Small and Large Signal Analysis of Photonic Crystal Fano Laser

Zali, Aref Rasoulzadeh; Moravvej-Farshi, Mohammad Kazem; Yu, Yi; Mørk, Jesper

Published in:
Journal of Lightwave Technology

Link to article, DOI:
10.1109/JLT.2018.2877816

Publication date:
2018

Document Version
Peer reviewed version

Link back to DTU Orbit

Citation (APA):
https://doi.org/10.1109/JLT.2018.2877816
Small and Large Signal Analysis of Photonic Crystal Fano Laser

Aref Rasoulzadeh Zali¹, Mohammad Kazem Moravvej-Farshi¹*, Senior Member, IEEE, Yi Yu², and Jesper Mork²

Abstract—We analyze the small- and large-signal response of a photonic crystal Fano laser (PhC-FL). Conventional current modulation, as well as modulation of the laser via the mirror, is investigated using a numerical approach as well as linear small-signal analysis. The results show that the amplitude modulation bandwidth of one of the laser ports (the through-port) and the frequency modulation (FM) bandwidth of another port (the cross-port) extend to the THz region. Large-signal simulations of the laser response to a 500 Gbit/s pseudo-random bit sequence modulation of the nanocavity are in good agreement with the predictions of the small-signal analysis. Finally, it is investigated how the design of the Fano mirror, in particular, the quality-factor (Q) of the nanocavity, affects the modulation properties.

Index Terms—Fano Laser, Laser modulation, Laser dynamics, Fano resonance, Photonic Crystal Laser, Large-signal, Small-signal

I. INTRODUCTION

In recent years, by introducing defects in photonic crystals (PhCs), high quality cavities and hence ultra-compact lasers have been realized [1–8]. Different types of PhC lasers have been realized, i.e., point-defect [3, 5], line-defect [6-9] and nano-beam [10] lasers, each with their advantages. Recently a different type of laser, a so-called Fano laser (FL), was suggested, where one of the laser mirrors is realized using a Fano interference between the continuum of waveguide modes and the discrete mode of a side-coupled nanocavity [11]. This Fano mirror only has a high reflectivity within a narrow bandwidth determined by the quality factor of the nanocavity and it was shown that the laser can be frequency modulated (FM) at frequencies largely exceeding the relaxation oscillation frequency by modulating the nanocavity resonance rather than the laser current [11, 12]. Recently, the Fano laser was experimentally demonstrated [13] and it was furthermore shown that the laser can operate in a regime of self-pulsation, where a train of short pulses is generated [13, 14].

In this paper, we extend the results of [11] by studying the large-signal modulation properties of the laser. The large-signal model is verified in the small-signal regime by comparison to semi-analytical small-signal results. Two different ways of modulating the Fano laser are investigated, i.e., via conventional current modulation or via modulation of the refractive index of the nanocavity, and the physical origin of the large difference in modulation bandwidth between the two schemes is explained. Both frequency modulation (FM) and amplitude modulation (AM) are being considered. Finally, the large signal response of the PhC-FL is investigated at bit-rates up to 500 Gbit/s by considering the response to a pseudo-random bit sequence. It is found that by proper choice of the output port of the Fano laser, both FM and AM modulation with good eye openings can be achieved at these extreme bitrates.

II. MODEL AND STEADY-STATE CHARACTERISTICS

A schematic of the PhC-FL under study is shown in Fig. 1. It is implemented in a III-V semiconductor membrane, where light confinement in the plane is due to the photonic crystal bandgap effect, realized by a pattern of airholes, and in the transverse direction light is confined by total internal reflection [15]. A line-defect PhC waveguide is terminated in one end by airholes while in the other end it is side-coupled to a point defect nanocavity giving rise to a Fano resonance [16]. The gain region is confined to the green region in the figure,
i.e., the nanocavity is passive, which can be realized using the buried heterostructure technology [6]. The nanocavity is also coupled to an upper PhC waveguide, which is denoted as the laser cross-port, while the lower output waveguide is the through-port. When it was shown in [11] that when the laser is operated at its minimum threshold gain, the power in the cross-port is much larger than that in the through-port, but bearing this in mind we shall here investigate the output properties of both ports. The bandwidth of the left mirror is very large and we shall for simplicity assume the reflectivity to be unity. The vertical dot-dashed line shows the horizontal position of the right mirror at which the nanocavity odd mode ($H_r$) interferes destructively with the waveguide modes, resulting in a Fano mode. The bandwidth of the left mirror is very large and we shall here investigate the output properties of both ports. The left mirror is very large and we shall for simplicity assume the reflectivity to be unity. The vertical dot-dashed line shows the horizontal position of the right mirror at which the nanocavity odd mode ($H_r$) interferes destructively with the waveguide modes, resulting in a Fano mode. A dynamical model for the FL can be derived by combining a transmission line model and coupled-mode theory [11-14].

