Towards Model-Based Control Design for Flexible Rotors Supported by Active Tilting Pad Bearings - Theory & Experiment

\[(EIv'')'' = q - \rho A\ddot{v}\]
Towards Model-Based Control Design for Flexible Rotors
Supported by Active Tilting Pad Bearings - Theory & Experiment

Jorge Andrés González Salazar

Kongens Lyngby 2016
DøCAMM Special Report No: S211
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DCAMM Special Report No.: S211
ISSN 0903-1685, ISBN 978-87-7575-463-3
This thesis is submitted in partial fulfilment of the requirement for obtaining the Ph.D. degree in Construction, Production, Civil Engineering and Transport at the Technical University of Denmark (DTU). The project was partly funded by the Chilean national council of science and technology “Conicyt” and DTU Mechanical Engineering. The research was conducted in the Solid Mechanics Section of DTU Mechanical Engineering Department (MEK-FAM) from the 1st of April 2012 to the 14th of April 2016 under the supervision of professor Dr.-Ing. Ilmar Ferreira Santos.

The research project dealt with the design, development and implementation of model-based controllers for improving the dynamic performance of flexible rotors supported by actively-lubricated bearings (ALBs). The main focus is to step further into the development of “smart mechatronic bearings” applied to a rotating machine close to an industrial size. The thesis consists of a collection of papers that summarizes the main findings obtained during the course of the research activities, to wit: exploring model-free controllers, identifying ALB coefficients, assessing the control basics of ALBs and designing and developing controllers based on mathematical models. My interest in this project arose after working many years on the reduction of machine lateral vibrations from a maintenance standpoint. This project gave me the chance to face such a subject from a machine design perspective through a multi-physics approach combined with control techniques. A great number of experimental campaigns and test rig improvements were needed in order to achieve the project goal. For these reasons, and considering my previous experience in international research groups, the external stay was cancelled, in agreement with the supervisor, to focus all the effort on the experimental part of the project.
Acknowledgements go to the persons who have contributed in many ways to the progress of this research project. To my supervisor Ilmar Ferreira Santos for giving me technical and personal support and always encouraging me to give my best to finalize this project. I feel very lucky to have been assigned to him. To Alejandro Cerda, my Chilean mate in distant Denmark, for always being available and willing to discuss personal and technical matters. To Alejandro de Miguel Tejada for making the last period of time in DTU more home-like. Thanks also go to the Ph.D. fellows: Christian Kim Christiansen, Shravan Janakiraman, Fabián Pierart, Jonas Lauridsen, Jon Larsen, Sebastian von Osmanski, Nikolaj A. Dagnæs-Hansen, Søren Enemark, the Brazilian team: Cesar Augusto da Fonseca and Geraldo de Souza. To Andreas J. Voigt and Bo Bjerregaard Nielsen with whom I shared the same stressful time period of finalizing the Ph.D. writing process. I also need to mention Martin Sønder Nielsen and Peter Svendsen for their help. I thank all of them for their invaluable contribution throughout many hours of talks on different technical and personal subjects. My apologies if I have forgotten to name somebody.

Last but not least, I would like to thank the closest and most beloved persons in my life for their unconditional support and company along this crazy and long period of life called “Ph.D. project”. Without their support, this would not have got to an end. To my unconditional wife Beatriz Rodriguez Torres and our children: Arturo and Isabel for being always by my side and my motivation. To my parents Beatriz and Jorge for always encouraging me to fly as high as I want. To my brothers Luis and Rafael for showing me the different ways that love can be expressed. To my grandmothers Amelia and Rosalía. Finally, I would like to dedicate this thesis to the memory of my grandfathers Rafael and Luis.

Kgs. Lyngby, August 31, 2016

Jorge Andrés González Salazar

“If you want to find the secrets of the universe, think in terms of energy, frequency and vibration.”

– Nikola Tesla
Tilting-pad journal bearings (TPJ Bs) are consolidated supporting elements for turbomachinery due to their undeniable stability properties among fluid-film bearings. However, the steadily increasing demand of manufacturing lines with larger production rate and extended life span, requires even more efficient and reliable machines. As a consequence, the TPJB, like many other sorts of bearings, have been redesigned in a mechatronic device. Under this active approach, by including sensing, actuating, processing and controlling capabilities, its properties can be modified aiming at improving its load carrying capacity and damping properties according to the operational requirements. Therefore, machines with reduced vibration levels and extended stability ranges can be employed under more demanding conditions. Further improvements such as self-diagnosis and maintenance free operation bearings are also intended under this approach. Among active TPJBs is the actively-lubricated bearing (ALB), which modifies the fluid-film pressure profile by injecting highly pressurized oil directly into the bearing clearance. This technology requires a servo hydraulic system and control laws to feature enhanced lube regimes. The feasibility of using ALBs on machines has already been proven in laboratory-size rigid rotor rigs via PID controllers, which are suitable as a post-manufacturing approach. The current maturity of the modelling of such bearings allows model-based controllers to be designed and thus the ALBs to be conceived in the early design stage of the whole machine.

The objective of this PhD research study is to bring ALBs closer to the industrial application by investigating, in a scaled-down industrial apparatus, the advantages and drawbacks of supporting flexible rotors by ALBs, theoretically and experimentally. The main original contributions of this research, contained
in a number of papers, are:
1) With regard to modelling, the coupled modelling of the ALB supporting a flexible rotor under passive, hybrid and active lubrication regimes is presented by using full linearized damping and stiffness matrices of the ALB that already contain the mechanical and hydraulic system link. This approach avoids characterizing the active force.
2) Regarding modelling validation, ALB stiffness and damping coefficients are experimentally identified and compared against simulations under all lubrication regimes to gain deeper insight into their calculation. On the other hand, the whole flexible-rotor bearing system dynamics is validated under passive and hybrid regimes via FRFs comparisons.
3) With regard to control, model-based LQG state-feedback controllers were designed, implemented and tested to develop active lubrication. Good agreement between the theoretical and experimental results is obtained for the known dynamics. The foundation dynamics is identified as an important system component to be further modelled. Additionally, model-free controllers were also designed and implemented to explore the position-dependent changes on the bearing dynamic properties via integral controllers and to reduce resonant vibrations through PD output feedback controllers. An application of integral controllers in ALBs when used as a shaker to maintain the equilibrium was proposed.

Finally, distinct pads for ALBs were proposed and investigated in a single-pad rotor system aiming at improving their design from a control basics perspective. Results suggest advantages in pads featuring nozzle-pivot offsets whilst they were partly validated for the pad with the nozzle-pivot centred and aligned.

Målet for dette ph.d.-studie er at bringe aktive lejer tættere på industriel anvendelse ved undersøgelse, i et nedskaleret industrielt setup, af fordele og ulemper ved at understøtte fleksible rotorer med aktive lejer, såvel teoretisk som eksperimentelt. De overordnede nyskabende bidrag fra dette studie, som er
indeholdt i et antal artikler, er:


2) Med hensyn til validering af modeller, er stivheds- og dæmpningskoeficienter eksperimentelt identificeret og sammenlignet med simuleringer under alle smøringsbetingelser for at opnå dybere indsigt i deres beregning. På den anden side, er det fulde system med fleksibel rotor og leje valideret under passive og hybridbetingelser via FRF-sammenligninger.


