Effective stresses and shear failure pressure from in situ Biot's coefficient, Hejre Field, North Sea

Stresses and shear failure pressure

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Effective stresses and shear failure pressure from in-situ Biot’s coefficient, Hejre Field, North Sea.


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Summary

We propose a combination of Biot’s equations for effective stress and the expression for shear failure in a rock to obtain an expression for minimum pore pressure in a stable vertical well bore. We show that a Biot’s coefficient calculated from logging data in the Hejre field, North Sea, is significantly different from 1. The log derived Biot’s coefficient is above 0.8 in the Shetland chalk Group and in the Tyne Group, and 0.6 to 0.8 in the Heno sandstone Formation. We show that the effective vertical and horizontal stresses obtained using the log derived Biot’s coefficient result in a drilling window for a vertical well larger than if approximating Biot’s coefficient by 1. The estimation of the Biot’s coefficient is straightforward in formations with a stiff frame, whereas in formations such as shales caution has to be taken. We discuss the consequence of assumptions made on the mineral composition of shales, as unphysical results could be obtained when choosing inappropriate mineral moduli.
Keywords

Borehole geophysics, Petrophysics, Rock physics.
Introduction

The Hejre field is situated in the Danish sector of the North Sea. Two wells were drilled in the field, one exploration well (Hejre-1) in 2001 and one appraisal well (Hejre-2) in 2004/05. The objective of both wells was to reach and investigate the sandstone reservoir in the Heno Formation. This is a deep (more than 5400 m) HPHT reservoir with formation pressure above 100 MPa and temperature above 160°C.

An accurate determination of the pre-production in-situ effective stresses is essential to better understand possible deformations, which might compromise borehole integrity during drilling and production (Zoback, 2007; Fjaer et al., 2008). The effective stresses are used to calculate the shear failure pressure (SFP) at the borehole wall (Yew and Liu, 1992; Zoback, 2007; Fjaer et al., 2008). SFP is one of the most important parameters when analyzing wellbore stability (Lang et al., 2011). SFP is a function of the effective principal stresses acting on the borehole wall, the rock properties and, in case of a deviated well, the well geometry (deviation from the vertical and, in case of horizontal stress anisotropy, azimuth with respect to the maximum horizontal stress). In the cases where SFP is larger than the pore pressure, SFP is used to determine the optimal well pressure of the drilling fluid (Yew and Liu, 1992; Fjaer et al., 2008), which mitigates the risk of borehole instabilities. The lower limit of the pressure exercised by the drilling fluid must be above SFP. The upper pressure limit is determined by the formations tensile strength or its ability to withstand the hydraulic fracturing. The onset of a fracture is noticed by a loss in circulated mud and the pressure level is called the leak off pressure (LOP). The lower limit (SFP) and the upper limit (LOP) define the “drilling window” or “drilling margins” which constitute the stable pressure interval when drilling.
The purpose of this study is to investigate the lower limit of the drilling window, the shear failure pressure from the data obtained from the drilling campaign of the two Hejre wells. As seen from the final well reports, the drilling window was estimated from experience and empirical relations (ConocoPhillips AS 2005) and no attempt was made to estimate shear failure pressure. The result was a narrow drilling window.
Background Theory

Effective stresses and principal stresses at the wall of a vertical well

The effective stresses in horizontal (h) and vertical (v) directions for an isotropic medium at a given depth were defined by (Biot 1941):

\[ \sigma_{h,\text{eff}} = \sigma_h \frac{(1-v)}{(1-2v)} - \sigma_v \frac{v}{(1-2v)} - \alpha P_p \]
\[ \sigma_{v,\text{eff}} = \sigma_v \frac{1}{(1-2v)} - \sigma_h \frac{2v}{(1-2v)} - \alpha P_p \]

where \( v \) is the drained Poisson’s ratio and \( \alpha \) is Biot’s coefficient. By introducing a total horizontal and vertical stress at given depth as:

\[ \sigma_{h,\text{TOT}} = \sigma_h \frac{(1-v)}{(1-2v)} - \sigma_v \frac{v}{(1-2v)} \]
\[ \sigma_{v,\text{TOT}} = \sigma_v \frac{1}{(1-2v)} - \sigma_h \frac{2v}{(1-2v)} \]

Equation (1) can be generalized to the familiar expression:

\[ \sigma_i,\text{eff} = \sigma_{i,\text{TOT}} - \alpha P_p, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1a) \]

Please note that both \( \sigma_h \) and \( \sigma_v \) (the overburden) are contributing to the estimate of each of the total stresses.

