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Ratchet due to broken friction symmetry

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A ratchet mechanism that occurs due to asymmetric dependence of the friction of a moving system on its velocity or a driving force is reported. For this kind of ratchet, instead of a particle moving in a periodic potential, the dynamics of which have broken space-time symmetry, the system must be provided with some internal structure realizing such a velocity- or force-friction dependence. For demonstration of a ratchet mechanism of this type, an experimental setup (gadget) that converts longitudinal oscillating or fluctuating motion into a unidirectional rotation has been built and experiments with it have been carried out. In this device, an asymmetry of friction dependence on an applied force appears, resulting in rectification of rotary motion. In experiments, our setup is observed to rotate only in one direction, which is in accordance with given theoretical arguments. Despite the setup being three dimensional, the ratchet rotary motion is proved to be described by one dynamical equation. This kind of motion is a result of the interplay of friction and inertia. We also consider a case with viscous friction, which is irrelevant to this gadget, but it can be a possible mechanism of rotary unidirectional motion of some swimming organisms in a liquid.

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I. INTRODUCTION

Inspired by the Smoluchowski-Feynman ratchet [1–3] and further pioneering studies [4–9], a variety of mechanisms for molecular motors have been suggested [10]. All these approaches involve rectifying motion of an overdamped Brownian particle in a spatially periodic structure due to nonequilibrium oscillations or fluctuations of zero average, resulting in a biased current, so-called ratchet. Besides this overdamped ratchet motion, Jung, Kissner, and Hänggi [11] have originated the whole class of deterministic ratchets for systems with finite inertia (see, e.g., related papers [12–20] and others [3]). The present paper also deals with an inertia ratchet—here the interplay of friction and inertia results in a unidirectional rotary motion.

The fundamental condition for the rectified transport to occur is that certain symmetries, associated with spatial or time reflection, are broken. On the other hand, a similar symmetry breaking can also be achieved parametrically, when at least one of the system parameters depends asymmetrically on external zero-mean forcing. However, for such a ratchet mechanism to occur, the system must possess some internal structure, i.e., it has to be a "machine" itself [21–23]. In this paper, we suggest a ratchet mechanism for a system with internal structure that admits altering intrinsic parameters of the system through a broken symmetry of the dependence of these parameters on its velocity or an external driving force. One of these parameters can be the friction in a motion of the system that depends asymmetrically on its velocity or the driving force. We call this type of rectifying motion, which occurs due to broken friction symmetry, a velocity- or force-dependent friction ratchet, respectively.

To demonstrate the ratchet mechanism that appears due to an asymmetry of the dependence of friction on an ac driving force, we have built a simple experimental setup that unambiguously shows rotary ratchet motion. Note that mechanical or electrical ratchet models can be useful for better understanding the physics of ratchet motion as well as for finding new ratchet mechanisms, similarly as it was performed for nonlinear dynamics studies in condensed matter physics, where mechanical devices and electrical circuits have been proved fruitful for modeling remarkable properties of nonlinear collective excitations in solids [24] (such as Scott's model [25], experimentally demonstrating the propagation of topological solitons, or the pendulum model suggested by Russell *et al.* [26] for visual illustration of discrete breathers).

The paper is organized as follows. In Sec. II, we describe a mechanical gadget, explaining intuitively the ratchet mechanism of rotations in this device with presentation of a corresponding general equation of motion. Explicit solutions of this equation in two particular cases of dry and viscous friction that demonstrate a rectified motion are given in Sec. III. Concluding remarks and a brief discussion on different particular cases of the general equation of motion of the underdamped oscillator a velocity (force) dependent friction are presented in Sec. IV. Derivation of the equation of motion for the mechanical gadget is described in the Appendix.

II. AN EXPERIMENTAL MODEL FOR RECTIFICATION OF ac FORCING

To demonstrate the rotary ratchet mechanism that appears due to an asymmetry of the dependence of friction on an ac driving force, we have made the simple experimental setup (gadget) shown as a photograph in Fig. 1. In experiments, this device shows unambiguously the rotary ratchet motion directed clockwise when viewed from above. It consists of two massive plates (weights), which are connected by two lateral springs, so that their geometric arrangement mimics a right-handed helical structure. It is important that the lateral

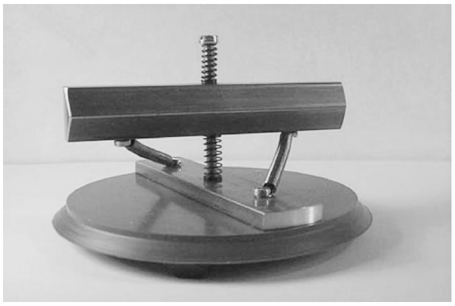

FIG. 1. The experimental setup that consists of two (upper and lower) helically rotating coupled plates. The supporting bottom plate creates friction for the lower plate. A driving oscillating or fluctuating force is applied to the upper plate, e.g., by hitting it slightly from various directions. Colored noise was created by the sound from a fog horn, acting on a horizontal membrane glued on top of the upper plate. In all cases of forcing vertically (from above or below), the helical system was observed to rotate only clockwise when viewed from above.

springs are very soft to bend, but hard to compress or stress (being in fact elastic rods).

