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The perfect match: structural optimization and additive manufacturing

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Abstract
Additive manufacturing (AM) and topology optimization (TO) is simply the perfect match. Where topology optimization, a computer based structural optimization method, is capable of producing lightweight components for e.g. airplanes and automotive applications, the resulting geometries are often extremely complex and therefore close to impossible to manufacture using subtractive methods. Therefore, to fully utilize the possible benefits of TO, and hence obtain reductions in e.g. raw material and fuel consumption, the new opportunities provided by AM are essential. That is, by tailoring the TO methodologies to match the geometric freedom provided by AM, it is possible to extend the applicability of TO in new ways that lead to further performance enhancements in a vast number different engineering applications.

Topology optimization, Additive manufacturing, Computational design, Gradient based optimization

1. Introduction

The combination of additive manufacturing (AM) and topology optimization (TO) is in many ways a perfect marriage. As in all successful marriages, both parties must adapt to his/hers partners weaknesses, accept compromises and evolve together while building on each other strengths. This is surely also the case for AM and TO. In this contribution we will cover a state-of-the-arts (SOTA) presentation for TO and AM, more specifically, this paper present some of the most important numerical approaches for performing structural optimization adapted for various AM methods. That is, first, the general topology optimization method is presented. Next, we present a design method with focus on the design of coated structures with isotropic, soft cores. Based on the findings obtained by this method we will highlight the comprise in stiffness maximization to improved buckling resistance. Secondly, a method allowing for stiffness optimized infill optimization presented. Finally, the findings are summarized, additional TO and AM possibilities are listed and possible future works are presented.

2. Density based topology optimization

The basic density based TO approach is cast as a mathematical programme. For the simplest of problems, i.e. stiffness maximization the optimization problems takes the following form.

$$\begin{aligned} \min_{x\in\mathbb{R}^n} \quad & \phi = \boldsymbol{u}^T\boldsymbol{F} \\ \text{s.t.} \quad & \boldsymbol{K}\boldsymbol{u} = \boldsymbol{F} \\ & V(\boldsymbol{x})/V^* - 1 \leq 0 \\ & 0 \leq x_i \leq 1, \quad i = 1, n \end{aligned} \tag{1}$$

The optimization problem in (1) states that we wish to determine the design, given by the design vector $\boldsymbol{x}$, that minimize the compliance (inverse of stiffness) expressed as the work done by the load, i.e. $\boldsymbol{u}^T\boldsymbol{F}$, where $\boldsymbol{u}$ is the displacement vector and $\boldsymbol{F}$ is the load vector. The displacement vector is obtained by solving a finite element problem based on the equations of linear elasticity, i.e. $\boldsymbol{Ku} = \boldsymbol{F}$, where $\boldsymbol{K}$ is the stiffness matrix arising from the standard Galerkin approach. In the density method we assign one design variable to each finite element and let the optimization procedure determine which of the design variables should be solid, $x_e = 1$, or void, $x_e = 0$. Continuing through equation (1) it is seen that we require that the design fulfils the equilibrium equation and that the material usage, $V(\boldsymbol{x})$, is constrained to be less than an a priori specified value, V^*. The final constraint states that the design variables are bound to be between zero and one. Of course, we need to ensure that the final design is purely zero and one, i.e. no intermediate densities, for which the SIMP stiffness interpolation rule is used, i.e. $\boldsymbol{K}_e = E_{min} + x_e^p(E_{max} - E_{min})\boldsymbol{K}_e^0$. Here, the factor p is used to penalize intermediate stiffness and usually $p = 3$ is used. The two elastic moduli E_{min} and E_{max} are used to describe the minimum stiffness, e.g. void, which must be non-zero but small in order to avoid singular stiffness matrices, while E_{max} simply refers to the actual elastic modulus of the candidate material. This optimization problem can be solved using standard mathematical programming methods which in turn requires the computation of gradients of objective and constraint functions wrt. the design variable $\boldsymbol{x}$. The gradients, also called sensitivities, can efficiently be computed using the adjoint method. To regularize the optimization problem, and hence obtain mesh-independent solutions, we need to introduce a design filtering scheme using methods from image processing. For details on the density based TO method the reader is referred to [1]. Finally, we note that the standard stiffness maximization problem can be implemented very efficiently on multiple platforms and that example codes can be found at www.topopt.dtu.dk and that interactive app's are available at the Apple App Store and Google Play [2]. For an example of the standard TO approach see Figure 2(a).