On the wall of a vertical wellbore, we define the axial stress, \( \sigma_a \), and split the horizontal stress field into a radial, \( \sigma_r \), and tangential, \( \sigma_t \), contribution (Fjær et al. 2008):

\[ \sigma_a = \sigma_v \quad \sigma_r = P_w \quad \sigma_t = 2 \sigma_h - P_w, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2) \]
where $P_w$ is the pressure inside the well.

**Estimating rock strength**

The set of Mohr-Coulomb failure parameters that define the strength of a rock, the failure angle $\beta$, and the uniaxial compressive strength, $C_o$, are normally obtained by compression tests in the laboratory. Here defined as (Fjaer et al. 2008):

$$2\beta = \frac{\pi}{2} + \phi; \quad C_o = 2S_c \tan \beta \quad \text{.................................} \quad (3)$$

In the absence of laboratory results, the angle of internal friction $\phi$, and the cohesion, $S_c$ in MPa, can be estimated from the compressive sonic log using Lal’s empirical correlations (Lal 1999):

$$\sin \phi = \frac{V_p - 1}{V_p + 1}; \quad S_c = \frac{5(V_p - 1)}{\sqrt{V_p}} \quad \text{.................................} \quad (4)$$

where $V_p$ is the elastic p-wave velocity, in km/s with units removed.

**Estimating Shear failure pressure in a vertical well**

The Mohr-Coloumb failure criterion is used to estimate the critical pressure at which, if exceeded, shear failure occurs:

$$\sigma_{1,\text{eff}} = C_o + \sigma_{3,\text{eff}} \cdot \tan^2(\beta) \quad \text{.................................} \quad (5)$$

where $\sigma_{1,\text{eff}}$ and $\sigma_{3,\text{eff}}$ are the effective maximum and minimum principal stresses,

The maximum and minimum among the principal stresses at the borehole wall $\sigma_r$, $\sigma_\theta$ and $\sigma_o$ are found as:
\[ \sigma_1 = \max \{ \sigma_r, \sigma_t, \sigma_a \} ; \sigma_3 = \min \{ \sigma_r, \sigma_t, \sigma_a \}, \] .............................. (6)

Where \( \sigma_r, \sigma_t \) and \( \sigma_a \) are the radial, tangential and vertical stress and the respective effective principal stresses are approximated as (Fjær et al. 2009):

\[ \sigma_{eff} = \sigma_i - \alpha P_p \] ................................. (7)

By finding the principal stresses by equation (2), the rock strength from equation (4) and determining the minimum and maximum effective stresses by equations (6) and (7), we can solve equation (5) in order to find the minimum well pressure \( P_w \) needed to avoid shear failure collapse of the borehole. This minimum well pressure is the Shear Failure Pressure, \( SFP \). For example, if \( \sigma_i = \sigma_r \) and \( \sigma_3 = \sigma_r \) then from (2), (3), (5), (6) and (7):

\[ P_w \equiv SFP = \frac{2 \sigma_r - c_o - \alpha P_p \left(1 - \tan^2(\beta)\right)}{1 + \tan^2(\beta)} , \] .......................... (8)

**Method for calculating Biot’s coefficient and elastic moduli from log data**

Biot’s coefficient can be calculated from bulk moduli (Biot 1941, Zimmermann 1991) as

\[ \alpha = 1 - \frac{K_{dry}}{K_{min}} , \] ................................. (9)

where \( K_{dry} \) is the bulk modulus of the dry rock frame and \( K_{min} \) is the bulk modulus of the mineral composing the rock frame. The saturated bulk modulus \( K_{sat} \) can be calculated from sonic log P-wave and shear velocities (\( V_p \) respectively \( V_S \)) and the bulk density as:
\[ K_{sat} = \rho_b \left( V_p^2 - \frac{4}{3} V_s^2 \right), \]  

and the dry rock frame modulus is calculated from the saturated modulus by the Gassmann’s fluid substitution (Gassmann 1951) as given in (Mavko et al. 2009):