The upper weight is resting on the lower weight with a lower vertical spring in between, while an upper vertical spring, for which the upper end is fixed, controls the pressure of the lower plate on a supporting plate. Therefore the vertical distance between the plates is fixed only by the vertical springs, and the lateral springs do not participate in the force balance. The ends of the lateral springs are attached to the plates rigidly, whereas the vertical springs are allowed to slide freely on the surfaces of the plates when they rotate. In equilibrium, both the vertical springs little pressed, and this construction allows us to exclude effectively the role of gravity in experiments. Indeed, when the gadget is oriented, e.g., horizontally, the upper plate will be found in another (shifted) equilibrium position, but still being pressed to the supporting plate, resulting in a nonzero friction. Therefore, gravity is not essential for the existence of a ratchet motion and both the weights can be considered as masses. Instead, the inertia of the upper plate is crucial for the sliding rotation of the lower plate on the supporting plate. In a rotation around the vertical axis, the friction of the lower weight depends on the current vertical position of the upper weight; that friction is larger as the weight (force) on the lower vertical spring is larger, so that when an external oscillating or fluctuating force is applied on the upper weight, the friction in a rotation of the lower weight sliding on the supporting plate depends strictly on the force applied to the upper weight. The pair of the lateral springs (elastic rods) converts the oscillating or fluctuating vertical force applied on the upper weight into a force changing the angle between the rotating plates. Because the frictional coupling between rotations of the plates is larger when the lower vertical spring is contracted than when it is extended, we get a rectification as the lateral springs transfer a vertical motion into a rotation in the horizontal plane.

In general, if we try formally to write the full system of equations of motion for our system, one obtains a set of three differential equations of the second order [27], the analysis of which is quite difficult. Numerical simulations of these equations show different sophisticated regimes that crucially depend on the system parameters. However, the geometry and the physical parameters of springs in our device can be fitted in such a way (see the Appendix) that the dynamics of plate B are governed by *one simple* dynamical equation of the type (A15). The friction term of this equation is of a quite general form, which in general can have no relevance to our specific gadget. Thus, the basic equation of our studies in this paper reads as follows:

$$\dot{\Omega}+\gamma(f)\sigma(\Omega)/\Omega=f, \tag{1}$$

where the dot stands for differentiation on time t, Ω is the angular velocity of the lower plate, $f=f(t)$ an external force, γ a friction coefficient that depends on external forcing, and the function $\sigma(\Omega)$ describes the type of friction. In the derivation of Eq. (1), the emphasis was placed on the dependence of its second (friction) term on the external force f and/or the velocity Ω. For simplicity, such effects in the dry friction dynamics as crossover from stick slip to steady sliding motion [28] and other effects (e.g., corrugation or pinning) were not modeled.

As expected intuitively, for any nonincreasing, but necessarily decreasing at least in some neighborhood of the point $f=0$ in the domain of the function $\gamma(f)$, the average velocity $J\equiv\langle\Omega(t)\rangle$ appears to be positive and this means that the lower plate rotates clockwise when vied from above, and this direction of rotation is indeed observed experimentally. With this property of the function $\gamma(f)$, the inequality $J>0$ will be proved below for the two cases $\nu=1,2$, and for some particular choices of the function $\gamma(f)$.

III. DIRECTED TRAJECTORIES DUE TO FRICTION ASYMMETRY

Now we can use the symmetry arguments of Flach *et al.* [12] to conclude that Eq. (1) is expected to support a ratchet motion driven by a periodic force $f(t+T)=f(t)$ under the following two conditions: either (i) $f(t+T/2)\neq-f(t)$ (broken time symmetry) and the dissipation function, i.e., the second term in this equation, is a nonlinear function of Ω, but $\gamma(-f)=\gamma(f)$ and $\sigma(-\Omega)=\sigma(\Omega)$; or (ii) $\gamma(-f)\neq\gamma(f)$ [or $\sigma(-\Omega)\neq\sigma(\Omega)$], but no conditions on the zero average force $f(t)$. The former condition results in the ratchet motion discovered by Vidybida and Serikov [29]. Here we consider the latter case when the function $\gamma(f)$ is a decreasing function of the force f, at least in some neighborhood of the point $f=0$.

The steady-state solution of the dynamical system (1), governed by a periodic force $f(t)$ with a frequency ω, is a periodic orbit $\Omega^*(\varphi)$, $0\leqslant\varphi\equiv\omega t\leqslant2\pi$. For this orbit, we define the average (global) velocity ("current") by the integral

$$J=\langle\Omega^*(\varphi)\rangle=\frac{1}{2\pi}\int_0^{2\pi}\Omega^*(\varphi)d\varphi. \tag{2}$$

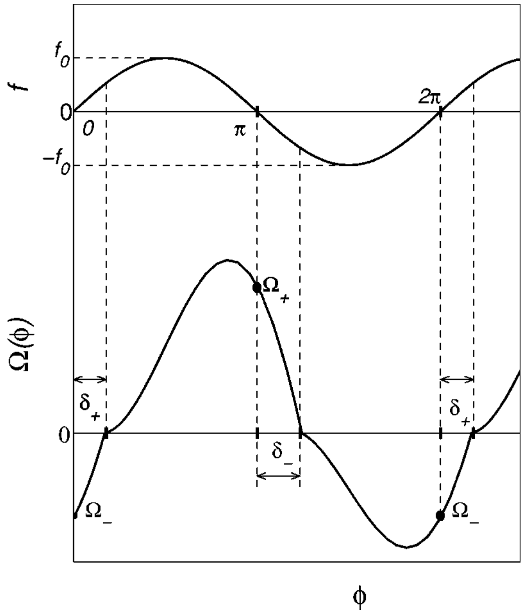

FIG. 2. Steady trajectory $\Omega^*(\varphi)$, $\varphi\equiv\omega t$ (arbitrary units), in the case of dry friction: the upper symmetric curve is input sinusoidal forcing $f(t)$ and the lower asymmetric one is output angular velocity $\Omega^*(\omega t)$. The computation has been performed for the exponential friction $\gamma(f)=\gamma_0\exp(-f/\lambda)$ with the driving force $f(t)=f_0\sin(\omega t)$, where $\gamma_0=1$, $\lambda=7$, $f_0=2$, and $\omega=1$.