The laser threshold current, $J_{th}$, versus the detuning is shown in Fig. 2(a). When the nanocavity is detuned from its optimum value, the threshold current increases since the laser now oscillates off the center of the nanocavity resonance in order to fulfill the phase matching condition [11]. The threshold current is asymmetric with respect to detuning due to the finite value of the alpha parameter [11, 14]. Moreover, the laser output power emerging from the through- and cross-port are given by

\[ P_t = 2\varepsilon_0 n_{c,t} c [A^+ (t) - A^- (t)] \]

and

\[ P_c = 2\varepsilon_0 n_{c,t} c [\gamma_c] [A^+ (t)]^2 / \gamma_c \]

respectively, in which $n_{c,t}$, $\varepsilon_0$, and $c$, are the effective refractive index, free space permittivity, and light velocity. $A^+ (t)$ and $A^- (t)$ represent the right- and left-propagating field envelopes at the reference plane adjacent to the Fano (right) mirror, with $a^+ \phi^+$ and $a^- \phi^-$ being the corresponding amplitudes and phases. Fig. 2(b) illustrates the dependencies of $P_t$ and $P_c$ on the nanocavity detuning for two different bias currents of 4.5 mA and 6.8 mA. For zero detuning, the laser frequency and the nanocavity resonance coincide, maximizing the destructive interference and hence minimizing the through-port power. The cross-port power is proportional to the optical energy stored in the nanocavity and is maximized for $\delta_r = 0$. As the nanocavity frequency is detuned, the power emerging from the through-port increases, corresponding to an increase of the differential quantum efficiency [11]. The corresponding variation of the laser frequency with detuning is shown in Fig. 2(c), displaying a quasi-linear dependence.

Next, we have analyzed the modulation properties of the laser using the standard small-signal (SMS) approach and compared the results with those obtained numerically. In the SMS approach, we used the first order perturbation for all the dynamical variables ($a^+ \phi^+$, $\phi^+$, $\phi^-$, and carrier density, $N$) around the steady-state values [17].

### III. SMALL-SIGNAL MODULATION

We consider two cases of modulation: (i) conventional current modulation defined by $J(t) = \Delta J \sin (2\pi f t)$ with $\Delta J$ and $f$ being the modulation amplitude and frequency; (ii) modula-

---

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameters used in simulations</strong> [11].</td>
</tr>
<tr>
<td><strong>Symbol</strong></td>
</tr>
<tr>
<td>$A_{wave}$</td>
</tr>
<tr>
<td>$\delta_r$</td>
</tr>
<tr>
<td>$L$</td>
</tr>
<tr>
<td>$n$</td>
</tr>
<tr>
<td>$N_0$</td>
</tr>
<tr>
<td>$Q$</td>
</tr>
<tr>
<td>$Q_{cross}$</td>
</tr>
<tr>
<td>$Q$</td>
</tr>
<tr>
<td>$R_0$</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\alpha_c$</td>
</tr>
<tr>
<td>$\Gamma$</td>
</tr>
<tr>
<td>$\lambda_c$</td>
</tr>
<tr>
<td>$\tau_0$</td>
</tr>
</tbody>
</table>

---

Fig. 2. (a) Threshold current versus nanocavity detuning. (b) Through-port and cross-port powers versus nanocavity detuning, for constant bias currents of 4.5 mA (circles) and 6.8 mA (diamonds). (c) Laser frequency (normalized to $\gamma_c$) as a function of nanocavity detuning.
By increasing the detuning from $\delta_c=0$ to 1, the threshold current increases, thus decreasing the ratio of the current modulation amplitude to the bias current. As a result, the low frequency component of the modulation index becomes smaller. Figs. 3(a) and 3(b) also reveal that the behavior of the FM amplitude and modulation index for the PhC-nanocavity lasers remain at room temperature with a nanocavity having a quality factor of $Q=500$ and a photon lifetime of $1.6\times10^4$. Using the SMS approach, we calculate the FM amplitude. Fig. 4(a) and 4(b) show the frequency dependence of the FM amplitudes for $\delta_c=0$, 0.5, and 1.