\[ K_{dry} = \frac{K_{sat} \left( \frac{\varphi \cdot K_{min}}{K_{fl}} + 1 + \varphi \right) - K_{min}}{\frac{\varphi \cdot K_{min}}{K_{fl}} + \frac{K_{sat}}{K_{min}} + 1 + \varphi}, \]  

where \( K_{fl} \) is the modulus of the pore fluid and \( \varphi \) is porosity. This method only works where shear velocity data are available. In the case where only \( V_p \) is available, an estimate of \( \alpha \) is possible because Biot’s coefficient can be estimated from the P-wave modulus of the rock frame, \( M_{dry} \) and P-wave modulus of the mineral, \( M_{min} \) (Fabricius 2014):

\[ \alpha \approx 1 - \frac{M_{dry}}{M_{min}}, \]  

and the P-wave modulus of the rock frame can be derived by an approximation of the Gassmann’s fluid substitution as suggested by (Mavko et al. 2009):

\[ M_{dry} \approx \frac{M_{sat} \left( \frac{\varphi \cdot M_{min}}{M_{fl}} + 1 + \varphi \right) - M_{min}}{\frac{\varphi \cdot M_{min}}{M_{fl}} + \frac{M_{sat}}{M_{min}} + 1 + \varphi}, \]  

The P-wave modulus and the bulk modulus are related through:

\[ K_{min} = M_{min} - \frac{4}{3} G_{min}, \quad K_{sat} = M_{sat} - \frac{4}{3} G \quad \text{and} \quad K_{dry} = M_{dry} - \frac{4}{3} G, \]  

…… (14)
where $G$ is the shear modulus, given as $G = \rho_b \cdot V_s^2$. Fluids have no shear component so the bulk modulus, $K_{fl}$ is equal to the P-wave modulus, $M_{fl}$:

$$K_{fl} = M_{fl}, \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\text{(15)}$$

The parameters to be estimated or determined are: $M_{sat}$ and where available $G$, the mineral moduli: $M_{min}$ and $G_{min}$, $K_{fl}$ and $\varphi$.

**Case study: Minimum well pressure at the Hejre field**

**Data**

We use well reports and logging data (kindly provided by DONG Energy) from the two wells Hejre-1 and Hejre-2. Both wells lack relevant log data in the upper section. In Hejre-2, the relevant logs (sonic, resistivity, density) were acquired only below 2500 m, while the corresponding Hejre-1 logs were acquired only below 4700 m. An overview of the log data from Hejre-2 is given in Figure 1.

Relevant formation pressure data on kicks and inflows from nearby wells as well as measurements of the reservoir pressure in Hejre-1 and Hejre-2 are summarized in Table 1. The data from the Norwegian wells were found at the Norwegian Petroleum Directorate website (npd.no/factpages). The data for the Danish wells (Mona-6, Hejre-1, and Hejre-2) were taken from the respective well reports (Chevron Petroleum Company of Denmark 1983, DONG Efterforskning og Produktion 2001, ConocoPhillips AS 2005).

The North Sea is an area of extensional stress regime, and the vertical stress ($\sigma_v$) is the maximum regional stress (Bartholomew et al. 1993). We assume an isotropic horizontal stress in the Hejre field...
(σ_h = σ_H < σ_v), and that the regional vertical and horizontal stresses are also principal stresses. We assume as well, that the sediments at any given depth have isotropic rock properties.

Data from the leak-off tests (LOT) in Hejre-1 indicate leak-off pressures of 33.7 MPa and 78.5 MPa at depth 1791 m and 3752 m respectively (DONG Efterforskning og Produktion 2001). In Hejre-2 at a depth of around 5420 m, dynamic losses were experienced at a well pressure of approximately 113 MPa (ConocoPhillips AS 2005).

 Calculation of Vertical/overburden stress

The vertical stress, σ_v (due to the weight of the sediments) can be estimated from the bulk density as:

\[ \sigma_v = g \sum \rho_b(z_i) (z_i - z_{i-1}) \]

where \( z_i \) is the depth of the i’th layer, \( \rho_b(z_i) \) is the corresponding bulk density derived from the density log, and \( g \) is the Earth’s acceleration of gravity (9.81 m/s^2). In the section from sea bed to 2500 meter, where log data are missing, we used a mean bulk density of 2.08 g/cm^3 (Japsen 1998). The density of the sea water was assumed to be 1.03 g/cm^3.