2.1. Coating approaches - multilayer filtering

Many AM methods have the possibility to allow the model to be printed with a hard shell and a softer (in the stiffness sense) infill. Here we present a TO method that allows to design coated structures with soft, isotropic infill. Note that the next Section presents a method to design the actual layout of the infill.

The key component in order to obtain a stable methodology allowing for coating design is to apply a hierarchy of filtering and projection operation. The outline of the coating filtering approach can be seen in Figure 1 and explained in detail in [3].

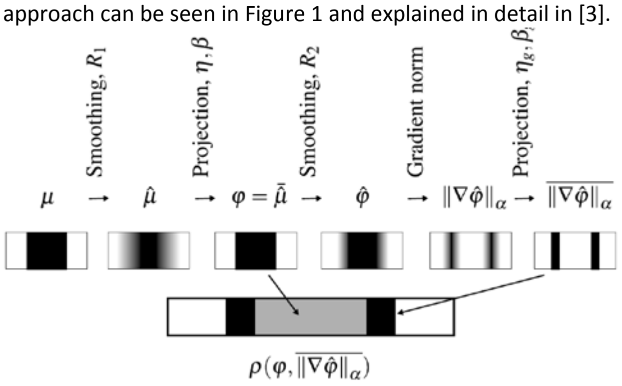

Figure 1. Illustration of the filter hierarchy used for the coating design approach. Image taken from [3].

In Figure 1 we note that the two material phases are obtained from a single design variable vector $\boldsymbol{\mu}$. By performing a filtering and a subsequent projection we obtain the base material describing the soft core. Continuing with a second filter operation, a gradient computation and a projection of the gradient norm we obtain the outer shell. Note that we can control the thickness of the shell through the second filtering operation radius R_2. An example of a design optimized with this method, as well as standard TO, is shown in Figure 2.

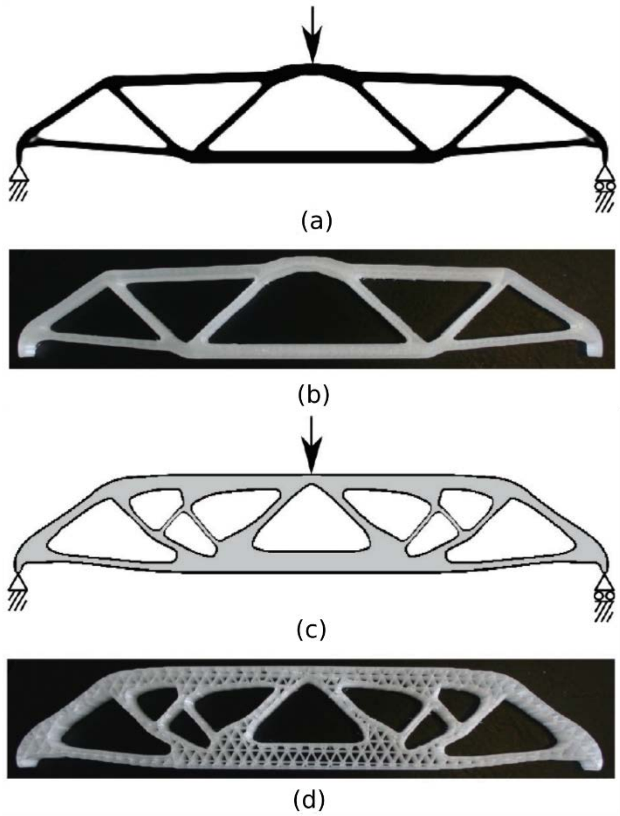

Figure 2. Result of standard TO (a) and an FDM printed specimen (b). Image (c) shows the result using the coating approach and its FDM specimen in (d). Image from [4].

