Theory of phase-sensitive measurement of photon-assisted tunneling through a quantum dot

Jauho, Antti-Pekka; Wingreen, Ned S.

Published in:
Physical Review B

Link to article, DOI:
10.1103/PhysRevB.58.9619

Publication date:
1998

Document Version
Publisher’s PDF, also known as Version of record

Citation (APA):
Phase coherence is the hallmark of all mesoscopic transport phenomena. Yet normal transport measurements yield information only about the magnitude of the transmission amplitude, and not its phase. In a groundbreaking set of experiments, Yacoby et al.\textsuperscript{1} and Schuster et al.\textsuperscript{2} recently demonstrated that a phase measurement is nevertheless possible in a mesoscopic double-slit geometry. Their experimental protocol can be summarized as follows: A magnetotransport measurement is performed on an Aharonov-Bohm ring with a quantum dot fabricated in one of its arms. If the quantum dot supports coherent transport, the transmission amplitudes through the two arms interfere. A magnetic field induces a relative phase change $2\pi\Phi/\Phi_0$ between the two transmission amplitudes $t_0$ and $t_{QD}$, leading to an oscillatory component to the conductance $g(B) = (e^2/h)\mathcal{T}(B)$, with

$$
\mathcal{T}(B) = \mathcal{T}^{(0)} + 2 \text{Re}\{t_0^* t_{QD} e^{2\pi\Phi/\Phi_0}\} + \ldots , \tag{1}
$$

where $\Phi$ is the flux threading the ring, $\Phi_0 = hc/e$ is the flux quantum, and where the ellipsis represents higher harmonics due to multiple reflections. The amplitudes $t_0$ and $t_{QD}$ give the coherent parts of the two sets of paths joining the emitter and the collector; the incoherent components lead to a structureless background signal, which can be neglected in the forthcoming analysis. In the experiments, an oscillatory component in magnetoresistance of this form was clearly observed thus demonstrating coherent transmission through the arm with the dot.\textsuperscript{1,2} In the experiment of Yacoby et al.,\textsuperscript{1} the Aharonov-Bohm phase could take on only two values 0 and $\pi$ as a consequence of microreversibility in a two-terminal geometry.\textsuperscript{3,4} The second generation of experiments,\textsuperscript{5} in a four-terminal geometry, allowed a determination of the continuous phase shift of the transmission amplitude through the dot. The success of these experiments gave rise to a number of other works which concentrated on refining the interpretation of the experimental results.\textsuperscript{5–6} Yet, the experiments also suggest application to other phase-coherent transport processes. One particular example, which has been of considerable recent interest both experimentally\textsuperscript{7–11} and theoretically,\textsuperscript{12–17} is photon-assisted tunneling. While photon-assisted tunneling (PAT) is intrinsically a coherent phenomenon, existing measurements of PAT are insensitive to the phase of the transmitted electrons, and do not directly demonstrate coherence in the presence of the time-dependent field. Here we propose a measurement of photon-assisted tunneling through a quantum dot in the mesoscopic double-slit geometry described above (see Fig. 1). In essence, we propose a combination of the experiments of Kouwenhoven and co-workers,\textsuperscript{8,11} where a microwave-modulated side-gate
voltage gave rise to photon-assisted tunneling through a quantum dot, and the interference experiments of Refs. 1 and 2.

For an experiment of this type, we calculate the coherent transmission amplitude through the quantum dot in the presence of an arbitrarily strong ac potential applied to the side gate. Our theoretical results indicate that phase-coherent absorption and reemission of photons can be unambiguously demonstrated via phase measurements at the sidebands of the main transmission resonance. In addition, for large driving amplitudes the phase shift associated with the main transmission resonance can be reversed from its usual behavior, providing a direct demonstration of coherence in a strong ac potential.

We focus on transport in the neighborhood of a single Coulomb oscillation peak associated with a single nondegenerate electronic level of the quantum dot. The effect of the ac side-gate voltage is described entirely through the time-dependent energy of this level,

\[ \epsilon(t) = \epsilon_0(V_s) + V_{\infty} \cos \omega t, \]  

where the static energy of the level \( \epsilon_0 \) depends on the dc side-gate voltage \( V_s \). All other levels on the dot can be neglected provided the ac amplitude \( V_{\infty} \) and the photon energy, \( \hbar \omega \), are small compared to the level spacing on the dot.

