 1)$ $(825, 200)$</td>
</tr>
<tr>
<td>AVERAGE $(G_{lc}[J/m^2], k)$</td>
<td>$G_{lc}((1 - k)\tan^4\Psi + 1)$ $(310, 0)$</td>
<td>$G_{lc}((k - 1)\sin^2\Psi + 1)^{-1}$ $(400, -0.49)$</td>
<td>$G_{lc}(k\tan^8\Psi + 1)$ $(725, 200)$</td>
</tr>
<tr>
<td>LOWER $(G_{lc}[J/m^2], k)$</td>
<td>-</td>
<td>-</td>
<td>$G_{lc}(k\tan^8\Psi + 1)$ $(625, 200)$</td>
</tr>
</tbody>
</table>
Chapter 5

Finite Element Based Debond Models at the Structural Level

In the previous chapter the tools for predicting the initiation of crack propagation have been presented and verified on small-scale test specimens. In this chapter these fracture mechanics tools will be implemented in a finite element program.

A 2-D crack propagation model has been produced in order to simulate propagation in a sandwich beam. Parallel to the 2-D model a similar 3-D residual strength model will be presented. The 3-D model only predicts initiation while the 2D model is also able to track the crack propagation. Both models use the CSDE-method in their respective versions in 2-D and 3-D.

5.1 2-D Crack Propagation Model

The 2-D crack propagation model can be described by an algorithm, see below. This algorithm, together with the 2-D version of the CSDE method, has been implemented in subroutines in the commercial finite element program ANSYS.

By means of this algorithm, it is possible to simulate a quasi-static traction-free propagation of a crack running in or just below the interface between face and core in a sandwich beam.

The routine is presently optimised for use in a sandwich interface, but could easily be transformed for application to delamination problems with an inter-laminar crack. The CSDE method can be directly used as long as new fracture toughness distributions are supplied for the inter-lamina crack. Investigations of this kind are found in the literature, see e.g. Beuth (1996).
The crack simulation routine can be described by the following algorithm:

1. A displacement controlled geometrical linear or non-linear finite element calculation is performed in two load steps, see Figure 5.1, where the second load step can be regarded as infinitesimal and controlled by an arbitrarily chosen load factor (e.g. 5%).

2. For each load step the corresponding Griffith-energy and mode-mixity are calculated by use of the 2-D version of the CSDE method.

3. The propagation displacement is determined by linearisation of the second infinitesimal load step, see Figure 5.1, using the fracture toughness for the calculated mode-mixity. If the fracture toughness does not lie in the interval bordered by the Griffith-energy calculated in the two load steps, the calculation is redone with the previous second load step as the first load. If the fracture toughness is still outside the interval after three recalculations the propagation displacement is determined by linear extrapolation using the two last load steps.

4. To determine if the crack will kink into the core at the present displacement load, the Griffith-energy of ten kinking angles to or from 45° is calculated numerically as described in the previous chapter. The possible kinking angle is found at the maximum Griffith-energy. The calculation is done at the propagation displacement.

5. The crack tip is moved an increment $\Delta a$, which is globally chosen for each crack problem, the model is remeshed and the routine is redone using the last propagation displacement load as the first load step. The routine is redone until catastrophic failure or crack arrest.

As it is seen in Figure 5.1, both stable and unstable crack growth can be simulated. Unstable crack growth is defined in this displacement controlled context and can be monitored, when the Griffith-energy is already crossed at the first load step. In this case the crack is moved and the same displacement value is used again until the structure suffers a catastrophic failure or the crack behaves in a stable manner again. In other words, unstable crack growth is present when the force-displacement curve has a vertical tangent. In all other cases stable crack growth is present.

In order to perform the remeshing automatically in the crack propagation simulation, remeshing routines have been produced. The routines are able to move the mesh both in the interface and out of the interface after kinking has taken place.

As described earlier, the model consists of primarily plane strain or stress isoparametric 8-noded elements, except in a small inner zone with a radius of three elements, where linear 4-noded elements have been used, see Figure 5.2. These elements are more robust when large distortions are encountered, and the solution in these elements is excluded from the

---

1 Not shown in this thesis, because no application showed kinking behaviour
CSDE method as described earlier. The rest of the crack tip mesh consists of two zones, of which the next zone surrounding the crack tip includes a number of element rings, where the actual Griffith-energy and mode-mixity determination is carried out. The last zone is just used to upscale gradually the mesh density from the structural to the near tip mesh density.

Figure 5.1: Crack propagation algorithm.

Figure 5.2: Crack tip mesh with linear near tip zone.
5.2 3-D Residual Strength Model

5.2.1 General Considerations

Efforts have previously been made to produce 3-D models for simulation of delamination initiation and propagation in fibre reinforced composite structures. The DEBUGS code is an example, which was produced in the mid 90s at Royal Institute of Technology, Sweden. The DEBUGS code is documented in Nilsson et al. (1997), Nilsson et al. (2001a), Nilsson et al. (2001b) and Asp and Nilsson (2002). It is a layered shell model with fracture mechanical propagation criteria, produced to simulate crack propagation between laminas in a laminate. The DEBUGS code has also recently been applied to debond problems in sandwich structures, thus stretching the validity of a shell model since the complex near tip interaction between the stiff face and the soft core is lost contrary to a solid model with medium to high near tip mesh density.

In this thesis a pure solid element model will be presented and used to simulate several different debond problems.

In order to expand the analysis to cover investigations of 3-D structures like panels and large sections of for example a sandwich vessel, several considerations have to be kept in mind. In summary these can be listed as follows:

- The number of elements is raised considerably, resulting in a drastic increase in demanded CPU time for carrying out the analysis.
- Not only one position has to be investigated for crack propagation, but the complete crack front has to be covered by the investigation.
- If crack propagation is to be included, the crack front has to be updated between each crack propagation increment, and the complete model has to be remeshed, which will lead to a massive increase in CPU time.
- Mode III deformations are now present.
- In practice the near tip mesh will be coarser in a 3-D model in order to minimise the global number of elements. This will also result in less accuracy in the prediction of the mode-mixity when the CSDE 3-D mode-mixity method is used.
- Kinking of the crack front into the core will be very complex, and just an investigation of whether the crack will kink or not will be very CPU expensive.
In order to avoid the worst problems mentioned above, the following choices have been made:

- Only rectangular panels with circular debonds will be investigated.
- When possible only the $1/4$-models will be used, thus exploiting symmetry. For cases of single-symmetric loading it will be necessary to use a $1/2$-model.
- Only a discrete number of positions along the crack front will be investigated for crack propagation.
- As mentioned earlier, mode III deformations will be neglected because they are considered small compared to mode I deformations.
- Crack propagation will not be simulated, and the panel will be regarded as failed, when crack propagation is monitored and registered along the crack front.
- Kinking, both simulation and monitoring, will not be carried out in 3-D.

The choice not to simulate crack propagation in 3-D is made, because the type of loading considered in this thesis is mainly in-plane loading of flat panels. In these cases it is expected that the panel in most cases will fail due to rapid debond propagation after or in combination with local buckling of the debonded face part.

The panel will thus be regarded as failed when the fracture toughness is reached in any position along the crack front.

This assumption is confirmed by experimental investigations presented later in this thesis. In these tests no propagation of the debond is seen prior to global failure of the panel either due to graduate or sudden outward debond buckling.

If needed, consecutive calculations can be made with increasing debond size, thus simulating the crack propagation path independent analogue to the approach presented for 2-D model. The debond shape just has to remain circular.

Furthermore, for crack front areas with at the same time high Griffith-energy values and positive mode-mixity, consecutive kinking analysis could be made as the risk of kinking out of the interface in these situations is high.

### 5.2.2 The Simulation Algorithm

Based on the above presented considerations, the 3-D residual strength model may briefly be described in the following manner:
1. The geometry is built by use of the rotational 2-D routines and the 3-D routines.

2. A displacement controlled geometrical non-linear finite element calculation is performed to introduce imperfections into the model, see below.

3. The displacement loads are introduced/scaled.

4. A displacement controlled geometrical non-linear finite element calculation is performed.

5. An arbitrary number of crack front positions are investigated using the 3-D version of the CSDE method.

6. If the fracture toughness is not exceeded in any crack front positions, the routine is carried on from step 3, scaling the displacement loads and using an incremental scaling factor.

7. If the fracture toughness is exceeded in any crack front position, the calculation stops and the panel is regarded as failed.

As described earlier routines have been produced to generate the geometry and mesh for 2-D models. These routines are parametrically based, and the parameters are used throughout the remaining routines, including the mode-mixity routines, in the complete simulation model. In 3-D it is the goal to reuse as many routines as possible from 2-D. It is therefore recommendable to use the same parameters in the geometry and mesh generation in 3-D.

In order to do this, an inner region is made where geometry and mesh are generated by rotation of the 2-D model. The rotation symmetric geometry generation is presented in Figure 5.4. The mesh is generated as a mapped mesh in this zone.

The outer region geometry is generated explicitly, but the mesh is still produced by means of mapped mesh generation. A 1/4-panel model with a circular central debond is seen in Figure 5.5.

With regard to the element types, a larger portion of linear elements is used in 3-D than in 2-D in order to minimise CPU time. Besides the near tip mesh zone of linear elements analogous to the 2-D model, some of the outer mesh region is in 3-D also linear elements.

The distribution is seen in Figure 5.4, where the blue zones represent the linear 8-noded solid element mesh zones. The remaining red zones are meshed with 20-noded parabolic solid elements.

During the experimental investigation of residual strength of debonded sandwich panels, later presented in this thesis, all tested panels had an initial debond opening, which was caused by degassing from the core producing a gas pressure inside the debond. This actually prevents the debond from closing during loading, and this has to be taken into account in the modelling of the panels. For that purpose a Debond Closure Prevention Device (DCPD) has been produced.
It is important that this device is not CPU expensive, but at the same time it has to be efficient. Therefore, a contact element approach has been abandoned, and a combination of tension and compression spar elements has been chosen. A tension or compression spar element only transfers loads in tension or compression respectively. If it is compressed (tension element) or in tension (compression element) the load is zero. The spar element is attached to the outermost top and bottom midpoint node at the panel centre and is only able to transfer translations. The connection points will therefore act as charniers.

The tension and compression only elements are then connected in the opposite edge with a DOF constraint tying their respective DOF’s to each other. Again these points will act as charniers, but furthermore they are only allowed to move in a vertical direction.

Initially, the tension only element will be prestretched corresponding to the wanted initial debond opening, and during the first load step mentioned above, the debond will be forced to open, and thus act as an imperfection, which is needed in a non-linear buckling analysis.

In the subsequent load steps the DCPD will prevent the debond opening from becoming smaller than the initial opening and in this way simulate the gas pressure inside the debond. Even if the total panel bends to the same side of the panel as the debond location, the combination of tension and compression only spar elements will prevent the debond from closing, because the stiffness of the elements has been chosen to be several magnitudes higher than the face stiffness.

The DCPD only acts on a single node pair in the centre of the debond, so the approach has its limitations, if the $t_f/D$-ratio becomes very small where $D$ is the debond diameter. In this case crack flank contact can occur even though the centre of the debond is kept contact-free. Nevertheless, the DCPD works well for all $t_f/D$-ratios encountered in this thesis.

The debond prevention device is presented schematically in Figure 5.3.

![Debond closure prevention device (DCPD)](image-url)
Figure 5.4: Distribution of linear (blue) and parabolic (red) elements.

Figure 5.5: Typical 3D 1/4-model simulating a debonded sandwich panel.
Chapter 6

Face Tearing Debonding in a Deck Structure of a Sandwich Vessel

The developed 2-D finite element crack propagation model is in this chapter tested on a typical debonding problem in a sandwich vessel.

The debond problem presented in this chapter has been chosen, because it does not include any difficult non-linear problems, e.g. local buckling of the face layer and global buckling of the sandwich panel. Therefore, this debond problem gives a clear picture of the behaviour of the model with respect to crack propagation and at the same time the model is applied to an important real life debond problem.

The theoretical part of this chapter served at an early stage of the project as an initial test of the model, where the developed fracture mechanics based model was compared to a damage mechanics model developed to simulate crack propagation in metal plates, Törnvist (2003). This work was also presented at the 6th International Conference on Sandwich Structures, Ft.Lauderdale, Florida, USA, 31 March to 2 April 2003, Berggreen et al. (2003b).

In order to verify the 2-D propagation model separate experimental test series have been carried out. This comparison is shown in the last part of this chapter. Furthermore, the experimental non-destructive test procedure and its application to crack front estimation is seen in Borum and Berggreen (2004).
Chapter 6. Face Tearing Debonding in a Deck Structure of a Sandwich Vessel

6.1 Theoretical Prediction of Crack Propagation

Crack propagation coupled to non-linear structural mechanics is the dominant mechanism in a large number of failure modes for sandwich vessels. The initial crack may be a result of production flaws, fatigue, impact or other causes. Thus, the configuration considered below is only one of many relevant ones for sandwich vessels.

The connection between the superstructure and the hull often causes structural problems due to various discontinuities. Such a connection is analysed in the present example to illustrate the potential of the developed simulation tool. Figure 6.1c shows the details of the considered connection between a vertical superstructure bulkhead, the deck and a bulkhead supporting the superstructure below the deck (hidden in Figure 6.1b). The real problem is assumed here to be two-dimensional (plane strain). The thicknesses of the superstructure...
and the bulkhead under the deck are $2e$ and $2c$, respectively. The initial debond has a length of $2a$.

When the hull girder is in a state of hogging the hull will tend to pull down and away from the front edge of the superstructure, as shown in Figure 6.1a. This induces a lifting, $\delta$, of the skin of the deck sandwich. At the same time, the deck is stretched due to the global bending of the hull girder. The simple model as shown in Figure 6.1a, assuming a rigid superstructure, captures this coupling between the lifting and the longitudinal stretching through the curvature $\kappa = 1/R$ of the hull beam:

\[
\delta = \left( \frac{1}{\kappa} + Z_d \right) \left( 1 - \sqrt{1 - \left( \frac{L_{ss}}{\frac{1}{\kappa} + Z_d} \right)^2} \right) \tag{6.1}
\]

\[
\Delta L = Z_d \kappa L \tag{6.2}
\]

where $Z_d$ is the distance from the neutral axis of the hull beam to the deck and $L_{ss}$ is a characteristic length of the superstructure. By using the two relations in (6.1) and (6.2), the complete crack problem can be controlled by the hull beam curvature.

6.2 Description of Finite Element Models

6.2.1 Geometry and Materials

The geometry of the model is seen in Figure 6.1c, and the corresponding quantities are given in Table 6.1.

When this study was carried out proper material data, both mechanical and fracture mechanical, were not yet available. Instead data from the literature was used.

The face material data is taken from Li and Carlsson (2001) and is assumed to be very similar to that used in Shipsha et al. (1999), who measured the critical Griffith-energy in mode I and mode II for an interface crack. The faces consist of glass/vinylester $[(0/45)/2 / (45/0)_3]$. The core is a cross linked PVC foam of the type Divinycell H100, for which material data is available from the manufacturer, DIAB (2000). The material properties are found in Table 6.2.

In Shipsha et al. (1999) the crack is located just below the interface, and the phase shift encountered from the homogeneous to the inhomogeneous (interface) crack is neglected here.
The critical interface Griffith-energy distribution is assumed to follow the distribution proposed by Hutchinson and Suo (1992):

\[
\Gamma (\Psi) = G_{lc} \left[ 1 + (1 - k) \tan^2 \Psi \right]
\]

and with the estimated values \( G_{lc} = 350 \text{ J/m}^2 \) and \( k = 0.853 \), the fracture toughness for mode I and mode II then becomes \( \Gamma (0^\circ) = 350 \text{ J/m}^2 \) and \( \Gamma (80^\circ) = 2000 \text{ J/m}^2 \) in accordance with Shipsha et al. (1999). The fracture toughness distribution is seen in Figure 6.2.

Table 6.1: Geometry parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam length</td>
<td>( L )</td>
<td>1000 mm</td>
</tr>
<tr>
<td>Crack length</td>
<td>( a )</td>
<td>25 mm</td>
</tr>
<tr>
<td>Support length</td>
<td>( c )</td>
<td>20 mm</td>
</tr>
<tr>
<td>Loading length</td>
<td>( e )</td>
<td>20 mm</td>
</tr>
<tr>
<td>Face thickness</td>
<td>( t_f )</td>
<td>3.6 mm</td>
</tr>
<tr>
<td>Core thickness</td>
<td>( t_c )</td>
<td>50 mm</td>
</tr>
<tr>
<td>Super structure length</td>
<td>( L_{ss} )</td>
<td>12500 mm</td>
</tr>
<tr>
<td>Neutral axis height</td>
<td>( Z_d )</td>
<td>variable</td>
</tr>
</tbody>
</table>

Table 6.2: Face (GFRP \([(0/45)_2 / (45/0)_3]\)) and core (Divinycell H100) mechanical parameter values from Li and Carlsson (2001) and DIAB (2000).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face in-plane Young’s modulus</td>
<td>( E_X )</td>
<td>21200 MPa</td>
</tr>
<tr>
<td>Face in-plane Young’s modulus</td>
<td>( E_Y )</td>
<td>12600 MPa</td>
</tr>
<tr>
<td>Face out-of-plane Young’s modulus</td>
<td>( E_Z )</td>
<td>11000 MPa</td>
</tr>
<tr>
<td>Face in-plane G-modulus</td>
<td>( G_{XY} )</td>
<td>8670 MPa</td>
</tr>
<tr>
<td>Face out-of-plane G-modulus</td>
<td>( G_{XZ} )</td>
<td>3510 MPa</td>
</tr>
<tr>
<td>Face out-of-plane G-modulus</td>
<td>( G_{YZ} )</td>
<td>3490 MPa</td>
</tr>
<tr>
<td>Face in-plane Poisson’s ratio</td>
<td>( \nu_{XY} )</td>
<td>0.539</td>
</tr>
<tr>
<td>Face out-of-plane Poisson’s ratio</td>
<td>( \nu_{XZ} )</td>
<td>0.313</td>
</tr>
<tr>
<td>Face out-of-plane Poisson’s ratio</td>
<td>( \nu_{YZ} )</td>
<td>0.469</td>
</tr>
<tr>
<td>Core Young’s modulus</td>
<td>( E_c )</td>
<td>105 MPa</td>
</tr>
<tr>
<td>Core G-modulus</td>
<td>( G_c )</td>
<td>40 MPa</td>
</tr>
<tr>
<td>Core Poisson’s ratio</td>
<td>( \nu_c )</td>
<td>0.3125</td>
</tr>
<tr>
<td>Core Fracture toughness</td>
<td>( \Gamma_{core} )</td>
<td>309 J/m(^2)</td>
</tr>
</tbody>
</table>
6.2 Description of Finite Element Models

6.2.2 Fracture Mechanics Model

A finite element model, based on the 2-D CSDE method, has been built in the commercial finite element code ANSYS (both described in a previous chapter) to simulate the crack propagation described in Figure 6.1c. A debonded sandwich cross-section, of the length $2L$ and face and core thickness $t_f$ and $t_c$, is taken from the deck as indicated in Figure 6.1b and treated as a 2-D plane strain problem.

An initial debonding of the length $2a$ is assumed below the superstructure front edge of the thickness $2e$. The debonded sandwich beam is supported at the beam midspan by an underlying bulkhead of the thickness $2c$ and assumed to be clamped into the two neighbouring bulkheads. The load is then introduced as a vertical displacement, $\delta$, of the superstructure front edge and a horizontal displacement, $\Delta L$, of the clamped beam ends, both controlled by the hull beam curvature, $\kappa$, through the coupling expressions given in Eqs. (6.1) and (6.2). The finite element calculation is therefore displacement controlled.

The model has been built according to the description given in the previous chapter, and the 2-D crack propagation algorithm is used in the calculations. The near tip mesh is seen in Figure 6.3.

Figure 6.2: Fracture toughness distribution formed on the basis of values achieved from Shipsha et al. (1999).
Figure 6.3: Finite element model with the three element zones used in fracture mechanics modelling, min. element size is 0.006 mm.

Figure 6.4: Finite element model used in damage mechanics modelling, min. element size is 1.0 mm.
6.2 Description of Finite Element Models

6.2.3 Damage Mechanics Model

The debond problem was also simulated by the explicit finite element program LS-DYNA, by means of a user-defined material model of continuum damage mechanics (CDM). The damage finite element model, see Figure 6.4, has been produced in order to compare with a separate and independent numerical method.

The CDM models are frequently used to study ductile fracture in metals, however, they can also be used in crack propagation simulations in different core materials. A reference volume element (RVE), see Figure 6.5, includes several cells in the core material and the idea is to reduce progressively the stiffness of the RVE as material between the cell fractures. When the fracture stress in the RVE is reached, the core material flows as perfect elasto-plastic material, where the damage function progressively reduces the stiffness of the RVE until total fracture occurs.

The CDM theory proposed by Bonora (1997) is a coupled model of elasto-plasticity and isotropic damage. The damage evolution is based on a continuous damage variable $D$:

$$D = 1 - \frac{E_{\text{eff}}}{E_0}$$

(6.4)

which is a non-linear function of the plastic strain. $E_{\text{eff}}$ and $E_0$ are the effective and the initial stiffness respectively.

Figure 6.5: The reference volume element (RVE).

Damage takes into account the progressive degradation of the material properties and the loss of stiffness due to the irreversible process of fracture between the core cells. The damage variable is based on the effect of damage on the elastic properties. Furthermore, Lemaitre (1985) made a hypothesis of strain equivalence, which stated that the strain behaviour of
the damaged material is represented by the constitutive equations of the virgin material \((D = 0)\). The effective stress \(\tilde{\sigma}\) is the stress calculated over the section that effectively resists the forces:

\[
\tilde{\sigma} = \frac{\sigma}{1 - D} \quad \varepsilon = \frac{\tilde{\sigma}}{E} = \frac{\sigma}{(1 - D)E}
\]  

(6.5)

The evolution of the damage variable, \(D\), is defined as (Bonora (1997)):

\[
dD = \alpha \frac{(D_{cr} - D_0)^{1/\alpha}}{\ln(\varepsilon_f/\varepsilon_{th})} f \left( \frac{\sigma_H}{\sigma_{eq}} \right) (D_{cr} - D)^{(\alpha-1)/\alpha} \frac{dp}{p}
\]

(6.6)

with the triaxiallity function defined as

\[
f \left( \frac{\sigma_H}{\sigma_{eq}} \right) = \frac{2}{3} (1 + \nu) + 3 (1 - 2\nu) \left( \frac{\sigma_H}{\sigma_{eq}} \right)^2
\]

(6.7)

\(D_{cr}\) and \(\varepsilon_f\) are respectively the critical value of the damage variable and the uniaxial strain for which ductile fracture occurs. The uniaxial damage strain threshold, \(\varepsilon_{th}\), indicates when void nucleation is initiated, when the material structure is considered similar to that of metals, which is a rough approximation for most foam materials. The damage exponent, \(\alpha\), characterises the non-linearity of the damage evolution.

When the damage parameter reaches the critical damage parameter values, the element is removed from the model. The model consists of 4-noded plane strain elements of element sizes of about 1x1 mm, which should approximately represent the RVE size of the core material.

As it is seen from Figure 6.4, a subinterface approach is adopted in the damage model contrary to the pure interface crack approach applied in the fracture mechanics model. In reality it has been observed (see the next section) that for lighter cores, the crack is always propagating as a subinterface crack just below the interface.