Estimation of Horizontal stress

In-situ horizontal stress and fracture pressure in an oil field are usually determined by hydraulic fracturing tests (the so called extended Leak-Off Tests, xLOT). In these tests, the pressure inside the wellbore is increased by pumping fluid in the well until the borehole wall breaks and part of the
pumped fluid is lost to the formation. The “breaking” point determines the so called Leak-Off Pressure, \( LOP \), i.e. the pressure at which a fracture is created. This pressure is often referred to as the fracture pressure and it depends also on the lithology (i.e. the rock strength). The pressure, at which the fracture is closing, is usually referred as the minimum horizontal stress (Brudy et al. 1997). In intact rock formations (i.e. no fractures or cracks) the fracture pressure is larger than the minimum horizontal stress (which is in fact a confining stress). Here, it is assumed that the horizontal stress, unlike the fracture pressure, is independent of lithology and rock type.

An upper limit for the horizontal stress \( (\sigma_{h,U}) \) was determined using the \( LOP \) values obtained from the two leak-off tests performed in the Hejre-1 well (DONG Efterforskning og Produktion 2001). The two data points define a line, which was extended from sea bed to \( TD \) (Total Drilled depth), assuming a constant slope. Since no digital data (pressure versus time or pumped volume) from the \( LOT \) tests were available, it was not possible to verify the accuracy of the reported values.

A lower limit of the horizontal stress \( (\sigma_{h,L}) \) was defined from losses experienced while drilling Hejre-2, just below the Heno Reservoir. The data were linearly extrapolated from sea bed to \( TD \), assuming the same slope as the one used for the upper limit.

**Estimation of Pore pressure profile**

Compaction trend analysis of the sonic and the resistivity logs is one of the most used methods in the oil industry to evaluate the pore pressure, \( P_p \). This analysis is based on the assumption that deviations of these logs from a normal compaction trend line can be used to estimate the actual pore pressure by empirical relations, e.g. Eaton’s relation (Eaton 1967). The so calculated \( P_p \) must always be calibrated to actual \( P_p \) data and experience gained in other wells, such as kicks, inflow, reservoir pressure
measurements, wellbore instabilities etc.. In order to define a normal compaction trend line from well log data, we need to have the above mentioned logs from sea bed to TD.

The lack of relevant logs (resistivity, sonic, density) from sea bed to 2500 m depth in both Hejre-1 and Hejre-2 did not allow us to define a normal compaction trend line and to apply trend analysis in order to evaluate the pore pressure.

Instead we used data on kicks, inflows and reservoir pressure data from Hejre-1, Hejre-2, and a number of near-by wells (Table 1). The depths of the pressure data points in the near-by wells were “stretched” to the local structure at the Hejre-2 location, i.e., the measured overpressure and the relative distance from the closest stratigraphic boundaries were preserved.

An upper \( P_{p,upper} \) and a lower limit \( P_{p,lower} \) for the pore pressure were defined in order to account for the spread in the pressure data. In both cases, it is assumed that the pore pressure is hydrostatic from sea bed to approximately 800 m–900 m. Below that depth, \( P_{p,upper} \) is assumed to increase with a constant slope down to TD, while the slope of \( P_{p,lower} \) is changing approximately at the top of the Chalk (Shetland Group).

**Results: Regional stresses and pore pressure**

The calculated regional stresses (the vertical, \( \sigma_v \), and the horizontal, \( \sigma_h \)) and the pore pressure profile, \( P_p \) in the Hejre field are shown in Figure 2. The lower limit of the pore pressure, \( P_{p,L} \), consists of three sections with increasing gradient Figure 2: one from sea bed to approximately 800 m depth, where the pore pressure is considered hydrostatic; the second one – from 800 m to the top of the Shetland Group (appr. 3000 m) and the third one – from the top of the Shetland Group to 5500 m. The slope of the middle section is defined from the kicks/inflows Table 1 in the lower part of the Hordaland Group.
(appr. 2400-2500 m) and at the base of the Rogaland / top of the Shetland Group. The kicks/inflows observed at the base of Rogaland, the upper parts of Cromer Knoll (appr. 3800 - 3900 m) and the Tyne (appr. 4500 m) Groups and the reservoir pressures in the Heno Formation determined the trend line of the lower section of $P_{p,L}$. The upper limit of pore pressure, $P_{p,U}$ is defined as consisting of two sections, the upper one (from sea bed to 800 m) is defined as hydrostatic, while the lower one (from 800 m to 5000 m) is defined as a trend line which accounts for the kicks observed in the upper part of the Hordaland Group and at the base of the Tyne Group.