As can be observed from Fig. 3(c), the nanolaser current injection level is in the range of mA. This may raise an important issue regarding the impact of thermo-optical effects. Nonetheless, a thorough systematic thermal analysis, using a full 3-D simulation as well as experimental results, it has been shown that InP based PhC lasers remain at room temperature under CW pumping.

In the second case, we investigate the response of the FL to modulation of the nanocavity frequency. This may be implemented by various approaches: (i) Dynamic modulation of nanocavity resonance frequency by applying voltage to the electrodes placed near it [19]; (ii) Modifying the nanocavity resonance frequency, without changing its quality factor, by moving a near-field probe vertically and laterally in the nanocavity [20]; (iii) Modulation of the nanocavity resonance frequency, by means of optical nonlinearities — i.e., via dispersion of the optically excited free carriers [21, 22]. Varying the modulation frequency in the range of $1\text{GHz} \leq f \leq 30\text{THz}$, we calculate the FM amplitude. Fig. 4(a) and 4(b) show the frequency dependence of the FM amplitudes for $A^+$ and $A^-$. Symbols represent the data obtained by the numerical approach and the solid curves depict the result of the SMS analysis.
where is almost flat up to ~1 THz. In other words, the are small signal phase response depends on the nanocavity decay rate ($\gamma_L$) for the FL with $Q = 500$ and nanocavity detuning $\delta = 0.05$. (b) Phase difference between the counter-propagating fields within the line-defect cavity region of the FL for $\varepsilon = 0.05$. Solid curves represent the corresponding data obtained via the SMS approach.

\[
\nu^r(\omega) = \frac{\gamma_L}{(\gamma_L + \gamma_T)(1 + i\omega/\gamma_L)}
\]

and

\[
\nu^d(\omega) = \frac{\gamma_L}{(\gamma_L + \gamma_T)(1 + i\omega/\gamma_L)}
\]

as shown in Figs. 4(a) and (b). So the DC component of the FM amplitudes for both fields are $\gamma_L/(\gamma_L + \gamma_T)$. As we can see from Fig. 4, for the PhC-FL with $Q = 500$ ($\gamma_L/2\pi = 194$ GHz) and $L = 5 \mu$m ($\gamma_L/2\pi = 1.34$ THz), the low frequency value of the FM amplitude is about 0.87, while, for the FL with $L = 10 \mu$m and $Q = 500$, the FM amplitude reduces to 0.78, and for the FL with $Q = 100$ and $L = 5 \mu$m, it decreases to 0.58. As shown in [11], the cut-off frequency (bandwidth) of the FM response depends on the nanocavity decay rate ($\gamma_L$) and the inverse of the waveguide cavity round-trip time ($\gamma_T$), i.e., $\omega_{\text{db}} = (\gamma_L + \gamma_T)$. Therefore, as $L$ increases, $\gamma_L$ and hence the modulation bandwidth, for a given $Q$ decreases. Moreover, as the nanocavity quality factor decreases, the total decay rate increases and the FM response bandwidth increases. Fig. 4(b) shows that the FM amplitude increases towards unity with the modulation frequency, reflecting that the cross-port field modulation always tracks the nanocavity field change instantaneously. We notice that the model becomes inaccurate for high modulation frequencies exceeding the intermode-spacing [12], but this limit occurs at frequencies much higher than the relaxation oscillation resonance frequency, safely allowing analysis in the high-frequency regime not accessible in conventional lasers. In the low-frequency limit, the FM response represents the slope of the steady-state lasing frequency versus nanocavity resonance frequency. These results show that in contrast to the case of current modulation, SMS modulation of the Fano mirror resonance changes the phase and frequency of the field in the nanocavity via the adiabatic wavelength conversion mechanism that lets the laser field to track the nanocavity field, directly, leading to an essentially pure FM signal. This is because, the modulation frequency is much faster than carrier dynamics timescale that limits the conventional lasers bandwidths [11, 23]. Consequently, the SMS analysis indicates that the usual bandwidth limitation imposed by relaxation oscillations in conventional current-modulated FP lasers is absent here [11].