The boundary and loading conditions are the same as in the fracture mechanics model. The geometry and material properties are given in Tables 6.1 and 6.2, and the additional damage parameters are given in Table 6.3.

To achieve the same load level as in the fracture mechanics model for \(Z_d = 2000\) mm, the tensile strength was assumed to be 3.5 MPa, which is close to the maximum standard value of 3.1 MPa, given by the manufacturer, DIAB (2000).
The Divinycell H-series PVC foams all behave brittle in tension, and the damage parameters in Table 6.3 have been chosen so that this behaviour is reproduced. Nevertheless, it has proved necessary to introduce a very small rigid-plastic region and a following stiffness reduction zone in the Bonora model just before failure to avoid stress wave effects in the dynamic finite element code, when the crack tip element is removed. In this way high stress peaks are filtered out and will have no effect on the overall crack propagation. However, just as it can be observed in experimental investigations (see the next section), the release of elastic energy when the crack propagates introduces dynamic effects into the system, which makes the crack propagate very fast and unstably. The small plastic and stiffness reduction zone introduced into the Bonora model, reduces these dynamic effects and makes the crack propagate stably and thus makes it possible to compare the fracture and damage models.

More details about this damage model and the application of the damage model to steel and aluminium structures are found in Törnqvist (2003) and Urban (2003).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
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<tr>
<td>Damage exponent</td>
<td>$\alpha$</td>
<td>0.2</td>
</tr>
<tr>
<td>Initial damage parameter</td>
<td>$D_0$</td>
<td>0</td>
</tr>
<tr>
<td>Critical damage parameter</td>
<td>$D_{cr}$</td>
<td>0.95</td>
</tr>
<tr>
<td>Damage strain threshold</td>
<td>$\varepsilon_{th}$</td>
<td>0.001</td>
</tr>
<tr>
<td>Uniaxial fracture strain</td>
<td>$\varepsilon_f$</td>
<td>0.02</td>
</tr>
</tbody>
</table>
6.3 Calculation Results

The fracture mechanics approach described above was implemented in ANSYS and the damage mechanics approach was implemented in LS-DYNA. None of the analyses predicted kinking. This seems intuitively correct since a direction perpendicular to the maximum principal, tensile stress indicates a kink into the face laminate, which is not feasible due to the high fracture toughness of the laminate. Furthermore, the mode-mixity varies between $-3^\circ$ and $-22^\circ$, which also indicates a kink direction into the face laminate.

Figure 6.6 shows the predicted crack length as a function of the lifting, $\delta$. Note that this result is independent of the initial crack length ($a > 0$). It is interesting to note that the non-linear model - which takes into account the membrane loads in the face skins at large deflections - predicts a nearly linear relationship between the lifting and the crack length. It is seen, for example, that if the superstructure lifts 10 mm away from the deck this will generate a crack length of $a = 180 \text{ mm}$, if the basis configuration with $Z_d = 2000 \text{ m}$, $G_{ic} = 350 \text{ J/m}^2$ is considered. Several variations from this basic configuration are shown in Figure 6.6. For example, the result of a linear analysis shows that this model works well up to a lift of approximately 5 mm ($a = 100 \text{ mm}$), but subsequently the non-linear membrane effects become important. For example, at a lift of 20 mm, the linear model under predicts the crack length by almost 40%.

The effect of fracture toughness is also investigated and is illustrated in Figures 6.6, 6.7 and 6.8. Especially in Figure 6.7, the effect on the crack length has been investigated for a reduction of $G_{ic}$ of 29% from 350 to 250 J/m$^2$. The crack length increases by approximately 11% (average) for all lifts, $\delta$. The problem is therefore quite sensitive to changes in the fracture toughness.

Likewise, the effect of the distance $Z_d$ to the neutral axis of the hull girder is investigated and is seen in Figures 6.6 and 6.8. $Z_d$ provides the link between the axial stretching of the deck skin and the vertical lift. The effect of $Z_d$ is observed to be minor for the considered configuration.

Figure 6.8 shows the vertical pulling force as a function of the vertical lift, $\delta$. Since the crack length is approximately proportional to the lift it can also be thought of as the force versus crack length. It is seen that if the configuration had been load controlled it would have been highly unstable, as the load diminishes rapidly after crack initiation. As above, the linear prediction only works at a very limited lift and the sensitivities noted above are observed once again. It is interesting to note that the linear model predicts a monotonously decreasing force after initiation, while the non-linear model predicts a stabilising behaviour with membrane stiffening where the load increases after a lift of approximately 4 mm and a crack length of about 90 mm.

In Table 6.4 the initiation forces have been compared for the different choices of $Z_d$ and $G_{ic}$. It is seen that varying the neutral axis height has very limited influence on the initiation force, which could also be expected as this parameter, as mentioned above, provides the link
between the lift and the axial stretching, which is very limited at initiation. Varying the fracture toughness through the $G_{Ic}$-parameter has a stronger effect, because this parameter directly governs when initiation takes place. The reduction of 29% from 350 to 250 $J/m^2$ results in a 15% reduction of the initiation force.

Finally, Figure 6.9 shows the predictions with the continuum damage mechanics model implemented in **LS-DYNA**. The erratic behaviour is due to the elastic wave propagation in the dynamic, explicit calculation. Otherwise, there is seen to be a remarkably good agreement with the fracture mechanics model, in view of the relatively simple procedure and implementation. Even the crack initiation forces are predicted with deviations of less than 15% from the fracture mechanics model, which can be seen in Table 6.4.

For future use this approach needs careful further investigation of course, because such local models are known to exhibit a strong mesh sensitivity so that the material parameters related to failure should be a function of the element size. But clearly, for propagation problems in 3-D, which are very complicated to solve with a fracture mechanics approach, as mentioned in the previous chapter, the local damage mechanics approach has a great potential.

**Table 6.4: Initiation forces from the fracture and damage mechanics models.**

<table>
<thead>
<tr>
<th>Model</th>
<th>$G_{Ic} [J/m^2]$</th>
<th>$Z_d [mm]$</th>
<th>$F_{initiation} [N/mm]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture, non-linear</td>
<td>350</td>
<td>2000</td>
<td>22.72</td>
</tr>
<tr>
<td>Fracture, linear</td>
<td>350</td>
<td>2000</td>
<td>23.00</td>
</tr>
<tr>
<td>Fracture, non-linear</td>
<td>350</td>
<td>4000</td>
<td>22.73</td>
</tr>
<tr>
<td>Fracture, non-linear</td>
<td>250</td>
<td>4000</td>
<td>19.33</td>
</tr>
<tr>
<td>Damage</td>
<td>-</td>
<td>2000</td>
<td>24.30</td>
</tr>
<tr>
<td>Damage</td>
<td>-</td>
<td>4000</td>
<td>25.60</td>
</tr>
</tbody>
</table>
Figure 6.6: Lifting as a function of crack length. The initial crack length is 25 mm.

Figure 6.7: Effect on the crack length by reducing the fracture toughness by approx. 30% from 350 J/m² to 250 J/m².
6.3 Calculation Results

Figure 6.8: Lifting force as a function of lifting.

Figure 6.9: Lifting force as a function of lifting. Comparison between fracture and damage mechanics models.
6.4 Experimental Validation

In the previous section the 2-D propagation model was verified against another numerical method, and excellent agreement was achieved.

In this section both the 2-D propagation model and the 3-D residual strength model will be validated against an experimental test series, using the same practical debond problem as presented in the previous section.

The test series was carried out by the author during a stay at RISØ National Laboratory. Furthermore, a new test rig was designed and produced in the same period, and special ultrasound NDI techniques were used in collaboration with the staff at RISØ National Laboratory.

6.4.1 Test Rig

A test rig has been produced and is presented schematically in Figure 6.10. It consists of a rigid overhanging beam construction, which simulates the bulkheads on each side of the deck panel and is assumed to be totally rigid in the tests. In order to obtain a perfect clamping of the beam ends, wood inserts have been introduced at the beam ends, and the beam is then bolted with four bolts onto the test rig. Two perfectly fitting bolts go all the way through the beam end with the wood insert and transfer the vertical forces from mainly the bottom face into the test rig. The two remaining bolts only introduce a compressive force on the bottom face and transfer the vertical forces into the test rig.

The arrangement around the middle of the beam on the debond side of the beam consists of two plates on each side of the face bolted together to ensure that the loading area remains straight throughout the test. For positioning of the plate on the core side of the face, a small part of the core has been removed, and the starter crack has been introduced with a razor blade extending 5 mm away from the plate. Furthermore these two plates are bolted together with a third underlying plate, which is bolted to a very stiff loading rod connected to the test machine.

A similar arrangement is present on the opposite side of the debond (the other side of the sandwich beam). The difference compared to the arrangement on the bottom side is that only the plate on the outside of the face is present, and in this case glued and screwed on. Furthermore, adjusting screws have been inserted between the two metal plates, making it possible to adjust the distance between the beam and the test rig.
Figure 6.10: Face tearing beam test rig.
Figure 6.11: Face tearing beam test rig in Instron 1333 without ultrasound scanning system.

Figure 6.12: Face tearing beam test rig in Instron 1333 with ultrasound scanning system.
6.4 Experimental Validation

The complete test rig is finally hanging in another rod and connected to the top loading head of the testing machine, and the connection between the rod and the test rig is effected by a spherical bearing. An Instron 1333 servo-hydraulic testing machine, with a maximum tension/compression capacity of 250 kN, is used with a separate 10 kN load cell, see Figure 6.11.

In the theoretical modelling in the last section, a stretch loading corresponding to the stretch of the deck was included. This stretch loading has been neglected in the experiments, because of practical problems with introducing the stretch loading in the same fashion. However, apart from this, exactly the same loading condition has been simulated in the experiments.

While the test is carried out, the force and the vertical movement of the loading heads are monitored. Furthermore, a strain gauge has been placed on the left side of the beam for verification purposes.

The crack propagation is monitored by visual inspection, as well as by in situ air-coupled ultrasonic equipment developed at RISØ National Laboratory, see Figure 6.12. The main advantage of this equipment compared to conventional ultrasonic equipment is that it avoids the disadvantages of the coupling liquid or coupling gel and the time-consuming cleaning after the inspection. Therefore, the non-contact ultrasonic technique is very attractive for in situ measurements in a mechanical test rig. Sandwich composites are inhomogeneous and anisotropic materials with extremely high sound attenuation. Through-transmission technique with separate receiver and transmitter transducers on opposite sides of the component is often used for testing. To overcome the high damping of the ultrasound at the interface air/sandwich material a special high-power transducer with integrated preamplifier is used. It consists of a composite system with impedance matching to air. A special X-Y-scanner has been built for this purpose. Two pairs of transducers are used: 50 kHz and 120 kHz. The diameter of the transducer is 19 mm, and the distance from the transducer to the specimen is approximately 15 mm. For an axial resolution of 0.3 mm the scanning speed is 20 mm/sec., which means that a typical scanning takes about 10 minutes. For practical reasons ultrasonic scanning monitors only the left crack.

Both the strain gauge and the ultrasonic transducers are schematically presented in Figure 6.10. More details about the scanning system can be found in Borum and Berggreen (2004).
6.4.2 Test Specimen Properties

In total ten specimens are produced by use of three core densities:

- H80 (80 \text{ kg/m}^3): Four specimens
- H130 (130 \text{ kg/m}^3): Three specimens
- H200 (200 \text{ kg/m}^3): Three specimens

The test specimen is shown in Figure 6.13 and the geometrical properties are given in Table 6.5. The beam width, which is 65 mm, should ensure pure plane strain in the midplane of the beam as indicated in CASE 1 in Chapter 4.

The face consists of hand layup glass fibre mats of the quadro-axial type DBLT1150 and chopped strand mat CSM450. The resin is polyester and the total thickness is approximately 4.5 mm.

The face properties are found by material testing at RISØ National Laboratory. Only specimens consisting of DBLT1150 have been used in these material tests, therefore the laminate properties of the faces are found by correcting the in-plane properties of the DBLT1150 laminates, using table data for the CSM450 mats, Zenkert (1997). The resulting mechanical face properties are shown in Table 6.6.

The interface fracture toughness distributions in Table 4.11 have been used as input to the models.

![Figure 6.13: Face tearing beam specimens with the core types H80, H130 and H200.](image)
### 6.4 Experimental Validation

Table 6.5: *Geometry parameter values for experimental test specimens.*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active beam length</td>
<td>$L$</td>
<td>430 mm</td>
</tr>
<tr>
<td>Overall beam length</td>
<td>$L_{oa}$</td>
<td>500 mm</td>
</tr>
<tr>
<td>Beam width</td>
<td>$B$</td>
<td>65 mm</td>
</tr>
<tr>
<td>Crack length</td>
<td>$a$</td>
<td>45 mm</td>
</tr>
<tr>
<td>Support length</td>
<td>$c$</td>
<td>40 mm</td>
</tr>
<tr>
<td>Loading length</td>
<td>$e$</td>
<td>40 mm</td>
</tr>
<tr>
<td>Face thickness</td>
<td>$t_f$</td>
<td>4.5 mm</td>
</tr>
<tr>
<td>Core thickness</td>
<td>$t_c$</td>
<td>50 mm</td>
</tr>
</tbody>
</table>

Table 6.6: *Mechanical properties of the GFRP face [CSM (0/45/90/−45)$_s$ CSM] and Divinycell H80, H130 and H200 PVC foam cores, DIAB (2000).*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face in-plane Young’s modulus</td>
<td>$E_X$</td>
<td>12270 MPa</td>
</tr>
<tr>
<td>Face in-plane Young’s modulus</td>
<td>$E_Y$</td>
<td>12270 MPa</td>
</tr>
<tr>
<td>Face out-of-plane Young’s modulus</td>
<td>$E_Z$</td>
<td>8400 MPa</td>
</tr>
<tr>
<td>Face in-plane G-modulus</td>
<td>$G_{XY}$</td>
<td>4491 MPa</td>
</tr>
<tr>
<td>Face out-of-plane G-modulus</td>
<td>$G_{XZ}$</td>
<td>2700 MPa</td>
</tr>
<tr>
<td>Face out-of-plane G-modulus</td>
<td>$G_{YZ}$</td>
<td>2700 MPa</td>
</tr>
<tr>
<td>Face in-plane Poisson’s ratio</td>
<td>$\nu_{XY}$</td>
<td>0.294</td>
</tr>
<tr>
<td>Face out-of-plane Poisson’s ratio</td>
<td>$\nu_{XZ}$</td>
<td>0.32</td>
</tr>
<tr>
<td>Face out-of-plane Poisson’s ratio</td>
<td>$\nu_{YZ}$</td>
<td>0.29</td>
</tr>
<tr>
<td>Core H80 Young’s modulus</td>
<td>$E_c$</td>
<td>85 MPa</td>
</tr>
<tr>
<td>Core H80 G-modulus</td>
<td>$G_c$</td>
<td>31 MPa</td>
</tr>
<tr>
<td>Core H80 Poisson’s ratio</td>
<td>$\nu_c$</td>
<td>0.35</td>
</tr>
<tr>
<td>Core H80 Fracture toughness</td>
<td>$\Gamma_{core}$</td>
<td>186 $J/m^2$</td>
</tr>
<tr>
<td>Core H130 Young’s modulus</td>
<td>$E_c$</td>
<td>175 MPa</td>
</tr>
<tr>
<td>Core H130 G-modulus</td>
<td>$G_c$</td>
<td>55 MPa</td>
</tr>
<tr>
<td>Core H130 Poisson’s ratio</td>
<td>$\nu_c$</td>
<td>0.35</td>
</tr>
<tr>
<td>Core H130 Fracture toughness</td>
<td>$\Gamma_{core}$</td>
<td>394 $J/m^2$</td>
</tr>
<tr>
<td>Core H200 Young’s modulus</td>
<td>$E_c$</td>
<td>310 MPa</td>
</tr>
<tr>
<td>Core H200 G-modulus</td>
<td>$G_c$</td>
<td>90 MPa</td>
</tr>
<tr>
<td>Core H200 Poisson’s ratio</td>
<td>$\nu_c$</td>
<td>0.35</td>
</tr>
<tr>
<td>Core H200 Fracture toughness</td>
<td>$\Gamma_{core}$</td>
<td>625 $J/m^2$</td>
</tr>
</tbody>
</table>
6.4.3 Results and Comparison with Numerical Model

Figures 6.14, 6.16 and 6.18 the results from the experimental investigation and the theoretical simulation are presented for the specimens with H80, H130 and H200 cores.

The general observation from the experimental investigation is as expected that very different propagation patterns are reported for the different core densities.

For the specimens with H80 cores, the crack propagates just above the glue interface on the core side, see Figure 6.15. On a microscale the crack tip continues to seek up into the interface because of the negative mode-mixity, but the fracture toughness of the H80 core is so low that the crack never reaches into the interface, but is instead forced to continue to propagate just below the interface on the core side. Furthermore, the propagation is very sudden and takes place in relatively large steps. The author believes this is influenced by dynamic effects when the crack propagates, as discussed earlier. The release of elastic energy as the crack starts propagating generates stress waves that make the crack propagate unstably, especially in the case of small crack lengths, where nearly unstable crack growth is already monitored by the theoretical models, without taking the dynamic effects into account.

This behaviour is also observed in the case of the lift force vs. lift displacement curves in Figure 6.14, where the sudden crack propagation generates curves with hard corners, when initiation of the crack propagation is reached and followed by a build-up of forces to the next point of propagation.

Furthermore, two crack tips are present in the specimen at the same time. This kind of crack propagation behaviour will introduce asymmetry in the system, since the propagation will not take place at the same time at both crack tips, because of imperfections in both the test rig and the test specimen. Especially for the initial debond propagation, this asymmetry has a large influence on the initiation force, because the asymmetry tilts the loading over to one of the crack tips, which results in premature crack propagation initiation.

For the specimens with H130 and H200 cores a very different crack propagation behaviour is observed, Figures 6.16 to 6.19. In the H130 specimens the propagation is for a small crack length similar to the one seen in the H80 specimens, but fibre bridging is quickly observed, because the stronger core makes the crack kink up into the interface between the core and the CSM mat, see Figure 6.17. The CSM mat consists of separate short chopped fibres, which are relatively easy to pull out, and thus large-scale fibre bridging can occur fairly easy.

For the H200 specimens, the fibre bridging occurs earlier, but the behaviour is subsequently very similar for H130 and H200 specimens, see Figure 6.19. It can also quite clearly be seen on the lift force vs. lift displacement curve in Figures 6.16 and 6.18 that the fibre bridging occurs between 5-10 mm lifting, because the lift force is increased rapidly. The individual, bridged fibres absorb a lot of energy and restrain the crack flanks from opening even more.
6.4 Experimental Validation

Figure 6.14: Lift force vs. lift displacement for the H80 case.

Figure 6.15: Typical subinterface crack propagation in H80 specimens.
Chapter 6. Face Tearing Debonding in a Deck Structure of a Sandwich Vessel

Figure 6.16: Lift force vs. lift displacement for the H130 case.

Figure 6.17: Typical interface crack propagation in H130 specimens.
6.4 Experimental Validation

Figure 6.18: Lift force vs. lift displacement for the H200 case.

Figure 6.19: Typical interface crack propagation in H200 specimens.
Figures 6.14 and 6.16 show results from the 2-D propagation model and the 3-D model. All three boundaries, upper, lower and average, and the average fibre bridging values from the fracture toughness distributions in Table 4.11 are used as input to the 2-D propagation model, while only the average values are used in the 3-D model. Furthermore, the 3-D model is used for crack lengths of 44.4 mm, 84.4 mm, 132.4 mm, 224.4 mm and 340.4 mm giving an almost complete span of the possible propagation length in the specimen.

For the H130 specimens a parameter study has furthermore been carried out with respect to the face thickness.

Generally, overall good correlation is observed between test and calculation up to fibre bridging. It is also observed that the experimental results confirm the results from the fracture mechanics material testing: that there is extensive scatter in the fracture toughness values, and that this scatter is included in the calculation results using the upper and lower fracture toughness border described earlier. It is observed in Figures 6.14, 6.16 and 6.18 for the H80, H130 and H200 specimens, where no fibre bridging is present, that the peaks of the experimental tests are nearly all included in the theoretical results from the 2-D propagation model using the upper and lower boundary as fracture mechanics input. This proves that there is a consistency between the fracture mechanics material data and the theoretically obtained values in this test. It should furthermore be noted that the results from the 3-D model agree well, both with regard to the propagation initiation value and when the experimental uploading path between each propagation is compared with the results from the 3-D model.

The overall good agreement between the numerical model and the experimental results for non-fibre bridging propagation is for very small crack lengths and lift displacements, below around 2 mm, while larger differences are seen between the theoretical and experimental results. As already mentioned, the crack propagation for small crack lengths occurs asymmetrically, which means that considerably smaller initiation forces can be expected. Furthermore, in this area the stiffness of the rather crude loading arrangement of the test rig has a large influence, and it cannot be expected that the initiation values will correlate between the theoretical model and the experimental results. The initiation value is overpredicted by the theoretical models.

The initiation forces from the experimental and numerical results are found in Table 6.7.

As mentioned earlier air-coupled ultrasonic scans have been carried out in situ during the testing to estimate the crack length at different lifts. One scan is approximately done for each 10 mm of crack propagation.

The quality of ultrasonic results is dependent on the core density. The sharpest picture of the crack front is obtained for the high-density cores. This is due to lower attenuation in the high-density cores. In order to be able to use the scans in the structural evaluation of the crack propagation, the scan plots have to be postprocessed. The postprocessing is mainly focused on two factors: Extrapolation and linearisation of the crack front, so that the propagation
6.4 Experimental Validation

Table 6.7: Initiation forces from experimental and numerical results.

<table>
<thead>
<tr>
<th>Model</th>
<th>H80 [kN]</th>
<th>H130 [kN]</th>
<th>H200 [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEST 1</td>
<td>1.620</td>
<td>2.637</td>
<td>4.004</td>
</tr>
<tr>
<td>TEST 2</td>
<td>1.932</td>
<td>2.280</td>
<td>3.645</td>
</tr>
<tr>
<td>TEST 3</td>
<td>1.624</td>
<td>2.658</td>
<td>4.399</td>
</tr>
<tr>
<td>TEST 4</td>
<td>1.965</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2-D LOWER</td>
<td>2.542</td>
<td>2.835</td>
<td>5.726</td>
</tr>
<tr>
<td>2-D AVERAGE</td>
<td>2.722</td>
<td>3.582</td>
<td>5.371</td>
</tr>
<tr>
<td>2-D UPPER</td>
<td>2.882</td>
<td>4.193</td>
<td>5.727</td>
</tr>
<tr>
<td>3-D AVERAGE</td>
<td>2.646</td>
<td>3.460</td>
<td>-</td>
</tr>
</tbody>
</table>

distance on the front and back sides of the beam can be determined. Identification of the crack lengths is based on the initial reference position scanning. In Figure 6.20 the linearised crack propagation lengths based on the ultrasonic scans are shown as red lines on the scan plots.

Figures 6.21 and 6.22 show the crack length (determined from the ultrasound scans) vs. the lift displacement. It is also here seen that the theoretical 2-D propagation model overpredicts the crack length up to a lift displacement of about 8 mm. This is even more distinct for the H130 and H200 specimens, where the complete prediction of the crack length is inaccurate, because of fibre bridging from about 5-10 mm lift displacement and forward. Additionally, the crack length estimation is also carried out visually, but very little difference is observed compared to the NDI values.