Endeligt er specifikke sko for aktive lejer fremsat og undersøgt i et enkeltsko rotorsystem med sigte på at forbedre deres design fra et grundlæggende kontrolperspektiv. Resultaterne indikerer fordele i sko med indspøjtningsdysen forskudt i forhold til vippepunktet, mens fordelene er delvist eftervist for skoen med sammenfaldende indspøjtningsdyse og vippepunkt.
Nomenclature

Acronyms
ALB   Actively-Lubricated Bearing
AMB   Active Magnetic Bearing
DAQ   Data Acquisition
DOF (dof) Degree-Of-Freedom
DTU   Technical University of Denmark
EHD   Elasto-Hydrodynamic
ETHD  Elasto-Thermo-Hydrodynamic
FRF   Frequency Response Function
LBP   Load-Between-Pads
LOP   Load-On-Pads
LQG   Linear Quadratic Gaussian Regulator
LQR   Linear Quadratic Regulator
MRE   Modified Reynolds Equation
PD    Proportional-Derivative
PI    Proportional-Integral
PID   Proportional-Integral-Derivative
THD  Thermo-Hydrodynamic
TPJB  Tilting-Pad Journal Bearing

**Greek Symbols**

- $\alpha$: Oil thermal diffusivity, $m^2/s$
- $\beta$: Viscosity law coefficient, $K^{-1}$
- $\gamma$: Coherence function
- $\kappa$: Oil thermal conductivity, $W/(mK)$
- $\mu$: Oil dynamic viscosity, $Pa \cdot s$
- $\mu_0$: Oil dynamic viscosity at injection orifice, $Pa \cdot s$
- $\mu_{\text{ref}}$: Oil reference viscosity, $Pa \cdot s$
- $\nu$: Oil kinematic viscosity, $m^2/s$
- $\Omega$: Shaft angular velocity, $rad/s$
- $\omega$: Excitation frequency, $Hz$
- $\omega_1$, $\omega_2$, $\omega_3$: Shaft natural frequencies, $Hz$
- $\omega_v$, $\omega_{vi}$: Servovalve natural frequency, $Hz$
- $\phi_{\text{in,r}}$: Inner diameter of rigid body, $m$
- $\phi_{\text{out,r}}$: Outer diameter of rigid body, $m$
- $\rho$: Oil density, $kg/m^3$
- $\sigma$: Standard deviation of Gaussian approach, $m$
- $\sigma_{FRF}$: Standard deviation of FRFs
- $\tau_a$: Servovalve amplifier time constant, $s$
- $\xi_v$, $\xi_{vi}$: Servovalve damping ratio

**Roman Symbols**

- $\Delta q_x$: Spool-driven flow, $m^3s^{-1}$
- $\Delta q_{P_L}$: Pressure-driven flow, $m^3s^{-1}$
- $\mathbb{D}$: Pad damping matrix. Solid domain
- $\mathbb{K}$: Pad stiffness matrix. Solid domain
<table>
<thead>
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<th>Description</th>
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<tr>
<td>$M$</td>
<td>Pad inertial matrix. Solid domain.</td>
</tr>
<tr>
<td>$\hat{q}$</td>
<td>Estimate of modal coordinate vector.</td>
</tr>
<tr>
<td>$D$</td>
<td>Modal reduced damping matrix of the pad.</td>
</tr>
<tr>
<td>$D_b$</td>
<td>Experimental bearing damping matrix.</td>
</tr>
<tr>
<td>$H_b$</td>
<td>Bearing impedance function.</td>
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<tr>
<td>$K$</td>
<td>Modal reduced stiffness matrix of the pad.</td>
</tr>
<tr>
<td>$K_b$</td>
<td>Experimental bearing stiffness matrix.</td>
</tr>
<tr>
<td>$M$</td>
<td>Modal reduced inertia matrix of the pad.</td>
</tr>
<tr>
<td>$q$</td>
<td>Modal coordinate vector.</td>
</tr>
<tr>
<td>$V, \tilde{V}$</td>
<td>Pseudo-modal matrix</td>
</tr>
<tr>
<td>$G(\dot{x}, \dot{z})$</td>
<td>Oil injection velocity weighting function</td>
</tr>
<tr>
<td>$h, h_i$</td>
<td>Fluid-film thickness</td>
</tr>
<tr>
<td>$K_{pq}$</td>
<td>Servo-valve load pressure-flow coefficient</td>
</tr>
<tr>
<td>$p, p_i$</td>
<td>Oil pressure</td>
</tr>
<tr>
<td>$p_{inj}$</td>
<td>Oil injection pressure</td>
</tr>
<tr>
<td>$q_v$</td>
<td>Servo-valve flow</td>
</tr>
<tr>
<td>$q_{inj}$</td>
<td>Oil injection flow</td>
</tr>
<tr>
<td>$q_{leak}$</td>
<td>Leakage flow</td>
</tr>
<tr>
<td>$R$</td>
<td>Journal radius</td>
</tr>
<tr>
<td>$S_{oil}$</td>
<td>Heat flux between oil-pad and oil-journal</td>
</tr>
<tr>
<td>$T$</td>
<td>Oil reference temperature</td>
</tr>
<tr>
<td>$T$</td>
<td>Oil temperature</td>
</tr>
<tr>
<td>$U$</td>
<td>Journal tangential velocity</td>
</tr>
<tr>
<td>$c_b$</td>
<td>Bearing clearance</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Pad clearance</td>
</tr>
<tr>
<td>$d_0$</td>
<td>Injection orifice diameter</td>
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<tr>
<td>$D_s$</td>
<td>Shaft element diameter</td>
</tr>
<tr>
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<td>Description</td>
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</tr>
<tr>
<td>$E$</td>
<td>Modulus of elasticity of foil</td>
</tr>
<tr>
<td>$i(t)$</td>
<td>Servovalve control current</td>
</tr>
<tr>
<td>$I_{P_r}$</td>
<td>Polar moment of inertia of rigid body</td>
</tr>
<tr>
<td>$I_{T_r}$</td>
<td>Transverse moment of inertia of rigid body</td>
</tr>
<tr>
<td>$k_a$</td>
<td>Servovalve amplifier gain</td>
</tr>
<tr>
<td>$l_0$</td>
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<tr>
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<td>$P_s$</td>
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<td>$r_0$</td>
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<tr>
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<td>Servovalve amplifier resistance</td>
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<tr>
<td>$R_{v}, R_{v_i}$</td>
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</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$Re^*$</td>
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<td>$S_0$</td>
<td>Injection area</td>
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<tr>
<td>$u(t)$</td>
<td>Servovalve control voltage</td>
</tr>
<tr>
<td>$v(t)$</td>
<td>Servovalve amplifier control voltage</td>
</tr>
<tr>
<td>$V_{inj}$</td>
<td>Oil injection velocity. Hagen-Poiseuille approach.</td>
</tr>
<tr>
<td>$V_{inj}^*$</td>
<td>Oil injection velocity. Gaussian approach.</td>
</tr>
<tr>
<td>$W_r$</td>
<td>Width of rigid body</td>
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The following publications, written during the research period, are part of this thesis:


The below publications were presented in international conferences and they constitute the starting point for the first three contributions listed above:


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1.1 Motivation

The beauty of lubrication theory can be found in everyday life, such as in the “kugel ball” fountains [1]. Such fountains are widely used for decorative purposes in malls, museums, squares and houses. These fountains are governed by the same working principles behind bearing lubrication theory. In fact, they can be seen as huge ball bearings where a remarkable thin fluid-film, of a few tenths of a millimetre, supports tons of granite or marble made spheres or wheels. Figure 1.1(a) portrays one of the sphere fountains found close to DTU. If a spinning motion is given to the sphere, it will take a long time for the free rotation to damp out due to the almost negligible shear forces to dissipate energy. Figure 1.1(b) shows a picture of a wheel fountain, whose governing equations are related closer to the physics of fluid-film bearings lubrication. Interestingly, the two mechanisms behind the load carrying capacity of such fountains, the hydrodynamic and mainly the hydrostatic pressure build ups are the same mechanisms behind the main study objective of this research: the actively-lubricated bearings.

In the case of the standard fluid-film bearing, the hydrodynamic lubrication is the principle that provides its main characteristics. It allows it, due to the pressure build up in the thin fluid-film, to support load and also to dissipate vibrating energy by squeezing the film. Additionally, it provides a means for cooling
the rotating parts down. This well developed and mature technology — with a long development history, which dates back to more than a century ago with the cornerstone work of Reynolds [2] — has been standardized and utilized in a wide range of applications. Turbo rotating machinery, such as turbo-generators, pumps, compressors and turbines, rely on the fluid-film bearings to support the machine and to avoid contact between the rotating and stationary parts, being vital machine elements. As any machine element, it has advantages and limitations. These limitations become more evident with the steadily increasing operational requirements and more demanding environmental regulations. The request of an energy dissipating source that allows machines to behave stably at higher angular velocities, has posed one of the major challenges in bearing design. However, with the vertiginous advances during the past decades in electronics and microprocessors, it became feasible to boost the integration of these technologies with mechanics, control and computational sciences to aid the design of enhanced fluid-film bearings. The harnessing of these multidisciplinary fields, referred to as “Mechatronics”\textsuperscript{1}, has rendered the fluid-film bearings “smart” and controllable, leading to bearings with improved damping properties, hence to machines with extended operational range. The first applications of the mechatronic concept to rotating machinery involved mainly the use of piezoelectric and magnetic actuators [5–7], to include lately other types such as those of a hydraulic nature [8]. A thorough review of the mechatronics development within the rotating machinery field, covering elements such as

\textsuperscript{1}Concept coined by Mr. Tetsuro Mori almost already fifty years ago [3]. “The term Mechatronics is a portmanteau of “Mechanics” and “Electronics” (MECHANics elecTRONICS). Mechatronics is the blending of mechanical, electronic, and computer engineering into an integrated design”. Quoted from [4].

\begin{figure}[h]
\centering
\begin{minipage}[t]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{spherical_fountain.png}
\caption{(a) Spherical fountain.}
\end{minipage}\hspace{1cm}
\begin{minipage}[t]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{cylindrical_fountain.png}
\caption{(b) Cylindrical fountain.}
\end{minipage}
\caption{Examples of the lubrication principles in everyday life: The granite fountains. (a) Ball fountain (3D) (b) Wheel fountain (2D).}
\end{figure}
dampers and bearings, is given in [3]². Among other benefits of rendering the bearing controllable are: the reduction of wear and starting torque, suppression of vibrations and the extension of periods between maintenance, thus of its lifetime.

1.2 Mechatronic Bearings

Mechatronic bearings represent the “active” counterpart to those “passive” solutions to dissipate energy in rotating machines. These solutions, such as squeeze film dampers, seal dampers, among others [10], have been studied already for decades. The development of different types of mechatronic bearings includes the magnetic bearings per excellence, oil-film journal and thrust bearings as well as gas, gas foil and gas bump foil bearings. One of the more important parts in developing mechatronic bearings has to do with the controller design. Indeed, in the case of magnetic bearings [11], the controller is responsible for closing the loop and making the bearing stable. In the case of active gas bearings, it represents the main damping mechanism in a bearing whose passive counterpart lacks it [12, 13]; making in some cases the crossing of the critical speeds prohibitive for rotors supported by such bearings. Oil-film bearings are also enhanced with the addition of controllers, although the lack of damping is not a problem as it is for the aforementioned bearings, their damping property can be further improved. As seen, mechatronic bearings can help to improve the behaviour of rotating machinery as a whole. Different control design methods can be used, but indisputably, whichever of them are used, it becomes more challenging when they are designed for flexible rotor-bearing systems.

1.3 State-Of-The-Art in Controllable Oil-Film Bearings

The controllable designs of fluid-film bearings have been developed upon tilting-pad journal bearings (TPJBs) due to their superior stability properties [14]. A comparison of controllable TPJBs based on hydraulic actuators was presented by Santos [15], who compared the hydraulic chamber and the radial oil injection systems. The last one has prevailed as the most suitable technology, hereafter referred indistinctly to as Actively-Lubricated Bearings (ALBs) or active TPJBs. Thenceforth, a large amount of investigation has been carried out in this field.

²Lately, the integration of tribology with mechatronics has started to be referred to as “Tribotronics” as suggested in [9].
The main achievements which establish the state-of-the-art are summarized next linked to the different test rig used, tracing a technology evolution. Test rig # 1: a TPJB with hydraulic chambers. Test rig # 2: an ALB of first generation. Test rig # 3: an ALB of second generation. Linked to the test rig #1, in Santos [15] some experimental results showing the effectiveness of the mechatronic device on reducing the bearing vibration amplitude were presented. The possibility of increasing the bearing damping properties and the stability margin by direct modification of the fluid flow within the journal-bearing clearance was observed. This analysis opened the door for the upcoming development of the radial oil injection system. In Santos [16] the active modification of the bearing force coefficients was demonstrated, theoretically and experimentally. Such a modification is possible due to the “flexible” chamber, which can affect the bearing gap. Additionally, the limitations of the hydraulic chamber system were clearly stated by pointing out their inability to adjust the bearing properties at high angular speeds due to the internal membrane flexibility restrictions. Going further, in Santos [17] the experimentally identified frequency dependent bearing force coefficients were compared against the theoretical counterpart. A generalized good agreement was obtained, except for the damping at higher rotational speed, a condition under which this property was underestimated. In Santos and Nicoletti [18], the self-excited vibration phenomenon in active bearing was addressed.