The energy dependence of the coherent part of the transmission amplitude \( \tilde{t}_{QD}(\epsilon) \) through the arm containing the quantum dot is determined by the transmission amplitude \( \tilde{t}_{QD}(\epsilon) \) through the dot, \( t_{QD}(\epsilon) \). In the absence of an ac potential, a suitable model for the dot transmission amplitude is the Breit-Wigner form

\[ t_{QD}(\epsilon) = \frac{-i \sqrt{i\Gamma_L \Gamma_R}}{\epsilon - \epsilon_0(V_s) + i\Gamma/2}, \]

where \( \Gamma = \Gamma_L + \Gamma_R \) is the full width at half maximum of the resonance on the dot due to tunneling to the left and right leads. Equation (3) implies a continuous phase accumulation of \( \pi \) in the transmission amplitude as the Coulomb blockade peak is traversed. (Note that the Breit-Wigner form is exact for a noninteracting system with \( \Gamma \) independent of energy.)

In the dynamic case, the simple Breit-Wigner description must be generalized, and the object to evaluate is the \( S \)-matrix element.\(^{19,20}\) Provided interactions in the leads can be neglected, the elastic transmission amplitude \( t_{QD}(\epsilon) \) can be written as the energy-conserving part of the \( S \) matrix between the left and right leads,

\[ \lim_{\epsilon' \to \epsilon} \langle \epsilon', R | S | \epsilon, L \rangle = \delta(\epsilon' - \epsilon) t_{QD}(\epsilon). \]

The \( S \) matrix is simply related to the retarded Green function of the level on the dot, including both tunneling to the leads and the ac potential,\(^{19}\)

\[ \langle \epsilon', R | S | \epsilon, L \rangle = -i \frac{\sqrt{\Gamma_L \Gamma_R}}{2\pi} \int dt dt' e^{i\epsilon' t - \epsilon t'} G'(t, t_1). \]

(5)

Combining Eqs. (4) and (5) allows us to write

\[ t_{QD}(\epsilon) = -i \sqrt{i\Gamma_L \Gamma_R} \langle A(\epsilon, t) \rangle, \]

(6)

where the brackets denote a time average, and where

\[ A(\epsilon, t) = \int dt_1 e^{i(\epsilon t_1 - t)} G'(t_1, t_1). \]

(7)

For the time-dependent energy level given by Eq. (2), we find\(^{20}\)

\[ G'(t, t_1) = -i \theta(t - t_1) \exp \left[ -\frac{i}{2} (t - t_1) - \frac{i}{\hbar} \int_{t_1}^t dt' \epsilon(t') \right], \]

(8)

so that

\[ \langle A(\epsilon, t) \rangle = \sum_{\epsilon_k} \frac{\int_k^2 (V_\infty / \hbar \omega) J_1^2(\epsilon_0(V_s) - k \hbar \omega + i \Gamma/2)}{\epsilon - \epsilon_0(V_s) - k \hbar \omega + i \Gamma/2}. \]

(9)

A combination of Eqs. (6) and (9), evaluated at the Fermi energy, gives the relevant transmission amplitude, and hence the amplitude of the Aharonov-Bohm oscillations at \( T = 0 \) K. At finite temperatures one must compute \( t_{QD} = \int d\epsilon \frac{f_0(\epsilon)}{\epsilon - \epsilon_0(V_s) - k \hbar \omega + i \Gamma/2} \), where \( f_0(\epsilon) \) is the Fermi function, and the final result is

\[ t_{QD} = \left( \frac{\Gamma}{4\pi T} \right) \sum_{\epsilon_k} \frac{\int_k^2 (V_\infty / \hbar \omega)}{\epsilon - \epsilon_0(V_s) - k \hbar \omega + i \Gamma/2} \psi' \left[ \frac{1}{2} - \frac{i}{2\pi T} \left( \mu - \epsilon_0(V_s) - k \hbar \omega + i \frac{\Gamma}{2} \right) \right], \]

(10)

where \( \psi' \) is the derivative of the digamma function, and \( \mu \) is the chemical potential in the leads.