The optimized designs and their 3D printed counterparts can be seen in Figure 2, which shows two designs with equal mass. The first design, Figure 2(a,b) is obtained from the standard TO approach and consists mainly of thin structural members, while the design obtained using the coating approach, i.e. Figure 2(c,d) is seen to be composed of thicker members. When comparing the stiffness performance of these two designs it is found that the standard TO approach leads to a ~10% stiffer design than the coating approach. This is in accordance with analytical predictions and elasticity theory. However, the cost of lower stiffness comes with improved buckling resistance as will be presented next.

2.2. Improved buckling resistance

It is clear from the visual inspection of the designs in Figure 2 that the coating approach leads to designs with thick structural members. Though this leads to a decrease in stiffness, the added second moment of inertia will lead to an increase in buckling loads. To verify this, a series of experiments has been conducted on the AM specimens, see [4] for details, and the main results are shown in Figure 3.

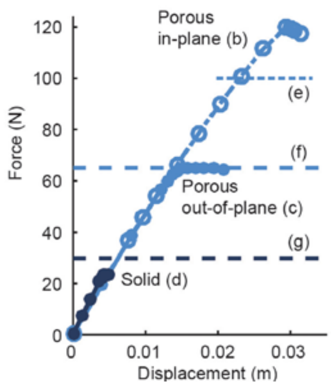

Figure 3. Buckling load for the designs in Figure 2. Image from [4].

From Figure 3 it is clear that the coating design has an increased buckling resistance compared to the standard design. The improvement amounts to approx. 225% and it should be noted that the first encountered buckling load is out-of-plane and hence an even greater improvement can be found when applying the design method to 3D design problems.

3. Infill design

In this Section we describe an easy method for obtaining an anisotropic and optimal infill pattern in the optimized designs. As the case with the coating approach, the key ingredient is a novel filtering scheme which entails a local volume constraint. The optimization problem can be stated as follows and the reader is referred to [5] for more details.

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & \phi = \boldsymbol{u}^T \boldsymbol{F} \\ \text{s.t.} \quad & \boldsymbol{K}\boldsymbol{u} = \boldsymbol{F} \\ & V(\boldsymbol{x})/V^* - 1 \leq 0 \\ & \|\hat{\rho}(\boldsymbol{\rho}) - \alpha\|_P \leq 0 \\ & 0 \leq x_i \leq 1, \quad i = 1, n \end{aligned} \tag{2}$$

In equation (2) it is clear that the only difference to the standard TO approach is the addition of an additional local volume constraint. The local volume constraints states through a p-norm (used to mimic the max-operator) that all sub-areas of the design must use a volume-fraction of α. In this way the structure is not allowed to include large holes and hence we can control the infill layout. A result using this approach is shown in Figure 4.

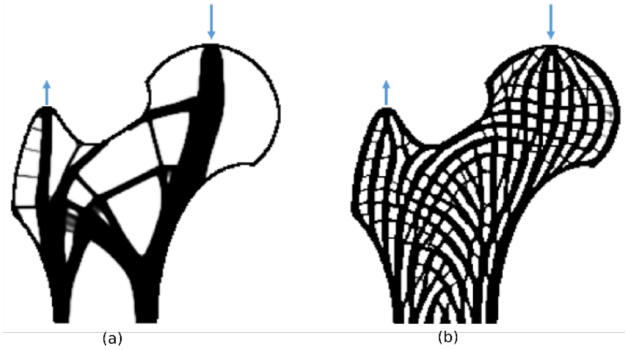

Figure 2. Result of standard TO (a) and the infill method (b) using a single load case. Image from [5].

From Figure 4(a) we see the result using the standard TO which does not give rise to infill patterns for the single load case considered. However, in Figure 4(b) the formation of an anisotropic infill pattern is evident, which leads to better performance wrt. local failure of internal structural members and changes in load conditions.

4. Conclusion & perspectives

The presented work clearly demonstrates the synergy obtained by combining AM and TO. Specifically, examples showing improved buckling resistance and anisotropic infill patterns were presented.

In the presentation of SOTA methods for AM and TO additional examples covering large scale methodoligies, design of heat sinks specifically for powder based metal printing, the design of jet engine brackets, full (multi)scale wing design, inclusion of overhang constraints will also be presented.

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