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Electrical Cable Optimization in Offshore Wind Farms - A review

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ABSTRACT
A state-of-the-art review of the optimization of electrical cables in Offshore Wind Farms (OWFs) is presented in this paper. One of the main contributions of this work is to propose a general classification of this problem, framed in the general context of the Offshore Wind Farms Design and Optimization (OWiFDO). The classification encompasses two complementary aspects. First, the optimum sizing of electrical cables, with the three main approaches used nowadays: static rated sizing, dynamic load cycle profile, and dynamic full time series, being conceptually analyzed and compared. The latest techniques and advances are described, along with the presentation of potential research areas not thoroughly addressed today, such as Dynamic Cable Rating, and cable’s lifetime estimation under time-varying conditions. Second, the network optimization of large OWFs is thoroughly presented, dividing the problem with a bottom-top approach: cable layout of collection system, Wind Turbines (WTs) allocation to Offshore Substations (OSSs), number and location of OSSs, and interconnection between OSSs and Onshore Connection Points (OCPs). A comparison among different methods is performed, taking into consideration the main engineering constraints. Global optimization, specifically, Binary Programming (BIP) or Mixed Integer Linear Programming (MILP), is envisaged as the best way to tackle this topic. The full combinatorial problem is found to be better addressed following a Top-Bottom approach, combining exact formulations with high level heuristics, or holistically with evolutionary algorithms.

INDEX TERMS Balance of plant, Cable sizing, Combinatorial optimization, Dynamic rating, Electrical cables, Heuristics, Global optimization, Mathematical programming, Metaheuristics, Offshore Wind Farm, Static rating.

I. INTRODUCTION

OFFSHORE Wind Farms (OWFs) represent one of the fastest and most steadily growing types of renewable energy technologies for electricity generation. The penetration level has increased almost five times in the last seven years, reaching a globally total installed power of nearly 19 GW [1]. This growth is mainly explained by reductions in costs of the technology [2]: the Levelized Cost of Energy (LCOE) has dropped recently from 240 USD/MWh to 170 USD/MWh, accounting for the last five years.

One of the main drivers for cost reduction is the economies of scale, which has been evident in the OWF industry by the accelerated increase of Wind Turbines (WTs) individual power, and consequently, the scaled up of the total installed capacity of state-of-the-start OWFs. The latter brings as side effect the increase of complexity for designing efficient and cost-effective infrastructure, such as the electrical systems, given that: i) the WTs are larger in power and number, being less uniformly scattered around the project area, ii) the Offshore Substations (OSSs) are built farther away from the Onshore Connection Point (OCP), increasing the export systems transmission length, and iii) more stiff, complete and complex requirements from the Transmission Systems Operators (TSOs) to the OWFs for providing auxiliary services are demanded. All added up, means the electrical infrastructure costs can go up to 15% compared to the total capital expenses (CAPEX) [3]. The last point together with the remarkable impact in terms of efficiency and reliability over the operational performance of OWFs (OPEX) [4], turn the electrical infrastructure into a cornerstone matter in designing the full system.

Between 2018 and 2028 more than 19000 km of cables for collection systems are prognosed to be installed worth £5.36bn [5], while longer and bigger cables for export are...
the trend. Hence, power cables represent an important aspect of the electrical infrastructure to be studied in the design of OWFs, not only because of its obvious weight on economic metrics, but also due to its impact in the overall availability of these type of projects.

The OWFs electrical infrastructure design and optimization is a multidisciplinary problem. This fact that has been proved by a comprehensive literature survey performed in the most important academic sources, detecting a wide variety of definitions, strategies, models, and frameworks to optimize performance metrics related to electrical infrastructure. Additionally, this is a relatively new research area, with no more than 15 years of studies by scientists from different fields, therefore plethora of methodologies and mathematical formulations have been proposed; this is reflected by a relatively large pallet of objectives and requirements identified in the scholar literature.

There is a lack of a comprehensive review articles summarizing, classifying and critically assess the current state of this topic, with only [6] published six years ago being identified. However this manuscript focuses on micrositing, collection systems optimization techniques, transmission systems, and briefly addresses other topics; consequently, improvable by deepening the scope of the cable sizing subject, incorporating new developments, and proposing a general classification of it. By virtue of the above, a literature review of the latest techniques for optimizing electrical cables in OWFs is performed in this paper, intending to provide a classification of the problem while underlying its most important aspects, highlighting the advancements, and finally identifying the open challenges according to the authors’ research.

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The current state of offshore wind farms

The earliest OWF venture known as Vindeby, located at the Danish waters of Lolland in the Baltic sea, was decommissioned last September 2017, after 25 years of operation and 243 GWh of energy produced [7]. Vindeby was located at a distance from shore of 2 km, and concrete foundations were installed above seabed with maximum depth of 4 m. The project consisted of 11 WTs of 450 kW (4.95 MW total installed power). By comparing the previous numbers with the largest OWFs under operation nowadays (see Table 1), the accelerated growth of the industry in a rather short time is becoming obvious, with total installed powers in the order of hundreds of MW, export route lengths close to 100 km, and maximum water depths across the projects’ area of almost 40 m. The escalation of those parameters means that some factors become more relevant and complex to handle, such as the electrical infrastructure, due to the increased investment, complexity in the designs, and the requirement for new technologies able to withstand such new environmental and operating conditions. For instance, on the one hand, the increase of WTs number causes the collection system design to scale up exponentially in terms of brute force design evaluation; on the other hand, the increase of WTs individual power, challenge the traditional voltage level used currently (35 kV), opening the door for new technologies of cables with higher insulation capacities (66 kV), and in contrast, lower power in WTs would require larger amounts of them, depending upon more sophisticated clustering techniques to allocate them in OSSs groups. See in the Table 1 London Array doubles in WTs number to Walney Extension, but the last one has bigger WTs to compensate such difference. Likewise, export route length is linked to decision-making problems such as choosing between AC and DC technology, voltage level, cables type, converters type, system structure, and so on. These trends are foreseen to remain, as presented in the Table 2 with OWFs projects already reaching the order of GW (both because of bigger WTs and larger amounts of them), distances between OSSs and OCPs of more than a hundred of kilometers, and water depths higher than 40 m. In fact, among the list of OWFs under proposal stage, projects in the order of several thousands of MW are waiting for consents to start construction stage. The previous trends mean that the share of the electrical infrastructure over economic metrics will be higher, putting these concepts in a major role to be taken into account when planning OWFs.

