Output Variability Caused by Random Seeds in a Multi-Agent Transport Simulation Model

7th International Workshop on Agent-based Mobility, Traffic and Transportation Models, Methodologies and Applications

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Random Seeds

- Normally random seeds do not receive much attention.

- But should they?
Motivation

- Transport model outputs are fundamental for policy support.
- Their results need to be trustworthy.
  - Do random numbers hinder this?
Outline

1. Literature Review
2. Methodology
3. Case Study & Results
4. Conclusions & Future Work
1 Literature Review
2 Methodology
3 Case Study & Results
4 Conclusions & Future Work
Literature Review

Selected Studies

**Nagel, 1997**
- Two seeds.
- TRANSims.
- Solely observes difference.

**Castiglione et al., 2003**
- 100 seeds.
- San Francisco model.
- Use partial means, and thus incomparable.

**Cools et al., 2011**
- 200 seeds.
- FEATHERS.
- Trip level: $C.V.'s \lesssim 1\%$.

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† Castiglione, Joe, Freedman, Joel, and Bradley, Mark (2003). “Systematic Investigation of Variability due to Random Simulation Error in an Activity-Based Microsimulation Forecasting Model”. In: Transportation Research Record: Journal of the Transportation Research Board 1831, pp. 76–88

More or less consensus in the literature that the Monte Carlo variability is a non-issue for link loads.

- Aditionally backed up by Veldhuisen et al., 2000*, Lawe et al., 2009† and Ziems et al., 2011‡.

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† Lawe, Stephen, Lobb, John, Sadek, Adel, Huang, Shan, and Xie, Chi (2009). “TRANSIMS Implementation in Chittenden County, Vermont”. In: Transportation Research Record: Journal of the Transportation Research Board 2132, pp. 113–121.

A Very Similar Study...

Horni et al., 2011 *

- Discovered after final submission deadline.
  - Thus not referenced in the proceedings paper.
- Actual study on MATSim.
- 30 random seeds.
- Link load average $C.V. \approx 4\%$.

What’s Missing?

Contributions

- Compare between-seed variation to within-seed variation.
- Analyse each seed separately.
- Larger dataset for link load analysis (network and number of seeds).
Outline

1. Literature Review
2. Methodology
   - MATSim
   - Experiment Design
3. Case Study & Results
4. Conclusions & Future Work
Methodology

The Multi-Agent Transport Simulation MATSim

MATSim
- Agent-based.
- Activity-based.
- Large-scale applicable.
- Scenarios around the world.
- Open source.

How Are Random Numbers Used in MATSim?

Used to Determine *which*...

- ... in-going links of a node to be handled first.
- ... agents use *which*...
  - ... plan mutation strategy.
  - ... plan selection strategy.
- ... plan an agent choose (from choice set).

For choosing between reasonable alternatives.
- MATSim ought note to be particularly prone to high output variability.
100 runs (100 different random seeds).
100 iterations in each run.
A total of $100 \times 100 \times |L|$ link loads produced.
$x_{i}^{s,i}$ is daily link load on link $l$ in iteration $i$ with seed $s$.

<table>
<thead>
<tr>
<th>Seed 1</th>
<th>lt. 1</th>
<th>lt. 2</th>
<th>\cdots</th>
<th>lt. 98</th>
<th>lt. 99</th>
<th>lt. 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{1}^{1,1}$</td>
<td>$x_{1}^{1,2}$</td>
<td>\cdots</td>
<td>$x_{1}^{1,98}$</td>
<td>$x_{1}^{1,99}$</td>
<td>$x_{1}^{1,100}$</td>
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<tr>
<td>$x_{1}^{2,1}$</td>
<td>$x_{1}^{2,2}$</td>
<td>\cdots</td>
<td>$x_{1}^{2,98}$</td>
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<td>\vdots</td>
</tr>
<tr>
<td>Seed 99</td>
<td>$x_{l}^{99,1}$</td>
<td>$x_{l}^{99,2}$</td>
<td>\cdots</td>
<td>$x_{l}^{99,98}$</td>
<td>$x_{l}^{99,99}$</td>
<td>$x_{l}^{99,100}$</td>
</tr>
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Experiment Design

- 100 runs (100 different random seeds).
- 100 iterations in each run.
- A total of $100 \times 100 \times |L|$ link loads produced.
- $x_{l,s,i}^{s,i}$ is daily link load on link $l$ in iteration $i$ with seed $s$.