Finally, the effect of changing the face thickness to 4.0 mm and 5.0 mm has been investigated on the H130 specimen as is seen in Figure 6.16, because the hand lamination production technique used for the test specimens results in a slightly varying face thickness. However, relatively weak overall dependence is observed. Furthermore, it has also been attempted to model the fibre bridging by using the higher average fibre bridging distribution from the fracture mechanics material tests, even though fibre bridging cannot be treated by use of linear fracture mechanics, which is the basis of the CSDE mode-mixity method. The result is shown in Figure 6.16, and it can be concluded that fibre bridging cannot be modelled even by using the high fracture toughness values from the material testing.

The overall conclusion on the face tearing beam specimens is that there is an overall good correlation between the theoretical and experimental results as long as there is no fibre bridging. In the case of fibre bridging the theoretical model predicts a conservative result. Comparison of the 2-D and 3-D model shows a very good correlation between the different models for both H80 and H130 specimens as is seen in Figures 6.14 and 6.16.
Figure 6.20: Ultrasonic scan results from in situ scanning. The upper left picture shows the reference scan used for absolute position reference. The other three pictures show scans of panels with different core densities and different shapes of the crack front. The scale on the plots is in dB and the debonded areas are indicated by the green/yellow colors.

Figure 6.21: Crack length vs. lift displacement for the H80 specimens.
Figure 6.22: Crack length vs. lift displacement for the H130 (a) and H200 (b) specimens.

6.5 Conclusion on Face Tearing Debonding

The first part of this chapter presented and compared two different methods for prediction of crack propagation in a sandwich structure: a fracture mechanics approach, where the CSDE mode-mixity method was used, and a local damage mechanics approach. The two methods were applied to a real-life example, where the superstructure in a vessel pulled the skin off the sandwich deck.

Excellent agreement was achieved between the two numerical approaches, and it is believed that both presented methods have a great potential in the evaluation of the residual strength of damaged sandwich structures for configurations where crack propagation and non-linear structural behaviour dominate the final collapse modes.

The last part of the chapter presented experimental tests carried out to verify the numerical fracture mechanics models. The comparison showed that as long as no fibre bridging is present good agreement is achieved between experimental and numerical results. In the case of fibre bridging, the numerical models predict a conservative result.

In this study this means that structures with core densities up to 80 kg/m$^3$ can be modelled with an accuracy acceptable for engineering applications. Heavier and stronger core materials cannot be modelled accurately - the predictions are however, conservative.
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Chapter 7

Residual Strength of Debonded Panel Types in Sandwich Vessels

Figure 7.1: Different panel load types in a vessel: Deck/bottom panel with uniform compression (right), side panel with non-uniform compression (middle) and a bottom panel with lateral pressure (left).

In a ship there are basically three different panel types, corresponding to their respective loading profiles. In Figure 7.1 these three panel types are shown. Combinations of all three loading types are common.

The three panel types and their corresponding loading profiles are:

- Bottom or deck panel - \textit{UNIFORM IN-PLANE COMPRESSION}
Chapter 7. Residual Strength of Debonded Panel Types in Sandwich Vessels

- Side panel - *NON-UNIFORM IN-PLANE COMPRESSION*
- Bottom panel - *LATERAL PRESSURE*

The loading on the panels in the vessel is usually a product of the global loading of the hull beam, resulting in either a uniform in-plane compression or tension profile of the deck and bottom panels and a non-uniform in-plane compression profile of the side panels. Additionally, local loading from water pressure and slamming incident acting on the bottom panels can occur. With respect to the global hull loading, this kind of loading may be a result of global hogging and sagging of the hull beam when it moves in the waves, but the deformations of the hull beam may also be a result of local loads such as slamming, so that two load cases can be combined. This is not investigated in the present thesis, but could be a subject of further investigations.

Based on the three different loading profiles shown in Figure 7.1, three experimental test series have been carried out on panels with circular debonds. Two of the test series, uniform compression and non-uniform compression, have also been investigated by the theoretical 3-D residual strength model using the 3-D version of the CSDE method, as described earlier. The lateral pressure test series have only been tested experimentally, while the theoretical modelling has been left for future work.

The goal of these test series is to show the validity of the theoretical models, but also to determine the residual strength of the different panel types by testing intact panels as well.

The three test series will subsequently be designated SERIES 1, SERIES 2 and SERIES 3, corresponding to the three cases shown in Figure 7.1.

SERIES 1 and SERIES 2 have been carried out at the DTU Strength Laboratory. Both these test series were partly carried out as a part of a B.Sc. and M.Sc. examination project, with the author as supervisor. SERIES 1 involved the design and construction of a complete, new test rig, which was done with great skill by two B.Sc. students. SERIES 2 involved modification of an existing test rig, The Giant Tearing Machine, which was originally designed and constructed to tear up to 20 mm steel plates without starter crack, Simonsen and Törnqvist (2004). The modification was likewise done with great skill by two M.Sc. students and included a modification of the test rig, so the loading direction was changed from tension to compression. The main parts of the test rig were reused, while most of the internal structure of the test rig around the panel was made anew. Construction details concerning the two test rigs are found in the two theses from the projects, Jørgensen and Christensen (2004) and Nøkkentved and Lundsgaard-Larsen (2004).

SERIES 3 has been carried out by the author in collaboration with the staff at the VTT Technical Research Center of Finland in an existing test rig, which will be described in this chapter.

The complete batch of test specimens for all three test series was produced by DANYARD AALBORG. Furthermore, both experimental and numerical results serve as input to inspection manuals, which are the end product of the saNDI-project.
7.1 Uniform Compression of a Bottom/Deck Panel

In this section SERIES 1, the uniform compression of a bottom/deck panel, is treated. The experimental testing is described followed by a short presentation of the specific SERIES 1 numerical model. Finally, the experimental and numerical results are compared and conclusions are drawn.

7.1.1 Experimental Setup

The Test Rig

As mentioned above a new test rig has been produced in order to introduce a uniform in-plane compression loading on a sandwich panel. The test rig is presented schematically with dimensions in Figure 7.2.

The test rig consists of two vertical towers dimensioned to withstand the horizontal forces, originating from Poisson's effect when the panel is compressed in the vertical direction. The forces are thus transferred into the underlying bottom plate, and the connection between the towers and the bottom plate is effected by four adjustment devices, also able to adjust the horizontal distance between the towers and make it possible to test panels with varying breadth. Furthermore, four vertical and adjustable steel ribs ensure that the panel edges are not able to move in the out-of-plane direction. Teflon tape is attached to the inside of the ribs, thus limiting friction between panel and steel ribs.

Finally, the loading is introduced by use of a horizontal steel beam attached to the test machine. It is specially shaped to be able to slide down between the steel ribs. The position of the beam can furthermore be adjusted in the out-of-plane direction.

The loading beam and the test rig inserted in the test machine are shown in Figure 7.3a. The test machine used in these tests is seen in Figure 7.3b. It is a Mohr & Federhaff universal testing machine with a maximum compression capacity of 200 tons. The tests are performed in displacement control, and the accuracy of the load cell is validated in connection with these tests and proves to be fairly accurate even for relatively limited forces below 10% of the maximum capacity.

Apart from the vertical load, the out-of-plane displacement of the midpoint of both sides of the panel is measured with two pin gauges attached to the towers. This makes it possible to measure the opening of a debond located at the centre of the panel.

The crosshead displacement is also measured but is less interesting, because this crosshead displacement also includes deformation at the load introduction contact points, which is not included in the numerical modelling. The displacement of the panel will instead be estimated with strain gauges on the panel itself, see below.
Figure 7.2: Schematic presentation of the SERIES 1 test rig for uniform in-plane compression loading.
Panel Specifications and Material Properties

Ten sandwich panel specimens are tested. Seven damaged panels with central, circular debonds and three intact panels are used with the specifications given in Table 7.1.

The face consists of hand layup glass fibre mats of the quadro-axial type DBLT850 and chopped strand mat CSM300. The resin is polyester and the total thickness is approximately 3.2 mm.

The face properties are found by material testing at RISO National Laboratory. Similar to the beam specimens described earlier, only specimens consisting of DBLT850 are used in these material tests. Therefore, the laminate properties of the faces are found by correcting the in-plane properties of the DBLT850 laminates, using table data for the CSM300 mats, Zenkert (1997). The resulting mechanical face properties are seen in Table 7.2, together with the table properties for the Divinycell H80 and H200 cores.

The panel geometry is shown in Figure 7.4 and the geometrical panel properties in Table 7.3. Because of the boundary condition enforced by the test rig, Figure 7.2, only a part
of the panel will be able to deflect out-of-plane. In this connection, note the reinforcement plywood inserts at the vertically loaded edges of the panel, used to avoid crushing of the core. In addition, horizontal steel ribs are bolted on the outside of the reinforced panel edges, to avoid debonding and propagation into the out-of-plane active panel between the faces and the plywood inserts.

In Figure 7.4 the positions of the four strain gauges are indicated.

Table 7.1: Distribution of test specimens on damage type and core density used in the SERIES 1 tests. All debond damages are central circular debonds. IDO=Initial Debond Opening for the A and B specimens, when measured and applicable.

<table>
<thead>
<tr>
<th>Panel Designation</th>
<th>Core Type</th>
<th>Damage type</th>
<th>Specimens</th>
<th>IDO A</th>
<th>IDO B</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPE S1.1</td>
<td>H80</td>
<td>Ø100 mm</td>
<td>2</td>
<td>2 mm</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S1.2</td>
<td>H80</td>
<td>Ø200 mm</td>
<td>2</td>
<td>2 mm</td>
<td>3.5 mm</td>
</tr>
<tr>
<td>TYPE S1.3</td>
<td>H80</td>
<td>Ø300 mm</td>
<td>2</td>
<td>3 mm</td>
<td>3 mm</td>
</tr>
<tr>
<td>TYPE S1.4</td>
<td>H80</td>
<td>Intact</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S1.5</td>
<td>H200</td>
<td>Ø200 mm</td>
<td>1</td>
<td>1 mm</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S1.6</td>
<td>H200</td>
<td>Intact</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.2: Mechanical properties of the GFRP faces [CSM (0/45/90/ -45)_s CSM] and Divinycell H80 and H200 PVC foam cores, DIAB (2000).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face in-plane Young’s modulus</td>
<td>$E_X$</td>
<td>12820 MPa</td>
</tr>
<tr>
<td>Face in-plane Young’s modulus</td>
<td>$E_Y$</td>
<td>12820 MPa</td>
</tr>
<tr>
<td>Face out-of-plane Young’s modulus</td>
<td>$E_Z$</td>
<td>8400 MPa</td>
</tr>
<tr>
<td>Face in-plane G-modulus</td>
<td>$G_{XY}$</td>
<td>4600 MPa</td>
</tr>
<tr>
<td>Face out-of-plane G-modulus</td>
<td>$G_{XZ}$</td>
<td>2700 MPa</td>
</tr>
<tr>
<td>Face out-of-plane G-modulus</td>
<td>$G_{YZ}$</td>
<td>2700 MPa</td>
</tr>
<tr>
<td>Face in-plane Poisson’s ratio</td>
<td>$\nu_{XY}$</td>
<td>0.295</td>
</tr>
<tr>
<td>Face out-of-plane Poisson’s ratio</td>
<td>$\nu_{XZ}$</td>
<td>0.32</td>
</tr>
<tr>
<td>Face out-of-plane Poisson’s ratio</td>
<td>$\nu_{YZ}$</td>
<td>0.29</td>
</tr>
<tr>
<td>Core H80 Young’s modulus</td>
<td>$E_c$</td>
<td>85 MPa</td>
</tr>
<tr>
<td>Core H80 G-modulus</td>
<td>$G_c$</td>
<td>31 MPa</td>
</tr>
<tr>
<td>Core H80 Poisson’s ratio</td>
<td>$\nu_c$</td>
<td>0.35</td>
</tr>
<tr>
<td>Core H200 Young’s modulus</td>
<td>$E_c$</td>
<td>310 MPa</td>
</tr>
<tr>
<td>Core H200 G-modulus</td>
<td>$G_c$</td>
<td>90 MPa</td>
</tr>
<tr>
<td>Core H200 Poisson’s ratio</td>
<td>$\nu_c$</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Figure 7.4: SERIES 1 panel geometry with debond location, strain gauge location and reinforcement approach.

Table 7.3: SERIES 1 panel dimensions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active panel height</td>
<td>A</td>
<td>510 mm</td>
</tr>
<tr>
<td>Active panel width</td>
<td>B</td>
<td>500 mm</td>
</tr>
<tr>
<td>Reinforcement length</td>
<td>$L_R$</td>
<td>55 mm</td>
</tr>
<tr>
<td>Support breadth</td>
<td>$s$</td>
<td>30 mm</td>
</tr>
<tr>
<td>Face thickness</td>
<td>$t_f$</td>
<td>3.2 mm</td>
</tr>
<tr>
<td>Core thickness</td>
<td>$t_c$</td>
<td>25 mm</td>
</tr>
<tr>
<td>Vertical SG position</td>
<td>$A_{SG}$</td>
<td>75 mm</td>
</tr>
<tr>
<td>Horizontal SG position</td>
<td>$B_{SG}$</td>
<td>150 mm</td>
</tr>
</tbody>
</table>
Chapter 7. Residual Strength of Debonded Panel Types in Sandwich Vessels

7.1.2 Numerical Model

The 3-D residual strength model will be used for the theoretical predictions of the propagation initiation and failure load. The model has already been described in detail earlier. Therefore, only a summary will be given here.

The main properties of the model used to simulate the failure of the SERIES 1 panels are as follows:

- The 3-D residual strength model on the basis of the CSDE 3-D mode-mixity method is applied, using input from the average interface fracture toughness distributions in Table 4.11
- The 1/4-model with a displacement consisting of linear and parabolic solid elements with 50,000-100,000 nodes, by use of the DCPD device, see Figure 7.5
- Refined near tip mesh, see Figure 7.6

As indicated earlier, only the active part of the panel will be modelled. In order to simulate the boundary conditions in the test rig as realistically as possible, the boundary conditions are chosen as presented in Table 7.4 by using the panel boundary definitions from Figure 7.4. The horizontal translation of the B1 and B2 panel edges is furthermore held when the horizontal movement reaches 0.15 mm. This choice is made on the basis of an assumption that it is not possible in practice to achieve full initial horizontal in-plane boundary restriction, because initial small scale deformation of the panel edge softens the edge restriction. Furthermore, an initial small gap may be present between the towers and the panel as a result of the horizontal adjustment of the towers in order to avoid introducing initial stresses in the panel by pressing the towers against the panel.

Table 7.4: Boundary conditions for the SERIES 1 panels.

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>$U_Z=0$, $U_X=0$ when $U_X=-0.15$</td>
</tr>
<tr>
<td>B2</td>
<td>$U_Z=0$, $U_X=0$ when $U_X=0.15$</td>
</tr>
<tr>
<td>B3</td>
<td>$U_X=U_Z=0$</td>
</tr>
<tr>
<td>B4</td>
<td>$U_X=U_Z=0$</td>
</tr>
<tr>
<td>B5</td>
<td>$U_X=U_Z=0$</td>
</tr>
<tr>
<td>B6</td>
<td>$U_X=U_Y=U_Z=0$</td>
</tr>
<tr>
<td>B7</td>
<td>$U_X=U_Y=U_Z=0$</td>
</tr>
<tr>
<td>B8</td>
<td>$U_X=U_Y=U_Z=0$</td>
</tr>
<tr>
<td>S1</td>
<td>$U_Z=0$</td>
</tr>
<tr>
<td>S2</td>
<td>$U_Z=0$</td>
</tr>
</tbody>
</table>
Figure 7.5: SERIES 1 1/4-model for the H80 Ø200 mm case in the deformed state (displacement scaling of x5 has been used).

Figure 7.6: Near tip zone in the 3 o’clock position for the H80 Ø200 mm case in the deformed state.
### 7.1.3 Comparison of Experimental and Numerical Results

In Figure 7.7 a typical failure scenario from the experimental investigation is seen. The specimen shown in these pictures is the H80 Ø200 mm panel. Similar presentations for the remaining test specimens are found in Appendix B.

Initially, only a small debond opening is present originating from degassing from the core, as is seen in Figure 7.7, and as the vertical load increases the debond opening will only grow slightly or not at all. At the debond buckling load, the debond opening is increased rapidly up to the debond propagation load. As soon as the debond propagation has started the crack front moves in a split second to the boundaries of the panel. The post-debonded panel is shown in Figure 7.7. This behaviour means that the assumption made in the modelling is justified for this type of panels, because the debond initiation propagation load can be regarded as the failure load of the panel.

Both conventional coin-tapping technique and air-coupled ultrasonic non-destructive inspection, described earlier, have been used to estimate the extent of the debond after failure. The post-failure debonding is shown in Figure 7.8, where it is quite clear that the initially circular-shaped debond has propagated into a debond-band across the panel width.

Furthermore, it should also be mentioned that the propagation does not take place at exactly the same moment on both the left and right side of the debond, but as soon as one side has propagated, the other side follows closely after.

This failure behaviour is observed for both the debonded H80 and H200 panels. It has not been possible to identify fibre bridging in any of the H80 or H200 panels. The same applies to kinking of the crack front into the core.

To be able to estimate the residual strength of the panel specimens, intact panels have also been tested. Common to all specimens is that they fail in compression in the faces. In Figure 7.9 the panel is seen before and after failure. As it is also clearly shown in Figure 7.10, the panel is subjected to large-scale debonding.

The compression strength of the multi-axial part of the faces has also been tested at RISO National Laboratory ($X_c = Y_c = -247$ MPa), and it indicated a compression failure load for the whole panel of 701 kN for the H80 case and 748 kN for the H200 case, which in both cases is considerably more than measured in the tests, see Table 7.5.

The global buckling strength of the intact panels has been investigated numerically by use of non-linear geometrical calculations, but yielded higher failure loads, both compared to the measured values and the theoretical compressive failure loads mentioned above.

Finally, the wrinkling failure load has been calculated analytically for the H80 and H200 intact panels and it yielded a failure loads of 594 kN and 1392 kN respectively. It is therefore assumed for the H80 case, in connection with the low intact failure loads and the relatively high degree of geometrical imperfections in the test panels, that the face compression failure is introduced by wrinkling failure of the faces.
Figure 7.7: The H80 panel with Ø200 mm debond (B specimen) before failure (left) and after failure (right).

Figure 7.8: The H80 Ø200 mm panel (B specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
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Figure 7.9: The H80 intact panel (B specimen) before failure (left) and after failure (right).

Figure 7.10: The H80 intact panel (B specimen) after testing, where the visual debond propagation is observed (left) and the current amount of debond (right) by use of air-coupled ultrasonic scanning. Note that debond damage to both the upper and lower interface cannot be distinguished by the air-coupled ultrasonic NDI technique.
Table 7.5: Measured experimental and theoretically predicted failure loads for SERIES 1 panels. (*) Analytically calculated wrinkling failure load. (**) Theoretical compression failure load.

<table>
<thead>
<tr>
<th>Panel designation</th>
<th>Core type</th>
<th>Damage type</th>
<th>Experimental failure load</th>
<th>Predicted failure load</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPE S1_1A</td>
<td>H80</td>
<td>Ø100 mm</td>
<td>222 kN</td>
<td>303 kN</td>
</tr>
<tr>
<td>TYPE S1_1B</td>
<td>H80</td>
<td>Ø100 mm</td>
<td>218 kN</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S1_2A</td>
<td>H80</td>
<td>Ø200 mm</td>
<td>162 kN</td>
<td>152 kN</td>
</tr>
<tr>
<td>TYPE S1_2B</td>
<td>H80</td>
<td>Ø200 mm</td>
<td>139 kN</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S1_3A</td>
<td>H80</td>
<td>Ø300 mm</td>
<td>140 kN</td>
<td>116 kN</td>
</tr>
<tr>
<td>TYPE S1_3B</td>
<td>H80</td>
<td>Ø300 mm</td>
<td>127 kN</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S1_4A</td>
<td>H80</td>
<td>Intact</td>
<td>270 kN</td>
<td>*594 kN</td>
</tr>
<tr>
<td>TYPE S1_4B</td>
<td>H80</td>
<td>Intact</td>
<td>370 kN</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S1_5A</td>
<td>H200</td>
<td>Ø200 mm</td>
<td>180 kN</td>
<td>221 kN</td>
</tr>
<tr>
<td>TYPE S1_6A</td>
<td>H200</td>
<td>Intact</td>
<td>414 kN</td>
<td>**748 kN</td>
</tr>
</tbody>
</table>

Figure 7.11: Measured experimental and theoretically predicted failure loads for SERIES 1 H80 specimens.
In Table 7.5 the failure load of both debonded and intact panels is seen together with the failure mechanism, and in Figure 7.11 the experimental results are compared with the calculated numerical results. It is quite obvious from these results, from an overall perspective, that the failure load is decreasing as the debond diameter is increased, as would also be expected, because the debond buckling load, which controls when the debond is opened, is decreasing with increasing debond diameter.

In Figures 7.12 to 7.14 the results from both the experimental testing and the theoretical simulation with the 3-D residual strength model are presented, for each of the SERIES 1 debonded H80 and H200 panels. Theoretical values for the intact panels are not included here, because the failure mechanism of the intact panels falls outside the scope of this thesis. In order to get reliable results refined analysis is required of the field of compression failure of intact laminates on a soft foundation (the core) in combination with the effect of production imperfections.

From the results for the H80 Ø100 case in Figures 7.11 and 7.12 it can be concluded that the theoretical results overestimate the buckling and the failure load of the panel, compared to the experimental results, by about 118-146% for the buckling load and 37-39% for the failure load, while the stiffness of the panel is quite accurately predicted as seen in Figure 7.12 (top). In order to explain these discrepancies, several factors must be addressed. First, it must be concluded that the theoretical prediction of the buckling load has a substantial influence on the failure load, as the buckling load governs when the debond is opened up and subsequently propagated. It is also widely known from normal buckling analysis that imperfections have a strong influence on the buckling load for short and fat beams, which may also apply to the debonded panel case. For debonds with a large \( t_f/D \)-ratio, where \( D \) is the debond diameter, the imperfection must have a big influence on the buckling load and subsequently also on the failure load. The difference in buckling load is shown in Figure 7.12 (bottom), where the buckling load can be identified, when the debond opening starts increasing.

Certain kinds of imperfections have also been investigated by use of the theoretical model - the initial debond opening has been varied as seen in Figure 7.12, but this seems to have a very small influence on the buckling and failure load, except for one case: When the initial debond opening is very close or exceeds the propagation opening. In this case the failure load is reached without any buckling behaviour, most likely because the debond must be reshaped after the initial opening governed by the DCPD.

When the propagation opening of the debond measured in the test and the corresponding opening from the theoretical model are compared, it must be concluded that there is a low degree of agreement. This could be explained by the fact that the crack front is relatively short, and the very local crack tip is dependent on the manufacturing of the artificial debonds and the resulting local shape of the crack front. For longer cracks length, these effects from production must be smaller, because the global buckling behaviour of the debond dominates.

Finally, the effect of un-even loading of the two faces during testing because of a tilting load surface on the specimen cannot be ruled out.
A comparison of experimental and theoretical results for the H80 and H200 cases in Figures 7.11 and 7.13 shows a much better agreement. For the H80 specimens the difference in failure load lies between 6-9% while it is 23% for the H200 specimens. Furthermore, it should be observed that the debond opening makes a jump at about 160 kN, which may be because the debond propagated on the left side first.