**Results: calculating Biot’s coefficient**

Resistivity logs from the highly porous section of the Tor formation (3250-3370 m) were used in the pore water salinity estimation. The deep resistivity log is free of mud invasion and we set the water saturation $S_w = 1$. We assume that the porosity from the neutron porosity log is valid in the clay free chalk section so by solving equation (12) with $\phi = 0.3, R_t = 0.4 \ \Omega m$ and $m = 2$ we find an apparent water resistivity of $R_{wa} = 0.0364 \ \Omega m$. For temperature $T = 80^\circ C$, the pore water salinity is then, $S = 90,000$ ppm. When calculating the pore water density from equation (A-6) with $T = 80^\circ C$, formation pressure $P_f = 45MPa$ and $S = 90,000$ ppm, we obtain a pore water density of $\rho_{br} = 1.05 \ g/cc$. We back calculate using equation (11) with $\rho_{min} = 2.71 \ g/cm^3$ for chalk, and a $\rho_b = 2.2 \ g/ \ cm^3$ and obtain a porosity of $\phi = 0.307$. The results are plotted in **Figure 3**.

The trend of increasing resistivity down the well bore in **Figure 1** suggests a possible change in the fluid composition. It cannot be explained by the change in cementation exponent or porosity. With
salinity being above 40,000 ppm, the excess conductivity of shale will be practically negligible (Clavier et al. 1984) and we dismiss the possibility of decreasing salinity with depth. Hence the only explanation for the increase in resistivity is the presence of hydrocarbons from depth 4100 m. We assume that the hydrocarbons constitute a light oil of $\rho_{\text{oil}} = 0.56\text{g/cm}^3$ as reported by (DONG Efterforskning og Produktion 2001). By continuous iteration we find that the water saturation varies between 0.4 and 0.8 down the wellbore. We use mineral density $\rho_{\text{min}} = 2.73\text{g/cm}^3$ for the Cromer Knoll Group and $\rho_{\text{min}} = 2.76\text{g/cm}^3$ for Tyne Group (Mbia et al. 2013). The brine density decreases with the increasing pressure and temperature, as given by equation (A-6). We find that an average fluid density of 0.85g/cm$^3$ is satisfying reasonably well both equation (16) and (A-2).

The effective bulk fluid modulus of mixed fluids is calculated with equation (15) and the equations given in Appendix. As mentioned above, the resistivity log suggests hydrocarbon presence below 4100 m. Thus, we assume the pore fluid to be brine (i.e. water saturation $S_w = 1$) down to 4100 m. Below that depth and down to the reservoir, $S_w$ decreases to 0.8. The brine fluid modulus is calculated with a salinity of 90,000 ppm (A-8), (A-9) and (A-10) and an oil of density 0.56 g/cm$^3$ or 121 API is used to estimate the oil fluid modulus (A-8) and (A-21). The fluid moduli profiles are shown in Figure 4.

Biot’s coefficient was then estimated for chalk, shale and sandstone using three different mineral moduli, $K_{\text{min}}, M_{\text{min}},$ and $M_{\text{min},E}$ (Table 3). We find that Biot’s coefficient calculated using the bulk modulus ($K_{\text{min}}$) is above 0.8 for the Shetland Group (chalk), above 0.8 for the Tyne Group (shales) and between 0.6 and 0.8 for the Heno sand Formation. We obtained no results for the Cromer Knoll Group due to lack of S-wave data. When Biot’s coefficient $\alpha$ is estimated by using the P-wave mineral modulus ($M_{\text{min}}$); results range between 0.7 and 0.9 in the Shetland and in the Tyne Group, between 0.4 and 0.8 in the Cromer Knoll Group and between 0.6 and 0.8 in the Heno sand Formation. We thus
found that by using $M_{\text{min}}$ instead of $K_{\text{min}}$, we obtained lower values for $\alpha$, but that both results follow the same trend. The mineral modulus $M_{\text{min-E}}$ differs from $M_{\text{min}}$ only for the shales in the Cromer Knoll Group, where it results in Biot’s coefficient being above 0.9. The estimated Biot’s coefficients are shown in Figure 4.