Design and optimization of offshore wind farms

The Offshore Wind Farms Design and Optimization problem (OWiFDO) can be defined as the body of decisions to be made in order to design reliable, secure, and efficient OWFs, while maximizing their performance through the evaluation of a quantifiable target. The definition of the set of modelling options, constraints, objective function, variables and parameters, is up to the OWF developers, according to established and particular practices. The OWiFDO is a non-linear, non-convex problem with integer and continuous variables, laying in the category of NP class [8]. Due to the mathematical complexity of the problem, the full picture of it can be split following a sequential divide-and-conquer approach, such as the one illustrated in the the Fig. 1 where four subsequent sub-problems are defined: i) macrositing, ii) micrositing, iii) infrastructures (including structural, electrical and civil design), iv) and definition of control, protection and operation schemes. The main inputs are: minimum and maximum number of WTs, minimum and maximum OWF’s total installed power, and definition of the objective, constraints, and other parameters. The macrositing problem encompasses: i) analysis of the available infrastructure (power system capacity at OCP, logistic resources, accessibility, etc), ii) evaluation of the environmental suitability (specially relevant in marine spatial planning), iii) wind resource potential assessment, and iv) geographical adequacy (most importantly maximum water depths). The main output of this block is the selection of the OWF site, and the upper bound of project’s area; important economic factors such as energy regulatory framework, financing and funding must be taken into consideration in this stage as well, in order to assess the financial sustainability of the project. Micrositing involves the OWF layout design, where
with the arrangement of the individual WTs is decided; in this sub-problem the number and geographical locations of the WTs along with their sizing are defined. After this, the electrical infrastructure is designed; each of the civil, structural and electrical designs has its own mathematical entity, hence in this paper the electrical infrastructure is studied individually. It is important to note at this point that in order to guarantee an optimum design (or near) of the OWF, the best possible solution in each block should be found, while balancing their effect on following blocks. For instance, when deciding the micrositing of certain conditions. The three main techniques used in OWFs cable sizing from the perspective of thermo-electrical conditions are presented in the following.

**Static rated sizing** represents the classic technique recommended in [10], [11], and [12] (industrial technical standards). It is a straight-forward approach, consisting only in a multi-parameter static equation for calculating the continuous current $I_t$, to be transmitted during infinite time, in order to obtain a continuous conductor temperature equal to $90^\circ$C. The smaller cable $t$ with $I_t$ equal or greater than the total current (including capacitive currents) at hot spot is selected. The aforementioned IEC and CIGRÉ standards consider static conditions at rated operation, however OWFs are characterized by low capacity factors and high power production variability.

### Optimization of electrical cables in offshore wind farms: A classification

The problem classification can be seen in the Fig. 2. In the left branch the topics corresponding to optimum sizing of electrical cables are presented. The definition of a cable’s nominal current must take into account the high variability of offshore wind power and its relatively low capacity factor [9]. This implies that a smaller nominal value can be chosen given certain conditions. The three main techniques used in OWFs cable sizing from the perspective of thermo-electrical conditions are presented in the following.

<table>
<thead>
<tr>
<th>OWF</th>
<th>Capacity [MW]</th>
<th>Turbines</th>
<th>Export Route Length [km]</th>
<th>Maximum Depth [m]</th>
<th>Commissioning Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walney Extension</td>
<td>650</td>
<td>40x8.25</td>
<td>75</td>
<td>23</td>
<td>2018</td>
</tr>
<tr>
<td>London Array</td>
<td>630</td>
<td>175x3.6</td>
<td>55</td>
<td>25</td>
<td>2013</td>
</tr>
<tr>
<td>Gemini</td>
<td>600</td>
<td>150x4</td>
<td>110</td>
<td>36</td>
<td>2017</td>
</tr>
</tbody>
</table>

The problem classification can be seen in the Fig. [2](#fig:2). In the left branch the topics corresponding to optimum sizing of electrical cables are presented. The definition of a cable’s nominal current must take into account the high variability of offshore wind power and its relatively low capacity factor [9]. This implies that a smaller nominal value can be chosen given certain conditions. The three main techniques used in OWFs cable sizing from the perspective of thermo-electrical conditions are presented in the following.

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#### FIGURE 1: Offshore Wind Farms Design and Optimization Problem (OWiFDO): An Overview.

#### FIGURE 2: OWFs electrical infrastructure problem classification.
In the last luustrum, an emerging topic in OWFs cable sizing is being studied: Dynamic Rating. The two main approaches using this technique are described in the following. Dynamic load cycle profile consists in finding the worst case dynamic load profiles, as presented in [13] (CIGRE: Working Group B1.40). This approach is taking into account the inherent variability of power production, representing a more sophisticated method that is emerging as industrial practice, as detailed in [13]. It consists of a four-step signal, calculated using the highest RMS values computed through different periods, sweeping through the yearly data set by means of a rolling RMS filter starting at each singular data point. A preprocessing analysis is needed to be carried out to find the set of periods of interest. In the study presented in [14], it has been concluded that the periods of 7 days, 10 days, 40 days, and 365 days capture reasonably the most representative windy days in a windy year. Thus, in a temporal sequential ordering, the pre-conditioning current is derived calculating the RMS value for the whole data set, lasting 308 days (remaining days after the periods 7, 10, and 40), then the greatest yearly RMS value using a period length of 7 days is obtained, keeping the same procedure for the periods of 10 days and 40 days, while not overlapping the periods between them. With the calculated four-step signal (including capacitive currents), the maximum conductor temperature is calculated, and similarly to the previous method, the smaller cable at maximum calculated temperature lower than 90°C is chosen. Note that this sequential arrangement represents a conservative criterion itself, because of the assumed steady state increase of current with time. Equivalent step-wise cyclic load profiles, as in [15] and [16], represent other approach to optimally size cables, by obtaining cyclic currents, followed by the application of the standard [11].

Finally, Dynamic full time series encompasses the use of full and high resolution time series for performing electro-thermal analysis. The previous two methods exclude reliability analysis, therefore new advances and strategies requiring time series information, such as, generated power, surface temperature, thermal parameters, among others, are required. Related to cables, so far work has been focused on: i) conductor temperature estimation, and ii) cables sizing considering a maximum instantaneous temperature never exceeding 90°C (which also is assumed in the previous two approaches). For conductor temperature estimation, several methods have been developed: Step Response (SR), Finite Method Analysis (FEM) and Thermo-Electrical Equivalent (TEE) model [17]. A TEE (1-D) represents the model with the best computation time-quality performance and is applicable for single-core and three-core type cables, albeit new 2-D models are under study and proposed as per [18]. Limiting the conductor temperature to 90°C is a well-established practice, as presented in [19], however a re-evaluation of the risk of exceeding this constraint in different time horizons becomes relevant and might be a way to avoid underuse of these components.