The buckling load is more difficult to compare because no definite buckling point can be observed in either the A or B specimen, but only a gradual transition into a buckled debond state. Again this behaviour can be addressed to the theoretical influence of imperfections, which decreases the ideal bifurcation buckling behaviour, resulting in this graduate out-of-plane bending of the debonded face laminate.

The initial debond opening has again been varied as in the Ø100 mm case, and again no influence is seen on either the buckling or failure load, see Figure 7.13.

Figures 7.11 and 7.14 show the results from the H80 Ø300 mm case. The difference in failure load is between 8-16%, which is small for this kind of tests. The buckling shape predicted by the theoretical model is as the other cases described above, a one-wave buckling shape. This is unfortunately in contrast to the experimental observations, which show a two-wave buckling shape, with an opening in the top of the debond and a completely closed debond with crack flank contact in the bottom. This behaviour has been investigated by an eigenbuckling analysis to determine if other buckling shapes lie near to the one-wave buckling mode, but the analysis shows that higher buckling shapes lie at considerably higher load values. The most likely explanation is that production imperfections along the crack front of the artificial debond restrain the debond from going into the theoretically determined least energy buckling shape with one wave. For instance these test specimen production defects could be that cured resin has entered the non-cured debond area and thus formed an area on the crack front with increased adhesion.

Fibre bridging was not observed in any of the above described debonded specimens. However, fibre bridging was actually observed in some of the initial trial specimens with the H200 core. These specimens proved to be very damage tolerant and thus support the observations gained in the face tearing beam tests, described in the previous chapter. Furthermore, as also mentioned earlier, the tested fracture toughness values all included fibre bridging for the H200 case. This fact could explain why the theoretical model in the H200 Ø200 case, where no fibre bridging is observed, overestimates the failure load.
Figure 7.12: Experimental and numerical results from specimens with H80 core and Ø100 mm debond, showing the vertical load vs. the vertical displacement (top) and the debond opening vs. the vertical load (bottom). $i_d$ is the amplitude of the initial imperfection and $t_f$ is the face thickness.
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Figure 7.13: Experimental and numerical results from specimens with H80 and H200 core and Ø200 mm debond, showing the vertical load vs. the vertical displacement (a) and the debond opening vs. the vertical load (b). $\delta_0$ is the amplitude of the initial imperfection and $t_f$ is the face thickness.
Figure 7.14: Experimental and numerical results from specimens with H80 core and Ø300 mm debond, showing the vertical load vs. the vertical displacement (a) and the debond opening vs. the vertical load (b). \( \text{id}_{0} \) is the amplitude of the initial imperfection and \( t_f \) is the face thickness.
In Figure 7.15 the *propagation initiation index* \(^1\), \(\Pi\), defined as

\[
\Pi = \frac{G}{\Gamma(\Psi)} \quad (7.1)
\]

has been plotted at the propagation load in a polar diagram along the complete crack front (360°) for all H80 panel cases, Ø100, Ø200 and Ø300 mm. The propagation index is normalised so that it will be equal to unity if the Griffith-energy in a certain crack front position and mode-mixity is equal to the fracture toughness for the present mode-mixity.

At propagation the \(\Pi\)-index distribution along the crack front is somewhat binocular-shaped, with peak positions at 0° and −180° and minimum value positions at −90° and 90°. This is also confirmed by experimental observations, because the initiation takes place around the 0° and −180° positions forming a propagation band across the panel to the panel edges. The minimum value positions, being at −90° and 90°, confirm the observations of a nearly or completely unloaded debond in the top and bottom crack front positions.

When the different debond diameters are compared in Figure 7.15, it is seen that the binocular shape becomes more extreme and more flat for large debonds, which means that the propagation initiation position becomes more localised for large debonds compared to the face thickness, or in terms of the \(t_f/D\)-ratio, the propagation initiation front become more localised for small \(t_f/D\)-ratios.

Figure 7.15: *Numerically obtained propagation initiation indexes, \(\Pi\), for the H80 panels with the debond diameters Ø100 mm, Ø200 mm and Ø300 mm.* Note that the result from each crack front position is connected with straight lines and the loading direction is vertical.

---

\(^1\)From now on designated the \(\Pi\)-index
Figure 7.16: Numerically obtained propagation initiation indexes, $\Pi$, plotted along the complete 360° circular crack front for different stages along the deformation behaviour path for the SERIES 1 panels: (a) H80 Ø100, (b) H80 Ø200, (c) H80 Ø300 and (d) H200 Ø200. The loading direction is for all cases vertical.
7.1 Uniform Compression of a Bottom/Deck Panel

In Figure 7.16 the Π-index has again been plotted in a polar diagram along the complete crack front for all panel cases, 100, 200 and 300 mm, but for different locations in the loading/deformation history path.

It is clearly seen that the Π-index goes from a circular distribution in the initial loading history to a compressed shape at the bifurcation buckling load and subsequently to the binocular shape as the loading increases further into the post-buckled regime. For the 300 case the point of vertical panel boundary contact with the test rig tower has also been included, but does not seem to have any influence on the Π-index distribution.

In order to investigate the influence of the mesh size in the outer regions of the panel a harsher mesh has been tested in the theoretical model, and the result is shown in Figure 7.12. However, it can be concluded that the harsh mesh in the outer regions has no influence on the debond buckling or the failure load.

The influence of the panel boundaries on the buckling and failure load has also been investigated numerically. The result is presented in Figure 7.12, where the type of panel boundary condition on the vertical panel edges has been varied between fully constrained and free in the horizontal direction. The influence is relatively weak. The buckling load is 286 kN and the failure load 296 kN in the fully constrained case and 311 kN and 316 kN in the free boundary case. This should be compared with the varying boundary conditions as described in Table 7.4, where the buckling load is 293 kN and the failure load is 303 kN.

In Figure 7.17 the influence of distance from the debond front to the panel boundaries has been investigated. The active panel area (panel inside the supporting ribs and loading edges) has been varied, as shown in Table 7.6, while the $t_f/D$-ratio has been held constant. Furthermore, in order to be able to compare the different panel sizes, the loading is treated as a global strain field measured in microstrain.

Different initial debond openings have been used, for numerical convergence reasons, but it can be concluded that only for a very small active panel area, below 127.5 x 125 mm, any influence in buckling and failure load can be observed. In general, this implies that (for this $t_f/D$-ratio) the loading in this debond case may be treated as a far field loading independent of the panel size.

It may be concluded that the experimental and theoretical results in SERIES 1 agree well in the Ø200 mm case for both failure and buckling load, but agree less well in the H80 Ø100 mm and Ø300 mm case, even though the failure loads in the Ø300 mm case agree relatively well. The most likely reason for the difference can in both cases be addressed to the combination of problems in the prediction of the debond buckling load and the imperfections in the crack front tip introduced by the artificial debond production method.
Figure 7.17: Debond opening vs. microstrain, panel size investigation, Ø100 and full side restriction.

Table 7.6: Panel dimensions used to investigate the influence of panel boundaries on the panel strength. The dimensions are chosen according to Figure 7.4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Unit</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active panel height</td>
<td>$A$</td>
<td>mm</td>
<td>1020</td>
<td>510</td>
<td>255</td>
<td>200</td>
<td>127.5</td>
</tr>
<tr>
<td>Active panel breadth</td>
<td>$B$</td>
<td>mm</td>
<td>1000</td>
<td>500</td>
<td>250</td>
<td>200</td>
<td>125</td>
</tr>
<tr>
<td>Debond diameter</td>
<td>$D$</td>
<td>mm</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Reinforcement length</td>
<td>$L_R$</td>
<td>mm</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Support breadth</td>
<td>$s$</td>
<td>mm</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
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<tr>
<td>Face thickness</td>
<td>$t_f$</td>
<td>mm</td>
<td>3.2</td>
<td>3.2</td>
<td>3.2</td>
<td>3.2</td>
<td>3.2</td>
</tr>
<tr>
<td>Core thickness</td>
<td>$t_c$</td>
<td>mm</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>
7.2 Non-Uniform Compression of a Side Panel

In this section SERIES 2, the non-uniform compression of a side panel, will be described. Just as in the previous chapter about the SERIES 1 tests, the experimental testing will be described first and then followed by a short presentation of the specific SERIES 2 numerical model. Finally, the experimental and numerical results will be compared and conclusions drawn.

7.2.1 Experimental Setup

The Test Rig

As mentioned in the introduction to this chapter, an existing test rig has been modified, in order to introduce a non-uniform in-plane compression loading on a sandwich panel. The test rig is shown in Figure 7.18 and schematically presented with dimensions in Figure 7.19.

The test rig consists of two horizontal beams connected by two vertical beams. The other ends of the horizontal beams are connected to the testing machine through bearings to vertical loading beams. In order to introduce the loads into the panel without introducing unwanted damages to the panel at the loading edges, and at the same time by use of a technique which will not introduce uncertainties into the load measurement, the following loading arrangement has been produced.

As in the SERIES 1 tests the panel edges are equipped with plywood reinforcements, and through these reinforcements the panel is bolted in between two load introducing plates. The plates are bolted to the reinforced panel edges with a specified torque, determined from compression tests on the reinforcement plywood, and thus exploit the friction between the loading plates and the panel as the main loading introduction mechanism. However, all bolts are close-tolerance bolts, designed to be able to transfer the total load as a second load introduction mechanism, if the friction is not adequate.

Between the two horizontal and the two vertical spacing beams an advanced arrangement has been produced, which is able to move independently of the rest of the test rig. It is only connected to the remaining test rig through the corner bearing, connecting the horizontal beams and the vertical spacing beams. The sole role of this arrangement is to restrict out-of-plane vertical panel boundary movement. The vertical panel boundaries are held with four steel ribs, equipped with PVC strips on the inside surfaces to minimise friction forces. Contrary to the vertical edges on the SERIES 1 panels, the SERIES 2 vertical edges are not restricted in the horizontal direction, as they are not in contact with the test arrangement in the horizontal direction.

As it is seen from Figure 7.19, the ribs nearly support the panel edges in the complete length, except for a small gap at the top and bottom, which is unavoidable because of
the movement of the horizontal top and bottom beam. The small gap is neglected in the numerical calculations.

The complete weight of the test rig is about 1 ton, and this weight will crush the panel if no attempts are made to balance it. As indicated in Figure 7.19, the test rig is therefore balanced very carefully with weights, through a wire system connected to the upper right corner of the test rig.

The rig inserted into the test machine is shown in Figure 7.18a. The test machine itself used in these tests is seen in Figure 7.18b. It is an Instron servo-hydraulic universal testing machine with a maximum compression capacity of 500 tons. The tests are performed in displacement control, and the accuracy of the load cell have been, as in the SERIES 1 tests validated, and proved to be accurate even for forces below 10% of the maximum capacity.

Apart from the vertical piston load, the out-of-plane displacement of the midpoint of both sides of the panel is measured with two pin gauges attached to the vertical spacing beams. This makes it possible to measure the opening of a debond located at the centre of the panel.

The maximum vertical displacement of the horizontal beams is also measured, but as in the SERIES 1 tests, it is less interesting because this displacement will also include deformation at the load introduction contact points, which is not included in the numerical modelling. The displacement of the panel will instead be estimated by strain gauges on the panel, see below.

Figure 7.18: The SERIES 1 test rig (left) in the DTU Strength Laboratory using the 500 ton Instron servo-hydraulic universal testing machine (right).
Figure 7.19: Schematic presentation of the SERIES 2 test rig for non-uniform in-plane compression loading.
Panel Specifications and Material Properties

As in SERIES 1, ten sandwich panel specimens were tested. Seven damaged panels with central, circular debonds and three intact panels were used with the specifications given in Table 7.1.

Exactly the same materials as in SERIES 1 are used in the specimens in SERIES 2, and the material properties are in Table 7.2.

The panel geometry is shown in Figure 7.21 and the dimensions in Table 7.8. As in SERIES 1, because of the boundary condition enforced by the test rig, Figure 7.19, only a small part of the panel will be active and able to deflect in an out-of-plane direction.

In Figure 7.20 the reinforcement plywood inserts at the vertically loaded edges of the panel are shown. The drilled holes through the plywood reinforcement, used for the loading introduction close-tolerance bolts, can also be observed.

In Figure 7.21 the position of the eight strain gauges is furthermore indicated.

![Figure 7.20: The plywood reinforced load introducing edge on a SERIES 2 panel.](image)

Table 7.7: Test specimens with damage type and core density used in the SERIES 2 test. All debond damages are central circular debonds. IDO=Initial Debond Opening for the A and B specimens, when measured and applicable.

<table>
<thead>
<tr>
<th>Panel designation</th>
<th>Core type</th>
<th>Damage type</th>
<th>Specimens</th>
<th>IDO A</th>
<th>IDO B</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPE S2_7</td>
<td>H80</td>
<td>Ø100 mm</td>
<td>2</td>
<td>0 mm</td>
<td>0 mm</td>
</tr>
<tr>
<td>TYPE S2_8</td>
<td>H80</td>
<td>Ø200 mm</td>
<td>2</td>
<td>1.8 mm</td>
<td>1.0</td>
</tr>
<tr>
<td>TYPE S2_9</td>
<td>H80</td>
<td>Ø300 mm</td>
<td>2</td>
<td>4.1 mm</td>
<td>1.3 mm</td>
</tr>
<tr>
<td>TYPE S2_10</td>
<td>H80</td>
<td>Intact</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S2_11</td>
<td>H200</td>
<td>Ø200 mm</td>
<td>1</td>
<td>1.3 mm</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S2_12</td>
<td>H200</td>
<td>Intact</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 7.21: SERIES 2 panel geometry with debond location, strain gauge location and reinforcement.

Table 7.8: SERIES 2 panel dimensions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active panel height</td>
<td>A</td>
<td>700 mm</td>
</tr>
<tr>
<td>Active panel width</td>
<td>B</td>
<td>500 mm</td>
</tr>
<tr>
<td>Reinforcement length</td>
<td>$L_R$</td>
<td>50 mm</td>
</tr>
<tr>
<td>Support breadth</td>
<td>$s$</td>
<td>40 mm</td>
</tr>
<tr>
<td>Face thickness</td>
<td>$t_f$</td>
<td>3.2 mm</td>
</tr>
<tr>
<td>Core thickness</td>
<td>$t_c$</td>
<td>45 mm</td>
</tr>
<tr>
<td>Vertical SG position</td>
<td>$A_{SG}$</td>
<td>70 mm</td>
</tr>
<tr>
<td>Horizontal SG position</td>
<td>$B_{SG}$</td>
<td>100 mm</td>
</tr>
</tbody>
</table>
7.2.2 Numerical Model

The 3-D residual strength model will again be used for the theoretical predictions of the propagation initiation and failure load.

The model is very similar to the one used in the SERIES 1 tests, except that in this case a 1/2-model has to be used because of the single symmetry dictated by the non-uniform loading profile.

Again, as indicated earlier, only the active part of the panel will be modelled, and the boundary conditions have been chosen as presented in Table 7.9 using the panel boundary definitions from Figure 7.21.

Contrary to SERIES 1, the non-uniform loading profile in SERIES 2 dictated by the test rig is more complicated to estimate. The loading profile can be estimated analytically based on the geometry of the test rig. However, it has turned out that even small tolerances in the bearing between the horizontal and vertical spacing beams may distort this theoretical loading profile to a large extent and thus completely change the loading behavior in the panel.

In order to estimate the actual loading profile used in the test, the results from the eight strain gauges have been used to estimate the displacement distribution along the upper and lower horizontal loading edges. This is done by assuming that the strain distribution is more or less constant down through the panel. This assumption has been tested by use of the theoretical loading profile and it has proved to be reasonable.

For all experimental tests performed, the measured loading profile is a function of the loading parameter (time). The top and bottom strain gauges are therefore used as input to the numerical model. Examples of loading profiles are shown in Figure 7.22a and c. The remaining loading profiles are described in Appendix B.

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>$U_Z=0$</td>
</tr>
<tr>
<td>B2</td>
<td>$U_Z=0$</td>
</tr>
<tr>
<td>B3</td>
<td>$U_X=U_Z=0$</td>
</tr>
<tr>
<td>B4</td>
<td>$U_X=U_Z=0$</td>
</tr>
<tr>
<td>B5</td>
<td>$U_X=U_Z=0$</td>
</tr>
<tr>
<td>B6</td>
<td>$U_X=U_Z=0$</td>
</tr>
<tr>
<td>B7</td>
<td>$U_X=U_Z=0$</td>
</tr>
<tr>
<td>B8</td>
<td>$U_X=U_Z=0$</td>
</tr>
<tr>
<td>S1</td>
<td>$U_Z=0$</td>
</tr>
<tr>
<td>S2</td>
<td>$U_Z=0$</td>
</tr>
</tbody>
</table>
In Figure 7.22b and d, the ratio between the signal from the left and right strain gauge is shown during the test. As it is clearly seen, the ratio varies between approximately 3 to 7 for the dominant part of the test. The finite element model is observed to reproduce the measurements very well, indicating that the load is introduced appropriately in the finite element model. The varying ratio between the left and right panel edge displacement can actually be compared with a uniformly loaded panel, where the length perpendicular to the loading direction is changed, which has a radical influence on the panel response.

Using the analytically determined loading profile would mean that this ratio is constant with the value 5.47, all throughout the test. This fact emphasises the importance of using the experimentally measured loading profiles in the numerical model. Failure to do so will result in a wrong panel response.

In Figure 7.23 the 1/2-model used in SERIES 2 is shown. Use of the 1/2-model combined with the experimentally achieved loading profiles means that for each panel case, two load profiles are tested in separate calculations. The interaction between two slightly different loading profiles has also been investigated for selected cases using a full model, see below.

![Figure 7.22: Averaged loading profiles (front and back) from the H80 Ø100 (a) and 200 mm (c) case. The ratio between the left and right strain gauge signal, Ø100 mm (b) and Ø200 mm bottom (d). Measured and numerical values.]](image)
7.2.3 Comparison of Experimental and Numerical Results

Figure 7.23: SERIES 2 1/2-model for the H80 Ø200 mm case in the deformed state (displacement scaling of x5 has been used).

Figure 7.24 shows a typical failure scenario from the tests in SERIES 2. The specimen in these pictures is again an H80 Ø200 mm panel. As in the SERIES 1 case, similar pictures of the other panels tested are in Appendix B.

The deformation history of the SERIES 2 panels is very similar to the one observed in SERIES 1, but the failure scenarios in SERIES 1 and 2 are different. The debond only propagates on the most loaded side of the panel, where the loading profile is highest, which could also be expected. The failed panel is seen in Figure 7.24b.

As mentioned above, the loading profile is changing throughout the test. There are examples of nearly triangular loading profiles during the complete loading history, but also examples of loading profiles changing from a triangular-shaped profile to a trapezoidal-shaped profile at the last load steps of the loading history.

Furthermore, the loading profiles have been used to estimate the pressure centre of the loading during the test. The force is measured at the piston in the test machine, so in order to predict the force acting on the panel, a moment equilibrium has to be carried out. When the loading profile, and hence also the pressure centre, is changing throughout the loading history, the moment arm in the moment equilibrium is varying too.
7.2 Non-Uniform Compression of a Side Panel

Figure 7.24: The H80 panel with Ø200 mm debond (B specimen) before failure (left) and after failure (right).

Figure 7.25: The H80 Ø200 mm panel (B specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
Figure 7.26: H80 intact panel (B specimen) before failure (left) and after failure (right).

Figure 7.27: The H80 intact panel (B specimen) after testing, where the visual debond propagation is observed (left) and the total amount of debond (right) by use of air-coupled ultrasonic scanning.
Finally, the front edge and the maximum in-plane displacement have been used in the estimation and the comparison of the panel stiffness with the theoretical models.

Both conventional coin-tapping technique and air-coupled ultrasonic non-destructive inspection have been applied to estimating the extent of debond after failure. The post-failure debonding is shown in Figure 7.25. The ultrasonic picture in Figure 7.25b quite clearly supports the visual observation that the debond has started to propagate on the left side first. It is also seen that propagation has started on the right side but has stopped, presumably when the left side propagated to the edge of the panel.

The rapid failure behaviour is again observed for both the debonded H80 and H200 panels. And again it is not possible to identify fibre bridging or kinking in any of the H80 or H200 panels. However, in one of the H80 Ø100 mm panels the panel does not fail due to debond propagation, but in face compression in the top left corner of the panel. This is discussed further in connection with the comparison with the numerical results.

In Figure 7.26 one of the H80 intact panels is seen before and after failure. It is again common to all intact specimens that they fail in face compression, but in the SERIES 2 case they fail, as the H80 Ø100 mm, in the top or bottom left corner, followed by a propagation across the panel, as shown in Figure 7.26 for the H80 Ø200 mm specimen (B). As it may also clearly be observed in Figure 7.27 and Appendix B for the remaining intact panels, the panel is as well subjected to large scale debonding, following the compression failure.

However, compared to SERIES 1 the theoretical prediction of the failure load of the intact SERIES 2 specimens proved to agree reasonably well, as seen in Table 7.10 and Figure 7.29. The theoretical results predicted wrinkling introduced failure, see Figure 7.28, due to high bending stresses in the post-wrinkled face for the H80 specimens at 317 kN, and face compression failure at 447 kN for the H200 specimen.

Figure 7.28: Local wrinkling predicted by the finite element calculations on the H80 intact B specimen. The displacements are scaled for clarity, and only a half model is used.
Table 7.10: Measured experimental and theoretically predicted failure loads for SERIES 2 panels. (*) Face compression failure calculated using values from Nøkkentved and Lundsgaard-Larsen (2004).

<table>
<thead>
<tr>
<th>Panel designation</th>
<th>Core type</th>
<th>Damage type</th>
<th>Experimental failure load</th>
<th>Predicted failure load</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPE S2_7A</td>
<td>H80</td>
<td>Ø100 mm</td>
<td>307 kN</td>
<td>340 kN</td>
</tr>
<tr>
<td>TYPE S2_7B</td>
<td>H80</td>
<td>Ø100 mm</td>
<td>318 kN</td>
<td>342 kN</td>
</tr>
<tr>
<td>TYPE S2_8A</td>
<td>H80</td>
<td>Ø200 mm</td>
<td>180 kN</td>
<td>153 kN</td>
</tr>
<tr>
<td>TYPE S2_8B</td>
<td>H80</td>
<td>Ø200 mm</td>
<td>171 kN</td>
<td>151 kN</td>
</tr>
<tr>
<td>TYPE S2_9A</td>
<td>H80</td>
<td>Ø300 mm</td>
<td>167 kN</td>
<td>100 kN</td>
</tr>
<tr>
<td>TYPE S2_9B</td>
<td>H80</td>
<td>Ø300 mm</td>
<td>173 kN</td>
<td>100 kN</td>
</tr>
<tr>
<td>TYPE S2_10A</td>
<td>H80</td>
<td>Intact</td>
<td>248 kN</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S2_10B</td>
<td>H80</td>
<td>Intact</td>
<td>286 kN</td>
<td>317 kN</td>
</tr>
<tr>
<td>TYPE S2_11A</td>
<td>H200</td>
<td>Ø200 mm</td>
<td>195 kN</td>
<td>221 kN</td>
</tr>
<tr>
<td>TYPE S2_12A</td>
<td>H200</td>
<td>Intact</td>
<td>333 kN</td>
<td>*447 kN</td>
</tr>
</tbody>
</table>

Figure 7.29: Measured experimental and theoretically predicted failure loads for the H80 SERIES 2 panels.
The wrinkling introduced failure was predicted using a linear finite element eigen-buckling analysis with the loading profile measured at failure in the test, shown in Figure 7.28. The model confirmed the assumption about local wrinkling, because the lowest buckling shapes from this analysis all showed local wrinkling. The loading also corresponded well to the measured force values, using an imperfection knockdown factor of 0.55, analogously to the imperfection factor used in Hoffmann’s wrinkling criteria. Furthermore, several mesh densities were used in order to investigate the mesh dependence, and the mesh shown in Figure 7.28 proved to be converged. The comparison with the experimental failure loads is seen in Table 7.10 and Figure 7.29.