Associated with the test rig #2, Santos and Russo [19] presented a completed mathematical model for TPJBs with an electronic radial oil injection system by linking an isothermal fluid-film model with a linear model of the hydraulic system and PID controller. This modelling, based upon the Modified Reynolds Equation (MRE), entails obtaining the pressure distribution in active bearing as a function of the servovalve control signal in addition to the Sommerfeld number parameters. In Santos and Nicoletti [20, 21] the variation of the viscosity due to the oil film temperature distribution was included in the modelling by means of the energy equation and an appropriated viscosity law, developing the so called thermo-hydrodynamic (THD) model for the active fluid-film bearings. The influence of the multi-orifices distribution on the bearing thermal and static behaviour was thoroughly studied, based upon the hybrid lubrication regime. In Santos and Scalabrini [22], the focus was put onto the design of the control system. The root locus curves were utilized as a tool for designing simple PD controllers, whose influence is directly accounted for in the variation of the stiffness and damping properties of the bearing. Nicoletti and Santos [23] explored linear and non-linear control techniques applied to active TPJBs. It resulted that, control laws with integral terms (PI and PID controllers) were not suitable for reducing vibration, since they lessened the bearing damping due to the centering of the journal. The same premise was kept in Nicoletti and Santos [24]. However, this does not represent a general rule for integral controllers,
since references other than the journal centre can be defined, which might lead to a vibration reduction. In Santos et al. [25], the feasibility of applying active lubrication to reduce vibrations – related to the first bending mode – in an industrial flexible-rotor gas compressor was theoretically explored. Bearing force coefficients for the active TPJBs were obtained considering an isothermal model and included in the bearing-rotor model as a function of the excitation frequency. The control law was developed considering, as in Nicoletti and Santos [23], servovalve input voltage and clearance restrictions. In Nicoletti and Santos [26] the analysis of the steady-state response in frequency domain was studied for a rigid rotor-tilting pad bearing system. The system response was compared under passive and active lubrication regimes. The most noteworthy contribution is related to the forms in which the active forces can be treated; two approaches were presented. In the first one, the active force is considered as an excitation to the system. It was implicitly assumed that the hydrostatic contribution does not produce a constant force, which might modify the system equilibrium, hence the bearing dynamic properties when compared to the passive lubrication. Additionally, the active force was characterized through a linear frequency independent function of the servovalve control voltage obtained quasi-statically. The importance of including the excitation frequency in the active force characterization was highlighted. The second approach, with better results, was to consider the active force as a part of the system. In Nicoletti and Santos [24], the authors designed a more advanced control system based on two output feedback schemes, to wit: the system outputs at the rotor centre and at the bearing positions. The aim was to reduce, by affecting the damping ratios, the lateral vibrations of the first two modes of a flexible rotor (compressor) supported by two actively lubricated TPJBs. The modelling of the hydrodynamic fluid-film forces was according to the isothermal modelling and the active hydrostatic force was treated as an external excitation to the system. Although the frequency dependency of the servovalve was recognized, it was included only as a linear relation between the active force and the control voltage, limiting its validity up to a maximum frequency. This due to the approach utilized to design the controller, which is based on linear models.

As seen, the investigations on the active lubrication associated with first generation of controllable bearings produced advances in two main branches: the bearing modelling and the design of control systems. Isothermal and THD models for calculating the fluid-film forces were investigated. On the other hand, PID and output feedback controllers were studied. A rigid rotor test rig was used to investigate theoretically as well as experimentally the active lubrication. Concerning flexible rotors supported by ALBs, only a few theoretical studies [24, 25] have been carried out at this stage. With the second generation of active TPJBs, i.e. with test rig #3, the focus was on the modelling improvements aimed at obtaining precise models which can accurately predict the bearing behaviour, so that model-based controllers can be designed. In Haugaard and Santos [27] the
Effect of the pad flexibility was introduced in the model, yielding the so-called elasto-hydrodynamic (EHD) approach to describe the ALBs. Therein, a finite element formulation was considered to solve the governing equations in the coupled solid-fluid domain. Likewise in Santos and Nicoletti [20, 21], multi-orifice pads and their effect on the bearing stability were also studied by Haugan and Santos [28, 29], but now from an elasticity standpoint. Lately, Varela et al. [30] and Varela and Santos [31] have contributed to the development of the modelling of TPJB with radial oil injection by including the thermo effects and subsequently evaluating its static and dynamic performance upon the elasto-thermo-hydrodynamic (ETHD) model. The main simplifications adopted to include the thermal effects were presented in Cerda et al. [32]. Additionally, stability analysis and a study on the performance improvement under different lubrication regimes, involving the injection of oil, were carried out by Cerda and Santos [33, 34] targeting flexible rotors. Finally, the use of active TPJBs as calibrated shakers was also studied in Varela and Santos [35].

1.4 This Research Project

The main goal of this research project is to reduce the lateral vibration to extend the operational range of flexible rotors supported by actively-lubricated TPJBs. To achieve this goal, model-based controllers are designed, developed and tested, aimed at increasing the damping property of the bearing. As a result, it is expected to bring the ALBs technology closer to industrial applications. This objective encompasses both theoretical and experimental investigations. From a theoretical perspective, topics such as: i) bearing modelling, which gathers fluid-structure interactions, tribology, solid mechanics and numerical resolution of differential equations as well as ii) classic and model-based control design, form the main background of this research. Other topics, such as finite element modelling of flexible rotors are also covered. It is important to highlight that this investigation focuses mainly on the controller design for active lubrication, relying on the current modelling of the flexible rotor and bearing systems. Therefore, all subjects involved in the determination of the controllable bearing properties are properly understood but not deepened in this investigation, even though some discussions are presented. In other words, the ETHD model for controllable bearing is used as a means to produce the relevant information needed in order to properly model the rotor-bearing system aiming at designing the model-based controllers. From an experimental perspective, challenges such as the run-out compensation, anti-aliasing filtering and fulfilment of the main assumptions need to be properly addressed. Furthermore, the fitting of the theoretical model with the experimental results became the biggest challenge to overcome, leading to exploring model updating techniques to close the
1.4 This Research Project

gap between the theoretical and experimental perspectives. Throughout the development of the project, unknown dynamics different from the rotor-bearing system took place, complicating the project success. As discussed in the conclusions, it is assumed that this was due to the influence of the foundation dynamics, which should be considered as a relevant system component for this apparatus. Despite all inconveniences, good results are obtained for the known and targeted dynamics.