**Results: Effective stress and Shear Failure Pressure**

Having found the Biot’s coefficient $\alpha$, we estimated the effective principle stresses and then, the shear failure pressure, i.e. the minimum well pressure needed to avoid shear failure collapse of the borehole. The estimated Shear Failure Pressures ($SFP$) for different pore pressures, horizontal stress and Biot’s coefficients are shown in Figure 5. $SFP$ estimated with the minimum value for $\alpha$ (corresponding to $M_{\text{min}}$) are lower than the $SFP$ estimated with $\alpha = 1$ by approximately 5 MPa.

When the effective stresses are estimated with $\alpha = 1$, the $SFP$ values become above the lower limit of $P_p$, $P_{p,\text{lower}}$ (Figure 5). This means, that if the pore pressure is $P_{p,\text{lower}}$, then the safe drilling window will be limited from $SFP$ to $\sigma_h$. When the effective stresses are estimated with $\alpha$ derived from the logs (right track, Figure 5), the $SFP$ values are below $P_{p,\text{lower}}$ and the drilling window is larger and ranging from $P_{p,\text{lower}}$ to $\sigma_h$.

Figure 6 shows how the $SFP$ is changing when using Biot’s coefficient below 1 with respect to well inclination. Only the lower section is represented (from 5000 m to 5400 m). If Biot’s coefficient of $\alpha = 1$ is used in the calculation, the $SFP$ is well above $P_{p,\text{lower}}$ for well inclination of 50° (from a vertical plane), which restricts the drilling window severely. When $SFP$ is calculated with a Biot’s coefficient $< 1$, $SFP$ is below $P_{p,\text{lower}}$ for the deviated well, thus allowing for much larger drilling window, which in this case is limited by $P_{p,\text{lower}}$. 
Discussion

Biot’s coefficient $\alpha$ estimated from petrophysical logs in the Hejre field (below 2500 m) ranges on average, depending on the moduli used, from above 0.7 in the Shetland Group (chalk), above 0.6 (with occasional low peaks to 0.4) in the Cromer Knoll Group (shales), and from above 0.8 in the Tyne Group (shales) to between 0.4 and 0.6 in the Heno Formation (sandstone). An $\alpha < 1$ is expected for stiff saturated formations (e.g. Shah and Shroff 2003), thus our estimates are in agreement with expectations. Gommesen et al. (2007) found a similar Biot’s coefficient of 0.8 for the Tor formation in North Sea chalk. The theoretical lower limit of $\alpha$ is the porosity (Fjær et al. 2008), which (on average) is 0.15. Our estimates show values well above the theoretical lower limit.

The chalk sediments in the Shetland Group (3109 m–3773 m) and the sandstone in the Heno Formation (5360 m–5411 m) are stiff formations compared to the shales is in the Cromer Knoll and in the Tyne groups. The stiffer rock-frames makes the fluid substitution relatively straight forward as the mineral modulus for the load bearing mineral in chalk (calcite) and in sandstone (quartz) are relative well determined. The stiff rock frame is dominating the bulk stiffness so changes in fluid modulus of 0.5 GPa has a negligible impact on the resulting Biot’s coefficient.

By contrast, in the much softer shale parts of Cromer Knoll and Tyne groups, caution is needed when choosing the fluid and mineral moduli, because fluid and mineral moduli can be chosen so as to obtain unphysical dry rock frame moduli. The elastic properties of minerals in shale are much debated (Cagatay et al. 1994, Mavko et al. 2009), and the reported mineral moduli range from 16 GPa (Simms 2007) to over 200 GPa (Chen and Evans 2006).
The chosen fluid modulus will dictate the solution space of the mineral modulus where a high fluid modulus will reduce the possible size of the mineral modulus. We chose to compute a fluid modulus from the fluid equations from (Batzle and Wang 1992) and this approach gave us an increasing fluid modulus with depth. We found that the temperature and pressure input in equation (A-6) and (A-8) could be changed with ± 10°C or ± 10 MPa for any depth point without any noticeable change in the calculated Biot’s coefficients, meaning that for this paper, a precision estimation of these parameters is not needed. However, the estimation of the salinity causes great trouble due the large temperatures and the low apparent water resistivity. By following the Schlumberger salinity chart, small changes in the range of 0.02 Ωm to 0.015 Ωm will at 100°C or more lead to changes in salinity in the size of 100,000 ppm. Such increase in salinity would cause the calculated fluid modulus of brine to be outside the range studied by (Batzle and Wang 1992) and as a result the fluid substitution becomes inaccurate. The fluctuations in the apparent water saturation are a result of the difficulties of balancing the input parameters in Archie’s equation. With no fluid samples to correlate with, the result from the logs becomes highly dependent on the log interpreters choices. It should be kept in mind, that Gassmann fluid substitution using the P-wave modulus, equation (12) and (13) instead of the bulk modulus, equation (9) and (11) is an approximation, which gives lower values for α as compared to the one estimated with the bulk moduli. All these assumptions and limitations lead to considerable uncertainty in the estimated values of the Biot’s coefficient α.