In Figures 7.30 to 7.32 the results from both the experiments and the theoretical simulation with the 3-D residual strength model are presented, for each of the debonded H80 and H200 panels.

By comparing experimental and numerical results for the H80 Ø100 mm case in Figures 7.29 and 7.30, it can be concluded, analogously to SERIES 1, that the theoretical results overestimate both the buckling and failure load of the panel compared to the experimental results, but in this case only by about 7-10% for the failure load, when the results from the top and bottom loading profile are averaged. This result is highly acceptable for this kind of testing. The buckling force is also visible in the A specimen, whereas it is more difficult to predict in specimen B, because the debond opening takes place more gradually. But in the A specimen the buckling force is quite close to the bifurcation buckling load predicted by the numerical model.

When the overall panel stiffness is compared for the H80 Ø100 mm specimens in Figure 7.30, larger differences may be found, especially for the last part of the loading history. A considerably stiffer response is seen in the numerical model. This is nevertheless expected, because a large amount of load is applied to the front corner of the panel, which is free in the horizontal direction, and on the very upper part of the panel edge, also an out-of-plane unsupported sandwich edge. Local bending of the sandwich due to imperfections in this area, which is not present in the theoretical model, may reduce the stiffness considerably. Furthermore, the free out-of-plane unsupported area, seen in Figure 7.24, between the vertical out-of-plane side restriction and the loaded top of the panel, is not modelled in the theoretical model, which further strengthens the assumption about local bending.

As mentioned above the H80 Ø100 mm specimen A did not actually fail by debond propagation, but in local wrinkling introduced face compression, in the same way as seen in the intact panels. However, as it is observed in Figure 7.30, debond opening is taking place at the failure load, and it is assumed that the failure just as well could have been debond propagation, as seen in specimen B. By comparison of the H80 intact and Ø100 mm specimens in Figure 7.29, it can be concluded that the Ø100 mm debond does not have any negative influence on the panel residual strength, as the Ø100 mm debonded specimens actually fail at higher loads compared to the intact panels. Whether this is just a coincidence or due to a stress redistribution in the debonded panels, because of the presence of a small debond in the center of the panel, is difficult to determine.
In the H80 and H200 Ø200 mm panel cases the same conclusions are drawn with regard to the panel stiffness, see Figure 7.31. The failure load is underpredicted by the theoretical model by 11-15% for the H80 specimens and overpredicted with 13% for the H200 specimen, which is again acceptable for this kind of analysis. With regard to the buckling load, in both H80 specimens the buckling load can be identified quite clearly, and the theoretical model gives a very good estimate for the B specimen, whereas the A specimen lies somewhat higher than the theoretical predictions. However, in both cases the buckling mode is predicted correctly by the models. For the H200 specimen, the buckling load is difficult to identify, because the buckling takes place gradually as in the H80 Ø100 mm specimen B.

Different initial debond openings are tested, but as in the SERIES 1 tests, it proves to have no influence on the failure load. However, the buckling load is slightly affected, but only with respect to the load at which the global buckling path is followed, as is seen in Figures 7.30, 7.31 and 7.32.

Figures 7.29 and 7.32 show the results from the H80 Ø300 case. In the experiments, two different buckling modes are observed. In the A specimen single wave buckling is observed, whereas a dual wave mode is observed in the B specimen. This is also observed in Figure 7.32, because the out-of-plane displacement measurement is taken from the centre of the debond. In the theoretical modelling only a single wave buckling mode is predicted, and through linear eigenvalue buckling analysis it is determined that the dual buckling mode lies at a considerably higher load. Furthermore, a very small initial debond opening, 0.05 $t_f$, is used to test whether it is possible to force the debond buckling mode into a dual wave path, but this does not prove to be successful.

When the numerical and experimental buckling loads are compared for the B specimen, the experimental buckling load is about two times the predicted load. But if the post-buckling responses are compared, it seems that the model is able to predict the same debond response behaviour. In other words, the buckling response is just shifted according to initiation of buckling. Furthermore, it is believed that the buckling load is very much dominated by the tip geometry, and how the crack front has been shaped during production of the artificial debond. This is relevant for both the A and B specimen, however during testing of the B specimen it was clearly seen that the debond only opened at a considerably smaller area compared to the full pre-produced debonded area. This observation can explain the high buckling load measured in the test. It seems likely that this imperfection has a great influence on the results, and that the failure load is underpredicted by 40-42% by the numerical model.
Figure 7.30: Experimental and numerical results from specimens with H80 core and Ø100 mm debond, showing the vertical load vs. the vertical displacement (a) and the debond opening vs. the vertical load (b).
Figure 7.31: Experimental and numerical results from specimens with H80 and H200 core and Ø200 mm debond, showing the vertical load vs. the vertical displacement (a) and the debond opening vs. the vertical load (b).
Figure 7.32: Experimental and numerical results from specimens with H80 core and Ø300 mm debond, showing the vertical load vs. the vertical displacement (top) and the debond opening vs. the vertical load (bottom).
Chapter 7. Residual Strength of Debonded Panel Types in Sandwich Vessels

Figure 7.33: Numerically obtained propagation initiation indexes, $\Pi [0, 0.1]$ (a) and $\Pi [0, 1]$ (b), plotted along the complete 360° circular crack front for different stages along the deformation behaviour path for the SERIES 2 H80 Ø200 mm panel. The loading direction is vertical.

In Figure 7.33 the propagation initiation index, defined in Eq. (7.1), is shown in a polar diagram for the H80 Ø200 mm case. In the close-up diagram it is seen that, similar to the SERIES 1 case, the $\Pi$-index distribution along the complete crack front (360°) goes from a completely circular distribution to a binocular-shaped profile. However, in the SERIES 2 the profile is oriented to the side of the highest in-plane loading and resulting largest opening of the debond. Furthermore, it is observed in Figure 7.33a that the tip loading in the areas close to $\pm 90°$ is decreasing, which indicates that the crack tip in these positions is closing during the loading history compared to the initial debond opening caused by the degassing from the core.

Both the unsymmetrical binocular-shaped profile tendency and the decreasing crack tip loading in the areas close to $\pm 90°$ are most dominant for the larger debonds, as the Ø200 mm and Ø300 mm cases, see Figure 7.34, where the $\Pi$-index is plotted for all three debond sizes. It is seen that the skewness (asymmetry) increases with increasing debond size. For the small debonds like the Ø100, the debond experiences something close to a uniform loading, similar to the one used in SERIES 1.

As mentioned earlier, full models have been built to investigate the influence of different loading profiles on the top and bottom loading edges. All H80 A specimens have been investigated, and the $\Pi$-index has afterwards been plotted in a polar diagram and compared
with the results from the half models. The result is seen in Figure 7.35, and very limited
difference is observed, only for the Ø100 mm case, there is a slight difference in the 9 o’clock
position (−180°). Furthermore, with regard to the failure load of the full models, again only
very limited and negligible difference is found. The investigations using the full models prove
and verify that the Π-index distribution, and therefore also the failure load of the panels, is
insensitive to small variations in the shape of the loading profile.

The B specimens have not been modelled, as the models will be doubled in size and are
therefore extremely CPU expensive to run.

It is concluded that the experimental and theoretical results in SERIES 2 agree well in the
100 mm and 200 mm cases for both failure and buckling load, but less well in the H80 300 mm
case, even though the post-buckling response seems to agree well for the B specimen. The
most likely reasons (similar to SERIES 1) for the difference can in both cases be addressed
to the imperfections in the crack front tip introduced by the artificial debond production
method, which causes uncertainty in the prediction of the buckling force and hence also the
failure load.
Figure 7.35: Numerically obtained propagation initiation indexes, $\Pi$, for the SERIES 2 panels, plotted along the complete $360^\circ$ circular crack front at the point of propagation by use of half and full models: (a) H80 $\Omega$100, (b) H80 $\Omega$200, (b) H80 $\Omega$300 and (d) H200 $\Omega$200. The loading direction is vertical.
7.3 Lateral Pressure Loading of a Bottom Panel

In this section SERIES 3, lateral pressure on a bottom panel, will be described. Contrary to the previous sections covering SERIES 1 and 2, SERIES 3 has only been investigated experimentally.

Numerical modelling of this kind of tests is possible with the 3-D residual strength model, but several problems will arise. First of all, the simple DCPD device is not sufficient anymore to prevent closure of the debond and following element overlapping. Therefore, contact elements are needed in order to prevent closure and overlapping. However, the use of contact elements is very expensive in calculation time as the number of numerical iterations is drastically increased. Secondly, even with the application of contact elements, the problem of modelling of friction between crack flanks at a closed crack front is not solved.

Numerical investigations of laterally loaded debonded sandwich panels have been carried out by Falk (1994) as discussed in the introduction and recently by Segercrantz (2004), but both these studies neglect the friction between the crack flanks under high mode II crack tip loadings.

However, to complete the experimental investigation in this thesis of the most important ship panel types and to get an overview of the importance of different debond locations in connection with the residual strength, eight panels have been tested experimentally, and, as already mentioned in the introduction to this chapter, by use of an existing test rig located in the VTT Technical Research Center of Finland.

The test rig and the results gained from these experiments are presented below.

7.3.1 Experimental Setup

The Test Rig

The test setup is composed of a rigid steel loading frame, which is loaded with a Robcon 400 kN universal testing machine, see Figure 7.36a and b. The panel is via its boundaries pressed against a water-filled cushion, lying on a base support, to create a uniformly distributed loading, see Figure 7.36c. The boundary conditions are close to simply supported with regard to both moment (M=0) and in-plane loads (N=0) and act in the vertical direction with respect to the panel surface. The boundary conditions are implemented by resting the panel on 15 mm wide and 8 mm thick 70 Sh polyurethane strips, see Figure 7.36d. These strips introduce the support reaction on a wider area than traditional steel rollers, while also allowing rotation with minor resistance. In Figure 7.37 the test panel including the support locations is shown. During the test the water pressure inside the cushion is measured together with the load introduced by the testing machine. Furthermore, a pin gauge is attached to the centre of the loading frame in order to measure the deflection of the midpoint of the panel.
Figure 7.36: VTT lateral pressure test rig in the Robcon 400 kN universal testing machine, (a) and (b). The lateral pressure introducing water-filled cushion (c) and the polyurethane strips (d), used to introduce the boundary conditions.
7.3 Lateral Pressure Loading of a Bottom Panel

Panel Specifications and Material Properties

Eight sandwich panel specimens have been tested, six damaged panels with circular debonds in various locations and two intact panels were used with the specifications given in Table 7.11.

Three debond locations have been chosen in areas believed to be critical with respect to different debond loading types:

- A panel *centre* location on the pressure side has been chosen to investigate if the compressive bending stresses combined are sufficient to provoke debond buckling and subsequent debond propagation

- A panel *edge* location (the midpoint and pressure side) just inside the load frame has been chosen in order to investigate the influence of the high out-of-plane shear forces near to the edge boundary condition

- A panel *corner* location on the pressure side and likewise just inside the load frame has been chosen to investigate the influence of high in-plane shear forces

Exactly the same materials are used as for the specimens in beam tearing tests, and the material properties are found in Table 6.6.

The panel geometry is presented in Figure 7.37 and the geometrical panel properties in Table 7.12. Note in this connection that the panels have a loaded part inside the loading frame and a panel overhang outside the loading frame.

Table 7.11: Distribution of test specimens on damage type, location and core density used in the SERIES 3 tests. All debond damages are circular debonds. IDO=Initial Debond Opening when measured and applicable.

<table>
<thead>
<tr>
<th>Panel Designation</th>
<th>Core Type</th>
<th>Damage type</th>
<th>Location</th>
<th>Specimens</th>
<th>IDO</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPE S3_13</td>
<td>H80</td>
<td>Ø200 mm</td>
<td>Centre</td>
<td>1</td>
<td>1.4 mm</td>
</tr>
<tr>
<td>TYPE S3_14</td>
<td>H200</td>
<td>Ø200 mm</td>
<td>Centre</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S3_15</td>
<td>H80</td>
<td>Ø100 mm</td>
<td>Edge</td>
<td>1</td>
<td>0.65 mm</td>
</tr>
<tr>
<td>TYPE S3_16</td>
<td>H200</td>
<td>Ø100 mm</td>
<td>Edge</td>
<td>1</td>
<td>1.9 mm</td>
</tr>
<tr>
<td>TYPE S3_17</td>
<td>H80</td>
<td>Ø200 mm</td>
<td>Corner</td>
<td>1</td>
<td>2.1 mm</td>
</tr>
<tr>
<td>TYPE S3_18</td>
<td>H200</td>
<td>Ø200 mm</td>
<td>Corner</td>
<td>1</td>
<td>5.6 mm</td>
</tr>
<tr>
<td>TYPE S3_19</td>
<td>H80</td>
<td>Intact</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S3_20</td>
<td>H200</td>
<td>Intact</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 7.37: SERIES 3 panel geometry with debond location shown for the three debond locations.

Table 7.12: SERIES 3 panel geometry values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total panel height</td>
<td>A</td>
<td>1120 mm</td>
</tr>
<tr>
<td>Total panel width</td>
<td>B</td>
<td>850 mm</td>
</tr>
<tr>
<td>Support height</td>
<td>$A_R$</td>
<td>980 mm</td>
</tr>
<tr>
<td>Support width</td>
<td>$B_R$</td>
<td>625 mm</td>
</tr>
<tr>
<td>Pressure area height</td>
<td>$A_P$</td>
<td>Varying</td>
</tr>
<tr>
<td>Pressure area width</td>
<td>$B_P$</td>
<td>Varying</td>
</tr>
<tr>
<td>Face thickness</td>
<td>$t_f$</td>
<td>4.5 mm</td>
</tr>
<tr>
<td>Core thickness</td>
<td>$t_c$</td>
<td>30 mm</td>
</tr>
<tr>
<td>Edge debond location</td>
<td>$L_E$</td>
<td>213 mm</td>
</tr>
<tr>
<td>Corner debond location</td>
<td>$L_{C1}$</td>
<td>220 mm</td>
</tr>
<tr>
<td>Corner debond location</td>
<td>$L_{C2}$</td>
<td>265 mm</td>
</tr>
</tbody>
</table>
7.3.2 Experimental Results

In this section the experimentally obtained results are presented. Figure 7.38 shows H80 panels with implanted debonds before testing at VTT. After testing, the panels were sent to RISO National Laboratory for ultrasound scanning to reveal the extent of damage. Especially in the H80 specimens a visual inspection can be difficult to carry out, because debond propagation is normally not connected with fibre bridging as described in the earlier sections.

In Figure 7.39 the testing of an H80 intact panel is shown. It is seen that extremely large deflections are obtained and the subsequent failure of the panel by core shear fracture results in a rapid crack propagation into the face-core interface, ending in a complete separation over the total panel area, see Figure 7.39.

A similar failure scenario is seen for the H80 central debond panel, and this panel actually fails under a slightly higher load than the intact panel, see Table 7.13, which implies that the central debond is non-critical for the failure of the laterally loaded panels.

Air-coupled ultrasound scanning has again been carried out at RISO National Laboratory on the tested panels after failure, and in Figure 7.40 the results from the intact and central debond panels are seen. It is very clear from the pictures that no fracture is visible from the central debond, and the failed ultrasound scanning gives approximately the same result for the intact and central debonded cases, if the initial central debonded area is neglected.

The H200 intact and central debond panels behave very similarly to the H80 panels with respect to panel response, but unfortunately the pressure limit for the rubber water bag used to introduce the distributed load was reached at 350 kPa for the central debond panel, and the test had to be stopped. For the intact panel a core butt joint in the panel results in premature failure, probably because of stress concentrations around the joint. The ultrasound scans are seen for both panels in Figure 7.40, where the core failure originating from the butt joint can be observed.

![Figure 7.38: SERIES 3 H80 test specimens with debond locations before testing. (a) Central Ø200 mm, (b) edge Ø100 mm and (c) corner Ø200 mm. The debond location and size in H200 specimens are the same.](image-url)
Figures 7.41 and 7.42 show the edge and corner debonded H80 and H200 panels after failure together with their respective ultrasound scans. Shear cracks with subsequent debonding can clearly be observed in the white areas extending from the support side of the initial debonds, which could also be expected because there is an increase in the shear forces in this area.

By comparison of the H80 and H200 it is concluded that fibre bridging is present in the H200 specimens (the white areas), whereas this is not the case in the H80 specimens, where no visible bridging can be observed visually from outside. Furthermore, the fibre bridging has a positive effect, seen from a structural point of view, that the debond propagation following the shear failure in the core is limited by the fibre bridging. In the H80 specimens, the debond propagation is more widespread, probably because of subinterface propagation of the crack front.

All edge and corner debonded H80 and H200 panels fail at relatively low loads compared to the intact and central debond panels, as is seen in Figure 7.43 and Table 7.13. The corner debond panels fail at a slightly higher load than the edge debonded panels for both H80 and H200.

The strength reductions for the H80 panels have been calculated and are presented in Table 7.13, and the relatively high strength knock-down for the edge and corner debonded panels is seen. Unfortunately, the same values cannot be achieved for the H200 panels, because of the premature failure of the H200 intact panel, but it is believed, based on the results from the edge and corner debonded panels, that very similar residual strength results are found for the H200 panels. The above strength reductions are all calculated on the basis of the ultimate collapse load of the intact panels. It could be argued that it would be more suitable to use the yield limit of the core, because this limit is used in practical design. In this case the strength reductions would be considerably lower.

Table 7.13: Measured experimental failure loads and residual strength factors for the SERIES 3 panels. (*) Premature failure due to core butt joint. (**) Test stopped at 350 kN.

<table>
<thead>
<tr>
<th>Panel designation</th>
<th>Core type</th>
<th>Damage type</th>
<th>Experimental failure load</th>
<th>Residual strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPE S3_13A</td>
<td>H80</td>
<td>Centre Ø200 mm</td>
<td>279.3 kN</td>
<td>108.1%</td>
</tr>
<tr>
<td>TYPE S3_14B</td>
<td>H200</td>
<td>Centre Ø200 mm</td>
<td>348.4 kN **</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S3_15A</td>
<td>H80</td>
<td>Edge Ø100 mm</td>
<td>64.26 kN</td>
<td>24.9%</td>
</tr>
<tr>
<td>TYPE S3_16B</td>
<td>H200</td>
<td>Edge Ø100 mm</td>
<td>158.4 kN</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S3_17A</td>
<td>H80</td>
<td>Corner Ø200 mm</td>
<td>90.09 kN</td>
<td>34.9%</td>
</tr>
<tr>
<td>TYPE S3_18B</td>
<td>H200</td>
<td>Corner Ø200 mm</td>
<td>218.1 kN</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S3_19A</td>
<td>H80</td>
<td>Intact</td>
<td>258.3 kN</td>
<td>-</td>
</tr>
<tr>
<td>TYPE S3_20B</td>
<td>H200</td>
<td>Intact</td>
<td>263.4 kN *</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 7.39: The H80 intact panel during testing (a) (note the large deflection of the panel inside the support frame), and after testing (b) where the extensive debonding can be observed.
Figure 7.40: Ultrasound scans of failed panels showing extent of damage. H80 intact (a), H200 intact (b), H80 central Ø200 mm (c) and H200 central Ø200 mm (d).
Figure 7.41: Visual impressions and ultrasound scans showing extent of damage in failed panels. H80 edge Ø100 mm (a) and (b), H80 corner Ø200 mm (c) and (d).
Figure 7.42: Visual impressions and ultrasound scans showing extent of damage in failed panels. H200 edge Ø100 mm (a) and (b), H200 corner Ø200 mm (c) and (d).
Figure 7.43: Total pressure force vs. midpoint panel deflection, H80 specimens (a), H200 specimens (b).
7.4 Manual Implementation

In order to implement the results presented in this thesis and future results based on theoretical predictions or experimental investigations of a parametric manual framework, it is necessary to clarify first of all, which parameters are dominant for the residual strength of a panel of a given debond size.

Table 7.14 lists input parameters necessary for both the theoretical 2-D propagation beam model and the 3-D residual strength panel model. Furthermore, the governing parameters for the residual strength are identified.

Beginning with the geometrical parameters it has been shown in the section on the SERIES 1 panel tests that the overall panel dimensions turned out to be less important to the investigated $t_f/D$-ratios. This statement of course needs to be further investigated for other $t_f/D$-ratios, but on the assumption that this statement is more or less global, the loading can be treated as a far nominal strain field.

In Table 7.14 all the mechanical properties have been listed as governing parameters, but in practice it is very difficult to assess especially the out-of-plane properties of the face laminate, because they demand advanced experimental testing if they are to be obtained. Nevertheless, these parameters are necessary in the model, because they have to be known in order to use the CSDE mode-mixity method. However, in practice these parameters can be estimated based on previous measured data for similar laminates, like the ones included in this thesis.

With regard to the fracture mechanical properties, all these parameters have to be known in order to obtain a usable estimate of the failure load of a debonded sandwich beam or panel. Each combination of face and core forms its own interface case with separate fracture toughness vs. mode-mixity curve.

In Table 7.15 a proposal is shown of how a parametric implementation could be done for the uniform compression case. Two non-dimensional parameters are implemented: A geometrical ratio, $t_f/D$, between the only governing geometrical parameters, the face thickness, $t_f$, and the debond diameter, $D$, and a parameter designating the interface type, $\beta^2$. These two parameters give residual strength factors to be used in the overall strength reduction factor for panel or global hull strength level, for which the principles are seen in the flow charts in Figure 7.44\textsuperscript{3}. A description of the damage tolerance approach developed in the saNDI-project can be seen in Hayman (2003).

\textsuperscript{2}Note that in this connection the 1-3 plane is the in-plane direction on the panel

\textsuperscript{3}Further details about this flow chart and similar flow charts with a higher degree of detail are found in the inspection manuals produced internationally in the saNDI-project (Inspection and Repair of Sandwich Structures in Naval Ships)
Table 7.14: Input parameters to the developed numerical model. The governing parameters for the debond failure of the panel are indicated.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>SI-Unit</th>
<th>Governing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometrical panel length</td>
<td>A</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Geometrical panel breadth</td>
<td>B</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Geometrical face thickness</td>
<td>$t_f$</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Geometrical core thickness</td>
<td>$t_c$</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Geometrical debond diameter</td>
<td>$D$</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Mechanical In-plane face E-modulus</td>
<td>$E_1$</td>
<td>Pa</td>
<td></td>
</tr>
<tr>
<td>Mechanical Out-of-plane face E-modulus</td>
<td>$E_3$</td>
<td>Pa</td>
<td></td>
</tr>
<tr>
<td>Mechanical In-plane face Poisson’s ratio</td>
<td>$\nu_{12}$</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Mechanical Out-of-plane Poisson’s ratio</td>
<td>$\nu_{13}$</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Mechanical Out-of-plane Poisson’s ratio</td>
<td>$\nu_{23}$</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Mechanical In-plane face G-modulus</td>
<td>$G_{12}$</td>
<td>Pa</td>
<td></td>
</tr>
<tr>
<td>Mechanical Out-of-plane face G-modulus</td>
<td>$G_{13}$</td>
<td>Pa</td>
<td></td>
</tr>
<tr>
<td>Mechanical Out-of-plane face G-modulus</td>
<td>$G_{23}$</td>
<td>Pa</td>
<td></td>
</tr>
<tr>
<td>Mechanical Core E-modulus</td>
<td>$E_c$</td>
<td>Pa</td>
<td></td>
</tr>
<tr>
<td>Mechanical Core Poisson’s ratio</td>
<td>$\nu_c$</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Mechanical Core G-modulus</td>
<td>$G_c$</td>
<td>Pa</td>
<td></td>
</tr>
</tbody>
</table>

Fracture mechanical

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation</th>
<th>SI-Unit</th>
<th>Governing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical mode 1 fracture toughness</td>
<td>$G_{1c}$</td>
<td>N/m</td>
<td></td>
</tr>
<tr>
<td>Fracture toughness distribution constant</td>
<td>$k$</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Characteristic length</td>
<td>$h$</td>
<td>m</td>
<td></td>
</tr>
</tbody>
</table>

The interface type is proposed to be parameterised by the 2nd Dundur parameter, Eq. (A.14), which is only one of several parameters used in interface fracture mechanics to characterise an orthotropic bimaterial. For isotropic bimaterials the interface may be described by only two parameters: The 1st and 2nd Dundur parameters, Eqs. (4.45) and (4.46).