1.4.1 Main Original Contributions

Six journal papers were produced during the course of the investigation. As a paper based thesis, the principal contributions of the investigation are presented within them and the thesis stands as an introductory and complementary document to steer and organize the information. In [P1], using a quasi-static system characterization, an integral controller is presented with the goal of assisting the hybrid lubrication in changing the equilibrium position in a controlled fashion to explore the effects on the bearing properties. As further contribution [P2], using a dynamic system characterization, a proportional-derivative controller was designed and implemented to develop the feedback-controlled lubrication to reduce the lateral vibration around the equilibrium. In both, a model-free approach through an experimental “plant characterization” was given. This characterization allows us to have a system model that can be used when a fast development and implementation of a controller is needed and also when the theoretical model is still far from accurately representing the system. In [P3], the bearing force coefficients of the ALB with light loading conditions were experimentally identified under different lubrication regimes. These conditions were set in order to replicate the operating conditions of some vertical machines. A comparison against the predicted ones by the ETHD model was also presented. The main challenge linked to this work was the implementation of an identification method suitable for flexible rotors, since most of them are meant for rigid ones. [P4] contributes mainly from a theoretical standpoint. In this work the stability, controllability and observability of a simplified ALB system was studied for different pad layouts obtained by modifying the pivot and injector line offsets. These fundamentals represent the basic properties to be studied for a system before generating model-based controllers. With the intention of promoting a clear understanding of these concepts a single pad configuration test rig was used. In [P5], the joint modelling of the flexible rotor with the ALB was presented for three possible lubrication regimes. It was also established how to calculate the bearing dynamic properties under such regimes by clarifying the contribution of the bearing and hydraulic subsystems to it. Theoretical and experimental results for the hybrid lubrication regime were compared. An integral controller, for influencing the journal bearing centre, based on model-
based standard tools was designed and tested. Reductions of around 30% were obtained at resonance. Finally, the publication [P6] achieves the goal imposed on this research project. Therein, model-based controllers were designed and implemented to develop the active lubrication in ALBs. For doing so, the previous model developed in the preceding publication was utilized to design an LQG regulator, which is an optimal state-feedback controller. Significant reduction of lateral vibrations at the resonant zone was obtained, of the order of 60%. However, unknown dynamics harmed the system response in closed-loop at low frequencies. The evidence strongly suggests that the apparatus foundation behaves so flexibly that its dynamics must be taken into account in future research.

1.5 Experimental Facilities

At the solid mechanic laboratory of DTU Mechanical Engineering Department there are two test rigs featuring actively-lubricated fluid-film bearings. Although the utilized pads in both bearings are conceptually the same in terms of material, dimension, geometry and nozzle placement, the bearing designs as a whole differ since they have been produced for different purposes.

The Small ALB Test Rig: which is shown in Figure 1.2(a), is a full instrumented rig built mainly to validate models of controllable bearings. The bearing is two rocker-pivoted pads supporting vertically a rigid rotor in a load-on-pad configuration. Thanks to a levered-arm, which can be fixed to restrict the rotor movements, the system can be studied either as a bearing or as an actuator, i.e. a “calibrated shaker”. More details on its design are found in [36] in which the design of the test rig is presented in depth. This test rig was used only in [P4].

The Big ALB Test Rig: which is shown in Figure 1.2(b), is comparable to a scaled-down industrial machine. Indeed, it resembles an overhung compressor characterized by a flexible rotor supported on one of its ends by an active TPJB. The controllable bearing is four bronze rocker-pivoted pads in a load-between-pads configuration. The test stand allows us to study the dynamic behaviour of flexible rotor-controllable bearing systems when different types of lubrication regimes are developed, such as the conventional, hybrid and the feedback-controlled lubrication regimes. Besides the ALB, which can also act as a calibrated exciter, two additional ways of exciting the rotor are available: 1) an AMB and 2) an electromagnetic shaker connected through an excitation bearing. Further design and operational parameters can be found in [37] where the test rig was first presented, and additionally in Chapter 2 within the framework of this research.
1.6 Structure of This Thesis Work

The thesis is split up into chapters, each one corresponding to a main topic. Additionally, the main contributions yielded by this research are included in the Appendix A. Further information is also appended.

Chapter 1 consists of an introduction through which the research is put into context. The concept of mechatronic bearing is introduced so the reader is acquainted with it. The state-of-the-art in controllable fluid-film bearing is reviewed.

Chapter 2 introduces the test rig utilized in the experimental investigation. Its main characteristics and possibilities are highlighted. Additionally, all the previous work carried out on this bench is summarized.

Chapter 3 describes the current approach for modelling the controllable bearing. It reviews the main contributions in modelling from the standard TPJB to the active TPJB. The results obtained in contribution [P3] are revised. In addition, theoretical and experimental bearing force coefficients obtained in another loading condition are provided.

Chapter 4 summarizes briefly the modelling of the flexible rotor and the inclusion of the active bearing properties in it. Some general considerations and assumptions are highlighted. Further theoretical results and the servovalve amplifier modelling are presented.

Chapter 5 presents the results obtained under the model-free approach. The main considerations are pointed out and the most interesting results highlighted.
Two contributions were produced, contribution [P1] linked to integral controllers and hybrid lubrication, and [P2] linked to proportional-derivative controllers and active lubrication.

Chapter 6 presents the results obtained under the model-based approach. This chapter is linked to the contribution [P4] where the control basics for the ALB technology considering different single-pad layouts were reviewed. It also summarizes the main results of the contributions [P5] and [P6], where LQR and LQG controllers were used for assisting the hybrid and developing the feedback-controlled lubrications, respectively. A complementary discussion on the main project results is also included.

In Chapter 7 conclusions and future aspects of the research project are summarized.
Chapter 2

The Test Rig

2.1 Test Rig Presentation

The test stand is presented in Figure 2.1 highlighting its primary parts and main instrumentation used in this work. The main design and operational characteristics are summarized in Table 2.1. The test rig was designed by [37] for studying the dynamic behaviour of flexible rotors when supported by ALBs. Currently, the active bearing is capable of operating under three different lubrication regimes, which shall be presented in section 2.2.4. After an exhaustive study in [38] on the dynamic of different foundations, the test rig was mounted on a cast iron T-slot baseplate. However, as shall be discussed later, the foundation is not rigid enough and some coupled dynamics remain.

The test rig, close to an industrial scale, is comparable to a compressor with overhung impellers. Up to three different rotor configurations can be set depending on the number of discs mounted on the flexible shaft: none, one or two discs. The whole rotor is supported by a ball bearing and the active TPJB at its driven and non-driven sides, respectively. The rotor is driven by a transmission-belt and a layshaft stand which supply the driven torque to the rotor through a flexible coupling. The AC motor is provided with a frequency driver with which the test stand may run up to 7000 rpm. Nonetheless, caution must be taken with regard to the different speed limits of the main rolling bearings, as presented in Table 2.2. Speeds above 5000 rpm are not recommended unless the
Figure 2.1: The active TPJB Test Rig at the Technical University of Denmark. 1) excitation bearing, 2) removable disc (up to two), 3) controllable ALB, 4) AMB, 5) proximity probe pedestal, 6) flexible coupling, 7) encoder, 8) AC motor, 9) layshaft pedestal, 10) ball bearing, 11) flexible shaft. a) flow meter, b) pressure manometer, c) pressure-temperature probe.

rear bearing is changed.