Bearing these assumptions and shortcomings in mind, in this study we showed, that accounting for α < 1 leads to lower shear failure pressures (SFP) as compared to the ones estimated assuming α = 1. Here, by “shear failure pressure” we denote the minimum well pressure (caused by the drilling fluid inside the well) needed to avoid shear failure collapse of the borehole wall. The effect is even larger for
inclined wells. Thus, using these “in-situ” values for $\alpha$ instead of a default value of 1, probably accounts better for the in-situ conditions. A value of $\alpha$ below one indicates a solid frame (which is the part that can break) that is probably able to support larger stresses and, thus, is more resistant to shear failure, than a loose packing of grains (which would be indicated by $\alpha = 1$).

In several published studies, it is assumed that the most appropriate definition of effective stress to be used in failure criteria is Terzaghi’s: $\sigma_{\text{eff}} = \sigma - P_p$, i.e. postulating $\alpha = 1$. For example, when measuring stress at failure by performing a set of triaxial experiments on Tavel limestone at three different pore pressures (Bouteca and Gueguen 1999) concluded that the effective stress at failure can be expressed as $\sigma_{i,\text{eff}} = \sigma_i - P_p$. From the geotechnical literature (e.g. Shah and Shroff, 2003) it is well known that for soft, saturated soils $\alpha = 1$ (and the Terzaghi’s formula is valid), while for stiff saturated soils $\alpha < 1$. Here, the shales in the upper section (from sea bed to the top of the chalk) can be considered as soft saturated soils with $\alpha = 1$, while the shales below the Chalk (i.e. below 3600 m) are much stiffer and $\alpha < 1$ is expected. Thus, we recommend to use $\alpha = 1$ from sea bed to the Chalk and the “in-situ” $\alpha < 1$, estimated from the log data to calculate the effective principal stresses and the corresponding shear failure pressure used in the wellbore stability analysis.

A serious limitation in this study was the availability and the quality of the log data. The lack of sonic, resistivity, density etc. logs in the upper well sections (from sea bed to approximately 2500 m) did not allow a more detailed evaluation of the pore pressure, salinity, water saturation, the different moduli, rock properties etc. The lack of digital data (pressures versus time or pumped volume) from the leak-off tests did not allow proper evaluation of the quality and the type of the measured leak-off pressure. The poor knowledge of the elastic moduli of minerals in shale limits how precise Biot’s
coefficient is determined. All these limitations affected the evaluation of the pore pressure and the horizontal stress.

All above discussed assumptions and limitations affects also the estimate of the shear failure pressure. The $SFP$ values depend also on the failure criteria used (Mohr-Coloumb, Lade etc.). Thus, it is indeed very hard to quantify the uncertainties.

**Conclusion**

In this study, we have shown a way to estimate the Biot’s coefficient and moduli from log data in an HPHT field. We have shown as well, that using a Biot’s coefficient different from 1 might lead to a better estimate of the in-situ effective stresses, which on their turn are used in the analysis of borehole stability in terms of shear failure pressure.

In a case of a narrow drilling margins (pore pressure close to the fracture pressure or minimum horizontal stress), using an $\alpha = 1$ instead of, for example, an $\alpha = 0.8$ might lead to an estimated shear failure pressure larger than the pore pressure and thus restricting the drilling window even more. Using a Biot’s coefficient below one for deep formations (below the Chalk Group) in the North Sea, leads to more correct estimate of the effective stresses acting on the borehole wall. This, on its turn, leads to improved estimate of the drilling margins in deep HPHT prospects.