Even though more than one parameter is theoretically necessary to designate an orthotropic bimaterial interface, it has been shown by RISO National Laboratory in connection with the fracture mechanics material testing that only the $\beta$- and $\lambda$-parameters vary significantly for the most used interface types in typical sandwich interfaces with foam cores. The variation of the remaining fracture mechanical parameters proves to be negligible.

Each of the residual strength factors will be dependent on the interface fracture toughness, $\Lambda$, which again will be dependent on the bimaterial case, $\beta$ (and $\lambda$). Furthermore, the
fracture toughness will also be dependent on the interface type present in the bimaterial. In this connection it is important to distinguish between interfaces with or without the use of CSM laminates in the interface. As indicated earlier, the effect of CSM mats in the interface will mean that fibre bridging is very likely to occur for heavy cores, with resulting high fracture toughness values as the crack propagates in the glue interface. In the case of no CSM laminates in the interface, more brittle fracture can be expected, either as subinterface propagation on the core side of the laminate for light cores or as direct interface propagation, but most likely without or only limited fibre bridging for heavier cores. Separate tables similar to the one shown in Table 7.15 have to produced for each of these interface types.

In the case of high in-plane orthotropy, problems will be encountered in connection with the calculation of the mode-mixity by the CSDE 3-D mode-mixity method, because it is assumed that the crack is parallel to the principal material directions. Normally, this is no problem because the face laminates used in most vessels are all quasi-isotropic, which for practical use can be considered as isotropic in the panel in-plane direction. But if the laminates are highly orthotropic, measures have to be taken to ensure that the material parameters parallel to the crack front position plane used. This can be done by normal lamination theory calculating the material properties as a function of the in-plane crack font position angle.

A secondary problem is furthermore that the assumption about plane strain conditions will be more vague and the implications of this assumption have to be further investigated.

The parametrical proposal shown in Table 7.15 could also be used for the lateral pressure case, but further information has to be incorporated, with regard to the debond location on the panel. In the non-uniform compression case the loading across the debond diameter furthermore has to be implemented. This could be done by introducing a ratio between the loading at the left and right edge of the debond.
Finally, the complete concept of using the geometrical ratio, $t_f/D$, and the interface parameter, $\beta$, to parameterise the residual strength factors has to be verified by a large number of debond cases using the theoretical model.

The residual strength prediction tool presented in this chapter is as mentioned earlier a part of a larger context. In Figure 7.44 the local panel and global hull residual strength approaches used in the inspection manuals are presented respectively. The inspection manuals are the end product of joint Nordic/Anglo three-year research project, saNDI. In both approaches the theoretical tool developed in the thesis is used to determine the local residual strength reduction factor $R_l$. Combining $R_l$ with a local location factor of the damage relative to the panel and a factor describing the load type on the panel gives the panel reduction factor $R_p$, which can be compared with an allowable panel strength reduction factor $R_{pa}$ in the local panel approach, as indicated in Figure 7.44. In the global hull strength approach, also seen in Figure 7.44, the panel strength reduction factor $R_p$ is combined with two global ship factors, including information about the global load case for the ship and location of the damage to the hull. The combination of these factors and the panel strength reduction factor gives a global ship strength reduction factor $R_s$, which again, as in the local panel approach, must be compared with an allowable strength reduction factor $R_{sa}$ in order to determine if the damage is critical or non-critical to the global hull strength.

![Figure 7.44: Residual strength approach to both local panel and global hull strength. Taken from the inspection manuals produced in the saNDI-project.](image-url)
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Chapter 8

Conclusion and Recommendations for Future Work

8.1 Conclusion

8.1.1 Damage Tolerance of Debonded Beams and Panels

A comprehensive discussion of the governing field equations for the near tip displacement field for linear fracture mechanics was carried out in this thesis. The displacement field formed the basis for defining Griffith-energy and mode-mixity for an interface crack. By using the derived displacement fields, the Griffith-energy and the mode-mixity expressions on the basis of linear interface fracture mechanics, three different mode-mixity methods from the literature were presented: The Virtual Crack Extension method, the modified Virtual Crack Closure Technique and the Crack Surface Displacement method. Advantages and disadvantages were found for all three methods, with special regard to application in foam core sandwich interfaces by using a finite element code with an automatic crack propagation routine. None of the investigated mode-mixity methods from the literature were found to possess qualities adequate for the applications mentioned above. Consequently, on the basis of the experiences gained in the investigation of the mode-mixity methods from the literature, a new mode-mixity method, based on extrapolation of crack flank displacement results, was derived and named the Crack Surface Displacement Extrapolation method (short CSDE).

To verify the new mode-mixity method and to illustrate the potential of the method compared to the best of the above-mentioned mode-mixity methods from the literature, namely the modified Virtual Crack Closure Technique, two test cases were chosen. One test case comprised a verification against a semi-analytical method but with isotropic bimaterials with a moderate stiffness ratio, another test case included orthotropic bimaterial with stiffness ratios comparable to the ones seen in practical sandwich structures. The CSDE method...
proved to be better suited for overcoming the numerical errors introduced into the system by the nodes of the first crack tip elements and proved to be reliable and robust for general structural use in a commercial finite element code.

To be able to use the CSDE method for prediction of initiation of crack propagation in three dimensional structures, the CSDE method was expanded into 3-D by use of a path independent contour integral (the J-integral) to predict the Griffith-energy instead of nodal extrapolation. The three dimensional version of the CSDE method was verified by two test cases using the two-dimensional version of the CSDE method with a high near tip mesh density. Even though the mesh was much coarser for the three-dimensional case it proved reliable and produced results acceptable for structural use in practical sandwich structures.

By use of experimentally obtained fracture toughness results from RISO National Laboratory, fracture toughness versus mode-mixity distributions were generated for sandwich interfaces with non-crimp quadro-axial GFRP faces and Divinycell PVC cross-linked foam cores with the densities 80, 130 and 200 kg/m$^3$.

By applying the two- and three dimensional CSDE methods respectively, two finite element based structural models have been produced. The two-dimensional model is able to simulate crack propagation in and out of a sandwich interface using the fracture toughness distributions described above. The model is furthermore able to simulate geometrical non-linearities as well as both stable and unstable crack propagation. The three-dimensional model is able to predict crack initiation along the crack front of a circular debonded area in a sandwich vessel under various loading conditions. Panel response up to the buckling load of the debonded face layer and the post-buckling response are simulated by the three-dimensional model. The panel is regarded as failed, when crack propagation is monitored at any point along the crack front. Similarly to the two-dimensional model, the fracture toughness distributions described above are likewise input to the three-dimensional model.

In order to verify and validate the two-dimensional model, a comparison with another numerical model and an experimental full-scale beam series has been carried out. In both cases an idealised real-life critical damage event was used as an application case for the two-dimensional model. The damage event chosen was face tearing in a deck superstructure corner connection in a ship exposed to global hull sagging and hogging. Comparison with an independent damage mechanics model using the Bonora damage model revealed a very good agreement for the two-dimensional model. The verification also showed that the model was relatively sensitive to variations in the fracture toughness. The example against an experimental beam series was carried out for core densities of 80, 130 and 200 kg/m$^3$. The comparison between the numerical model and the experimental results showed that as long as no fibre bridging was present, which was the case for the beam specimens with H80 core, good agreement was obtained. In the case of H130 and H200 cores, the numerical model yielded highly conservative results as soon as fibre bridging occurred.

The residual strength of three common ship panel types with circular debonds has been investigated both experimentally and numerically using the three-dimensional model. The
chosen panel types were: A bottom or deck panel with pure in-plane uniform compression, a side panel with non-uniform in-plane compression (combined in-plane bending and compression) and a bottom panel with lateral pressure\(^1\). Additionally to the residual strength investigation the three-dimensional model was validated against the experimental results from both in-plane test series.

It was common to the first two panel test series that the in-plane loading of the panel resulted in outwards buckling of the debonded face in a local buckling mode. This failure behaviour was seen both in experimental and numerical investigations. The outward buckling happened in all cases gradually, and in almost all cases the local buckling ended with a very rapid propagation of the debond front to the edge of the panel. Furthermore, this crack propagation was located just below the glue interface in the core, as no large-scale fibre bridging was observed either visually or through air-coupled ultrasonic scans.

In Figure 8.1 the average residual strength factor has been plotted for varying debond sizes, based on the results from the experimental and numerical investigations. For each debond diameter the results from the two identical specimens are averaged. Additionally, for the experimental results the variations from the average value at each debond size are indicated. In SERIES 1 the residual strength is made non-dimensional by the analytically calculated wrinkling strength of the intact panel. In SERIES 2 the residual strength factors are made non-dimensional by the failure load obtained from the numerical modelling of the wrinkling introduced compressive failure of the intact panels.

From a comparison of the experimental and numerical average residual strength factors in Figure 8.1a for the SERIES 1 H80 panels under uniform loading, it can be concluded that numerical predictions of the residual strength for intact and panels with small debonds yield non-conservative results, but both numerical and experimental results are in good agreement in terms of residual strength and show considerable strength reductions with average residual strength factors around 20-25\% for debond diameters around Ø200-300 mm. For smaller debond diameters, the experimental results show average residual strength factors around 37\%, whereas the numerical results yield a conservative 51\%. As shown earlier, considerable differences in the absolute values between the numerical and the experimental results were observed for the small debond sizes, which is most likely due to large influence from imperfections on the debond buckling for high \(t_f/D\)-ratios.

In Figure 8.1b similar results are plotted for the non-uniformly loaded SERIES 2 panels with H80 core. In this case it can first of all be concluded that both experimental and numerical investigations show that small debonds below at least Ø100 mm are not critical to this panel case, as the residual strength factors are above 1 for the numerical results and very close to 1 for the experimental results. However, as mentioned earlier, the failure of the two Ø100 mm panels proved that the failures caused by wrinkling introduced compression and debond propagation were quite close, as the two panels failed in the two failure mechanisms. As in the uniformly loaded case both experimental and numerical results show considerable

\(^1\)The latter panel case was only experimentally investigated
strength reduction for larger debonds with average residual strength factors around 32-55%. For debond diameters from approximately Ø150 mm and up the numerical model yields increasingly conservative results compared to the experimental values. The increasing conservatism may be explained by problems in the numerical model for predicting the correct buckling mode under influence of production introduced imperfections, as discussed in the previous chapter. However, for engineering purposes, the conservative results are applicable.

Finally, it must be concluded from Figure 8.1 that the largest variations from the average residual strength values and between the experimental and theoretical results in both SERIES 1 and 2 are seen for the intact panels. This emphasises the need for performing better experimental predictions of the intact panel strength, but also that imperfections may have a considerable influence on the residual strength factors, especially when the failure mechanism is highly imperfection sensitive, as experienced in the present investigation where wrinkling introduced compression failure was observed. Residual strength factors should therefore be used with great care, especially for practical structures where imperfections are typically bigger compared to laboratory test specimens. However, it should also be noted that in practical design, these variations in the intact panel failure loads are to some degree taken into consideration through safety factors used in the determination of the maximum allowable loading of the structure. Likewise, safety factors should be applied to for example fracture toughness distributions, which also exhibit some variations as described earlier in this thesis, to obtain residual strength factors based on failure loads for both intact and damaged panels where a certain level of safety is included.

For the laterally loaded SERIES 3 case it is not possible on the basis of the performed experimental investigations to produce plots similar to the ones presented in Figure 8.1, as the residual strength factors in this case also depend on a debond location parameter and only one debond diameter has been investigated for each location. But as indicated in the previous chapter, the central debond proved to be non-critical whereas the edge and corner debonds yielded residual strength factors of 25% and 35% respectively.

In all three panel cases the main part of the test specimens consisted of panels with H80 cores, but it is assumed, based on the few specimens investigated, that the residual strength factors for the H200 panels will yield similar results as long as no fibre bridging takes place, as observed in the experimental panel test series presented in this thesis.

As mentioned in the introduction, the main objective of the work described in this thesis is to present a theoretical method for prediction of residual strength factors for a wide range of debond damages. Therefore, the residual strength factors dealt with in this thesis are only examples of debond cases and only a limited interval of governing parameters has been investigated. In order to get a more complete overview of the residual strength factors, a wider parameter study has to be carried out using the numerical models presented in this thesis, backed by more experimental testing.
8.1 Conclusion

Figure 8.1: Average residual strength factors from experimental and numerical tests on panels with circular central debonds, uniform compression with H80 core (a) and non-uniform compression with H80 core (b). It should be noted for the experimental results that the individual variations at each debond diameter are based on the average theoretical intact strength.
However, residual strength factors for the chosen loading types have been presented in this thesis, and it has been demonstrated how the produced theoretical model is able to predict failure loads in most cases with acceptable accuracy for structural analysis, taking level of production imperfections and uncertainty of material parameters and fracture mechanics input data into account.

8.1.2 Non-Linearity of Curved Panels

Several finite element models and programs were used in this thesis to investigate the non-linear behaviour of curved composite sandwich panels. A novel test arrangement was developed to produce experimental data.

The comparisons between experiments and predictions showed that actually only one finite element model captured the panel response and failure with a satisfying level of accuracy. There are several reasons for the differences between the different finite element models and experiments. Detail differences in boundary conditions affect the results between different finite element models and tests considerably. The current test arrangement allowed the panel corners to lift off the support, which was prevented in the implicit finite element models and modelled in the explicit models. Preventing corner lift-off in models leads to smaller deflections than those observed in the test and a different distribution of strains within the panel. It should be noted though that a panel in a ship structure, aircraft structure etc. is normally not subject to complex contact boundary conditions and could possibly be modelled well by simpler models. Furthermore, the approximate estimation of material strength at lamina level influences the results to some degree.

The use of an explicit code in a quasi-static analysis also introduces some inaccuracy in the response. The best approach to obtaining consistent results for the problem would probably be to use an implicit finite element code with the corner lift-off allowed.

Linear implicit FE analysis was found to be completely inapplicable to predictions of the panel behavior and could in worst case lead to an underdimensioned structure. Because of this fact, linear finite element calculation in connection with curved sandwich panels should be avoided.

The conclusion on the investigation of non-linear response of curved sandwich panels is that the experimental results showed fair agreement with the analysis, if the modelling was performed at a sufficiently high level of detail. Furthermore, the used novel test arrangement was found to work well.

Previous studies have shown that the weight of curved sandwich panels may be reduced, if the membrane effect is taken properly into account in the structural analysis. It has, however, never been made clear how much weight can actually be saved and also what extra requirements the structure around the panel should fulfil, since it has to carry an
additional in-plane load. The analysis presented here considered a curved bottom panel in a representative medium-size vessel.

In the first part of the chapter concerning non-linearity of curved panels, the bottom panel was designed for minimum weight both by use of DNV’s Rules for High Speed Light Craft and by direct calculations. The rules are based on linear theory for a plane panel\(^2\), with the usual idealised boundary conditions. The direct calculations were carried out for a large section of the hull in order to model the boundary conditions correctly. Furthermore, the direct calculations were carried out both for the current curved panel of the vessel and for a plane panel with the same dimensions embedded in the same structure. The analysis showed that by taking into account the panel curvature in the structural analysis, approximately 20% of weight could be saved. This weight reduction is sufficiently high to be utilised in truly weight critical ship structures.

The weight reduction is made possible by activation of compressive membrane forces in the panel. Therefore, in order to obtain this weight reduction, the structure around the panel must be able to hold these forces. The purpose of the second part of the chapter concerning non-linearity of curved panels was to investigate how close to a free boundary the panel could be placed without losing the potential of weight reduction. The analysis showed that there must be a significant in-plane stiffness of the structure around the panel in order to reach the weight reduction. For the considered panel, the width of the plate strip around the panel must be comparable in size to the panel itself in order to reach the 10-15% of weight reduction. Remarkably, the analysis showed that the difference in weight between a panel that was fully clamped and one with only a small strip around it was 40-50% for certain loads. Similar results may apply to plane panels with large deflections. This means that previously published analyses, which are typically based on an assumption of full fixation against in-plane movement of the boundaries, should be followed up by a critical assessment of the realism of this assumption.

### 8.2 Recommendations for Future Work

The reliability of the structure is closely linked to the uncertainty of the loads. At present the Rules prescribe uniform pressure loads. For a plane panel the stresses and deflections are not very sensitive to the distribution of the lateral pressure. When the panel curvature is utilised, on the other hand, the pressure distribution may have a large effect on the response (cf. loading of an egg shell), so the load requirements should be revised to investigate the criticality of a non-uniform pressure distribution.

A natural next step for future application of the two numerical damage tolerance models presented in this chapter is to establish a connection between the work carried out with regard

\(^2\)Recently an updated version, DNV (2003), has been made available, which includes non-linearities for plane panels, which can be exploited to some extent.
to curved panels and debonded plane panels. Experimental investigations with debonded curved beams have already been carried out by Layne and Carlsson (2002), and it would be highly relevant to use the two-dimensional propagation model presented in this thesis to simulate these test results. Furthermore, by means of the test rig located at the VTT Technical Research Center of Finland, used in this thesis for both the full-scale curved panel response tests and the lateral pressure testing of debonded plane panels, it will be highly relevant to combine these two test series and investigate the influence of panel curvature on the residual strength of debonded sandwich panels with various debond locations. Additionally, the three-dimensional model, presented in this thesis, can be applied to this debond problem.

As indicated earlier, the \( t_f/D \)-ratios and panel types, investigated in this thesis, are only a small part of the panel cases necessary to achieve a complete set of elementary panel cases for different loading types. More panel cases have to be investigated for other core and face laminate types for manual implementation. Moreover, a larger section of the vessel should be modelled using shell elements, but with the debonded panel modelled by the same solid element approach as presented in this thesis, thus being able to investigate both possible stress redistribution to the surrounding panels and more realistic loading and boundary conditions. An investigation like the latter would be highly relevant to vessel-specific inspection manuals, which are the next step following the saNDI-project described earlier.

Earlier in this thesis it was furthermore shown that a precise prediction of the buckling load is of great importance, because it is governing for the failure load for especially high \( t_f/D \)-ratios. More experimental investigations with regard to debonded panels with these properties are therefore needed in order to validate the numerical results. Moreover, special care has to be taken in order not to introduce unwanted imperfections along the crack front, when the artificial debond is produced in the test specimens. Furthermore, as mentioned above, there is a present need for performing more precise experimental investigations of the intact strength. Future testing must aim at minimizing the level of imperfections in the test specimens themselves, but also at limiting the errors introduced by the test rig. The developed test arrangements proved highly suitable for testing debond damaged panels, but proved in some cases too crude for precise intact panel testing.

In connection with the face tearing beam tests presented in this paper it was experienced that especially for heavier core types considerable large-scale bridging was taking place, which removes the justification for applying linear fracture mechanics to the sandwich interface. To be able to predict crack propagation in interfaces with bridging behaviour by use of the two-dimensional propagation model, application and implementation of cohesive laws are needed. Senior scientist Bent F. Sørensen and coworkers from RISØ National Laboratory have expanded the knowledge in this field to a large extent, especially for glue interfaces in single skin laminates. Among the publications in this field are: Sørensen and Jacobsen (1998), Sørensen and Jacobsen (2000), Jacobsen and Sørensen (2001) and Sørensen (2002).

Finally, in this thesis only monotonic loading has been investigated, which is a reasonable starting point for developing theoretical models. However, in real-life sandwich structures
the main source of debond spreading is most likely cyclic loading of the debonded panels. An expansion of the present models into the fatigue area is highly relevant, but should be validated against a large-scale experimental test series, possibly with the same beam and panel materials and geometries as used in this thesis. The knowledge gained in this thesis forms a firm foundation for theoretical and experimental investigation of crack propagation in structures exposed to repeated cyclic loading, as eg. ships, wind generator blades, aircraft structures, trains.
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References


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Appendix A

Orthotropic Bimaterial Interface
Crack Tip Displacement Field

A.1 Derivation of Complex Numbered Expressions

The displacement and the stress field close to the crack tip can be described by the Lekhnitskii-Eshelby-Stroh (LES) formulation in 2-D, Suo (1990): (similar representations exist for the stress components)

\[
\begin{align*}
    u_1^1 &= 2 \Re \left[ \sum_{k=1}^{2} A_{1k}^1 f_1k (Z_k) \right] \\
    u_2^1 &= 2 \Re \left[ \sum_{k=1}^{2} A_{2k}^1 f_2k (Z_k) \right]
\end{align*}
\]

Material 1

\[
\begin{align*}
    u_1^2 &= 2 \Re \left[ \sum_{k=1}^{2} A_{1k}^2 f_1k (Z_k) \right] \\
    u_2^2 &= 2 \Re \left[ \sum_{k=1}^{2} A_{2k}^2 f_2k (Z_k) \right]
\end{align*}
\]

Material 2

(A.1)

where \( A_{1k}^1 \) and \( A_{2k}^2 \) depend on material parameters for material 1 and 2 respectively (the upper index on the A-parameter corresponds to the material), \( \Re \) refer to the real part and \( f_1k \) and \( f_2k \) are two material-dependent holomorphic functions, which can be found by solving the partial differential equation given according to Charalambides and Zhang (1996):

\[
\frac{\partial^4 F}{\partial x^4} + 2\rho\lambda^{1/2} \frac{\partial^4 F}{\partial x^2 \partial y^2} + \lambda \frac{\partial^4 F}{\partial y^4} = 0
\]

(A.2)

1Complex differentiable
where $\lambda$ is defined in (A.7) and $F(x, y)$ is a stress function and can be represented as

\[
F(x, y) = 2\Re \left[ F_1(Z_1) + F_2(Z_2) \right] \tag{A.3}
\]

where the two holomorphic functions, $f_1$ and $f_2$, correspond to $f_{1k}$ and $f_{2k}$ in (A.1) and are then given as

\[
f_1(Z_1) = \frac{dF_1}{dZ_1} \quad f_2(Z_2) = \frac{dF_2}{dZ_2} \tag{A.4}
\]

where $Z_1$ and $Z_2$ are two complex variables:

\[
Z_1 = x + \mu_1 y \quad Z_2 = x + \mu_2 y \tag{A.5}
\]

$x$ and $y$ describe the position in a coordinate system located at the crack tip, see Figure 4.5. $\mu_1$ and $\mu_2$ are two distinct complex numbers with positive imaginary part, and can be found as roots to the characteristic equation resulting from (A.2). Depending on the sign and magnitude of the material constant, $\rho$, the complex roots, $\mu_\alpha$, are given as:

\[
\mu_1 = \begin{cases} 
  i\lambda^{-1/4} (n + m) & \text{for } \rho > 1, \\
  \lambda^{-1/4} (in + m) & \text{for } -1 > \rho > 1.
\end{cases} \tag{A.6}
\]

\[
\mu_2 = \begin{cases} 
  i\lambda^{-1/4} (n - m) & \text{for } \rho > 1, \\
  \lambda^{-1/4} (in - m) & \text{for } -1 > \rho > 1.
\end{cases}
\]

where $n, m, \rho$ and $\lambda$ are non-dimensional orthotropic bimaterial constants given in terms of the compliances of either material 1 or 2:

\[
\begin{align*}
  n &= \sqrt{\frac{1}{2} (1 + \rho)} \\
  m &= \sqrt{\frac{1}{2} (1 - \rho)} \\
  \lambda &= \frac{S_{11}}{S_{22}} \\
  \rho &= \frac{1}{2} \frac{S_{12} + S_{66}}{\sqrt{S_{11}S_{22}}} \tag{A.7}
\end{align*}
\]
The compliances are given in plane stress as

\[
\begin{align*}
S_{11} &= \frac{1}{E_1} \quad S_{12} = S_{21} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2} \\
S_{22} &= \frac{1}{E_2} \\
S_{66} &= \frac{1}{G_{12}}
\end{align*}
\] (A.8)

and can be transformed to plane strain by

\[
S_{ij}^* = S_{ij} - \frac{S_{3i}S_{3j}}{S_{33}}
\] (A.9)

Furthermore, the material terms from Eq. (A.1), \(A_{1i}^k\) and \(A_{2i}^k\), are defined as

\[
\begin{align*}
A_{1\alpha} &= S_{11} \mu_\alpha^2 + S_{12} \\
A_{2\alpha} &= S_{21} \mu_\alpha + \frac{S_{22}}{\mu_\alpha} = \frac{1}{\mu_\alpha} \left( S_{21} \mu_\alpha^2 + S_{22} \right)
\end{align*}
\] (A.10)

In order to derive explicit expressions for the displacement field given in Eq. (A.1), some derivations have to be carried out.