Important progress has been done in this topic, however there is still room for new advancements, such as the application of lifetime methods and probabilistic techniques for sizing these components. The dynamics of the system must be considered holistically, being able to estimate fatigue factors to which a real operating cable is exposed to. This can be achieved by developing new lifetime models, validating their parameters by real experiments in the frame of accelerated tests, based on historical information and forecast values of produced power, seabed temperature, soil thermal parameters, and so on. Real time control of OWFs while abiding thermal constraints can be also merged with these concepts. In general, the previous statements can be extrapolated to other power components, like transformers [20], filters, and compensations units. In fact, not much work has been found related to the latter, therefore representing a potentially important research topic in the short to medium term.

The right branch of the Fig. 2 represents the combinatorial optimization problem related to the electrical infrastructure in OWFs. The main objective is to achieve an optimized, in terms of length and/or investment costs, cable layout. Given the mathematical complexity, computational optimization is required; however project-specific particularities must be taken into account. For instance, the number of OSSs can be defined in function of the number of constructing stages of the project, or the WTs clustering can obey to practical reasons such as power balancing or standardization. Special attention is given to this topic as it has been addressed more intensively in the scientific papers; Section III deals with these aspects.

II. NETWORK OPTIMIZATION

The set of activities shaping the problem of the topological network optimization can be seen in the right branch of the Figure 2: i) WTs collection system design, ii) WTs allocation to OSSs, iii) Number and location of OSSs, and finally, iv) the interconnection between OSSs to OCPs. It should be noticed that the selected classification assumes classic OWFs design: large AC OWFs (MV at collection system level and HV at transmission system, operating at \( f_n = 50 \) Hz), and HVDC connected OWFs (with AC collection system, and the AC/DC station next to the OSS), because these are the types planned, built and under operation nowadays.

A. WTS COLLECTION SYSTEM DESIGN

This problem resembles to historical mathematical problems such as Minimum Spanning Tree (MST) and its constrained version, the Capacitated Minimum Spanning Tree (C-MST), which classifies under the category of NP-hard class [21], and the Travelling Salesman Problem (TSP) with all its variants [22], also NP-hard. Problems from other fields map to this one, like telecommunication networks design back in the 60’s and 70’s [23], or network planning [21]: however in the case of OWFs, Ad hoc methods are necessitated in function of particular spatial (nature reserve or occupied areas, seabed bathymetry, among others), planarity (non-crossing of cables, trenching requirements, and so on), and technical (stochas-
ticity on power generation, cables capacities, topological structure, ancillary services support, etc) constraints.

Based on the literature review, the set of actions used for the collection system design and optimization are represented in the flow chart of the Fig. 3 starting from the data related to variables (binary, integer, continuous, etc), parameters (unitary costs, bounds, etc), objectives, and constraints definition, and continuing with selecting the topological network type. The range of options span networks ensuing different patterns: i) radial, ii) radial plus star, iii) radial plus star plus splices, iv) single looped, and v) others, as illustrated with sample schemes in the same figure. Each topological type. The range of options span networks ensuing different fields. Global optimization requires external solvers (usually used as black box), which use algorithms such as Branch-and-Cut, or Benders Decomposition. Combining different paradigms lead to new hybrid methods, which mix their strengths in order to palliate the weaknesses. An example of that is merging global optimization with heuristics for accelerating the convergence into global minimum, resulting in new formulations called Matheuristics. A qualitative comparison between the fundamental versions of the methods is presented in the Table 3. The main functional advantage of heuristics over the other methods is their polynomial running time; this allows for obtaining solution points very quickly. However, typically, those solutions are considerably far away from the global minimum, and robust algorithms proposed so far, optimize mostly for cables total length (L). The stochastic nature of the operators in metaheuristic methods, improve heuristic by offering better quality solutions. Similarly to heuristics, metaheuristics are self-implmentable, excluding the need to use external black-box solvers. However, metaheuristics do not quantify the quality of the calculated solutions. In order to cope with the last drawback, global optimization, by means of mathematical programming, provide with a dual value during the computation, which is translated into an assessment of the solution quality. Nonetheless, external solvers are necessitated, and the capacity to model the physics behind is rather limited (inherent to the mathematical program). In function of the priorities established by the developer, any solution method could be categorized as best, hence a clear distinction

![FIGURE 3: OWFs collection systems design and optimization: Decision flowchart.](image)

![FIGURE 4: OWFs collection systems design and optimization: Solution methods.](image)
between brightsides and drawbacks must be delimited prior to the selection of the method.

TABLE 3: OWFs collection systems design and optimization: Qualitative comparison among the methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Brightsides</th>
<th>Drawbacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heuristics</td>
<td>• Available bound for worst-case behavior.</td>
<td>• Primal bound is mostly weak.</td>
</tr>
<tr>
<td></td>
<td>• Polynomial running time.</td>
<td>• Purpose-built algorithms.</td>
</tr>
<tr>
<td></td>
<td>• No need of external solvers.</td>
<td>• Optimizes mostly for total length.</td>
</tr>
<tr>
<td>Metaheuristics</td>
<td>• Frameworks allow high modeling flexibility.</td>
<td>• No theory on quality and running time.</td>
</tr>
<tr>
<td></td>
<td>• Provides good primal bound.</td>
<td>• No worst-case analysis.</td>
</tr>
<tr>
<td></td>
<td>• No need of external solvers.</td>
<td>• Computationally expensive.</td>
</tr>
<tr>
<td>Global Optimization</td>
<td>• Dual bound available during computation.</td>
<td>• Uncertainty on computational time and memory.</td>
</tr>
<tr>
<td></td>
<td>• Transparent formulation.</td>
<td>• External solver is required.</td>
</tr>
<tr>
<td></td>
<td>• Similar to graph theory problems.</td>
<td>• Lacks modelling flexibility.</td>
</tr>
</tbody>
</table>

The distribution on the application of these methods in addressing the OWFs collection systems optimization problem in the scholarly literature (using the web search engine Google Scholar) is presented in Fig. 5. It can be noted that the largest portion of the works apply GA (34%), followed by BIP and MILP global optimization formulations (25%), and heuristics - solely or combined with clustering techniques (19%). Minorities are defined by other metaheuristics and global mathematical modelling types. Different conclusions can be drawn from this illustration. Firstly, GA shapes as the most preferred metaheuristic algorithm, given its flexibility and wide application in different fields [37]. Secondly, due to their fast computation time, heuristics represent a good way of finding initial feasible points. They are also subject to the creativity of the designer to come up with different implementations. Lastly, global optimization represents only the third part of the options spectrum (BIP, MILP, MIQP and MINLP), therefore a significant potential in this area is identified, especially on the proposition of efficient formulations and how to integrate high fidelity models with them. Hybrid methods combining global optimization with heuristics and metaheuristics are scarce (apart of embedded heuristics included in commercial solvers such as [38]), albeit potentially being able to solve large (between 80 and 100 WT) and very large instances (more than 100 WT). Consequently, particular focus is directed towards global optimization in this paper.