However, the quality and the availability of the input data were a considerable issue and put severe limitations to this study. No laboratory results were available at the time of this study and comparing or calibrating the results of this study with laboratory studies would be an interesting next step.
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References


Appendix A – Methodology with log data.

Porosity

The porosity, \( \phi \), is calculated from the density log:

\[
\phi = \frac{\rho_b - \rho_{\text{min}}}{\rho_f - \rho_{\text{min}}} , \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldo
\[ m = -0.7533 \cdot V_{sh} + 2 \quad \text{..................................} \quad (A-3) \]

where \( V_{sh} \) is the volume fraction of shale as estimated from the gamma ray log:

\[ V_{sh} = \frac{GR_{\text{log}} - GR_{\text{base}}}{GR_{sh} - GR_{\text{base}}} \quad \text{..................................} \quad (A-4) \]

and where \( GR_{sh} \) is the gamma log signal of pure (100%) shale (corresponding to the maximum GR value from the log), \( GR_{base} \) is the gamma ray signal of pure (100%) sand (i.e. the lowest GR value) and \( GR_{\text{log}} \) is the actual gamma ray log reading.

The fluid modulus of brine is dependent on pressure, temperature and salinity equation (A-9), and the modulus of oil is dependent on pressure, temperature and API degree equation (A-11). A temperature gradient of \( \Delta T = 40^\circ C/km \) for formation below 2500 m was established based on the temperature readings from the final well reports (DONG Efterforskning og Produktion. 2001, ConocoPhillips AS. 2005). The modulus of a fluid composed of two or more components is calculated on the basis of their fractional amounts Batzle and Wang (1992):

\[ K_{f,\text{mix}} = \left( \frac{S_w}{K_{br}} + \frac{1 - S_w}{K_{oil}} \right)^{-1} \quad \text{..................................} \quad (A-5) \]

where \( K_{br} \) is bulk modulus of pore water and \( K_{oil} \) is bulk modulus of oil.

**Fluid properties**

Fluid properties can be predicted as follows (Batzle and Wang 1992). The brine density, \( \rho_{br} \) in g/cm\(^3\), can be estimated as:
\[
\rho_{br} = \rho_w + S \cdot \left( 0.668 + 0.44S + 10^{-6} \cdot \left[ 330P - 2400P \cdot S + T \cdot \left[ 80 + 3T - 3000S + 13P + 47P \cdot S \right] \right] \right)
\]

where \( P \) is water pressure in Megapascal, \( T \) is temperature in celcius, and \( S \) is salinity in fractions of one (ppm x 10^{-6}).

The density of fresh water \( \rho_w \) is given as:

\[
\rho_w = 1 + 10^{-6} \left( -80T - 3.3T^2 + 0.00175T^2 + 489P - 2T + 0.013T^2 P - 1.3 \cdot 10^{-5} T^3 P - 0.333P^2 - 0.002T \cdot P^2 \right), \ldots \tag{A-7}
\]

The procedure for calculating the modulus of a fluid is similar to the one for a solid. However, the fluid does not have a shear component so the bulk modulus of a fluid equals the P-wave modulus:

\[
K_\beta = \rho_\beta \cdot V_\beta^2. \quad \ldots \tag{A-8}
\]

The P-wave velocity of a brine is found as:

\[
V_{\beta} = V_w + S \cdot \left( 1170 - 9.6T + 0.055T^2 - 8.5 \cdot 10^{-5} \cdot T^3 + 2.6P - 0.0029T \cdot P - 0.0476 \cdot P^2 \right) + S^{1.5} \left( 780 - 10P + 0.16P^2 \right) - 820S^2, \ldots \tag{A-9}
\]

where the velocity of fresh water \( V_w \) is found from:

\[
V_w = \sum_{i=0}^{4} \sum_{j=0}^{3} W_{ij} \cdot T^i \cdot P^j, \quad \ldots \tag{A-10}
\]

and where the computational constants is found in Table A-1:
The fluid modulus of oil is calculated from equation (A-8) where the fluid velocity is given as (Mavko et al. 2009):

\[
V_{oil} = 15450 \cdot (77.1 + API)^{-1/2} - 3.7T + 4.64P + 0.0115T \cdot P \cdot \left(0.36API^{1/2} - 1\right).
\]

(A-11)