Suo (1990) gives the following general expressions for the two potentials \(f_1 = (f_{11}, f_{12})\) and \(f_2 = (f_{21}, f_{22})\):

\[
\begin{align*}
L_1 f_1' (Z) &= \frac{e^{\pi\varepsilon K} Z^{i\varepsilon} w + e^{-\pi\varepsilon K} Z^{-i\varepsilon} w}{2 (2\pi Z)^{1/2} \cosh \pi \varepsilon} = g_1 (Z) \quad \text{Material 1} \\
L_2 f_2' (Z) &= \frac{e^{-\pi\varepsilon K} Z^{i\varepsilon} w + e^{\pi\varepsilon K} Z^{-i\varepsilon} w}{2 (2\pi Z)^{1/2} \cosh \pi \varepsilon} = g_2 (Z) \quad \text{Material 2}
\end{align*}
\] (A.11)

where ‘ indicates differentiation with respect to \(Z_i^2\) and the eigenvector \(w\) and the material matrix \(L_j\) (index \(j\) indicate the material number) are given as

\[
w = \begin{bmatrix} -\frac{1}{2}^i \\ \frac{1}{2} \sqrt{\frac{H_{11}}{H_{22}}} \end{bmatrix} \quad L_j = \begin{bmatrix} -\mu_1 & -\mu_2 \\ 1 & 1 \end{bmatrix}
\] (A.12)

\(^2\)The index \(i\) have been left out in (A.11).
and $K$ is the complex stress intensity factor given as $K = K_1 + iK_2$.

$H_{11}$, $H_{22}$ and $\beta$ are bimaterial constants, again depending on the material compliances:

\[
H_{11} = \left[2n\lambda^{1/4}\sqrt{S_{11}S_{22}}\right]_1 + \left[2n\lambda^{1/4}\sqrt{S_{11}S_{22}}\right]_2
\]
\[
H_{22} = \left[2n\lambda^{-1/4}\sqrt{S_{11}S_{22}}\right]_1 + \left[2n\lambda^{-1/4}\sqrt{S_{11}S_{22}}\right]_2
\]

\[
\beta = \frac{\left[\sqrt{S_{11}S_{22}} + S_{12}\right]_2 - \left[\sqrt{S_{11}S_{22}} + S_{12}\right]_1}{\sqrt{H_{11}H_{22}}}
\]

and $\varepsilon$ is the oscillatory index given as

\[
\varepsilon = \frac{1}{2\pi} \ln \left(\frac{1 - \beta}{1 + \beta}\right)
\]

By re-arranging Eq. (A.11), the following system of equations can be achieved. Here it is shown here for material 1:

\[
g_{11} (Z_1, Z_2) = -\mu_1 f_{11}' (Z_1) - \mu_2 f_{12}' (Z_2)
\]
\[
g_{12} (Z_1, Z_2) = f_{11}' (Z_1) + f_{12}' (Z_2)
\]

This system can be solved for the differentiated holomorphic functions:

\[
f_{11}' (Z_1) = -\frac{g_{11} (Z_1, Z_2) + \mu_2 g_{12} (Z_1, Z_2)}{\mu_1 - \mu_2}
\]
\[
f_{12}' (Z_2) = \frac{g_{11} (Z_1, Z_2) + \mu_1 g_{12} (Z_1, Z_2)}{\mu_1 - \mu_2}
\]

and if Suo’s expressions for $g_{11}$ and $g_{12}$ are inserted together with the conjugated and non-conjugated eigenvectors from Eq. (A.12), the differentiated holomorphic functions can be written in full length as:
A.1 Derivation of Complex Numbered Expressions

\[ f'_{11} (Z_1) = \frac{-1}{\mu_1 - \mu_2} \frac{1}{2} (2\pi)^{1/2} \cosh \pi \varepsilon \]
\[ \left[ e^{\pi \varepsilon K} Z_1^{-1/2+ie} \left( \frac{1}{2} \mu_2 \sqrt{\frac{H_{11}}{H_{22}}} - \frac{1}{2} i \right) + e^{-\pi \varepsilon K} Z_1^{-1/2-ie} \left( \frac{1}{2} \mu_2 \sqrt{\frac{H_{11}}{H_{22}}} + \frac{1}{2} i \right) \right] \]
\[ f'_{12} (Z_2) = \frac{1}{\mu_1 - \mu_2} \frac{1}{2} (2\pi)^{1/2} \cosh \pi \varepsilon \]
\[ \left[ e^{\pi \varepsilon K} Z_2^{-1/2+ie} \left( \frac{1}{2} \mu_1 \sqrt{\frac{H_{11}}{H_{22}}} - \frac{1}{2} i \right) + e^{-\pi \varepsilon K} Z_2^{-1/2-ie} \left( \frac{1}{2} \mu_1 \sqrt{\frac{H_{11}}{H_{22}}} + \frac{1}{2} i \right) \right] \]

(A.18)

If these expressions are integrated with respect to \( Z_1 \) and \( Z_2 \) respectively, explicit expressions are achieved for the holomorphic functions and hence also for the displacement field given in Eq. (A.1):

\[ f_{11} (Z_1) = \frac{1}{2 (\mu_1 - \mu_2)} \left[ -\sqrt{\frac{H_{11}}{H_{22}}} \mu_2 \left[ D_1 (Z_1) + D_2 (Z_1) \right] + i \left[ D_1 (Z_1) - D_2 (Z_1) \right] \right] \]
\[ f_{12} (Z_2) = \frac{1}{2 (\mu_1 - \mu_2)} \left[ \sqrt{\frac{H_{11}}{H_{22}}} \mu_1 \left[ D_1 (Z_2) + D_2 (Z_2) \right] - i \left[ D_1 (Z_2) - D_2 (Z_2) \right] \right] \]

(A.19)

where

\[ D_1 (Z) = 2R e^{\pi \varepsilon K} (1 - 2i\varepsilon) Z^{1/2+ie} \]
\[ D_2 (Z) = 2R e^{-\pi \varepsilon K} (1 + 2i\varepsilon) Z^{1/2-ie} \]

(A.20)

and

\[ R = \frac{1}{2 (2\pi)^{1/2} \cosh \pi \varepsilon} \frac{1}{1 + 4\varepsilon^2} \]

(A.21)

\( f_{21} (Z_1) \) and \( f_{22} (Z_2) \) for material 2 can be obtained by replacing \( \pi \) with \( -\pi \) everywhere.
A.2 Real-Numbered Expressions

In this appendix section real-numbered expressions for the orthotropic bimaterial interface crack tip displacement field are presented.

The displacement field is seen in its general complex form in (A.1), and by use of the expressions for the holomorphic functions, (A.19), real-numbered expressions for the $x$- and $y$-displacements have been derived, with an incremental choice of either the mode 1 or 2 stress intensity factors.

The expressions are dependent on the value of the material parameter, $\rho$, and will be treated in two intervals with consequently different displacement field solutions.

The displacement field presented in this appendix acts as input to the virtual crack extension mode-mixity method, and in order to implement this method in a non-complex environment (like ANSYS), the real-numbered displacement field expressions presented in this chapter are necessary.
\section*{A.2 Real-Numbered Expressions}

\subsection*{A.2.1 $\rho > 1$}

\begin{align*}
\Delta u_x^{(1)}|_{K_2=0} &= \frac{R}{m} \left[ \Delta K_1 \left( \sqrt{r_1 (F_{11} + M_{21})} + \sqrt{r_2 (F_{32} + M_{42})} \right) \right] \\
\Delta u_x^{(2)}|_{K_1=0} &= \frac{R}{m} \left[ \Delta K_2 \left( \sqrt{r_1 (-F_{11}' - M_{21}')} + \sqrt{r_2 (-F_{32}' - M_{42}')} \right) \right]
\end{align*}

\begin{align*}
\Delta u_y^{(1)}|_{K_2=0} &= \frac{R}{m} \left[ \Delta K_1 \left( \sqrt{r_1 (M_{51} + F_{61})} + \sqrt{r_2 (M_{72} + F_{82})} \right) \right] \\
\Delta u_y^{(2)}|_{K_1=0} &= \frac{R}{m} \left[ \Delta K_2 \left( \sqrt{r_1 (M_{51} + F_{61})} + \sqrt{r_2 (M_{72} + F_{82})} \right) \right]
\end{align*}

$F_{ij}$, $F'_{ij}$, $M_{ij}$ and $M'_{ij}$ are all given in Table A.1 by use of Tables A.2 and A.3.

$r_i$ and $\phi_i$ are modulus and argument for the complex field variables, $Z_1$ and $Z_2$ given in (A.5).

\begin{align*}
r_1 &= \sqrt{x^2 + \lambda^{1/2} (n + m)^2 y^2} \quad \phi_1 = \arctan \left[ \frac{\lambda^{-1/4} (n + m) y}{x} \right] \\
r_2 &= \sqrt{x^2 + \lambda^{1/2} (n - m)^2 y^2} \quad \phi_2 = \arctan \left[ \frac{\lambda^{-1/4} (n - m) y}{x} \right]
\end{align*}

$\beta_{ij}$ is a function of the oscillatory index and modulus of the complex field variables and is given as

\begin{align*}
\beta_{11} &= \cos (\epsilon \ln r_1) + 2 \epsilon \sin (\epsilon \ln r_1) \quad \beta_{12} = \cos (\epsilon \ln r_2) + 2 \epsilon \sin (\epsilon \ln r_2) \\
\beta_{21} &= \sin (\epsilon \ln r_1) - 2 \epsilon \cos (\epsilon \ln r_1) \quad \beta_{22} = \sin (\epsilon \ln r_2) - 2 \epsilon \cos (\epsilon \ln r_2)
\end{align*}

Finally, $\delta_{ij}$ where $j$ corresponds to the material number and (+) is chosen for material 1 and (−) for material 2 is defined as

\begin{align*}
\delta_{j1} &= e^{(\pm \pi - \phi_1) \epsilon} \\
\delta_{j2} &= e^{(\pm \pi - \phi_2) \epsilon}
\end{align*}
### Table A.1: $F_{ij}$, $F_{ij}'$, $M_{ij}$ and $M_{ij}'$.

<table>
<thead>
<tr>
<th>Index</th>
<th>$F_{ij}$</th>
<th>$F_{ij}'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>$\beta_{11}\gamma_{11}\cos\frac{\phi_1}{2} - \beta_{21}\gamma_{11}'\sin\frac{\phi_1}{2}$</td>
<td>$\beta_{21}\gamma_{11}\cos\frac{\phi_2}{2} + \beta_{11}\gamma_{11}'\sin\frac{\phi_2}{2}$</td>
</tr>
<tr>
<td>32</td>
<td>$\beta_{12}\gamma_{32}\cos\frac{\phi_2}{2} - \beta_{22}\gamma_{32}'\sin\frac{\phi_2}{2}$</td>
<td>$\beta_{22}\gamma_{32}\cos\frac{\phi_2}{2} + \beta_{12}\gamma_{32}'\sin\frac{\phi_2}{2}$</td>
</tr>
<tr>
<td>61</td>
<td>$\beta_{11}\gamma_{61}\cos\frac{\phi_1}{2} - \beta_{21}\gamma_{61}'\sin\frac{\phi_1}{2}$</td>
<td>$\beta_{21}\gamma_{61}\cos\frac{\phi_2}{2} + \beta_{11}\gamma_{61}'\sin\frac{\phi_2}{2}$</td>
</tr>
<tr>
<td>82</td>
<td>$\beta_{12}\gamma_{82}\cos\frac{\phi_2}{2} - \beta_{22}\gamma_{82}'\sin\frac{\phi_2}{2}$</td>
<td>$\beta_{22}\gamma_{82}\cos\frac{\phi_2}{2} + \beta_{12}\gamma_{82}'\sin\frac{\phi_2}{2}$</td>
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</table>

<table>
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<th>$M_{ij}$</th>
<th>$M_{ij}'$</th>
</tr>
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<td>21</td>
<td>$\beta_{11}\gamma_{21}\cos\frac{\phi_1}{2} - \beta_{21}\gamma_{21}'\sin\frac{\phi_1}{2}$</td>
<td>$\beta_{21}\gamma_{21}\cos\frac{\phi_2}{2} + \beta_{11}\gamma_{21}'\sin\frac{\phi_2}{2}$</td>
</tr>
<tr>
<td>42</td>
<td>$\beta_{12}\gamma_{42}\cos\frac{\phi_2}{2} - \beta_{22}\gamma_{42}'\sin\frac{\phi_2}{2}$</td>
<td>$\beta_{22}\gamma_{42}\cos\frac{\phi_2}{2} + \beta_{12}\gamma_{42}'\sin\frac{\phi_2}{2}$</td>
</tr>
<tr>
<td>51</td>
<td>$\beta_{11}\gamma_{51}\cos\frac{\phi_1}{2} - \beta_{21}\gamma_{51}'\sin\frac{\phi_1}{2}$</td>
<td>$\beta_{21}\gamma_{51}\cos\frac{\phi_2}{2} + \beta_{11}\gamma_{51}'\sin\frac{\phi_2}{2}$</td>
</tr>
<tr>
<td>72</td>
<td>$\beta_{12}\gamma_{72}\cos\frac{\phi_2}{2} - \beta_{22}\gamma_{72}'\sin\frac{\phi_2}{2}$</td>
<td>$\beta_{22}\gamma_{72}\cos\frac{\phi_2}{2} + \beta_{12}\gamma_{72}'\sin\frac{\phi_2}{2}$</td>
</tr>
</tbody>
</table>

### Table A.2: $\gamma_{ij}$ and $\gamma_{ij}'$.

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<th>$\gamma_{ij}'$</th>
<th>Index</th>
<th>$\gamma_{ij}$</th>
<th>$\gamma_{ij}'$</th>
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</thead>
<tbody>
<tr>
<td>11</td>
<td>$Q_1\delta_{j1} + \frac{Q_1}{s_{j1}}$</td>
<td>$Q_1\delta_{j1} - \frac{Q_1}{s_{j1}}$</td>
<td>51</td>
<td>$Q_5\delta_{j1} + \frac{Q_5}{s_{j1}}$</td>
<td>$Q_5\delta_{j1} - \frac{Q_5}{s_{j1}}$</td>
</tr>
<tr>
<td>21</td>
<td>$Q_2\delta_{j1} + \frac{Q_2}{s_{j1}}$</td>
<td>$Q_2\delta_{j1} - \frac{Q_2}{s_{j1}}$</td>
<td>61</td>
<td>$Q_6\delta_{j1} + \frac{Q_6}{s_{j1}}$</td>
<td>$Q_6\delta_{j1} - \frac{Q_6}{s_{j1}}$</td>
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<tr>
<td>32</td>
<td>$Q_3\delta_{j2} + \frac{Q_3}{s_{j2}}$</td>
<td>$Q_3\delta_{j2} - \frac{Q_3}{s_{j2}}$</td>
<td>72</td>
<td>$Q_7\delta_{j2} + \frac{Q_7}{s_{j2}}$</td>
<td>$Q_7\delta_{j2} - \frac{Q_7}{s_{j2}}$</td>
</tr>
<tr>
<td>42</td>
<td>$Q_4\delta_{j2} + \frac{Q_4}{s_{j2}}$</td>
<td>$Q_4\delta_{j2} - \frac{Q_4}{s_{j2}}$</td>
<td>82</td>
<td>$Q_8\delta_{j2} + \frac{Q_8}{s_{j2}}$</td>
<td>$Q_8\delta_{j2} - \frac{Q_8}{s_{j2}}$</td>
</tr>
</tbody>
</table>

### Table A.3: $Q_i$.

- $Q_1 = \lambda^{-1/4} \sqrt{\frac{H_{11}}{H_{22}}} (S_{11}\lambda^{-1/2} (n + m)^2 - S_{12}) (n - m)$
- $Q_2 = S_{12} - S_{11}\lambda^{-1/2} (n + m)^2$
- $Q_3 = \lambda^{-1/4} \sqrt{\frac{H_{11}}{H_{22}}} (S_{12} - S_{11}\lambda^{-1/2} (n - m)^2) (n + m)$
- $Q_4 = S_{11}\lambda^{-1/2} (n - m)^2 - S_{12}$
- $Q_5 = \sqrt{\frac{H_{11}}{H_{22}}} (S_{21}\lambda^{-1/2} (n + m)^2 - S_{22}) \frac{n - m}{n + m}$
- $Q_6 = \lambda^{1/4} (S_{22} - S_{21}\lambda^{-1/2} (n + m)^2) (n + m)^{-1}$
- $Q_7 = \sqrt{\frac{H_{11}}{H_{22}}} (S_{22} - S_{21}\lambda^{-1/2} (n - m)^2) \frac{n + m}{n - m}$
- $Q_8 = \lambda^{1/4} (S_{21}\lambda^{-1/2} (n - m)^2 - S_{22}) (n - m)^{-1}$
A.2 Real-Numbered Expressions

\textbf{A.2.2} \quad -1 < \rho < 1

\[ \Delta u_x^{(1)} \bigg|_{\Delta K_2 = 0} = \frac{R \lambda^{1/4}}{m} \Delta K_1 \]
\[ \frac{\sqrt{r_1}}{m} \left( F_{91} - M'_{101} + M_{111} + F'_{121} \right) + \sqrt{r_2} \left( F_{132} - M'_{142} + M_{152} - F'_{162} \right) \]
\[ \Delta u_x^{(2)} \bigg|_{\Delta K_1 = 0} = \frac{R \lambda^{1/4}}{m} \Delta K_2 \]
\[ \frac{\sqrt{r_1}}{m} \left( -F_{91} - M_{101} - M'_{111} + F_{121} \right) + \sqrt{r_2} \left( -F_{132} - M_{142} - M'_{152} - F_{162} \right) \]  
(A.28)

\[ \Delta u_y^{(1)} \bigg|_{\Delta K_2 = 0} = \frac{R \lambda^{1/4}}{m} \Delta K_1 \]
\[ \frac{\sqrt{r_1}}{m} \left( F_{172} - M'_{181} + M_{191} - F'_{201} \right) + \sqrt{r_2} \left( F_{212} + M'_{222} + M_{232} - F'_{242} \right) \]
\[ \Delta u_y^{(2)} \bigg|_{\Delta K_1 = 0} = \frac{R \lambda^{1/4}}{m} \Delta K_2 \]
\[ \frac{\sqrt{r_1}}{m} \left( -F_{171} - M_{181} - M'_{191} - F_{201} \right) + \sqrt{r_2} \left( -F_{212} + M_{222} - M'_{232} - F_{242} \right) \]  
(A.29)

\( F_{ijk}, F'_{ijk}, M_{ijk} \) and \( M'_{ijk} \) are all given in Table A.4 by use of Tables A.5 and A.6.

\( r_i \) and \( \phi_i \) are modulus and argument for the complex field variables \( Z_1 \) and \( Z_2 \) given in (A.5).

\[ r_1 = \sqrt{(x + \lambda^{-1/4}my)^2 + n^2\lambda^{-1/2}y^2} \quad \phi_1 = \arctan \left[ \frac{n\lambda^{-1/4}y}{x + \lambda^{-1/4}my} \right] \]  
(A.30)

\[ r_2 = \sqrt{(x - \lambda^{-1/4}my)^2 + n^2\lambda^{-1/2}y^2} \quad \phi_2 = \arctan \left[ \frac{n\lambda^{-1/4}y}{x - \lambda^{-1/4}my} \right] \]  
(A.31)

\( \beta_{ij} \) and \( \delta_{ij} \) are given in (A.26) and (A.27).
## Table A.4: $F_{ij}$, $F'_{ij}$, $M_{ij}$ and $M'_{ij}$.