According to the literature review, the distribution of the objective functions as per the used solution methods is displayed in the Fig. 6. Total length (L), Investment (I), and Investment plus total electrical losses (IL) have been handled using heuristics, metaheuristics, and global optimization methods; on the other hand, when reliability is involved as a target for the system, mathematical formulations emerge as the best approach, due to the flexibility, easiness, and exactness offered by analytical expressions integrated into the framework. Reliability assessment weighted out in the objective function was addressed in [38] through a GA, nevertheless the collection system was based on a grid-based pattern while limiting the possible connections to pre-established alternatives (radial, looped, etc.). After selecting the solution method, the modelling choices must be carried out, taking into account the inherent biased caused by the chosen methodology. Mainly five aspects needs to be examined: i) wake effects, ii) wind stochasticity, iii) power flow, iv) electrical losses, and v) reliability, as schematized in Fig. 7. Wake effects are mostly important for micrositing optimization, one of the most used models being the Katic-Jensen, given its linear behaviour and simplicity, as implemented in the pioneer work of; more more advanced and accurate models based on Computational Fluid Dynamics (CFD) are coming up, nonetheless they are expensive computationally and hard to couple with computational optimization techniques. [39]. The impact of wake effect on the collection system optimization has not yet been addressed on the literature, hence assumed to be negligible. Wind stochasticity deals with modelling the wind power production, basically done by applying...
either discretization of typical probability density functions depending on particular sites [40], or by simulating time series using computational models and historical information [41]; the main two variables in this matter are wind speed and direction. The effect of change on the wind direction on the collection system optimization has not yet been addressed in the literature. Power flow models, going from low complexity to high complexity are: transportation (Kirchhoff’s 2nd law), DCPF (assuming nominal voltages, ignoring losses and reactive power), ACPF (full system of non-linear equations), and Point-to-Point (in-detail transmission line modelling using an infinitesimal lumped model with non-trivial differential equations) [42]. The power flow model drives to different options for modelling the electrical losses, as seen in Fig. 7. In this figure is expressed the interrelation between several blocks. For instance, a DCPF assumes no losses, however they can be computed by doing iterative methods [43]. Likewise, a transportation model can ignore losses or allows integrating it by means of linear or quadratic approximations. AC power flow inherently includes quadratic losses, but can be ignored in the objective function formulation. Point-to-Point flows yield to exact losses calculations [44]. Reliability can be modelled using deterministic and probabilistic approaches [45].

The solution method selection and modelling choice are highly interrelated. Generalizing, mathematical formulations have the enormous advantage of being able to provide certified optimum solutions when the problem is convex; however, their application is subject to the use of commercial solvers, and certain mathematical knowledge of the problem is required for formulating it efficiently and taking advantage of its structure. Therefore, heuristics and metaheuristics can be important because of their easiness in implementation without the need of external solvers using highly efficient programming languages. Additionally, formulations like MILP do not allow explicit (pre-processing techniques can be used as approximation) quadratic modelling of losses (therefore limiting power flow to either DCPF or transportation models), and probabilistic reliability approaches are handled by scenario numeration in a tree fashion, or may be simply ignored. MIQP formulations can include quadratic losses in the objective function, however they are less efficient computationally compared to BIP or MILP. MINLP can capture to a high degree the complexity of the problem, but having the risk of formulating a non-convex problem. On the other hand, heuristics and metaheuristics are more flexible in that sense, due to, in theory, the possibility to consider all the complexity on physical modelling, at expense of not having an optimality guarantee. The proper balance between solution method complexity and model fidelity, represents one of the main challenges for the designer, and compromises have to be adopted within certain assumptions. Each of the different topological network options according to the literature, are discussed in the following, describing the most sound solution methods, modelling choices, and spatial/planarity constraints handling.

1) Radial topology

As presented in Fig. 3, a radial network is that for which branching is not allowed in the nodes corresponding to WTs; mathematically it represents a tree graph with degree equal to 1 or 2 for all vertices belonging to the WTs set. Two publications dealing with this issue are analyzed in the Table 4 and Table 5. Two approaches are developed in [46], one by means of modifications to the probably best-known Vehicle Routing Planning (VRP) heuristic (\(O(|V_c|^2 \log |E| + |E||V_c| + |V_c|C)\)), the Clarke and Wright savings, and the other, through the formulation of an BIP model using a straight-forward hop-indexed formulation with planarity constraints (this is very practical for instances where the largest cable capacity \(C\) is between 5 and 10 nodes as in OWFs [47]). These two methods were compared in 18 different instances for three OWFs (Barrow with 30 WTs and 1 OSS, Sheringham Shoal with 88 WTs and 2 OSSs, and Walney 1 with 51 WTs and 1 OSS). The BIP model takes up to 20 minutes for solving to optimality all the evaluated instances, while the best heuristic requires less than 0.060 seconds, albeit generating solutions on average 2% more expensive. Those are positive results, however the level of complexity for the models is rather limited, as presented in the Table 4 ignoring wake effects, wind/reliability stochasticity, and power losses. Additionally, the available set of cables is restricted to only 1, and computational experiments point out the escalation on computation time when the capacity \(C\) and number of allowed cables are augmented. An important aspect about the heuristics is their inability to optimize the total investment when choosing the cable type, therefore only applying to minimizing total length (L); this limitation is overcome by the BIP formulation. Similar choices are made in [48], but applying a GA algorithm. Comprehensive cost models have been used in this work, encompassing the cost of cables, WTs transformers, and OSSs, however advantages over exact formulations can not be embodied. Nevertheless,
enhancements over heuristics are achieved by including the cable type selection and improving solutions quality. Regarding the spatial and planarity constraints handling, a way to model non-crossing cables for mathematical formulations is by implementing a lazy constraint callback, consisting in stating the corresponding constraints during the construction of the branch-and-cut tree when incumbents are found at any node. By means of this procedure, finding of the maximal cliques in the graph is circumvent. Both works neglect spatial modellings such as seabed bathymetry and restricted zones.

2) Radial plus star topology

A collection system network considering simultaneously radial and star structures is presented in Fig. [3]. This problem is equivalent to a C-MST considering the capacitated constraint to be equal to the power capacity of the largest available cable (U), and it is a superset of the radial version. As demonstrated in [31] and [49], branched trees perform better in terms of total trenching length compared to non-branched trees, however the decision of whether permitting branching at WTs or not, from an optimization point of view, depends strongly on the cost models assumed for cables, WTs switchgears, and other electrical components [50]. A comprehensive summary of publications related to this problem is given in Table [6] and Table [7], and based on this survey, it has been identified a lack of consideration about the previous point, meaning papers has not implemented any strategy for accounting these expenses, perhaps due to the scarce information about cost functions of all electrical components involved in the design. Likewise, the impact of wake effects on the electrical collection systems has not yet been addressed applying exact mathematical formulations; this includes the effects of wind direction.