Table 2.1: Test rig main design and operational characteristics.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC motor power</td>
<td>3</td>
<td>kW</td>
<td>Bearing load / no disc</td>
<td>400</td>
<td>N</td>
</tr>
<tr>
<td>Rotor max. speed</td>
<td>7000</td>
<td>rpm</td>
<td>Bearing load / one disc (80 mm)</td>
<td>880</td>
<td>N</td>
</tr>
<tr>
<td>Shaft length</td>
<td>1150</td>
<td>mm</td>
<td>Bearing load / two discs</td>
<td>1440</td>
<td>N</td>
</tr>
<tr>
<td>Shaft weight</td>
<td>49.5</td>
<td>kg</td>
<td>Lubricant</td>
<td>ISO VG22</td>
<td>-</td>
</tr>
<tr>
<td>Disc 80 mm weight</td>
<td>37.3</td>
<td>kg</td>
<td>Oil Viscosity</td>
<td>1.892 \cdot 10^{-2}</td>
<td>kg/ms</td>
</tr>
<tr>
<td>Disc 100 mm weight</td>
<td>46.0</td>
<td>kg</td>
<td>Oil flow (2 bar)</td>
<td>1.3</td>
<td>L/min</td>
</tr>
</tbody>
</table>

Table 2.2: Speed limits for rolling bearings of the test rig.

<table>
<thead>
<tr>
<th>Bearing</th>
<th>Type</th>
<th>Limit Speed</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation Bearing</td>
<td>SKF 6008-2RS1</td>
<td>6300</td>
<td>rev/ min</td>
</tr>
<tr>
<td>Rear Pedestal Bearing</td>
<td>SKF 2210 E-2RS1TN9</td>
<td>4800</td>
<td>rev/ min</td>
</tr>
<tr>
<td>Layshaft Pedestal Bearing</td>
<td>SKF 6208-2Z</td>
<td>9000</td>
<td>rev/ min</td>
</tr>
<tr>
<td>Layshaft Pedestal Bearing</td>
<td>SKF 6207-2Z</td>
<td>9000</td>
<td>rev/ min</td>
</tr>
<tr>
<td>Encoder Bearing</td>
<td>Scancon SCA50</td>
<td>12000</td>
<td>rev/ min</td>
</tr>
</tbody>
</table>
2.1.1 The Test Rig Foundation

One of the big inconveniences of the test rig setup has to do with its iron cast T-slot baseplate. Despite it being chosen as the best option for mounting the flexible rotor-bearing system [38], it seems that its dynamics still contributes to the dynamics of the whole foundation-mechanical system, especially at low frequencies. This fact was earlier recognized by authors such as [39 41], who addressed it theoretically and experimentally. Appendix F summarizes an experimental modal testing carried out over the baseplate in order to identify the frequencies and modeshapes involved in the frequency range of interest. It was discovered that some frequencies related to torsional modeshapes lie within the study range, which might couple with the rotor-bearing system dynamics. Additionally, it was seen that the first bending modeshapes of the baseplate move significantly at the ALB position. Although this last should not affect the machine from a dynamic point of view, it influences its static behaviour. Indeed, since the displacement probes measure relative distances, any baseplate deformation may affect the reading of their gap (DC value), which in turn, can produce erroneous determination of the shaft equilibrium positions. Trying to overcome this problem, two main experimental actions were attempted:

- The ball bearing housing was stiffened as was previously done with the ALB in order to get a more rigid structure.

- A significant amount of mass was added to the baseplate free-end, approximately half of a ton. This intended to lower as much as possible the plate natural frequencies, taking them out from the studied frequency range. Although it was possible to reduce the first bending frequency below 10 Hz, a strongest dynamic coupling between the test rig and the foundation was occasioned.

![Figure 2.2: Suggested installation of the T-slot plate according to manufacturers.](image)
From a theoretical perspective, model updating techniques were tried in order to fit theory with experiment, but this approach worked only for some of the system configurations (one disc configuration) and most of the time yielding unrealistic parameter values. In accordance with some manufacturers, this type of T-slot base plate should be installed on a metal-concrete foundation with a proper anchor system (Figure 2.2(a)) when dynamic and high dynamic loadings are expected, or on a ribbed floor (rib pattern) to improve its stability (Figure 2.2(b)).

2.1.2 General Instrumentation

The test rig is instrumented with:

1. Ten displacement sensors for monitoring the lateral movement of the shaft. Only four of them were considered for controller design purpose. They are placed in a pedestal such as element 5) of Figure 2.1. The complete distribution is presented in the modelling section.

2. Digital temperature and pressure meter at the inlet low pressure line. Element c) of Figure 2.1.

3. Digital flowmeter at the high pressure feeding line. Element a) of Figure 2.1.

4. Analogue pressure meter at the high pressure feeding line. Element b) of Figure 2.1.

5. Encoder for determining the shaft angular position. Element 7) of Figure 2.1.

The calibration of the proximity probes is presented in Appendix C. Therein the explanation why only some of the probes were considered for controlling purposes is included. Also important is the runout compensation in this type of transducer. This is reviewed in Appendix D. It is worth mentioning that, the DAQ system does not have an anti-aliasing (analogue) filter. Therefore, a high sampling frequency of around 6 kHz (six times the Nyquist frequency) was utilized in order to avoid aliasing phenomenon in the frequency range of interest, i.e below 500 Hz. However, this solution puts a high computing load on the DAQ system, even though signals can be down sampled. The test rig is also instrumented with an in-house made current measurement system. This is connected to the servovalve amplifier and used to record the drive current governing the servovalves. More details can be found in the Appendix C. All
Figure 2.3: The test rig connected to the shaker (Horizontal configuration). 1) Electromagnetic shaker. 2) stinger. 3) Dynamic force transducer. 4) Excitation bearing (Ball bearing). 5) Proximity probe pedestal.

signal are recorded with a dSPACE DS1104 DAQ system, which also generates the servovalve control signals and the excitation signal for the shaker, all at the same time.

2.1.3 System Excitation Possibilities

Although the system can also be excited through an active magnetic bearing (AMB) placed next to the active TPJB, in all the reported experimental campaigns for the contributions the system was excited via an electromagnetic shaker. The standard layout is presented in Figure 2.3. The shaker is connected to the excitation bearing placed at the rotor free-end through a stinger and a piezoelectric force transducer which can sense the applied dynamic load. This unidirectional dynamic force is applied either horizontally or vertically with the help of a shaker supporting frame, which allows quick direction changes to be made in the two orthogonal directions without any stinger disconnection. The frame-shaker setup presents two main advantages: i) test campaigns are carried out in both directions without losing the geometrical and thermal equilibria due to an easy direction adjustment while the rotor is still running, and ii) it ensures a better grade of repetitiveness in the experiments.
The Test Rig

2.2 The Actively-Lubricated Bearing (ALB)

2.2.1 The ALB Design

The controllable bearing is a tilting-pad journal bearing with 4 bronze pads in a LBP configuration, see Figure 2.4. The pads are centrally pivoted on a rocker type pivot and the injector is aligned with the pivoting line. Further design parameters are included in Table 2.3. The advantages of the tilting-pad journal bearings over the conventional journal bearings rely on the better stability properties due to their almost null cross force coefficients [14]. The number of pads normally varies from 3 to 7, with 5 being a widely used pad arrangement. A lower number of pads, like 4 pads, helps to reduce the pad temperature during operation due to the load-between-pads configuration. This pad arrangement increases the minimum film thickness and from a dynamic standpoint, it provides more symmetrical stiffness and damping coefficients. With a symmetric bearing, vibration orbits are circular and smaller compared to elliptical ones of asymmetric bearings. Preload for the current bearing was calculated to be around 25%, based on the cold-clearance measurement, reported in the Appendix B. Measurements of hot-clearance, as suggested by [42], were not possible with the available testing equipment. Another important characteristic is the offset, in this case 0.5. Although it yields a smaller load capacity than pads with slightly larger offsets, it has the advantage that the rotor can run clockwise and counterclockwise. A deeper insight into the geometry aspects of conventional TPJBs

Figure 2.4: 4 rocker LBP Active Tilting-Pad Journal Bearing. (1) Conventional lubrication inlet, (2) High response servo-valve, (3) displacement sensors, (4) pad, (5) conventional lubricant feeding nozzle (6) High pressure oil injection orifices, (7) High pressure oil injectors.
can be found in [43, 44].