Below we will illustrate the existence of a nonzero current J [even if $\langle f(t)\rangle=0$] in some particular cases of the function $\gamma(f)$. To do this, we will need to calculate the phase shifts of the nodes of the periodic function $\Omega^*(\varphi)$ with respect to the nodes of the external force $\varphi=0,\pi$. These shifts are defined by the equations

$$\Omega^*(\delta_+)=0 \quad \text{and} \quad \Omega^*(\pi+\delta_-)=0. \tag{3}$$

For convenience we introduce

$$\Omega_-=\Omega^*(0) \quad \text{and} \quad \Omega_+=\Omega^*(\pi). \tag{4}$$

Figure 2 illustrates the phases $\delta_\pm$ and the velocities $\Omega_\pm$ in the case of harmonic driving.

In the present paper, we restrict ourselves only to the two particular cases of the basic equation (1): $\sigma(\Omega)=|\Omega|^\nu$ with $\nu=1$ (dry friction) and $\nu=2$ (viscous friction). As regard the function $\gamma=\gamma(f)$, it is assumed to be nonincreasing, but necessarily decreasing at least in some neighborhood of the point $f=0$. For some particular cases of this function, the global velocity J can be calculated explicitly and the inequality $J>0$ proved rigorously. Below we will consider the cases with dry and viscous friction separately.

A. Dry friction

In the case of dry friction ($\nu=1$) we have $\sigma(\Omega)=|\Omega|$ and the general equation (1) takes the form

$$\dot{\Omega}+\mathrm{sgn}(\Omega)\,\gamma(f)=f. \tag{5}$$

The attractor of this equation is an asymmetric orbit and the velocities (4) can be expressed as a result of integration,

$$\Omega_-=-\int_0^{\delta_+/\omega}[f(t)+\gamma(t)]dt=\int_{(\pi+\delta_-)/\omega}^{2\pi/\omega}[f(t)+\gamma(t)]dt,$$

$$\Omega_+=\int_{\delta_+/\omega}^{\pi/\omega}[f(t)-\gamma(t)]dt=-\int_{\pi/\omega}^{(\pi+\delta_-)/\omega}[f(t)-\gamma(t)]dt. \tag{6}$$

Eliminating from these equations the velocities $\Omega_\pm$, in the case of the harmonic forcing (A14), one finds the two equations for the phase shifts $\delta_\pm$,

$$2f_0(\cos\delta_+ +\cos\delta_-)=\int_0^{2\pi}\gamma(f_0\sin\varphi)d\varphi, \tag{7}$$

$$\int_{\delta_+}^{\pi+\delta_-}\gamma(f_0\sin\varphi)d\varphi=\int_{\pi+\delta_-}^{2\pi+\delta_+}\gamma(f_0\sin\varphi)d\varphi. \tag{8}$$

Rewriting the last equation in the form

$$\left(\int_{\delta_+}^{\pi}-\int_0^{\delta_+}\right)\gamma d\varphi=\left(\int_{\pi+\delta_-}^{2\pi}-\int_{\pi}^{\pi+\delta_-}\right)\gamma d\varphi, \tag{9}$$

one can prove that $\delta_+<\delta_-$. Indeed, since $\gamma(f)$ is a nonincreasing, but strongly decreasing at least in some neighborhood of the point $f=0$, function, for the integral equality (9) to be valid, the difference length of the intervals $[\pi-\delta_+,\pi]$ and $[0,\delta_+]$ must exceed the difference length of the intervals $[\pi+\delta_-,2\pi]$ and $[\pi,\pi+\delta_-]$, resulting in $\delta_+<\delta_-$. Using this inequality in Eqs. (6), one finds immediately that $\Omega_+>-\Omega_->0$.

In a neighborhood of the point $f=0$, a continuously decreasing function $\gamma(f)$ can be approximated by a linear dependence,

$$\gamma(f)=\gamma_0-\gamma_1 f, \quad \gamma_{0,1}>0. \tag{10}$$

In the particular case of the harmonic force (A14), with a sufficiently low amplitude ($f_0<\gamma_0/\gamma_1$, otherwise the friction γ occurs to be negative and, therefore, meaningless), Eqs. (6)–(8) are reduced to an explicit form and, therefore, the attractor of Eq. (5) reads

$$\Omega^*(\varphi)=\frac{f_0}{\omega}\times\begin{cases}(1+\gamma_1)(\cos\delta_+-\cos\varphi)+\gamma_0(\delta_+-\varphi)\\ \qquad \text{if} \quad \delta_+\leqslant\varphi\leqslant\pi+\delta_+\\ (\gamma_1-1)(\cos\delta_-+\cos\varphi)+\gamma_0(\varphi-\pi-\delta_-)\\ \qquad \text{if} \quad \pi+\delta_-\leqslant\varphi\leqslant 2\pi+\delta_+,\end{cases} \tag{11}$$

where the phases $\delta_\pm$ satisfy the two equations

$$\cos\delta_+ +\cos\delta_-=\pi\gamma_0/f_0, \quad \delta_- -\delta_+=\pi\gamma_1. \tag{12}$$

