Investigation of the Lower Limit of the Applicability of Effective Medium Approximation for Hyperbolic Metamaterials

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Hyperbolic metamaterials (HMMs) are hailed as one of the main advances in nano-optics in general and metamaterials in particular. Generally, HMMs are multi-layered structures where it is assumed that, due to geometric reasons, one can describe them as a uniform material with effective properties [1]. Their extraordinary characteristics come precisely from the extremely different values of their effective parameters. While the assumption of assigning material properties to these structures may be valid for a very large number of layers, it will start to fail when the number of layers diminishes. In practice, the number of periods these structures have is generally below 10, thus the investigation of the lower limit of such assumption is necessary.

In this work, we investigate the limits of applicability of the effective medium approximation (EMA) in the case of a multilayer structure aimed at working in the visible and near-IR regime and having the period at least 20 times smaller than the wavelength it is measured at.

The structures were fabricated on silica wafers. Each period is of 22 nm nominally and is composed of 10 nm Al2O3 layer, 1 nm 3-Aminopropyl trimethoxysilane (APTMS) adhesion layer [2], 10 nm Au layer and another 1 nm APTMS layer. The structure is then repeated for up to 10 periods. After each period, the structure is investigated to check for Au layer quality. The plasmon propagation characteristics of the structures were measured using a classical Otto configuration with a ZnSe semi-cylinder as high index prism [3].

Two types of theoretical studies were performed. We used both effective medium approach (EMA) and rigorous simulations based on scattering matrix method (SMM) [4] to calculate the plasmon polariton dispersion. The measurements and the simulations were done in the 500 to 1750 nm range.

We then compared the experimental results with both theoretical approaches, as well as the theoretical approaches between themselves using for comparison the mean square standard deviation of reflectance: $MSDR = \sqrt{1/_N \sum_{\lambda,\phi}[R_1(\lambda,\phi) - R_2(\lambda,\phi)]^2}$, where R_1 and R_2 are the reflectivity maps of the experimental measurements and/or theoretical calculations.

The results show (fig 1) that, in this case, the EMA matches the experimental approach better than 5% only starting from 4 periods for our structure. The rigorous approach predicts the reflectivity within 5% mismatch starting from 1 period. Theoretical investigations for structures with different periods show that the larger the period, the bigger the number of periods to reach good matching between EMA and SMM methods is needed.

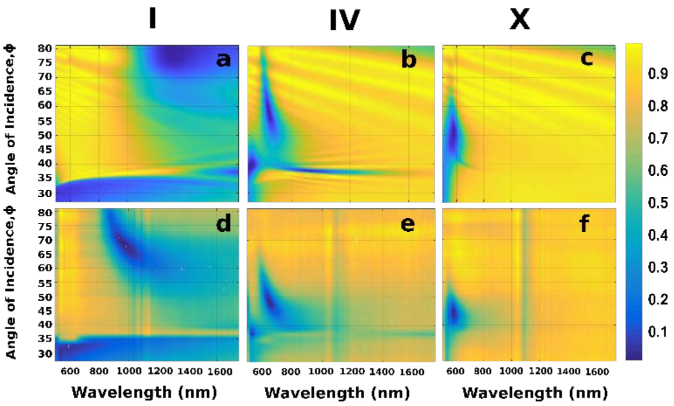

Fig. 1 Dispersion diagram comparison of HMM modes for ZnSe prism-SiO2 (50 nm)-HMM-SiO2 (500 µm) structures. 3D EMA simulations (a-c) and measurements (d-f) with 1, 4 and 10 periods, respectively. The MSDR values are 15.9, 3.6 and 3.5% respectively.

References

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We investigate the limits of the effective medium theory with respect to the minimum number of periods and its dimensions. Both the theoretical and experimental values match showing the importance of establishing this limit.