The active or controllable feature of the bearing is developed by an electronic radial oil injection system as proposed by [8], This injection system superimposes hydrostatic pressure over the hydrodynamic pressure distribution by injecting pressurized oil between the journal-pad clearance through a nozzle

Table 2.3: Conventional and controllable design parameters of the ALB.

<table>
<thead>
<tr>
<th>Conventional Design Parameters</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journal radius ($R$)</td>
<td>49.89</td>
<td>mm</td>
</tr>
<tr>
<td>Pad inner radius ($R_p$)</td>
<td>50</td>
<td>mm</td>
</tr>
<tr>
<td>Pad aperture angle ($\alpha_p$)</td>
<td>69</td>
<td>°</td>
</tr>
<tr>
<td>Pad width ($L$)</td>
<td>100</td>
<td>mm</td>
</tr>
<tr>
<td>Pad thickness ($t$)</td>
<td>14</td>
<td>mm</td>
</tr>
<tr>
<td>Nominal radial clearance ($C_p$)</td>
<td>110</td>
<td>pm</td>
</tr>
<tr>
<td>Assembly radial clearance ($\bar{C}_p$)</td>
<td>83</td>
<td>pm</td>
</tr>
<tr>
<td>Pad offset</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Lubrication oil type</td>
<td>ISO VG22</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Controllable Design Parameters</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Servovalve type</td>
<td>MOOG E760-912</td>
<td>-</td>
</tr>
<tr>
<td>Servovalve configuration</td>
<td>4 way, spool valve</td>
<td>-</td>
</tr>
<tr>
<td>Cut-off frequency (210 bar)</td>
<td>350</td>
<td>Hz</td>
</tr>
<tr>
<td>Damping ratio (210) bar</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>Cut-off frequency (100 bar)</td>
<td>260</td>
<td>Hz</td>
</tr>
<tr>
<td>Injection crifice diameter ($d_0$)</td>
<td>3.3</td>
<td>mm</td>
</tr>
<tr>
<td>Injection crifice length ($L_0$)</td>
<td>21</td>
<td>mm</td>
</tr>
</tbody>
</table>
placed in the pad’s middle. Figures 2.5(a) and (b) compare two pads with and without such a nozzle. The high pressure oil flow is controlled by two high-frequency response servovalves, each one coupled to a pairwise of counter pads. The lubricant is supplied for the conventional and for the active lubrication by a low (max. 2 bar) and a high (max. 100 bar) pumping units, which are described next. A scheme of the ALB including the reference frames is presented in Figure 2.5(b).

2.2.2 The Hydraulic System

The hydraulic system associated with the ALB is shown schematically in the layout of Figure 2.6. It is an in-house assembled system which comprises three different units: the high pressure, low pressure and return units. All of them are connected to a unique reservoir.

![Diagram of the hydraulic system](image)

**Figure 2.6:** Hydraulic system layout. High pressure (HP) unit. Low pressure (LP) unit and return unit.
2.2.2.1 High Pressure Hydraulic Unit

The high pressure unit is the one which provides the lubricant to the servovalves at a defined supply pressure. This pressure sets the dynamic behaviour of the servovalve. The larger the pressure, the wider the frequency range of the linear response. The unit is mainly composed of a positive displacement pump which generates a pumping frequency of around 175 Hz. The pump can raise the pressure slightly over 200 bar. However, due to sealing limitations of the injector seal, the working range goes from 12 bar min. to 100 bar max. As part of the pumping unit, a pressure gauge, a filter and a relief valve are also included. The pump is driven by a 15 HP electric motor. Additionally, the supply line has been instrumented with a flow meter and a pressure gauge upstream of the servovalves in order to know the supply conditions. These are used as boundary conditions when determining the bearing properties theoretically. Finally, a return line is provided to connect the tank port of the servovalve.

2.2.2.2 Low Pressure Hydraulic Unit

The low pressure unit ensures a minimum supply of oil for developing the conventional lubrication through 4 flooding injectors, see Figure 2.4. The working supply pressure can be set up to 2.4 bar providing a maximum lubricant flow of 1.4 L/min approximately. In Figure 2.7 the flow-pressure relationship for this unit is plotted. In addition to a pressure gauge, the feeding line is instrumented with a pressure-temperature transducer to monitor these variables of the lubricant.

![Figure 2.7: Flow-pressure relationship for the low pressure unit.](image)
2.2.3 The Servovalve Description

The actuator corresponds to a servovalve MOOG E760-912. It is a high performance electro-hydraulic flow control valve commonly used in very demanding applications. It is an old version of the servovalve already replaced by the Moog D765 series, but completely suitable for the active lubrication application. Among its main characteristics are: it is an underlapped, high-response, 4-way spool valve, without electrical feedback for PID regulation. Since the amplifier is not incorporated in its electronics, the servovalve is driven by a current control signal instead of a voltage one, unlike those more modern ones. Due to this, an external servo-amplifier is required to control it. This amplifier introduced a phase lag in the system which can be either modelled or compensated as will be explained later in the modelling section. The dynamics of the electro-hydraulic servovalve is highly dominated by nonlinear phenomena. However, it is standard practice to linearise its behaviour around an operation point, normally the spool centre position. As shown in Figure 2.8, there are three main stages of the servovalve which contribute the most to its general behaviour, to wit: the torque motor, the flapper and the spool stage, with the latter one dominating its dynamic characteristics. The linearisation of the flow driven by the servovalve produces [46]:

\[ q_v = q_{\text{leak}} - \Delta q_{P_L} + \Delta q_{x_v} \]  

(2.1)

where \( q_v \) is the flow delivered by the servovalve, \( q_{\text{leak}} \) is the leakage flow due to the servovalve underlapped configuration, \( \Delta q_{x_v} \) is the spool driven flow and \( \Delta q_{P_L} \) is the pressure driven flow. The leakage flow \( q_{\text{leak}} \) is constant while the pressure driven flow is a variable flow which is proportional to the pressure load in the form of \( \Delta q_{P_L} = K_{pq} \Delta P_L \). The last term \( \Delta q_{x_v} \) corresponds to the flow

![Figure 2.8: Contributions to the servovalve dynamic behaviour. Taken from [45].](image-url)
2.2 The Actively-Lubricated Bearing (ALB)

driven by the spool position $x_v$ and satisfies the $2^{nd}$ order linear equation:

$$
\Delta \ddot{q}_{x_v} + 2\xi_v \omega_v \Delta \dot{q}_{x_v} + \omega_v^2 \Delta q_{x_v} = \omega_v^2 R_v u
$$

(2.2)

Table 2.4 summarizes the characteristic parameters of servovalve 1 obtained theoretically and experimentally in [47], but the servovalve damping $\xi_v$, which is assumed constant after experiments presented in [48]. These parameters are presented for two supply pressures $P_s$. What can be drawn from this table is that:

- The servovalve natural frequency $\omega_v$ decreases markedly with a decrease in the supply pressure $P_s$ from its nominal value of 350 Hz at 210 bar to 260 Hz at 100 bar, approximately.
- The servovalve damping factor $\xi_v$ and the leakage flow $q_{\text{leak}}$ remain constant.
- The control input proportional gain $R_v$ reduces with a decrease of $P_s$, so that smaller active forces are applied.
- The load pressure proportional gain $K_{pq}$ slightly increases with a decrease of the supply pressure meaning that a slightly larger back flow is developed between two counter pads due to their pressure difference.