Figure 5 (a) shows the frequency dependence of the modulation indices of the cross-port and through-port powers under an injection current of $J = 5 J_0$ and modulation amplitudes of $\varepsilon = 0.05$ (circles) and $\varepsilon = 0.25$ (crosses). The numerical results (markers) agree quantitatively with the results of the SMS analysis. However, by increasing the modulation amplitudes to $\varepsilon = 0.25$, the deviation between the SMS and the numerical results starts to be apparent (see inset in Fig. 5(a)). The modulation spectra for the cross-port signals, with peaks observed around the relaxation oscillation frequencies, behave similarly to that of a conventional FP laser. Besides, as the modulation amplitude increases, the modulation index increases. Furthermore, for constant modulation amplitude, since the cross-port power is higher than that of the through-port, the intensity modulation index of the through-port power is larger.

As seen in Fig. 5(a), the intensity modulation bandwidth for the through-port power is in great contrast to that of the cross-port. The bandwidth of the modulation index of the through-port power can be extended to the THz range. To understand this, let us compare the expressions for their corresponding small signal fluctuations. A simple calculation reveals that $dP_C$ (small signal fluctuation of cross-port power) has no phase dependence (on $\phi^+$ and/or $\phi^-$) whereas $dP_T$ (small signal fluctuation of through-port power) is related to the phase difference of the counter-propagating signals through $(d\phi^+ - d\phi^-)\sin(\phi^+ - \phi^-)$, where $d\phi^\pm$ are small signal phase fluctuations. This dependence, flattens out the modulation index of the through-port power at high modulation frequencies, as can be seen from Fig. 5(b) where the spectrum $(d\phi^+ - d\phi^-)$ is almost flat up to ~1 THz. In other words, the characteristic relaxation rate for the phase difference is governed by the total rate ($\gamma_L + \gamma_T$) by which the phase coherence can decay, resulting in a THz bandwidth and circumventing the usual limitation imposed by the relaxation oscillation frequency [11].

IV. LARGE-SIGNAL MODULATION

Finally, to examine the capability of digital data transfer from electrical to the optical domain by the PhC-FL, we investigate the large signal behavior. In Ref. [11], it was shown that when the large-signal modulation amplitude exceeds a certain critical limit, short-pulses may be generated due to the build-up of a large carrier density during the part of the modulation cycle where the laser is below the threshold. In order to avoid these strong nonlinear effects, the modulation amplitude should be limited. Here we limit ourselves to positive detuning...
of the nanocavity and consider two cases, i.e., $0 \leq \delta_c(t) \leq 1$ and $0 \leq \delta_c(t) \leq 0.5$. We investigate the laser temporal responses under a pseudo-random binary sequence (PRBS) modulation of the nanocavity detuning, at 500 Gbit/s, as illustrated in Fig. 6(a). This PRBS bit rate is chosen to be close to the 3-dB bandwidth of the SMS response of the FL. In order to evaluate the quality of the output modulation, we consider the corresponding eye diagrams. The eye diagram opening is thus a measure of the suitability of the FL for high bit rate modulation [24].

Figures 6(b)-(d) show simulated eye diagrams for the FM amplitude of the cross-port field, through-port power, and cross-port power of the PhC-FL with $Q=500$ and $L = 5 \mu m$ for $J=5 J_{th}$, in response to a 64-bit PRBS modulation of amplitude $0 \leq \delta_c \leq 1$ (Fig. 6(a)). As can be seen, the eye diagrams for the FM amplitude are wide open, confirming the THz wide FM modulation bandwidth predicted by the SMS analysis. However, the eye diagram for the cross port power exhibits a power variation of 40% which is undesirable since we are looking for pure FM modulation without any power variation. On the other hand, the eye diagram for through-port power exhibits an open pattern, verifies its large 3-dB bandwidth which extends to THz frequencies and predicted by SMS analysis.

By decreasing the modulation amplitude to $0 \leq \delta_c(t) \leq 0.5$, the difference between the power levels 0 and 1 decreases, as depicted in Fig. 7(a) and (c). Consequently, the variation of the cross-port power decreases to 11% (Fig. 7(b)), improving the purity of the signal. Optimization of the modulation properties of the laser is beyond the scope of this paper, besides, noting that we also need to take into account the demodulation techniques used for detecting the FM-modulated signal.