A straightforward calculation gives the global (average) velocity,

$$J=(f_0/2\omega)(1+\gamma_1^2)(\cos\delta_+-\cos\delta_-)$$
$$+(\gamma_1 f_0/\pi\omega)(\sin\delta_+ +\sin\delta_-)>0. \qquad (13)$$

Let us consider now another case of interest, when the $\gamma(f)$ has only one discontinuity at $f=0$, being the step function

$$\gamma(f)=\begin{cases}\gamma_+ & \text{if} \quad f\geqslant 0,\\ \gamma_- & \text{if} \quad f<0.\end{cases} \qquad (14)$$

Then the attractor of Eq. (5) is given by

$$\Omega^*(\varphi)=\frac{f_0}{\omega}\times\begin{cases}(\cos\delta_+-\cos\varphi)+\gamma_+(\varphi-\delta_+)\\ \qquad \text{if} \quad 0\leqslant\varphi\leqslant\delta_+\\ (\cos\delta_+-\cos\varphi)+\gamma_+(\delta_+-\varphi)\\ \qquad \text{if} \quad \delta_+\leqslant\varphi\leqslant\pi\\ -(\cos\delta_-+\cos\varphi)+\gamma_-(\pi+\delta_--\varphi)\\ \qquad \text{if} \quad \pi\leqslant\varphi\leqslant\pi+\delta_-\\ -(\cos\delta_-+\cos\varphi)+\gamma_-(\varphi-\pi-\delta_-)\\ \qquad \text{if} \quad \pi+\delta_-\leqslant\varphi\leqslant 2\pi,\end{cases} \qquad (15)$$

where the phase shifts $\delta_\pm$ defined by Eqs. (3) satisfy the equations

$$\cos\delta_+ +\cos\delta_- =\pi(\gamma_+ +\gamma_-)/2f_0, \qquad (16)$$

$$\gamma_+(\pi/2-\delta_+)=\gamma_-(\pi/2-\delta_-). \qquad (17)$$

Explicitly, the velocities at the force nodes (4) become

$$\Omega_\pm=\pm[f_0(1-\cos\delta_\mp)+\gamma_\mp\delta_\mp]/\omega. \qquad (18)$$

Therefore, in the case of the friction asymmetry, e.g., when $\gamma_+<\gamma_-$, we have a nonzero current (global velocity) $J(\gamma_+,\gamma_-)=-J(\gamma_-,\gamma_+)>0$ that vanishes in the limit $\gamma_+\to\gamma_-$. This velocity is given by

$$J=[f_0(\cos\delta_+-\cos\delta_-)+(\gamma_-\delta_-^2-\gamma_+\delta_+^2)]/2\omega. \quad (19)$$

In a general case, any decreasing function $\gamma(f)$ can be represented as a limit of the sums of step functions of the type (14). Using such an expansion, one can extend the proof of positivity of the current J for any nonincreasing, but decreasing at least at one point, function $\gamma(f)$.

Since the case of the friction coefficient $\gamma(f)$ defined in the Appendix by the rational function (A13) seems to be the most realistic one for our gadget, but it cannot be treated analytically, it is interesting to find numerically the steady solution to Eq. (5). As shown in Fig. 3, after starting numerical simulations, the solution $\Omega(t)$ approaches the attractor Ω^* very fast.

This figure clearly demonstrates that plate B practically never steps backwards, like the recent molecular motor model [21], and this behavior is indeed observed visually in experiments with our gadget.

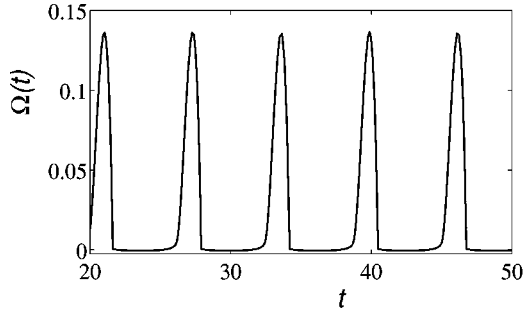

FIG. 3. Rotational velocity $\Omega(t)$ (arbitrary units) in the case of dry friction with rational friction dependence $\gamma(f)=\gamma_0(z_1-f)/(f-z_2)$ and the driving force $f(t)=f_0\sin(\omega t)$, computed numerically for $\gamma_0=1$, $z_1=-z_2=1$, $f_0=1/2$, and $\omega=1$.

B. Viscous friction

In the particular case of viscous friction ($\nu=2$), the basic equation (1) is reduced to the equation for an underdamped particle moving in a viscous liquid,

$$\dot{\Omega}+\gamma(f)\Omega=f. \qquad (20)$$

Although the viscous friction is not the case of relevance to our gadget, nevertheless overdamped equations of the type (20) are ubiquitous in mechanochemistry and biology [3–10]. As already mentioned, for some systems the friction term in Eq. (20) is assumed to depend nonlinearly on the velocity [29], resulting in a unidirectional motion. Here we impose γ to depend on the forcing and it is intuitively obvious that this can lead to a ratchet effect. To confirm this statement analytically, we study the properties of a steady solution of Eq. (20).

First we consider the overdamped case, when the first (inertial) term in Eq. (20) is ignored and the force $f(t)$ has another origin, not related to the gadget described above. This case may be of biological relevance, describing a rotary motion of some bacteria [30,31] or complex filaments [32] in a liquid, the surface friction of which can depend on the direction of the driving force $f(t)$.