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<th>...</th>
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<th>It. 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{l,1}^{1,1}$</td>
<td>$x_{l,2}^{1,2}$</td>
<td>...</td>
<td>$x_{l,98}^{1,98}$</td>
<td>$x_{l,99}^{1,99}$</td>
<td>$x_{l,100}^{1,100}$</td>
<td></td>
</tr>
<tr>
<td>Seed 2</td>
<td>$x_{l,2,1}^{2,1}$</td>
<td>$x_{l,2,2}^{2,2}$</td>
<td>...</td>
<td>$x_{l,98}^{2,98}$</td>
<td>$x_{l,99}^{2,99}$</td>
<td>$x_{l,100}^{2,100}$</td>
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<tr>
<td>...</td>
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<td>...</td>
</tr>
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<td>Seed 99</td>
<td>$x_{l,99,1}^{99,1}$</td>
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<td>...</td>
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</tr>
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</tr>
</tbody>
</table>

- We (primarily) use those from the last iteration.
Our initial analysis measure is the coefficient of variation,

\[ c_{vl}^{\chi_l} = \frac{\text{Standard Deviation}}{\text{Mean}} = \sqrt{\frac{1}{|S| - 1} \sum_{s \in S} \left( x_{l,s,100}^{s} - \bar{x}_l \right)^2} \]

\[ = \frac{\sum_{s \in S} x_{l,s,100}}{|S|}, \quad l \in L. \]

With \( \bar{x}_l \) being the mean link load of link \( l \in L \),

\[ \bar{x}_l = \frac{\sum_{s \in S} x_{l,s,100}}{|S|}, \quad l \in L. \]
Measures II – Proportions

- We introduce additional measures:
  - Proportion of links that deviate more than $q \cdot 100\%$ from their mean when using seed $s \in S$,
    \[
    r^s_q = \frac{\sum_{l \in L} \left[ \left| \frac{x^{s,100}_l - \bar{x}_l}{\bar{x}_l} \right| > q \right]}{|L|}, \quad s \in S, \; q > 0.
    \]
  - Proportion of seeds in which the link load of link $l \in L$ deviates more than $q \cdot 100\%$ from the mean,
    \[
    r^l_q = \frac{\sum_{s \in S} \left[ \left| \frac{x^{s,100}_l - \bar{x}_l}{\bar{x}_l} \right| > q \right]}{|S|}, \quad l \in L, \; q \geq 0.
    \]
Finally, we introduce two auxiliary measures:

- Between-seed variation:
  \[
  B_l = \frac{1}{|S| - 1} \sum_{s \in S} \left( x_{l,100}^s - \bar{x}_l \right)^2 , \quad l \in L.
  \]

- Within-seed variation:
  \[
  W_l = \frac{1}{|S|} \sum_{s \in S} \left( x_{l,99}^s - x_{l,100}^s \right)^2 , \quad l \in L.
  \]
### Measures IIIb – Between-Seed vs Within-Seed Variation

#### Between-seed mean

<table>
<thead>
<tr>
<th></th>
<th>lt. 1</th>
<th>⋯</th>
<th>lt. 99</th>
<th>lt. 100</th>
<th>Between-seed mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seed 1</td>
<td>$x_{i}^{1,1}$</td>
<td>⋯</td>
<td>$x_{i}^{1,99}$</td>
<td>($x_{i}^{1,100} - \bar{x}_i$) $^2$</td>
<td></td>
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</table>

#### Within-seed mean

<table>
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<td>($x_{i}^{100,99} - x_{i}^{100,100}$) $^2$</td>
<td></td>
</tr>
</tbody>
</table>
These allow calculating the importance of the between-seed variation,

\[ \tilde{R}_l = \sqrt{\frac{B_l + W_l}{W_l}}, \quad l \in L. \]

- \( \tilde{R}_l \rightarrow 1 \) \( \Rightarrow \) Not important.
- \( \tilde{R}_l = \sqrt{2} \) \( \Rightarrow \) Equally important.
- \( \tilde{R}_l \gg \sqrt{2} \) \( \Rightarrow \) Strongly dominates the within seed-variation.

Thus, \( \tilde{R}_l \) can indicate whether it is better to...

- ... run more iterations (low \( \tilde{R}_l \)) or
- ... run more seeds (high \( \tilde{R}_l \)).
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Case Study

Santiago de Chile Open Data Scenario v2b*

- 10% Population Sample: 665,201 agents.
- 22,981 link network.
- Schedule based public transport.