<table>
<thead>
<tr>
<th>Index</th>
<th>$F_{ij}$</th>
<th>$F'_{ij}$</th>
<th>$M_{ij}$</th>
<th>$M'_{ij}$</th>
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</thead>
<tbody>
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<td>$\beta_{11}\gamma_{91} \cos \frac{\delta_{1}}{2} - \beta_{21}\gamma'<em>{91} \sin \frac{\delta</em>{1}}{2}$</td>
<td>$\beta_{21}\gamma_{91} \cos \frac{\delta_{2}}{2} + \beta_{11}\gamma'<em>{91} \sin \frac{\delta</em>{2}}{2}$</td>
<td>$\beta_{21}\gamma_{101} \cos \frac{\delta_{1}}{2} + \beta_{11}\gamma_{101} \sin \frac{\delta_{1}}{2}$</td>
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</tr>
<tr>
<td>121</td>
<td>$\beta_{11}\gamma_{121} \cos \frac{\delta_{1}}{2} - \beta_{21}\gamma'<em>{121} \sin \frac{\delta</em>{1}}{2}$</td>
<td>$\beta_{21}\gamma_{121} \cos \frac{\delta_{2}}{2} + \beta_{11}\gamma'<em>{121} \sin \frac{\delta</em>{2}}{2}$</td>
<td>$\beta_{21}\gamma_{111} \cos \frac{\delta_{1}}{2} + \beta_{11}\gamma'<em>{111} \sin \frac{\delta</em>{1}}{2}$</td>
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<tr>
<td>132</td>
<td>$\beta_{12}\gamma_{132} \cos \frac{\delta_{2}}{2} - \beta_{22}\gamma'<em>{132} \sin \frac{\delta</em>{2}}{2}$</td>
<td>$\beta_{22}\gamma_{132} \cos \frac{\delta_{2}}{2} + \beta_{12}\gamma'<em>{132} \sin \frac{\delta</em>{2}}{2}$</td>
<td>$\beta_{22}\gamma_{122} \cos \frac{\delta_{2}}{2} + \beta_{12}\gamma'<em>{122} \sin \frac{\delta</em>{2}}{2}$</td>
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<tr>
<td>162</td>
<td>$\beta_{12}\gamma_{162} \cos \frac{\delta_{2}}{2} - \beta_{22}\gamma'<em>{162} \sin \frac{\delta</em>{2}}{2}$</td>
<td>$\beta_{22}\gamma_{162} \cos \frac{\delta_{2}}{2} + \beta_{12}\gamma'<em>{162} \sin \frac{\delta</em>{2}}{2}$</td>
<td>$\beta_{22}\gamma_{111} \cos \frac{\delta_{2}}{2} + \beta_{11}\gamma'<em>{111} \sin \frac{\delta</em>{2}}{2}$</td>
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<td>$\beta_{11}\gamma_{171} \cos \frac{\delta_{1}}{2} - \beta_{21}\gamma'<em>{171} \sin \frac{\delta</em>{1}}{2}$</td>
<td>$\beta_{21}\gamma_{171} \cos \frac{\delta_{2}}{2} + \beta_{11}\gamma'<em>{171} \sin \frac{\delta</em>{2}}{2}$</td>
<td>$\beta_{21}\gamma_{201} \cos \frac{\delta_{2}}{2} + \beta_{11}\gamma'<em>{201} \sin \frac{\delta</em>{2}}{2}$</td>
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<tr>
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<td>$\beta_{11}\gamma_{201} \cos \frac{\delta_{1}}{2} - \beta_{21}\gamma'<em>{201} \sin \frac{\delta</em>{1}}{2}$</td>
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<tr>
<td>242</td>
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<td>$\beta_{22}\gamma_{242} \cos \frac{\delta_{2}}{2} + \beta_{12}\gamma'<em>{242} \sin \frac{\delta</em>{2}}{2}$</td>
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## Table A.5: $\gamma_{ij}$ and $\gamma'_{ij}$.

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<th>$\gamma'_{ij}$</th>
<th>Index</th>
<th>$\gamma_{ij}$</th>
<th>$\gamma'_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>91</td>
<td>$Q_9\delta_{j1} + \frac{Q_9}{\delta_{j1}}$</td>
<td>$Q_9\delta_{j1} - \frac{Q_9}{\delta_{j1}}$</td>
<td>171</td>
<td>$Q_{17}\delta_{j1} + \frac{Q_{17}}{\delta_{j1}}$</td>
<td>$Q_{17}\delta_{j1} - \frac{Q_{17}}{\delta_{j1}}$</td>
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<tr>
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<td>$Q_{10}\delta_{j1} - \frac{Q_{10}}{\delta_{j1}}$</td>
<td>181</td>
<td>$Q_{18}\delta_{j1} + \frac{Q_{18}}{\delta_{j1}}$</td>
<td>$Q_{18}\delta_{j1} - \frac{Q_{18}}{\delta_{j1}}$</td>
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<td>111</td>
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<td>$Q_{11}\delta_{j1} - \frac{Q_{11}}{\delta_{j1}}$</td>
<td>191</td>
<td>$Q_{19}\delta_{j1} + \frac{Q_{19}}{\delta_{j1}}$</td>
<td>$Q_{19}\delta_{j1} - \frac{Q_{19}}{\delta_{j1}}$</td>
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<tr>
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<td>$Q_{12}\delta_{j1} - \frac{Q_{12}}{\delta_{j1}}$</td>
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<td>$Q_{20}\delta_{j1} + \frac{Q_{20}}{\delta_{j1}}$</td>
<td>$Q_{20}\delta_{j1} - \frac{Q_{20}}{\delta_{j1}}$</td>
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<tr>
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<td>$Q_{13}\delta_{j1} + \frac{Q_{13}}{\delta_{j1}}$</td>
<td>$Q_{13}\delta_{j1} - \frac{Q_{13}}{\delta_{j1}}$</td>
<td>212</td>
<td>$Q_{21}\delta_{j1} + \frac{Q_{21}}{\delta_{j1}}$</td>
<td>$Q_{21}\delta_{j1} - \frac{Q_{21}}{\delta_{j1}}$</td>
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<td>$Q_{14}\delta_{j1} + \frac{Q_{14}}{\delta_{j1}}$</td>
<td>$Q_{14}\delta_{j1} - \frac{Q_{14}}{\delta_{j1}}$</td>
<td>222</td>
<td>$Q_{22}\delta_{j1} + \frac{Q_{22}}{\delta_{j1}}$</td>
<td>$Q_{22}\delta_{j1} - \frac{Q_{22}}{\delta_{j1}}$</td>
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<td>152</td>
<td>$Q_{15}\delta_{j1} + \frac{Q_{15}}{\delta_{j1}}$</td>
<td>$Q_{15}\delta_{j1} - \frac{Q_{15}}{\delta_{j1}}$</td>
<td>232</td>
<td>$Q_{23}\delta_{j1} + \frac{Q_{23}}{\delta_{j1}}$</td>
<td>$Q_{23}\delta_{j1} - \frac{Q_{23}}{\delta_{j1}}$</td>
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<td>$Q_{16}\delta_{j1} + \frac{Q_{16}}{\delta_{j1}}$</td>
<td>$Q_{16}\delta_{j1} - \frac{Q_{16}}{\delta_{j1}}$</td>
<td>242</td>
<td>$Q_{24}\delta_{j1} + \frac{Q_{24}}{\delta_{j1}}$</td>
<td>$Q_{24}\delta_{j1} - \frac{Q_{24}}{\delta_{j1}}$</td>
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### Table A.6: \( Q_i \)

<table>
<thead>
<tr>
<th>( Q_i )</th>
<th>Expression</th>
</tr>
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<tr>
<td>( Q_9 )</td>
<td>( \lambda^{-1/4} \sqrt{\frac{H_{12}}{H_{22}}} m \left( S_{12} + S_{11} \lambda^{-1/2} (n^2 + m^2) \right) )</td>
</tr>
<tr>
<td>( Q_{10} )</td>
<td>( \lambda^{-1/4} \sqrt{\frac{H_{12}}{H_{22}}} n \left( S_{11} \lambda^{-1/2} (n^2 + m^2) - S_{12} \right) )</td>
</tr>
<tr>
<td>( Q_{11} )</td>
<td>(-2\lambda^{-1/2} S_{11} nm )</td>
</tr>
<tr>
<td>( Q_{12} )</td>
<td>(- \left( S_{12} + S_{11} \lambda^{-1/2} (m^2 - n^2) \right) )</td>
</tr>
<tr>
<td>( Q_{13} )</td>
<td>( \lambda^{-1/4} \sqrt{\frac{H_{12}}{H_{22}}} m \left( S_{12} + S_{11} \lambda^{-1/2} (n^2 + m^2) \right) = Q_9 )</td>
</tr>
<tr>
<td>( Q_{14} )</td>
<td>( \lambda^{-1/4} \sqrt{\frac{H_{12}}{H_{22}}} n \left( S_{12} - S_{11} \lambda^{-1/2} (n^2 + m^2) \right) = -Q_{10} )</td>
</tr>
<tr>
<td>( Q_{15} )</td>
<td>(-2\lambda^{-1/2} S_{11} nm = Q_{11} )</td>
</tr>
<tr>
<td>( Q_{16} )</td>
<td>(- \left( S_{12} + S_{11} \lambda^{-1/2} (m^2 - n^2) \right) = Q_{12} )</td>
</tr>
<tr>
<td>( Q_{17} )</td>
<td>(- \sqrt{\frac{H_{12}}{H_{22}}} \left( \frac{S_{22} + S_{21} \lambda^{-1/2} (m^2 - n^2)}{m^2 + n^2} \right) \left( n^2 - m^2 - 4\lambda^{-1/2} S_{21} n^2 m^2 \right) )</td>
</tr>
<tr>
<td>( Q_{18} )</td>
<td>(- \sqrt{\frac{H_{12}}{H_{22}}} \left( \frac{S_{22} + S_{21} \lambda^{-1/2} (m^2 - n^2)}{m^2 + n^2} \right) \left( n^2 - m^2 - 4\lambda^{-1/2} S_{21} n^2 m^2 \right) )</td>
</tr>
<tr>
<td>( Q_{19} )</td>
<td>( \frac{\lambda^{1/4}}{m^2 + n^2} \left( S_{22} n + S_{21} \lambda^{-1/2} n (m^2 - n^2) - 2\lambda^{-1/2} S_{21} n^2 m^2 \right) )</td>
</tr>
<tr>
<td>( Q_{20} )</td>
<td>( \frac{\lambda^{1/4}}{m^2 + n^2} \left( S_{22} m + S_{21} \lambda^{-1/2} m (m^2 - n^2) - 2\lambda^{-1/2} S_{21} n^2 m^2 \right) )</td>
</tr>
<tr>
<td>( Q_{21} )</td>
<td>( \sqrt{\frac{H_{12}}{H_{22}}} \left( \frac{S_{22} + S_{21} \lambda^{-1/2} (m^2 - n^2)}{m^2 + n^2} \right) \left( n^2 - m^2 - 4\lambda^{-1/2} S_{21} n^2 m^2 \right) = -Q_{17} )</td>
</tr>
<tr>
<td>( Q_{22} )</td>
<td>( \sqrt{\frac{H_{12}}{H_{22}}} \left( \frac{S_{22} + S_{21} \lambda^{-1/2} (m^2 - n^2)}{m^2 + n^2} \right) \left( n^2 - m^2 - 4\lambda^{-1/2} S_{21} n^2 m^2 \right) = -Q_{18} )</td>
</tr>
<tr>
<td>( Q_{23} )</td>
<td>( \frac{-\lambda^{1/4}}{m^2 + n^2} \left( S_{22} n + S_{21} \lambda^{-1/2} n (m^2 - n^2) - 2\lambda^{-1/2} S_{21} n^2 m^2 \right) = -Q_{19} )</td>
</tr>
<tr>
<td>( Q_{24} )</td>
<td>( \frac{-\lambda^{1/4}}{m^2 + n^2} \left( S_{22} m + S_{21} \lambda^{-1/2} m (m^2 - n^2) - 2\lambda^{-1/2} S_{21} n^2 m^2 \right) = Q_{20} )</td>
</tr>
</tbody>
</table>
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Appendix B

Additional Results from the In-Plane Compression Tests

In this appendix additional results from the two compression test series, SERIES 1 and 2, are found. The appendix is not meant for reading, but only as a backup for the main report, where the results from testing the different panel specimens are discussed.

B.1 Uniform Compression

Pictures of the tested panels before and after failure are found below together with results from the air-coupled ultrasonic scans performed on the panels after failure.

Results from eight panel specimens are described below. The results from the two remaining specimens, H80 Ø200 mm (B specimen) and H80 intact (B specimen), are found in the main report.
Appendix B. Additional Results from the In-Plane Compression Tests

B.1.1 H80 Ø100 mm (A Specimen)

Figure B.1: The H80 panel with Ø100 mm debond (A specimen) before failure (left) and after failure (right).

Figure B.2: Air-coupled ultrasonic scanning showing the failed H80 Ø100 mm panel (A specimen).
B.1 Uniform Compression

B.1.2 H80 Ø100 mm (B Specimen)

Figure B.3: The H80 panel with Ø100 mm debond (B specimen) before failure (left) and after failure (right).

Figure B.4: The H80 Ø100 mm panel (B specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
B.1.3 H80 Ø200 mm (A Specimen)

In this case unfortunately only pre-failure pictures are available, except for pictures from the ultrasonic scans performed after panel failure.

It should be noted in B.5 (left) how the loading device is shaped. Furthermore, the perfectly machined plane top edge of the panel and the bolted reinforcement ribs on the plywood edges of the panel are clearly seen.

Figure B.5: The H80 panel with Ø100 mm debond (B specimen) before failure (left) and by air-coupled ultrasonic scanning after failure (right).
B.1 Uniform Compression

B.1.4 H80 Ø300 mm (A Specimen)

Figure B.6: The H80 panel with Ø300 mm debond (A specimen) before failure (left) and after failure (right).

Figure B.7: The H80 Ø300 mm panel (A specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
Appendix B. Additional Results from the In-Plane Compression Tests

B.1.5 H80 Ø300 mm (B Specimen)

Figure B.8: The H80 panel with Ø300 mm debond (B-specimen) before failure (left) and after failure (right).

Figure B.9: The H80 Ø300 mm panel (B specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
B.1 Uniform Compression

B.1.6 H80 Intact (A Specimen)

Figure B.10: The H80 intact panel (A specimen) before failure (left) and after failure (right).

Figure B.11: The H80 intact (A specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
Appendix B. Additional Results from the In-Plane Compression Tests

B.1.7 H200 Ø200 mm (A Specimen)

Figure B.12: The H200 panel with Ø200 mm debond (A specimen) before failure (left) and after failure (right).

Figure B.13: The H200 Ø200 mm panel (A specimen) after testing, where the debond propagation has been estimated by coin-tapping (right) and air-coupled ultrasonic scanning (left).
B.1.8 H200 Intact (A Specimen)

Figure B.14: The H200 intact panel (A specimen) before failure (left) and after failure (right).

Figure B.15: The H200 intact (A specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
Appendix B. Additional Results from the In-Plane Compression Tests

B.2 Non-Uniform Compression

In this section pictures of the tested panels before and after failure are presented, together with results from the air-coupled ultrasonic scans performed on the panels after failure.

Results from eight panel specimens are described below. The results from the two remaining specimens, H80 Ø200 mm (B specimen) and H80 intact (B specimen), are found in the main report.

Additionally, polynomial trend curves describing the vertical displacement in the left and right strain gauge positions\(^1\) for the top and bottom gauges are included for later numerical implementation. The polynomial trend curves are seen in Figure 7.22 together with the measured values.

In total four polynomial trend curves, one for each strain gauge position, are used to describe the vertical panel edge displacement through a linear extrapolation over the panel edge.

The horizontal panel movement is calculated analytically on the assumption that the length of the panel edge remains constant. The vertical panel edge displacement acts therefore as input to this analytical prediction. The assumption about no stretching of the panel edge seems fair, as the panel is bolted to a rigid steel construction, see Figure 7.19.

The four polynomial trend curves for the vertical displacements in the strain gauge positions are given as a sixth order polynomial:

\[
U_Z = a_6 t^6 + a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 \tag{B.1}
\]

where \(t\) is time, and \(U_Z\) is measured in mm.

The coefficients \(a_i\) are found in Tables B.1 to B.5 in the last subsection for all panel cases.

It should be noted that the polynomial trend curves are only valid up to the end time given in the tables. However, using the polynomial trend curves to estimate the loading profiles makes it possible to model the dynamic response of the panel if an explicit finite element code is applied, eg. LS-DYNA.

\(^1\)The signal from the front- and backside strain gauges are averaged
B.2 Non-Uniform Compression

B.2.1 H80 Ø100 mm (A Specimen)

Figure B.16: The H80 panel with Ø100 mm debond (A specimen) before failure (left) and after failure (right).

Figure B.17: Air-coupled ultrasonic scanning showing the failed H80 Ø100 mm panel (A specimen).
Appendix B. Additional Results from the In-Plane Compression Tests

B.2.2 H80 Ø100 mm (B Specimen)

Figure B.18: The H80 panel with Ø100 mm debond (B specimen) **before** failure (left) and **after** failure (right).

Figure B.19: The H80 Ø100 mm panel (B specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
B.2 Non-Uniform Compression

B.2.3 H80 Ø200 mm (A Specimen)

Figure B.20: The H80 panel with Ø200 mm debond (A specimen) before failure (left) and after failure (right).

Figure B.21: The H80 Ø200 mm panel (A specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
B.2.4 H80 Ø300 mm (A Specimen)

Figure B.22: The H80 panel with Ø300 mm debond (A specimen) **before** failure (left) and **after** failure (right).

Figure B.23: The H80 Ø300 mm panel (A specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
B.2.5 H80 Ø300 mm (B Specimen)

Figure B.24: The H80 panel with Ø300 mm debond (B specimen) **before** failure (left) and **after** failure (right).

Figure B.25: The H80 Ø300 mm panel (B specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
B.2.6 H80 Intact (A Specimen)

Figure B.26: The H80 intact panel (A specimen) before failure (left) and after failure (right).

Figure B.27: The H80 intact (A specimen) after testing, where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
B.2 Non-Uniform Compression

B.2.7 H200 Ø200 mm (A Specimen)

Figure B.28: The H200 panel with Ø200 mm debond (A specimen) before failure (left) and after failure (right).

Figure B.29: The H200 Ø200 mm panel (A specimen) after testing, where the debond propagation has been estimated by coin-tapping (right) and air-coupled ultrasonic scanning (left).
B.2.8 H200 Intact (A Specimen)

Figure B.30: The H200 intact panel (A specimen) before failure (left) and after failure (right). Note the face compression failure on the backside of the panel.

Figure B.31: H200 intact (A specimen) after testing (front side), where the debond propagation has been estimated by coin-tapping (left) and air-coupled ultrasonic scanning (right).
### B.2 Non-Uniform Compression

#### B.2.9 Load Profile Coefficients

Table B.1: **H80 Ø100 mm**.

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Appendix B. Additional Results from the In-Plane Compression Tests
1961  Strøm-Tejsen, J.
    Damage Stability Calculations on the Computer DASK.

1963  Silovic, V.
    A Five Hole Spherical Pilot Tube for three Dimensional Wake Measurements.

1964  Chomchuenchit, V.
    Determination of the Weight Distribution of Ship Models.

1965  Chislett, M.S.
    A Planar Motion Mechanism.

1965  Nicordhanon, P.
    A Phase Changer in the HyA Planar Motion Mechanism and Calculation of Phase Angle.

1966  Jensen, B.
    Anvendelse af statistiske metoder til kontrol af forskellige eksisterende tilnærmelsesformler og udarbejdelse af nye til bestemmelse af skibes tonnage og stabilitet.

1968  Aage, C.
    Eksperimentel og beregningsmæssig bestemmelse af vindkræfter på skibe.

1972  Prytz, K.
    Datamatorienterede studier af planende bådes fremdrivningsforhold.

1977  Hee, J.M.
    Store sideportes indflydelse på langskibs styrke.

1977  Madsen, N.F.
    Vibrations in Ships.

1978  Andersen, P.
    Bølgeinducerede bevægelser og belastninger for skib på lægt vand.

1978  Rømeling, J.U.
    Buling af afdtvede pladepaneler.

1978  Sørensen, H.H.
    Sammenkobling af rotations-symmetriske og generelle tre-dimensionale konstruktioner i elementmetode-beregninger.

1980  Fabian, O.
1980  Petersen, M.J.  
Ship Collisions.

1981  Gong, J.  
A Rational Approach to Automatic Design of Ship Sections.

1982  Nielsen, K.  
Bølgeenergimaskiner.

1984  Nielsen, N.J.R.  
Structural Optimization of Ship Structures.

1984  Liebst, J.  
Torsion of Container Ships.

1985  Gjersøe-Fog, N.  
Mathematical Definition of Ship Hull Surfaces using B-splines.

1985  Jensen, P.S.  
Stationære skibsølger.

1986  Nedergaard, H.  
Collapse of Offshore Platforms.

1986  Yan, J.-Q.  
3-D Analysis of Pipelines during Laying.

1987  Holt-Madsen, A.  

1989  Andersen, S.V.  

1989  Rasmussen, J.  
Structural Design of Sandwich Structures.

1990  Baatrup, J.  
Structural Analysis of Marine Structures.

1990  Wedel-Heinen, J.  
Vibration Analysis of Imperfect Elements in Marine Structures.

1991  Almlund, J.  
Life Cycle Model for Offshore Installations for Use in Prospect Evaluation.

1991  Back-Pedersen, A.  
Analysis of Slender Marine Structures.
1992  Bendiksen, E.
  Hull Girder Collapse.

1992  Petersen, J.B.
  Non-Linear Strip Theories for Ship Response in Waves.

1992  Schalck, S.
  Ship Design Using B-spline Patches.

1993  Kierkegaard, H.
  Ship Collisions with Icebergs.

1994  Pedersen, B.
  A Free-Surface Analysis of a Two-Dimensional Moving Surface-Piercing Body.

1994  Hansen, P.F.
  Reliability Analysis of a Midship Section.

1994  Michelsen, J.
  A Free-Form Geometric Modelling Approach with Ship Design Applications.

1995  Hansen, A.M.
  Reliability Methods for the Longitudinal Strength of Ships.

1995  Branner, K.
  Capacity and Lifetime of Foam Core Sandwich Structures.

1995  Schack, C.
  Skrogudvikling af hurtiggående færger med henblik på sødygtighed og lav modstand.

1997  Simonsen, B.C.
  Mechanics of Ship Grounding.

1997  Olesen, N.A.
  Turbulent Flow past Ship Hulls.

1997  Riber, H.J.
  Response Analysis of Dynamically Loaded Composite Panels.

1998  Andersen, M.R.
  Fatigue Crack Initiation and Growth in Ship Structures.

1998  Nielsen, L.P.
  Structural Capacity of the Hull Girder.

1999  Zhang, S.
  The Mechanics of Ship Collisions.

1999  Birk-Sørensen, M.
  Simulation of Welding Distortions of Ship Sections.
1999  Jensen, K.
    Analysis and Documentation of Ancient Ships.

2000  Wang, Z.
    Hydroelastic Analysis of High-Speed Ships.

2000  Petersen, T.
    Wave Load Prediction—a Design Tool.

2000  Banke, L.
    Flexible Pipe End Fitting.

2000  Simonsen, C.D.
    Rudder, Propeller and Hull Interaction by RANS.

2000  Clausen, H.B.
    Plate Forming by Line Heating.

2000  Krishnaswamy, P.
    Flow Modelling for Partially Cavitating Hydrofoils.

2000  Andersen, L.F.
    Residual Stresses and Deformations in Steel Structures.

2000  Friis-Hansen, A.
    Bayesian Networks as a Decision Support Tool in Marine Applications.

PhD Theses
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Technical University of Denmark · Kgs. Lyngby

2001  Lützen, M.
    Ship Collision Damage.

2001  Olsen, A.S.
    Optimisation of Propellers Using the Vortex-Lattice Method.

2002  Rüdinger, F.
    Modelling and Estimation of Damping in Non-linear Random Vibration.

2002  Bredmose, H.
    Deterministic Modelling of Water Waves in the Frequency Domain.

2003  Urban, J.
    Crushing and Fracture of Lightweight Structures.
2003  Lazarov, B.S.  
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2003  Törnqvist, R.  
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2003  Nielsen, K.B.  
*Numerical Prediction of Green Water Loads on Ships.*

2004  Folso, R.  
*Comfort Monitoring of High Speed Passenger Ferries.*

2004  Fuhrman, D.R.  
*Numerical Solutions of Boussinesq Equations for Fully Nonlinear and Extremely Dispersive Water Waves.*

2004  Dietz, J.S.  
*Application of Conditional Waves as Critical Wave Episodes for Extreme Loads on Marine Structures.*

2004  Berggreen, C.  
*Damage Tolerance of Debonded Sandwich Structures.*