Let us represent an oscillating or fluctuating force $f(t)$, $0\leqslant t<\infty$, with $\langle f(t)\rangle=0$, as the sum of its positive and negative parts: $f(t)=f_+(t)-f_-(t)$ where $f_\pm(t)\geqslant 0$. Then for any decreasing function $\gamma(f)$, we immediately find from Eq. (20), in the limit $\dot{\Omega}\to 0$, the following chain of inequalities:

$$J=\langle f_+(t)/\gamma[f_+(t)]\rangle-\langle f_-(t)/\gamma[f_-(t)]\rangle>\langle f_+(t)/\gamma(0)\rangle$$
$$-\langle f_-(t)/\gamma(0)\rangle=0, \qquad (21)$$

because due to $\langle f(t)\rangle=0$, it follows that $\langle f_+(t)\rangle=\langle f_-(t)\rangle$.

Intuitively, in the underdamped case of Eq. (20), the phase of a steady orbit will be delayed in comparison with the phase of a periodic forcing $f(t)$, similarly to the plot shown in Fig. 2. This can be confirmed again for the case of the step function (14). Indeed, in a general case, one can represent a steady solution of Eq. (20) in the following form:

$$\Omega^*[\varphi(t)]=\Omega_\mp A_\pm(t)+B_\pm(t), \qquad (22)$$

where the functions $A_{\pm}(t)$ and $B_{\pm}(t)$ are defined by

$$A_{\pm}(t)=\exp\left[-\int_{t_{\pm}}^{t}\gamma(\tau)d\tau\right],$$

$$B_{\pm}(t)=\int_{t_{\pm}}^{t}\exp\left[-\int_{\tau}^{t}\gamma(\tau')d\tau'\right]f(\tau)d\tau, \tag{23}$$

with $t_{+}=0$ and $t_{-}=\pi/\omega$. Using the periodicity of the orbit (22), we find the velocities $\Omega_{\pm}$ [see their definition (4)],

$$\Omega_{\pm}=(\alpha_{\pm}\beta_{\mp}+\beta_{\pm})/(1-\alpha_{+}\alpha_{-}), \tag{24}$$

where $\alpha_{\pm}$ are given as follows:

$$\alpha_{\pm}=A_{\pm}(t_{\pm}+\pi/\omega),\quad \beta_{\pm}=B_{\pm}(t_{\pm}+\pi/\omega). \tag{25}$$

In the particular case of the harmonic forcing (A14) and the step friction (14), a direct calculation yields

$$A_{\pm}(t)=\exp[-\gamma_{\pm}(t-t_{\pm})],$$

$$B_{\pm}(t)=\frac{f_0}{\omega^2+\gamma_{\pm}^2}[\gamma_{\pm}\sin(\omega t)-\omega\cos(\omega t)\pm\omega A_{\pm}(t)], \tag{26}$$

and, therefore [see the definitions (25)], one obtains

$$\alpha_{\pm}=\exp(-\pi\gamma_{\pm}/\omega),\quad \beta_{\pm}=\pm f_0\omega\frac{1+\alpha_{\pm}}{\omega^2+\gamma_{\pm}^2}. \tag{27}$$

Using the expression (27), we get

$$\Omega_{\pm}=\pm\frac{f_0\omega}{(\omega^2+\gamma_{+}^2)(\omega^2+\gamma_{-}^2)}\times\left[\omega^2+\gamma_{\pm}^2+\frac{(1+\alpha_{\pm})(\gamma_{\mp}^2-\gamma_{\pm}^2)}{1-\alpha_{+}\alpha_{-}}\right]. \tag{28}$$

Some limiting cases of Eqs. (28) can be considered. In particular, when $\gamma_{-}\to\infty$, but γ_{+} is finite, we have

$$\Omega_{+}\to\frac{f_0\omega(1+\alpha_{+})}{\omega^2+\gamma_{+}^2}>0,\quad \Omega_{-}\to 0. \tag{29}$$

Also, $\Omega_{+}=\Omega_{-}=f_0\omega/(\omega^2+\gamma^2)$ if $\gamma_{+}=\gamma_{-}\equiv\gamma$. Next, using Eqs. (22), (26), and (27), the integral (2) can be calculated, resulting in the expression

$$\begin{aligned}J(\gamma_{+},\gamma_{-})&=-J(\gamma_{-},\gamma_{+})\\&=\frac{f_0}{2\pi}\left\{\frac{\omega^2}{1-\alpha_{+}\alpha_{-}}\left(\frac{1}{\omega^2+\gamma_{+}^2}-\frac{1}{\omega^2+\gamma_{-}^2}\right)\right.\\&\quad\times\left[\frac{(1-\alpha_{+})(1+\alpha_{-})}{\gamma_{+}}+\frac{(1+\alpha_{+})(1-\alpha_{-})}{\gamma_{-}}\right]\\&\quad\left.+2\left(\frac{\gamma_{+}}{\omega^2+\gamma_{+}^2}-\frac{\gamma_{-}}{\omega^2+\gamma_{-}^2}\right)\right\}.\end{aligned} \tag{30}$$

If $\gamma_{+}<\gamma_{-}$, the first term in Eq. (30) is positive, but the second one can be either positive or negative. Since the function $\gamma/(\omega^2+\gamma^2)$ has a maximum at $\gamma=\omega$, we obtain

$$\frac{\gamma_{+}}{\omega^2+\gamma_{+}^2}-\frac{\gamma_{-}}{\omega^2+\gamma_{-}^2}>-\frac{\omega}{\omega^2+\omega^2}=-\frac{1}{2\omega}. \tag{31}$$

Using this inequality, we derive the positivity of the averaged velocity,

$$J>J(\gamma_{+}=0,\gamma_{-}=\omega)=\frac{f_0}{4\omega}\coth\frac{\pi}{2}>0. \tag{32}$$

Note, in the particular case $\gamma_{+}=0$ and $\gamma_{-}>0$, the expression (30) is simplified to $J=(\Omega_{+}+\Omega_{-})/4$, and furthermore, if $\gamma_{-}\to\infty$, we obtain $J\to f_0/2\omega$.