One of the main contributions in [51] is the proposition of heuristics (with proof of admissibility) for optimizing investment of WTs interconnections with multiple cables selection, however being feasible to implement for up to 14 WTs; in this work the exponential time complexity of BIP models is also remarked. Large instances of OWFs collection system design applying heuristic is proposed in [52], taking into consideration cable choices and power losses; nevertheless formalities such as Big O notation and proofs are dispensed. Metaheuristics have been widely used in the literature, such as the work in [53], where results compared to deterministic heuristics are considerably improved by means of a PSO framework. Exact mathematical formulations encompass basically 4 different types: i) BIP, ii) MILP, iii) MIQP, and iv) MINLP.

BIP and MILP modelling represent the most basic approaches and cover problems instances where is required to optimize length or investment. Many scientists agree on the fact that BIP and MILP present the best balance between solution quality and computation time. The challenge consists on finding efficient strategies for adapting high fidelity models into those programs, for incorporating, for instance, electrical power losses or reliability; pre-processing, decom-position, or heuristics shape up as the way to go to cope with this issue.

The base formulations of BIP and MILP models are presented. These are able to cope with an arbitrary number of WTs, \( w_n \), and considering only one OSS; although with further modifications can be extended for multiple OSSs. Let the OSS define the set \( N_{\alpha} \), such as \( N_{\alpha} = \{1\} \); likewise, for the WTs, \( N_w = \{2, \ldots, 1 + w_n\} \). The \( I^2 \) norm between points \( i \) and \( j \) is defined as \( d_{ij} \).

These sets and parameters are condensed as a weighted directed graph \( G(\mathcal{N}, \mathcal{A}, \mathcal{D}) \), where \( \mathcal{N} \) represents the vertex set \( (\mathcal{N} = N_{\alpha} \cup N_w) \), \( \mathcal{A} \) the set of available arcs arranged as a pair-set, and \( \mathcal{D} \) the set of associated weights for each element \( a_{ij} \in \mathcal{A} \), where \( i \in \mathcal{N} \cap j \in \mathcal{N} \). For instance, for \( a_{ij} = (i, j) \), \( d = d_{ij} \), where \( d \in \mathcal{D} \). In general, \( G(\mathcal{N}, \mathcal{A}, \mathcal{D}) \) is a complete directed graph. Additionally, a predefined list of available cables types is defined. Let the set of cables be \( T \). In this sense, let the capacity of a cable \( t \in T \) be \( u_t \) (in terms of number of supportable WTs connected downstream). Hence, let \( U \) be the set of capacities sorted as in \( T \). Furthermore, each cable type \( t \) has a cost per unit of length, \( c_t \), in such a way that \( u_t \) and \( c_t \) describe a perfect positive correlation. The set of metric costs is defined as \( C \). In this sense, the parameter \( c_{ij}^t \) defines the cost of installing a cable \( t \) between \( i \) and \( j \). The set \( \chi \) stores pairs of arcs \( \{(i, j), (u, v)\} \), which are crossing between each other; this constraint also includes the inverse arcs of those elements. This constraint is a practical restriction in order to avoid hot-spots and potential single-points of failure caused by overlapped cables.

**BIP**

\[
\min \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}_w} \sum_{t=1}^{|T|} u_t \cdot c_{ij}^t \cdot y_{ij}^t = 1 \quad (1)
\]

subject to:

\[
\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}_w} y_{ij}^t = 1 \quad \forall j \in \mathcal{N}_w \quad (2)
\]

\[
\sum_{i \in \mathcal{N}} \sum_{t=1}^{|T|} u_t \cdot k \cdot y_{ij}^t - \sum_{i \in \mathcal{N}_w} \sum_{t=1}^{|T|} u_t \cdot k \cdot y_{ji}^t = 1 \quad \forall j \in \mathcal{N}_w \quad (3)
\]

\[
x_{ij} + x_{ji} + x_{uv} + x_{vu} \leq 1 \quad \forall \{(i, j), (u, v)\} \in \chi \quad (4)
\]

\[
\sum_{t=1}^{|T|} u_t \cdot y_{ij}^t - x_{ij} \leq 0 \quad \forall (i, j) \in \mathcal{A} \quad (5)
\]

\[
x_{ij} \in \{0, 1\} \quad y_{ij}^t \in \{0, 1\} \quad (6)
\]

\[
\forall (i, j) \in \mathcal{A} \land t \in \{1, \ldots, |T|\} \land k \in \{1, \ldots, u_t\}
\]

**MILP**

\[
\min \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}_w} \sum_{t=1}^{|T|} c_{ij}^t \cdot y_{ij}^t = 1 \quad (7)
\]
subject to:

\[ \sum_{i \in N} f_{ij} - \sum_{i \in N_w} f_{ji} = 1 \quad \forall j \in N_w \]  
\[ \sum_{i \in N} \sum_{t=1}^{T} y_{ij}^t = 1 \quad \forall j \in N_w \]  
\[ \sum_{t=1}^{T} u_t \cdot y_{ij}^t \geq f_{ij} \quad \forall (i,j) \in A \]  
\[ x_{ij} + x_{ji} + x_{uv} + x_{vu} \leq 1 \quad \forall \{(i,j), (u,v)\} \in \chi \]  
\[ \sum_{t=1}^{T} y_{ij}^t - x_{ij} \leq 0 \quad \forall (i,j) \in A \]  
\[ f_{ij} \geq 0 \quad x_{ij} \in \{0,1\} \quad y_{ij}^t \in \{0,1\} \quad \forall (i,j) \in A \land t \in \{1,\ldots,T\} \]

The main features of the basic BIP formulation are:

- Variables: Two set of binary variables are required. On the one hand, \( x_{ij} \) is equal to one if an arc with head in \( j \) is selected; on the other hand, \( y_{ij}^t \) is one if the arc \((i,j)\) is selected, using the cable type \( t \) and connecting \( k \) WTs downstream (including the one in \( j \)). Thus, the worst-case maximum number of variables is \( |N|^2 + U \cdot |T| \cdot |N_w|^2 \).

- Constraints: Constraint (3) permits keeping the topology of the solution, and selecting only one cable type per arc. Constraint (5) is the flow conservation equation, this along with the variables definition, allows for respecting the cables capacity constraints. Constraint (4) and Constraint (5) avoid the use of crossing cables. Finally Constraint (6) defines the variables type. Excluding the crossing constraints which are not fundamental restrictions, the number of constraints is given by \( 2 \cdot |N_w| \).

