Integration of different CHP steam extraction modes in the stochastic unit commitment problem

Blanco, Ignacio; Song, Hyoung-Yong; Guericke, Daniela; Morales González, Juan Miguel; Park, Jong-Bae; Madsen, Henrik

Published in:
IEEE Transactions on Power Systems

Publication date:
2018

Document Version
Peer reviewed version

Citation (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.
Appendix to Integration of different CHP steam extraction modes in the stochastic unit commitment problem

I. INTRODUCTION TO THE IMPROVED HYBRID DECOMPOSITION

In this document we explain in detail how the suggested improvements for the scenario partition and decomposition method, variant 1 (SPDA1) proposed in [1] are carried out. The improvements consist in applying heuristics to find a suitable number of scenario partitions or clusters for the specific problem and find a partly fixed first stage-decision to initialize the problem solution. These heuristics are based in the Progressive Hedging algorithm and rounding techniques. The Progressive Hedging algorithm was first introduced by [2] and has been applied to solve large-scale stochastic programming problems in different applications such as forest planning [3], resource allocation problems [4] and unit commitments problems [5]. The Progressive Hedging is an iterative process in which first the problem is solved for each scenario individually and the solutions obtained for the first-stage decisions are averaged for all scenarios. From these solutions a multiplier is created and afterwards, the problem is solved again for each scenario including this multiplier as a penalty in the objective function. Using a squared proximal term to calculate the distance between the first stage decision vector of each scenario and the average term of these, we can determine if the algorithm should stop. Later on, we use these values to initialize the solution of the two-stage stochastic programming problem. In our solution approach, in order to determine a suitable number of partitions, we take from Progressive Hedging the way of averaging the first-stage decision, making use of the squared proximal term to stop the algorithm. Furthermore, to initialize the solution of the problem, we use the rounding technique proposed in [6] where just those values close to 1 and 0 are fixed to initialize the solution. These values depend on two thresholds that we will name \( \alpha \) and \( \beta \).

II. FORMULATING THE HYBRID UNIT COMMITMENT

In this section, the stochastic unit commitment (4a)-(4c) is reformulated to the hybrid unit commitment following the work done in [1]. The finite set of scenarios \( \Omega \) is divided into \(|P|\) different partitions. Therefore, the entire set of scenarios \( \Omega \) is divided into different subsets named \( \Omega_p \), which is comprised of all the scenarios \( \omega \in \Omega \) that belong to partition \( p \in P \). The hybrid unit commitment writes as follows.

\[
\min_{x^i, y^i, \gamma_p} \sum_{t \in T} \sum_{g \in G} \left( a_g y^i_{g,t} + C^{SU}_g y^i_{g,t} + C^{SD}_g z^i_{g,t} \right) + \sum_{p \in P} \rho_p \gamma_p
\]

s.t. \( \gamma_p \geq \sum_{t \in T} \sum_{g \in G} b_g p^i_{g,t,\omega} + \sum_{t \in T} \sum_{m \in M} C^L_{s,\omega} \) (6a)

\[
+ \sum_{t \in T} \sum_{g \in G} (\sum_{m \in M} \omega) g,m \cdot y^i_{g,m,t,\omega} + \sum_{t \in T} \sum_{m \in M} \omega) g,m \cdot y^i_{g,m,t,\omega} + \sum_{t \in T} \sum_{g \in G} (\sum_{m \in M} \omega) g,m \cdot y^i_{g,m,t,\omega} + \sum_{t \in T} \sum_{m \in M} \omega) g,m \cdot y^i_{g,m,t,\omega}
\]

\((\forall \omega \in \Omega_p)\) (1a) − (1n), (2a) − (2v) \((\forall \omega \in \Omega_p)\) (7a)

The auxiliary variable \( \gamma_p \) equals the worst-case system cost for partition \( p \) and therefore the second term in the objective function (6a) represents the expected value of the worst-case scenarios at each partition \( p \in P \). To formulate the decomposition algorithm, we need to distinguish between the master problem and the subproblems. Both are formulated as in [1]. The master problem (MP) is formed by both first-stage and second-stage decisions. It solves one per scenario \( p \in P \) and for iteration \( i \) it writes as follows.

\[
\min_{x^i, y^i, \gamma_p} \sum_{t \in T} \sum_{g \in G} \left( a_g y^i_{g,t} + C^{SU}_g y^i_{g,t} + C^{SD}_g z^i_{g,t} \right) + \sum_{p \in P} \rho_p \gamma_p
\]

s.t. \( \gamma_p \geq \sum_{t \in T} \sum_{g \in G} b_g p^i_{g,t,\omega} + \sum_{t \in T} \sum_{m \in M} C^L_{s,\omega} \) (7b)

\[
+ \sum_{t \in T} \sum_{g \in G} (\sum_{m \in M} \omega) g,m \cdot y^i_{g,m,t,\omega} + \sum_{t \in T} \sum_{m \in M} \omega) g,m \cdot y^i_{g,m,t,\omega} + \sum_{t \in T} \sum_{m \in M} \omega) g,m \cdot y^i_{g,m,t,\omega} + \sum_{t \in T} \sum_{m \in M} \omega) g,m \cdot y^i_{g,m,t,\omega}
\]