IV. CONCLUDING REMARKS

Thus, we have suggested a ratchet mechanism that appears due to an asymmetry of the dependence of the dissipation function of the system on its velocity or external forcing. This mechanism appears to be described by the standard underdamped equation of motion for a particle moving in a periodic potential $U(x)$, with $x=x(t)$ being a coordinate of the particle, generalized to include some additional dynamical properties such as nonlinear friction, dependence on external forcing, etc. and written in the form [compare with Eq. (1)]

$$\ddot{x}+U'(x)+G(\dot{x},f)/\dot{x}=f. \tag{33}$$

Here the friction term is given in a generalized form, through the dissipative function $G(t)$. Besides a dependence on the velocity $\dot{x}(t)$, which in general may be nonlinear [29], the function G in Eq. (33) is imposed to depend also on an external oscillating or fluctuating force $f(t)$ with zero average $[\langle f(t)\rangle=0]$. Equation (33) can be considered as a more generalized version of the gadget model, when the surface corrugation of the supporting plate is modeled by a spatially periodic potential.

Following the symmetry arguments formulated recently by Flach *et al.* [12], one can classify the following four particular types of Eq. (33), with

$$G(\dot{x},f)=\gamma(f)\sigma(\dot{x}),\quad \sigma(\dot{x})\geqslant 0, \tag{34}$$

and γ being a force-dependent friction coefficient, each of which admits ratchet dynamics:

(i) The friction coefficient is a symmetric function with respect to an external force, i.e., $\gamma(-f)=\gamma(f)$, including the usual case, when this coefficient does not depend on the force at all. Also, the dissipation is assumed to be viscous, i.e., $\sigma(\dot{x})=\dot{x}^2$. The rectification in this case occurs either due to broken spatial $[U(-x)\neq U(x)]$ or time (e.g., harmonic mixing) symmetry [3,12].

(ii) The periodic potential is absent $[U(x)\equiv 0]$ and $\gamma(-f)=\gamma(f)$, or γ does not depend on *f*. The rectification occurs due to nonlinearity of the function $\sigma(\dot{x})$ (e.g., if σ

$=\dot{x}^2+\dot{x}^4$) and broken time symmetry (e.g., due to harmonic mixing in the force f) as shown by Vidybida and Serikov [29].

(iii) The periodic potential is absent [$U(x)\equiv 0$], the function $\sigma(\dot{x})$ does not necessarily result in a nonlinear friction term in the equation of motion (33), and the time symmetry is not broken. The rectification occurs, as shown in this paper, due to broken symmetry of the force-dependent friction, i.e., when $\gamma(-f)\neq\gamma(f)$.

(iv) The overdamped limit of the case (iii), when the first (inertia) term in Eq. (33) is omitted, can be adopted as a particular case. Here the rectification also occurs due to broken symmetry of the force- or velocity-dependent friction, i.e., when $\gamma(-f)\neq\gamma(f)$ or $\sigma(-\dot{x})\neq\sigma(\dot{x})$, respectively.

The present paper focuses on the case of force-dependent friction, which can exist in both underdamped and overdamped limits. In order to support the idea of the ratchet as a result of broken friction symmetry, we have constructed an experimental device—a mechanical diode that changes the friction coefficient, while an external force is applying to the system. The simplest solution of this problem seemed to convert the force applied normally to a sliding weight. To make the asymmetry, which could drive a unidirectional rotation of the sliding plate, we used a helical asymmetry being a basic feature of biomolecular structure. However, the full system of corresponding exact equations of motion for such a system is too sophisticated: three coupled nonlinear dynamical equations have been derived previously [27]. This set of equations appears to be very difficult for analytical analysis and numerical simulations do not clarify the physics of their evolution. Different regimes, including regular and chaotic behavior, reversals of directed motion, and others were found numerically, and this is not a surprise because even the ratchet dynamics of one underdamped equation of motion of the type (33) are not yet fully understood as shown by recent studies [11,17,19]. Therefore, there was a technical problem to construct a gadget in such a way that it would be possible somehow to reduce the set of three dynamical equations to only *one* equation of motion and to demonstrate clearly a ratchet motion. The gadget shown in Fig. 1 indeed demonstrates the rotary motion illustrated by Fig. 3, being a representative of motors that never step backwards [21]. The essence of this device is that its dynamics can be described by the simple equation (1), which can easily be treated analytically. Therefore the basic equation of motion (1) with the force-dependent friction coefficient of the rational type given below by Eq. (A13) is of a realistic type because it results in the solution shown in Fig. 3 confirmed with experiments on the gadget.