Finally, the cavity coupling rate and the quality factor of the nanocavity can be engineered with important consequences for the laser dynamics. To investigate the effect of the nanocavity quality factor on the temporal response of the large-signal modulation of the PhC FL, we consider three different nanocavity quality factors of $Q=500$, 1000, and 1500 under a fixed bias current of $J=2.3$ mA. Here we consider modulation of the nanocavity with an amplitude of $0 \leq \delta_c \leq 0.5$. Fig. 8(a)-8(c) shows that by decreasing the quality factor of the nanocavity via an increased coupling to the waveguide, the average power increases due to the increase of the maximum Fano mirror reflectivity and the corresponding reduction of the threshold current. In addition, by increasing the quality factor of the nanocavity from 500 to 1500, the power variation decreases from 11% to 0.8%, due to the smaller absolute variation of the nanocavity resonance frequency. Thus, the power modulation variation decreases consequently, which leads to a more pure FM modulated signal. As expected from the SMS analysis, as the nanocavity quality factor increases, the 3-dB bandwidth decreases since $Q \propto \gamma_r^{-1}$. Correspondingly, the eye diagrams start to close (the difference between 0 and 1 levels decreases) as seen in Figs. 8(d)-(f).

V. CONCLUSION

In conclusion, we have studied small- and large-signal modulation properties of photonic crystal Fano lasers. In particular, conventional current modulation has been compared to the case of modulating the resonance frequency of the nanocavity governing the Fano mirror, which controls the wavelength and intensity of the laser. It was shown that the Fano laser has the prospect of being modulated at frequencies of several hundred gigahertz when considering modulation of the nanocavity. Furthermore, the effect of the nanocavity quality factor ($Q$) on the modulation properties was investigated, showing that this provides an important design parameter.
Future work should analyze different methods for modulating the nanocavity as well as optimizing the performance under consideration of the actual detection technique.

REFERENCES


Aref Rasoulzadeh Zali was born in Tabriz, Iran in 1986. He received the B. Sc. and the M. A. degrees in Electrical engineering from Tabriz University, Tabriz, Iran, in 2010, and Tarbiat Modares University, Tehran, Iran, in 2012, respectively. He is currently working on his PhD project. His research interests include photonic devices such as photodetectors, lasers, and SOAs.

Mohammad Kazem Moravej-Farshi (2005 SM) received the B. Sc. and the M. A. degrees in physics from Sharif University of Technology (SUT), Tehran, Iran, in 1976, and the University of Southern California (USC), Los Angeles, California, in 1978, respectively, the M. Sc. and the Ph. D degrees in
electronics from the University of California at Santa Barbara (UCSB), in 1980, and the University of New South Wales (UNSW), Sydney, Australia, in 1987, respectively. From 1980 to 1984, he was a member of research staff with the Division of Microwave, Iran Telecommunication Research Center (ITRC). He joined Tarbiat Modares University (TMU) in 1987, where he is currently a Professor of Electronics and head of Nano Plasmo-Photonic Research Group. Professor Moravvej Farshi is currently a senior member of IEEE and a senior member of the Optical Society of America (OSA). He is also one of the founders of the Optics and Photonics Society of Iran.

Yi Yu received the B.Sc. and M.Eng. degrees from Huazhong University of Science and Technology, China, in 2008 and 2011, respectively, and his Ph.D. degree from Technical University of Denmark (DTU), Denmark, in 2015. Since then he worked as a Postdoc at DTU Fotonik and now at the Institute of Physics in École Polytechnique Fédérale de Lausanne, Switzerland. His current research focuses on nano-cavity lasers and quantum nanophotonic structures.

Jesper Mørk received the M.Sc., Ph.D., and Dr. Techn. degrees from the Technical University of Denmark (DTU), Lyngby, in 1986, 1988, and 2003, respectively. Since 2002 he is a Professor in semiconductor photonics and since 2008 he is heading the Nanophotonics Section at DTU Fotonik, Technical University of Denmark. He is the author of more than 245 papers in refereed journals and around 350 contributions to international conferences, including more than 80 invited talks. His current research interests include semiconductor quantum photonics, photonic crystal structures, slow light, nano- and micro-cavity lasers and integrated photonics. Jesper Mørk is a Fellow of OSA and serves as Associate Editor of Optica.