Equation (10) is the main result of this paper, and in what follows we shall evaluate it in several cases of interest. We emphasize that a conventional conductance measurement would yield information only about the time average of the square of the transmission amplitude, and the double-slit geometry is necessary in order to probe the phase. Figure 2 shows the computed magnitude of \( t_{QD} \) (bottom) and its

![Figure 2](image-url)
phase (top), as a function of the level energy $\epsilon_0(V_s)$. As compared to the time-independent case (shown as a dotted line), several features are noteworthy. The magnitude of $t_{QD}$ shows photonic sidebands, reminiscent of those seen in transmission through a microwave modulated quantum dot. However, there is an important difference from the usual case of photon-assisted tunneling. The amplitude of the Aharonov-Bohm oscillation is sensitive only to the time average of the transmission amplitude $t_{QD}$. Hence only elastic transmission through the dot contributes, i.e., the net number of photons absorbed from the ac field must be zero. The sideband at, say, $\epsilon = \epsilon_0(V_s) - \hbar \omega$ corresponds to a process in which an electron first absorbs a photon to become resonant at energy $\epsilon_0(V_s)$, and subsequently reemits the photon to return to its original energy.

Perhaps most interesting are the features appearing in the phase: the phase shift shows a nonmonotonic behavior, with pronounced resonances located at the energies corresponding to the photonic sidebands. The strengths of these phase resonances are strongly dependent on the ac amplitude $V_{ac}$, and in Fig. 3 we show the computed signal as a function of both $\epsilon_0(V_s)$ and the amplitude of modulation. In Fig. 4 we highlight another interesting consequence of Eq. (10): it is possible...
sible to *quench* the main transmission peak (bottom panel) 
entirely, or *change the sign* of the slope of the phase at 
resonance by adjusting the ratio $V_{ac}/\hbar \omega$ to coincide with a 
zero of the Bessel function $J_0$ (top). This phenomenon is 
mathematically analogous to the recently observed absolute 
negative conductivity in THz-irradiated superlattices; in our 
case, however, it is the *phase* rather than the current that 
displays this behavior.

In summary, we have proposed an experiment to probe 
phase coherence in a quantum dot driven by a strong ac 
potential. The phase measurement relies on the mesoscopic 
double-slit geometry pioneered in Refs. 1 and 2. The amplitude 
of Aharonov-Bohm oscillations reflects the amplitude for coherent transmission through the dot with zero net ab-
sorption of photons. We find that coherent absorption and 
reemission of photons can be unambiguously detected via 
phase measurement at sidebands of the main transmission 
resonance through the quantum dot.

The authors acknowledge useful comments from Karsten 
Flensberg, Ben Yu-Kuang Hu, and Andreas Wacker.

---

2 R. Schuster, E. Buk, M. Heiblum, D. Mahalu, V. Umansky, and 
5 G. Hackenbroich and H. A. Weidenmüller, Phys. Rev. Lett. 76, 
(1996).
7 P. S. S. Guimaraes, B. J. Keay, J. P. Kaminsky, S. J. Allen, Jr., P. 
F. Hopkins, A. C. Gossard, L. T. Florez, and J. P. Harbison, 
8 L. Kouwenhoven, S. Jauhar, J. Orenstein, P. L. McEuen, Y. Nagamune, J. Motohisa, 
10 S. Zeuner, B. J. Keay, S. J. Allen, Jr., K. D. Maranowski, A. C. 
53, 1717 (1996). A detailed theoretical analysis of these experi-
ments is presented in A. Wagner, A.-P. Jauho, S. Zeuner, and S. 
J. Allen, *ibid.* 56, 13268 (1997); note, however, that the tunnel-
ing in these experiments is *sequential*, and would not support the 
interference effects discussed in the present paper.
van der Vaart, and C. J. P. M. Harmans, Phys. Rev. Lett. 78, 
1536 (1997).
17 C. A. Stafford and N. S. Wingreen, Phys. Rev. Lett. 76, 1916 
(1996).
18 U. Meirav and E. B. Foxman, Semicond. Sci. Technol. 10, 255 
40, 11834 (1989).
50, 5528 (1994).
21 A detailed analysis of the zero-temperature quenching of trans-
mission through an ac-modulated quantum well is available in 