- Flexibility: In this context flexibility is defined as the capability to reformulate the objective function without altering the nature of the whole formulation, if total electrical active losses are required to be optimized simultaneously with investment (IL). In this case the following term can be added into the objective (1):

\[ \sum_{i,j \in N \setminus \{1\}} \sum_{t=1}^{T} y_{ij}^t \cdot w_f \]

where \( R_t \) is the resistance per unit of length of cable \( t \), \( I_n \) is the nominal current of a single WT, and \( w_f \) is the weighting factor to quantify the losses in the same domain as the investment.

Correspondingly, the features of the basic MILP formulation are:

- Variables: Three set of variables are required. \( f_{ij} \) is linear and models the flow in the arcs, \( x_{ij} \) is one if the arc \((i,j)\) is considered in the solution, and \( y_{ij}^t \) if a cable \( t \) is used in that arc. Thus, the worst-case maximum number of variables is \( 2 \cdot |N|^2 + |T| \cdot |N|^2 \).

- Constraints: The Constraint (5) is the flow conservation equation, and avoids also cycles (loops). Constraint (6) allows keeping the topology of the solution. Constraint (10) ensures not exceeding the cables capacity. Constraint (11) and Constraint (12) avoid the use of crossing cables. Constraint (13) defines the variables type. The number of fundamental constraints is \( |N|^2 + 2 \cdot |N_w| \).

- Flexibility: There is no possibility to include total electrical active losses without reformulating the model. By including a quadratic term utilizing the flow variables, a MIQP formulation can be obtained; additionally, a linear loss function can be considered and through the multiplication of \( y_{ij}^t \) and \( f_{ij} \) a MINLP model is formulated (may be linerizable).

Comparing the basic BIP and MILP formulations, one can conclude that the number of variables are in the same order, whilst the number of constraints are considerable less in BIP than in MILP. Other important advantage of the binary formulation over the mixed one, is its flexibility to be adapted for IL problems. The advantage of modelling explicitly the flow in the arcs in MILP formulations may be useful when trying to optimize taking into account the wake losses in the WTs, when different WTs models are considered, and when including the active power losses in the computation of upstream WTs.

It must be stated that further simplification to both models can be proposed, either by strategies to reduce the number
of variables [54], by means of pre-processing techniques, or by adding valid inequalities or cuts [55]. Particularly, it has been shown in [56] how to include total active losses using a MILP formulation through pre-processing, and how to smartly limit the search space to speed up the termination under certain optimality conditions. These hybrid methods combining classical global optimization with heuristics rules are called Matheuristics. It is still an open research question to prove analytically in this context which formulation is better than the other. Given a set \( X \subseteq \mathbb{R}^n \), and two formulations \( P_1 \) and \( P_2 \) for \( X \), \( P_1 \) is a better formulation than \( P_2 \) if \( P_1 \subseteq P_2 \) [57]. This means that if \( P_1 \) is a strict subset of \( P_2 \), then \( P_1 \) is a better formulation than \( P_2 \) because the feasible set is smaller. Suppose \( P_1, P_2 \) are two formulations for the program: \( \min\{cx : x \in \mathbb{Z}^n\} \) with \( P_1 \) a better formulation than \( P_2 \). If \( z_1^P = \min\{cx : x \in P_1\} \) for \( i = 1, 2 \) are the values of the associated linear programming relaxations, then \( z_1^P \geq z_2^P \). Nevertheless the required solution time and memory capacities for both programs may not have any link with this fact, given the unpredictability of combinatorial problems. Further experiments need to be implemented to come up with a representative statistical sample to infer about which formulation solves more efficiently.

Another action to include losses in a linear model is by an iterative method as in [43], where the structure of the problem is exploited by Benders’ Decomposition reducing drastically computation time. Wind power stochasticity and reliability can be included in linear formulations with scenario numeration as in [43] and [45], where a Markov model is used to calculate the states probabilities of cables, and finally a tree is formed numbering all possible operating scenarios. The latter opened the door for stochastic optimization in the collection system problem, concepts that is also applied in transmission expansion planning, or unit commitment, for example. The model is formulated as a two-stage stochastic problem, where the first-stage variables represent the set of connections to build, and the second ones power and energy curtailment. The resulting system considers parallel cables installed in the same trench, which is not common in practice. MIQP formulation permits including quadratic approximation of electrical losses embedded in the optimization model, incorporating probabilistic modelling of the stochasticity of wind and reliability in the same framework, but its computational efficiency is low compared to linear formulations [58], and is dependant on the positive semidefinite nature of the objective function to be a convex problem. Finally, a MINLP formulation [59] captures the full complexity of wind stochasticity, power flow and electrical losses, and with Benders decomposition solving iteratively a master and subproblems, the feasibility of the final solution may be obtained by adding constraints. The main drawback is the non-convex nature of these problems and the possibility to fall into local minimals. Modelling the complexity of terrains initially by a WRP in onshore cases (see Table 7) is proposed in [51], where the terrain is modelled as a grid and each cell is attributed with a coefficient to modify the cables’ length. This may be the way to go in OWFs. For exact mathematical formulations the most used way to add up cables crossing constraints is by following the explained lazy constraint callback approach, although additional experiments are required to infer about the possibility to exploit these constraints as valid inequalities in small problem instances; concerning metaheuristics, penalization strategies when detecting the crossings using specialized algorithms such as Bentley-Ottmann [64] are generally applied.

Modelling restricted zones is transparently achieved by adding up fixed edges in order to approximate the areas with polygons as in [56] and [61]. The main disadvantage of the latter is that for complex shapes useful areas can be disconsidered forcing the final solution to be non-optimal, therefore in [62] (linearized MINLP) a more detailed approach with Delaunay triangulation and shortest path algorithms is proposed [65].

3) Radial plus star plus splices topology
Collection systems with optional intermediate nodes (splices for connecting cables outside of WTs switchgears) is illustrated in Fig. 5. An exhaustive handbook of Steiner tree problem is available in [66] for NP-Complete variants. Currently there are no OWFs implementing this approach, not even under planning stage, however it is presented in this paper as a way to show its applicability from an academic perspective. Steiner tree formulation has been used in [67], [68], and [69]. In all these articles, the objective function is the total trenching length of cables, this being the biggest advantage of Steiner tree formulations. However, this does not necessarily means that the total investment cost is optimized. Likewise, all these articles use greedy heuristics for solving the problem, which provide good solutions but without proofs of correctness and optimality. Regarding modelling choices the simplest approaches were chosen: no wakes, deterministic wind speed, transportation power flow, and no reliability considered, since the adopted solution methods were not tailored for OWFs application.