In should be mentioned that our gadget demonstrates the rotary ratchet motion, using *dry* friction. However, this idea can be extended to systems with *viscous* friction and this type of ratchet can play an essential role in rectification of bacterial swimming [30]. Thus, strains of the cyanobacterium *Synechococcus* are known [31] to swim in seawater at speeds of up 25 μm/s, demonstrating very high efficiency of a unidirectional motion. They are rod-shaped organisms with about 1 μm in diameter and 2 μm long. *Synechococcus* swim in the direction of their long axis, following an irregular helical track. Their means of locomotion are not known, and they have no flagella, either external or internal. As far as one can see by light microscopy, they do not change shape. Under certain growth conditions, long asymmetric cells appear, but these just roll rigidly around an axis parallel to their long axis, the direction of locomotion [31]. An ion turbine mechanism has been proposed for other bacteria [33] but it has been ruled out for *Synechococcus* [34]. Based on this knowledge, a so-called self-propulsive mechanism of cyanobacterial swimming has been suggested and corresponding theories have been developed in a number of works (see, e.g., Refs. [31,35,36]). According to these theories, the swimming is a result of surface tangential or normal waves, or their combination, that travel along the outer cell membrane. The type of surface oscillations and the direction of wave propagation determine the direction of motion for cyanobacteria. For instance, for tangential surface waves, a spherical organism swims in the same direction as the surface wave. However, it is not clear yet why the direction of wave propagation occurs in one direction, but not in the other one. Some kind of *internal* dynamical asymmetry can be involved to explain a unidirectional motion [22]. On the other hand, a surface asymmetry of bacteria seems to play an essential role in rectification of their rotary or linear motion. In particular, helical tracks observed in these motions confirm this point of view. Note that the helical asymmetry is ubiquitous in biology and it can be a source of a broken symmetry in the dependence of surface friction of bacteria on their direction of motion, i.e., their linear or rotary velocity. Combination of self-propulsion with the asymmetric dependence of the friction on internal or external forcing or the velocity seems to a reasonable idea for further studies of bacterial swimming that can provide an insight into the physics of microbiological motility.

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APPENDIX: EQUATIONS OF MOTION FOR THE EXPERIMENTAL MODEL

Using the intuitive arguments of Sec. II, here we derive the equation of motion (1) for the experimental model, the photograph of which is shown in Fig. 1. For notations used in this derivation, a schematic figure (Fig. 4) is presented. Here the upper and the lower plates are denoted with A and B, respectively. Next, the lateral springs and the lower and the upper vertical springs are numbered with 1, 2, 3, and 4, respectively. The positive direction of a rotation of plate B is shown by the arrow.

Let R_0 be the radius of the helical backbone of the setup, and M_A and M_B be masses of plates A and B, respectively. The equilibrium state of the system is given by the dimensionless (measured in units of R_0) vertical distance h be-

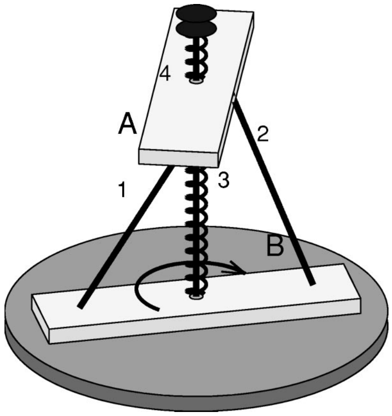

FIG. 4. Schematic picture of the setup: the two lateral springs (acting as elastic rods) 1 and 2 are shown with thick solid lines.

tween plates A and B and the angle $\phi<\pi/2$ that determines the relative equilibrium orientation of the plates. Under different motions of plates A and B, the length l of springs 1 and 2, given by

$$l=\sqrt{h^2+2(1-\cos\phi)}, \tag{A1}$$

practically does not change and, therefore, they create a constraint in the system. The supporting plate in the setup (see Figs. 1 and 2) generates a sliding friction for a rotary motion of plate B. This friction depends on the normal response force N_B directed upwards and created by the supporting plate. In its turn, this response depends on how spring 3 is pressed (or stretched), and this dependence is governed by an external normal force $F_n(t)$, acting from plate A through spring 3 as well as through the lateral springs, acting as a constraint. Any tangential forcing that exceeds the friction of rest, results in a rotation of plate B.

One of the equations of motion can be derived for the angular variables $\theta_A(t)$ and $\theta_B(t)$, instantaneous deviations of plates A and B from their equilibria. We denote the moments of inertia by $aM_AR_0^2$ and $bM_BR_0^2$, with a and b being (dimensionless) geometric form factors for plates A and B, respectively. Since the lateral springs are soft to bend, the interaction of A and B through these springs in the vertical direction can be ignored. This technical point essentially simplifies the full system of dynamical equations, which in general take a very complicated form [27]. Therefore in this setup, one can account for only a tangential response force T_t that appears due to the constraint created by the lateral springs. In general, except for the force T_t, an external tangential force F_t may be applied, so that the tangential equation of motion for plate A is

$$aM_AR_0\ddot{\theta}_A=T_t+F_t. \tag{A2}$$

The second equation that governs the friction dynamics of plate B can be written in the form

$$bM_BR_0\ddot{\theta}_B+G(\dot{\theta}_B,N_B)/R_0\dot{\theta}_B=-T_t, \tag{A3}$$

where the dissipation function G for the lower plate depends on the angular velocity $\dot{\theta}_B$ and the response force N_B, acting from the side of the supporting plate. Next, the response force T_t can be excluded from the equations of motion (A2) and (A3) and, as a result, we obtain one tangential equation for the angle variables θ_A and θ_B:

$$aM_A\ddot{\theta}_A+bM_B\ddot{\theta}_B+G(\dot{\theta}_B,N_B)/R_0^2\dot{\theta}_B=F_t/R_0. \tag{A4}$$

The equation of motion that describes the vertical dynamics of plate A driven by the normal force $F_n(t)$ reads

$$M_A\ddot{z}+\eta_AM_A\dot{z}+KV'(z)=F_n/R_0, \tag{A5}$$

where z is the displacement of plate A from its vertical equilibrium position given by springs 3 and 4 (and weight A if the gadget is placed vertically). The (dimensionless) function $V(z)$, with a minimum at $z=0$, describes the strain energy of the vertical springs. The string parameters η_A and K stand for the friction and the stiffness, respectively. The solution of Eq. (A5) uniquely determines the variable $z(t)$ as a function of the normal force $F_n(t)$.