\((\forall \omega \in \Omega_p)\) (1a) − (1n), (2a) − (2v) \((\forall \omega \in \Omega_p)\) (7b)

Where \( X^i = \{ x^i_{g,t}, y^i_{g,t}, z^i_{g,t} \} \) and \( Y^i = \{ u^i_{g,m,t,\omega}, v^i_{g,m,t,\omega} \} \).
solved determining the second-stage decision variables.

\[
\begin{align*}
\min_{\mathcal{Y}^D} & \sum_{t \in T} \sum_{g \in \mathcal{G}} b_g p_{g,t} + \sum_{t \in T} \sum_{n \in \mathcal{N}} C^k L_{m,t}^{\text{shed},i} + \sum_{t \in T} \sum_{g \in \mathcal{G}} a_{g,m} x_{g,m,t} + \sum_{t \in T} \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} \sum_{I} \sum_{\omega} b_C^{g,m} (p_{g,m,t} + \varphi_{g,m,t}^{C^{ip},i} + \varphi_{g,m,t}^{C^{ip},i}) \\
\text{s.t.} \ (1f) - (1n), (2a) - (2v).
\end{align*}
\]  

(8a) where \( \mathcal{Y}^D = \{ p_{g,m,t} \} \) for each partition \( p \) that the master problems (7a)-(7c) and subproblems (8a)-(8b) for each partition \( p \in P \) are solved in parallel and that they are called instances of the SPDA1 algorithm.

1) Initialize iteration \( j = 0 \). Select the initial number of partitions \( k^0 \) applying hierarchical clustering to the set of scenarios \( \Omega^0 \).
2) Create \( k^0 \) parallel instances of the SPDA1 algorithm.
3) Initialize iteration \( i = 0 \) and set \( \Omega^i_p = 0 \).
4) Solve the master problem and return the optimal solution found for the vector of first stage decisions \( \mathcal{X}^i_p \). Obtain the Lower Bound (LB) as \( \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} \sum_{t} (a_{g,m,t} + C_{g,m,t}^{SU} g_{m,t} + C_{g,m,t}^{SD} g_{m,t} + \gamma_{p}) \).
5) Solve the subproblems (SP) with the first-stage decision variables fixed at \( \mathcal{X}^i_p \). Once all the subproblems are solved, obtain \( \mathcal{X}^i_p \) that yields the lowest system cost. Include this scenario in the reduced set of worst-case scenarios \( \Omega^i_{p_{\text{rd}}} \) such that \( \Omega^i_{p_{\text{rd}}} = \Omega^i_p \cup \{ \mathcal{X}^i_p \} \) and obtain the Upper Bound (UB) as \( \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} \sum_{t} (a_{g,m,t} + C_{g,m,t}^{SU} g_{m,t} + C_{g,m,t}^{SD} g_{m,t} + \gamma_{p}) \).
6) Check convergence. If \( |\text{UB} - \text{LB}| \leq \xi \), where \( \xi \) is the tolerance value, the iterative process \( i \) stops. If \( |\text{UB} - \text{LB}| > \xi \) then \( i := i + 1 \) and go to step 4.
7) Once all partitions have converged, we obtain the first-stage decision vector for each partition \( \mathcal{X}^i_p \).
8) Increase iteration number \( j := j + 1 \). Calculate the average value for the first-stage commitment decisions over all partitions \( \mathcal{X}^j_p = \frac{\sum_{p \in P} p \mathcal{X}^j_p}{\mathcal{X}^j_{p_{\text{rd}}}} \). Obtain squared distance \( \sigma^j = || \mathcal{X}^j - \mathcal{X}^j_{p_{\text{rd}}} ||^2 \) (where \( \mathcal{X}^j = 0 \)). If \( \sigma^j \leq \varepsilon \) we stop the iteration process for \( j \) and move a step forward. If \( \sigma^j > \varepsilon \), we increase the number of partitions \( k^j := k^{j-1} + 1 \) and go step 2.
9) Obtain the partly fixed commitment decisions using the rounding technique:

\[
\mathcal{X}_{\text{round}} = \begin{cases} 
1 & \text{if } \mathcal{X}^j \geq 1 - \alpha \\
0 & \text{if } \mathcal{X}^j \leq \beta \\
\{0, 1\} & \text{if } \beta < \mathcal{X}^j < 1 - \alpha 
\end{cases}
\]

10) Solve (6a)-(6c) for the scenarios finally retained in the set of worst-cases scenarios \( \Omega^j_p \) using \( \mathcal{X}_{\text{round}} \) as partly fixed commitment decisions.

### REFERENCES