4) Single looped topology
London Array OWF described in Table 1 follows a single looped collection system pattern as the one presented in Fig. 3. Tailored methods for single looped design have been proposed in [70] and [71]. A set of different algorithms combined and nested are used in [70] in a hierarchical fashion, having as most internal block a Multiple Traveling Salesman Problem (MTSP) solver, which is an extension of the classical Travelling Salesman Problem (TSP), but with \( M \) salesmen \( \{s_1 \cdots s_M\} \) who have to visit each city \( \{c_1 \cdots c_M\} \). There are plenty of heuristics to obtain good quality solutions for the TSP, as Farthest Insertion algorithm and the Chained Lin Kernigan algorithm [72], the latter presents a gap with the Held-Karp lower bound of 0.2%. The objective function of this particular sub-problem is the...
TABLE 6: Radial plus Star Collection Systems Papers: Objective, Solution Method, and Modelling Choices

<table>
<thead>
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<td>DCPF</td>
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<td>Katic-Jensen</td>
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<td>No</td>
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<td>Transportation</td>
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<td>Transportation</td>
<td>Linear</td>
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<td>[62]</td>
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<tr>
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<td>Simulation Wind</td>
<td>ACPF</td>
<td>Quadratic</td>
<td>No</td>
<td>[59]</td>
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<td>L</td>
<td>Heuristics: Enhanced MST.</td>
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<td>Deterministic</td>
<td>Transportation</td>
<td>No</td>
<td>No</td>
<td>[63]</td>
</tr>
<tr>
<td>IL</td>
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<td>Deterministic</td>
<td>Transportation</td>
<td>Other: Preprocessing</td>
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<tr>
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<td>Transportation</td>
<td>Quad.</td>
<td>Probabilistic</td>
<td>[45]</td>
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TABLE 7: Radial plus Star Collection Systems Papers: Spatial and Planarity constraints handling

<table>
<thead>
<tr>
<th>Seabed bathymetry</th>
<th>Non-crossing cables</th>
<th>Restricted zones</th>
<th>Paper</th>
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<td>No</td>
<td>[43]</td>
</tr>
<tr>
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<td>No</td>
<td>[51]</td>
</tr>
<tr>
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<td>Steiner nodes</td>
<td>[50]</td>
</tr>
<tr>
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<td>No</td>
<td>No</td>
<td>[52]</td>
</tr>
<tr>
<td>No</td>
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<td>No</td>
<td>[53]</td>
</tr>
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<td>No</td>
<td>No</td>
<td>[60]</td>
</tr>
<tr>
<td>No</td>
<td>Lazy Constraint Callback</td>
<td>No</td>
<td>[49]</td>
</tr>
<tr>
<td>No</td>
<td>Lazy Constraint Callback</td>
<td>Steiner nodes</td>
<td>[61]</td>
</tr>
<tr>
<td>No</td>
<td>Lazy Constraint Callback</td>
<td>Delaunay triangulation/Shortest Path</td>
<td>[62]</td>
</tr>
<tr>
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<td>No</td>
<td>[59]</td>
</tr>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>[63]</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>No</td>
<td>[54]</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>No</td>
<td>[45]</td>
</tr>
</tbody>
</table>

cable’s total trenching length and the simplest modelling options are considered. Special attention must be paid to the forbidding of cables crossing, which must be also handled by the strategy for clustering the feeders.

5) Others topologies
The proposition of different types of topologies not mapping to mathematical formulations, but carried out following particular criteria of the OWF developers, has also been addressed in the literature, in articles such as [73], [74], and [75]. Those designs are the result of a case-by-case analysis where limited networks are studied by means of economics, power flow, reliability, and dynamic analyses, applying specialized software. The main output of these studies is the best topology for a particular OWF from a set of finite self-designed networks. Examples of these special designs are: star design, single-sided ring design, combined double-sided half design, single-sided ring design, modified double-sided half ring design, and others.
B. WT ALLOCATION TO OSS

In Section II-A, the assumption is that there is only 1 OSS with a specified geographical location. In this Section, we consider one step up in the complexity layer (bottom-up approach): Given a large OWF with pre-defined number of OSSs (greater than 1) and WTs (typically greater than 100 WT), with given geographical locations, design the collection systems in such a way, that a WT is allocated unequivocally only to one OSS (i.e., no direct electrical coupling from one WT to more than one OSS), while guaranteeing the OSS capacities (in terms of nominal power) are not exceeded.

To solve this, there are three alternatives: i) a single approach, where this is solved simultaneously with the collection system problem, ii) a multi-step approach, where as a first step the WT are clustered, and then each WT-OSS problem is solved individually by means of one of the methods explained in the previous Section, or iii) a nested approach, which basically consists on an iterative calculation process, where in the outer loop the WT allocation problem is addressed, and in the inner loop in turn the collection system is tackled.

In the single approach, mathematical formulations can be used transparently to leave the optimization set up to deal with the full problem, as in [46] or [56], however, for large OWFs this may be computationally expensive as presented in [51]. Nevertheless this is the exact way to solve the problem to optimality. Given the flexibility of metaheuristics methods, they can be easily designed to handle this issue, but due to the combinatorial complexity it may be challenging to be adapted as explained in [31]. Using a multi-step approach helps handling the problem complexity, with two ways to split up the WT in OSSs groups: a) mathematically, by formulating the problem using network theory, for instance as a Minimum Cost Flow Problem (MCFP) [76], or b) by applying clustering algorithms like QT, K-means, FCM, among others. MCFP allows shaping the problem with a BIP mathematical formulation, and the network simplex algorithm can be applied to solve it to optimality by exploiting the problem structure and the duality conditions. Finally, an iterative approach similar to the one used in [63], where using pattern search, the WT clustering can be updated based on iterative calculations of collection systems searching for a cheaper solutions.

C. NUMBER AND LOCATION OF OSS

In the Section II-B it was considered the number and location of OSSs are defined as inputs, however, when these points are part of the decision-making problem, there are different alternatives to go through it. As a generalization, authors have regarded this problem including the WT allocation to OSSs, therefore one could consider it as a variant with the added complexity of deciding the OSSs number and geographical location. Three different alternatives of this problem have been found: i) variable OSSs number and variable OSSs location, ii) fixed OSSs number and variable OSSs location, and iii) variable OSSs number and fixed OSSs position.

Variable number and location of OSSs has been coped by means of a multi-step approach in [77] (MILP formulation), [78] (GA), [79] (FCM plus Prim algorithm), and [80] (immune GA). The work of [77] may be the first work dealing with this issue (onshore case) using mathematical models, dividing the full problem into wind farm production optimization model, and wind farm infrastructure optimization model; the potentials of MILP formulations for solving this problem were pointed out, although basic physical modelling choices were selected. The impact of different number and location of OSSs is studied in [78], but the set of potential alternatives were rather limited, and the collection system is assumed to be symmetrically distributed in terms of WTs. A FCM algorithm for clustering WT into OSSs and finding their location considering the shapes centroid (the maximum number of OSSs must be pre-defined), followed by a FCM algorithm to group the WT into feeders is used in [79].