The last equation of motion results from the constraint imposed on plates A and B by the stiff lateral springs (acting in fact as rods), leading to a geometric relation between the variables θ_A, θ_B, and z. Since the length of the lateral springs in our setup is practically unchanged (because they are too hard to compress or stress), approximately, one finds the dependence $z=z(\theta_A-\theta_B)$,

$$z=\sqrt{2\cos(\phi-\theta_A+\theta_B)-2\cos\phi+h^2}-h. \tag{A6}$$

For our analysis it is sufficient to use the linear approximation of Eq. (A6),

$$z=h^{-1}(\theta_A-\theta_B)\sin\phi. \tag{A7}$$

Using Eq. (A7), the variable $\theta_A(t)$ can be eliminated in Eq. (A4), leading to the equation,

$$\dot{\Omega}+G(\Omega,N_B)/MR_0^2\Omega=-\mu\ddot{z} \tag{A8}$$

with respect to the angular velocity $\Omega\equiv\dot{\theta}_B$ of plate B. Here $M=aM_A+bM_B$ and $\mu=ahM_A/M\sin\phi$ are system parameters. The tangential force F_t appears to be not involved into the ratchet mechanism and, therefore, it is omitted in Eq. (A8). Without loss of generality, we assume that the dissipation function G in Eq. (A8) can be factorized as

$$G(\Omega,N_B)=M_BR_0^2\Gamma(N_B)\sigma(\Omega), \tag{A9}$$

where Γ is the friction coefficient being a function of only the variable z and $\sigma(\Omega)$ is a function of the angular velocity Ω depending on the type of friction. In the particular case of our gadget, we deal with dry friction and, therefore, here one can put $\sigma(\Omega)=|\Omega|$.

As mentioned above, the response N_B depends on how spring 3 is stretched or compressed, i.e., on the displacement z of plate A from its equilibrium. More precisely, the dependence $N_B=N_B(z)$ is given by

$$N_B = -\begin{cases} KR_0 V'(z-z_1) & \text{if} \quad z \leqslant z_1 \\ 0 & \text{if} \quad z > z_1, \end{cases} \tag{A10}$$

where z_1 is the distance, by which spring 3 is pressed down to get equilibrium due to the strain of spring 4 and weight A.

The next step is to determine the dependence of the coefficient Γ on N_B. It is reasonable to assume this dependence to be of the exponential type. Then, according to Eq. (A10), we have

$$\Gamma(N_B) = \overline{\Gamma_0} \exp(N_B/\Lambda) = \overline{\Gamma_0} \exp[-V'(z-z_1)/\overline{\lambda}], \tag{A11}$$

where $\overline{\lambda} = \Lambda/KR_0$ is a dimensionless characteristic vertical displacement of plate A. In the case, when springs 3 and 4 are harmonic $[V(z) = z^2/2]$, the function (A11) is simplified to

$$\Gamma(z) = \Gamma_0 \exp(-z/\overline{\lambda}), \tag{A12}$$

where the factor with z_1 has been absorbed into Γ_0, resulting in the exponential behavior against the z displacement [37]. However, the dependence (A12) does not account for surface corrugation, friction of rest, the disappearance of friction when the displacement z of plate A up exceeds a certain critical length, and other more sophisticated friction phenomena [28]. Consequently, the z dependence should be modified and, therefore, it is more realistically to use in our gadget a rational function as follows:

$$\Gamma(z) = \Gamma_0 \times \begin{cases} \infty & \text{if} \quad z \leqslant z_2 \\ (z_1 - z)/(z - z_2) & \text{if} \quad z_2 < z < z_1 \\ 0 & \text{if} \quad z \geqslant z_1, \end{cases} \tag{A13}$$

where z_2 is a critical value (friction at rest), below which the friction becomes infinite.

In the particular case of a sinusoidal force $F_n(t)$ with a frequency ω, the steady-state solution (trajectory or attractor) of Eq. (A5) is also a sinusoidal function with the amplitude z_0 being proportional to the force amplitude F_0 [38]. Next, we denote the right-hand side of Eq. (A8) by $f(t)$, so that in this particular case,

$$f(t) = f_0 \sin(\omega t), \tag{A14}$$

with $f_0 = \mu z_0 \omega^2$, becomes an external force for plate B. Then Eq. (A8) can be rewritten in the form of Eq. (1),

$$\dot{\Omega} + \gamma(f)\sigma(\Omega)/\Omega = f, \tag{A15}$$

where the dimensionless friction coefficient γ is normalized by $\gamma(0) \equiv \gamma_0 = \Gamma_0 M_B/M$. Particularly, for exponential behavior (A12), we have $\gamma(f) = \gamma_0 \exp(-f/\lambda)$ with $\lambda = \mu\omega^2\overline{\lambda} = \mu\omega^2\Lambda/KR_0$.

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