Default “Out-of-the-Box” Configurations

- Mode choice: Walk, public transport, car.
- 100 iterations.
- After 80 iterations:
  - Choice sets are locked.
  - Plan level MSA initialised.

Coefficients of Variation: All links.

- Generally not high – somewhat in accordance with earlier studies.
  - 2/3 of links have a coefficient of variation $\leq 5\%$.
  - $\sim 15\%$ of links have a coefficient of variation $> 10\%$.

\[
c_{V}^{x_l} = \sqrt{\frac{1}{|S|-1} \sum_{s \in S} \left( x_{l,s,100} - \bar{x}_l \right)^2} , \quad l \in L.
\]
The 50 busiest links seem more stable.

- Coefficient of variation \( \approx 1\% \).
Are Some Seeds Worse than Others?

- All seeds perform more or less equally well.
  - ... i.e. 42 will do just fine.

\[
 r^s_q = \frac{\sum_{l \in L} \left( \frac{|x_l^{s,100} - \bar{x}_l|}{\bar{x}_l} > q \right)}{|L|}, \quad s \in S, \ q > 0.
\]

<table>
<thead>
<tr>
<th>q</th>
<th>1%</th>
<th>2.5%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>25%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>66.28%</td>
<td>42.16%</td>
<td>23.08%</td>
<td>10.19%</td>
<td>6.02%</td>
<td>3.18%</td>
<td>1.49%</td>
</tr>
<tr>
<td>SD</td>
<td>0.65%</td>
<td>0.70%</td>
<td>0.52%</td>
<td>0.33%</td>
<td>0.21%</td>
<td>0.13%</td>
<td>0.06%</td>
</tr>
</tbody>
</table>
How Often Does It Go Wrong? I

- ~20% of links have a relative error of \( \geq 5\% \) for \( \geq 50\% \) of the seeds.
- ~10% of links have a relative error of \( \geq 15\% \) for \( \geq 20\% \) of the seeds.

\[
\begin{align*}
\text{CDF}(r_q) = \sum_{s \in S} \left[ \frac{\left|x_l^{s,100} - \bar{x}_l\right|}{\bar{x}_l} > q \right] / |S|, \quad l \in L, \quad q \geq 0.
\end{align*}
\]
How Often Does It Go Wrong? II

- Generally worst for the smallest links.
  - However, many busy links are quite volatile.

\[
q = 1%
\]
Practically all links have $\tilde{R}_I \geq 4$.

- Between seed-variation dominates within seed-variation.
- Despite only using 100 iterations.

$$\tilde{R}_I = \sqrt{\frac{B_I + W_I}{W_I}}, \quad I \in L.$$
Problem intensifies for busiest links.

\[
\tilde{R}_l = \sqrt{\frac{B_l + W_l}{W_l}}, \quad l \in L.
\]
Conclusions & Future Work

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   - Conclusions
   - Future Work
Conclusions & Future Work

Conclusions

Findings

- Coefficient of variation is generally low, and a decreasing function of link load.
- Large relative errors do happen – also for the busiest links.
- No bad seeds.
- Between-seed variation is much larger than within-seed variation.

Future Work
Conclusions & Future Work

Conclusions II

How to Deal with It?

- Use multiple seeds.
  - May be better to run $10 \times 100$ iterations than $1 \times 1,000$.

- Average out results across all seeds.
  - But point estimates may not correspond to an actual solution.

- Present results as a distribution.
  - As suggested in Chapter 48* of the MATSim book.
  - Contributes to deeper understanding of uncertainties.

Future Work

Spatial Analysis
- Current study ignores spatial interdependencies.
  - Visual overview.
  - Actual geostatistical analysis (e.g. Kriging).

Network Sensitivity
- Sensitivity to small changes in the network.
  - Does the output variability overshadow the effects of infrastructural changes?
  - Extremely relevant for scenario analysis.
Thank You for Your Attention

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Random Number Generation in MATSim?

**Linear Congruential Generator (LCG)**

Pseudo random numbers are constructed in MATSim using Java’s default LCG with minor modifications,

$$X_{n+1} = aX_n + c \mod m.$$  

- $X_n =$ latest draw.  
- $X_{n+1} =$ next draw.  
- $a = 25, 214, 903, 917.$  
- $c = 11.$  
- $m = 2^{48}.$

In order to get a uniformly distributed number $X_{n+1}$ is divided by $m.$