Listing heuristically the possible set of OSSs number is carried out in [80], this is accompanied by the subsequent design of the WT collection system. A nested approach has been applied in [32] and in [70]. An external layer using a PSO algorithm, deciding the number of OSSs and location, is designed with an internal layer clustering the WT into OSS groups by means of a FCM algorithm in [32]. In every iteration the OSSs number and location are updated and this is followed by an internal recalculation of the WT division and collection system. Likewise, a nested hierarchical design is proposed in [70].

Fixed OSSs number with variable position is more typically integrated in single approaches using mathematical formulations like in [43], and [81], although of course using metaheuristics is also possible. A multi-step approach is proposed in [62], where a Capacitated Centred Clustering Problem (CCCP) and a heuristic is used to find the OSS location. Lastly, a nested approach can be found in [63].

Variable OSSs number with fixed position is not so common, the work in [82] is considered to follow this procedure, because it is clear the number of OSSs is encoded in the GA but not their positioning, therefore it is assumed they are fixed.

D. INTERCONNECTION OF OSS TO OCP

Let divide this problem into two variants: i) point-to-point interconnection between a single (or few) OSS to a single (or few) OCP, and ii) interconnection between multiple OSSs and multiple OCPs (large OWFs spread out in a large area).

Regarding the first problem, it basically consists on finding the proper balance between the collection system design (including the OSS positioning), and the transmission system design (export system to connect the OSS to the OCP), given that the shorter distance between OSS to OCP, the more expensive the collection system, but the cheaper the transmission system. Most of the authors assume negligible the influence of OSSs location to the transmission system costs inside of a given range. Nevertheless, in works like [83] (onshore case), [84], multi-fidelity and heuristics approaches, respectively, are considered to analyze the trade-off between...
these two costs. Other works taking into consideration simultaneously the collection system design (with OSSs location), and the transmission system design are: [43], [32], [85], [70], [82], [86], [87], and others.

When the problem is seen from a broader perspective, the interconnection between multiple OSSs and multiple OCPs gets more interesting. In this case, the OWFs are seen in an aggregated way, disregarding the collection system design and calculating the total installed power of the OWF. A GA algorithm to support decision-makers about OWF transmission system developing, and long-term offshore grid planning is proposed in [88]; in this work, the objective is to provide a ranking sorted by their total lifetime costs, of different electrical options to interconnect OWFs between each other, and to OCPs, using either HVAC or HVDC technology, and supporting radial, ring, and meshed designs. At the end, decisions of whether new connections or reinforcements in the onshore grid are required are also provided. One of the main disadvantages of this work is that the implementation has to be improved to apply the developed methodology to larger power systems, and also other types of technologies for electricity generation are not considered, which can be very important for a holistic offshore grid planning. The latter aspects has been handled in the series of publications [89], [90], and [91]. A MILP model to solve the transmission expansion problem accounting for fluctuations in wind power generation and load is proposed in [89], where not only offshore energy is considered, but also other types such as hydro, gas, among others. The tool helps to decide about the feasibility to install DC breakers compared to AC breakers in meshed systems. This work was extended in [90] including clustering of OWFs into larger groups in order to reduce computation time, followed by an offshore grid optimization. The tool seeks to find a balance between new OWFs project and the integration with new or reinforced interconnectors between countries, while having present other types of electricity generation connected to OCPs. Lastly, the designed tool was applied to the case study of Baltic Sea in the time horizon 2030 in [91]. Main results indicate that radial connections are preferred when OWFs are highly scattered between each other, in contrast to meshed grids, which are more beneficial for agglomerated OWFs in a given area. The latest works can be improved considering optimal power flow (AC or DC), and integrating more sophisticated platforms to forecast the energy produced in different time horizons.

### III. CONCLUSIONS

A detailed review regarding the optimization of electrical cables in Offshore Wind Farms (OWFs) is carried out in this article. As a result, the full picture of the problem is divided in two main branches: optimum sizing of electrical cables, and network optimization.

Regarding the former, the three main techniques available today in the industry practices and scientific literature are presented. They span from a lower to higher level of complexity as follows: static rated sizing, dynamic load cycle profile, and dynamic analysis with full time series. The most commonly used today is the dynamic load cycle profile, given its simplicity and representability of more realistic power generation scenarios. It intends to exploit the high variability and low capacity factor of the power production (no superior to 50%). Likewise, the third technique embodies an important topic that is shaping up as crucial: Dynamic rating of electrical components; further studies through this technique yields to lifetime estimation, which can be done offline with models calibrated in laboratories, or online by sensing in real time variables such as power, external temperature, or mechanical stresses. In order to do so, novel models encompassing thermal analysis of cables (extendable to other electrical components), including detailed physical components modelling, while not compromising the computation requirements, are necessitated. Development of lifetime models of electrical cables represents an important research area, investigating the impact of real static and dynamic operation conditions, such as total length, and system’s dynamics. In general, one can say that the trend is towards combining dynamic sizing with lifetime estimation, ensuring that the chosen solution does not adversely impacts either.

Related to topological network optimization, the cable layout in collection system sub-problem is envisaged to be addressed by applying BIP and/or MILP formulations, given that this approach brings a proper balance between solution quality and computation time, with a physical modelling choices inside of a permitted level of complexity. In the reviewed works applying this methodology, it was not found the inclusion of wake effects, and the power losses are computed either by pre-processing strategies, linearization, or iterative processes. Models with dynamic location of Offshore Substations are required. Details experiments for assessing the efficiency between different mathematical programs are highly encouraged. Given the trend in OWFs towards larger and larger projects, focus must be directed into this aspect, by proposing methodologies able to provide solutions in reasonable computation time.

There is also space for new global optimization formulations including a probabilistic approach for reliability assessment to obtain looped networks, instead of installing parallel cables in the same trench. The effects of the Wind Turbines (WT) fatigue over the cable layout design is becoming increasingly important. More elaborated cost models are required as well, including more components: cables, transformers, switchgears, and installation costs. None of the reviewed works deal with modelling the seabed bathymetry, which can have a great impact over final collection systems as this can significantly impact the cables’ length. Case studies comparing the new proposed collection system voltage (66 kV) with the classic one (33 kV) can be interesting as well.

In the impossibility to use external solvers (hence exact mathematical formulations), metaheuristics seem to be the best choice to solve this problem stage, despite the fact the door is open for designing heuristics with cable choice and
accounting for electrical losses, with time and quality bound proofs. The full picture of the topological network optimization has been addressed until today, by means of multistep and nested approaches, however evolutionary algorithms mixed with mathematical formulations might be an appealing option. Finally, the two branches can be combined, for instance, a resultant collection system network can use dynamic rating with probabilistic lifetime models of cables for sizing such elements, in order to provide a more tuned stage after the convergence of combinatorial algorithms.

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