Recognizing that the dynamics of both servovalves are most likely different and assuming slight differences, the same parameters are used for servovalve 2 due to the lack of the characterization of it.

Table 2.4: Servovalve 1 characterization for two different supply pressures $P_s$.

<table>
<thead>
<tr>
<th>$P_s$ (bar)</th>
<th>$\omega_v$ (Hz)</th>
<th>$\xi_v$</th>
<th>$R_v$ (mm$^3$/Vs)</th>
<th>$q_{\text{leak}}$ (mm$^3$/s)</th>
<th>$K_{pq}$ (mm$^5$/Ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>260</td>
<td>0.73</td>
<td>7300</td>
<td>165</td>
<td>407</td>
</tr>
<tr>
<td>60</td>
<td>220</td>
<td>0.73</td>
<td>3943</td>
<td>160</td>
<td>525</td>
</tr>
</tbody>
</table>

* Nominal value of 350 Hz for 210 bar of supply pressure.
† From experiments in [48].

2.2.4 Lubrication Regimes with the ALB

The addition of the high pressure hydraulic unit to the bearing allows us to develop further lubrication regimes due to the option of injecting additional lubricants into the pad-journal clearance. Basically, three different lubrication
regimes are identified. The way in which they are named can be basically
distinguished according to either the underlying lubrication mechanism or from
a control point of view. These terms are used herein indistinctly.

**The Conventional Lubrication Regime**  It is the usual lubrication regime,
in which the hydrodynamic effect provides the load carrying capacity of the
bearing and its dynamic properties. For control purposes, and since there is not
a controller, this regime is also referred to as passive lubrication.

**The Hybrid Lubrication Regime**  In this regime, the hydrostatic effect is
activated. Depending on the servovalve’s spool position, the high pressurized
oil can be injected through different injection layouts. The spool position is
driven by a constant control signal so that a permanent lubricant injection is
obtained. Even when there is no control signal and the spool is centred, a small
leakage flow is still injected due to the tolerances of the underlapped design.
In general terms, the higher the pressure the more pronounced the hydrostatic
effect in the bearing, yielding a modification of the journal equilibrium position
as a consequence of the extra constant hydrostatic force applied.

**The Controllable Lubrication Regime**  If the signal driving the servovalve
is defined by a control law with well-tuned gains, then the whole or a part of
the hydrostatic force exerted over the journal can be actively-controlled. By
using the journal position as a feedback signal, control laws can be implemented
in hardware with real time processing capabilities with the aim of varying the
hydrostatic force in response to the journal movement. For control purposes,
this regime is named the feedback-controlled or active lubrication regime.

## 2.3 The Characteristics of the Test Rig

An overview of the operational parameters for carrying out the experiments
are summarized in Table 2.5. Their influence on the system temperature and
dynamic response is briefly reviewed. Limitation to the maximum rotor speed
is imposed due to the speed limits of the bearings, as informed in Table 2.2.
Likewise, the limitation to the high pressure unit is imposed by the maximum
pressure able to support the o-ring sealing of the nozzle. Figure 2.9 pictures the
changes of the oil temperature for the different operational conditions set by the
lubrication regimes. Although this temperature is measured at the inlet of the
conventional lubrication pipe, see Figure 2.6, it is considered representative as an
Figure 2.9: Inflow lubricant temperature for the conventional (hydrodynamic) and hybrid (hydrodynamic+hydrostatic) lubrication regimes.

overall indicator. It is clearly seen that the temperature rises with both the shaft angular velocity and the pressure of the hydraulic system. Figure 2.10 shows the viscosity variation of the Mobil DTE 22 lubricant used with the bearing alongside its approximation law curve. In addition to this the temperature range within which the lubrication regimes are developed is also shaded in gray scale. A significant viscosity drop is identified of approximately 50%.

To determine whether the flow corresponds to a laminar, turbulent or lies in the transition zone, the Reynolds number is calculated. Typically, Re below 1000 is considered laminar and above 1500 is considered turbulent. In between these limits a transition flow is assumed. For a bearing, the Reynolds (Re) and alternatively the reduced Reynolds (Re*) numbers are defined in terms of the

Table 2.5: Common operational conditions for the lubrication regimes tested.

<table>
<thead>
<tr>
<th></th>
<th>LP unit</th>
<th></th>
<th>HP unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Speed</td>
<td>Pressure</td>
<td>Pressure</td>
</tr>
<tr>
<td></td>
<td>[rev⁻¹]</td>
<td>[bar]</td>
<td>[bar]</td>
</tr>
<tr>
<td>Conventional</td>
<td>1000-4000</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Hybrid</td>
<td>1000-4000</td>
<td>2</td>
<td>12-100</td>
</tr>
<tr>
<td>Controllable</td>
<td>1000-4000</td>
<td>2</td>
<td>12-100</td>
</tr>
</tbody>
</table>
Figure 2.10: Viscosity of Mobil DTE 22 (ISO VG22).

characteristic velocity ($V$), length ($L$) and the fluid kinematic viscosity ($\nu$) as:

$$\text{Re} = \frac{VL}{\nu} = \frac{\rho \Omega D c_b}{\mu}; \quad (2.3)$$

$$\text{Re}^* = \frac{\rho \Omega D c_b}{\mu} \left(\frac{c_b}{D}\right) = \text{Re} \left(\frac{c_b}{D}\right) = \frac{\rho \omega c_b^2}{\mu} \quad (2.4)$$

where $\rho$ denotes the oil density, $\omega$ is the shaft rotational speed, $D$ is the shaft diameter, $c_d$ is the bearing clearance and $\mu$ is the oil dynamic viscosity. In obtaining the Re number, the shaft tangential velocity $\omega D$ and the bearing clearance $c_d$ are considered characteristic properties.

By taking into account the variation of the viscosity as a function of the oil temperature for the different shaft rotational speeds summarized in Figures 2.9 and 2.10, the shaft diameter of 99.78 mm, a bearing clearance of 84 $\mu$m and an oil density of 865 kg/m$^3$, the Table 2.6 summarizes the Re values. It can be seen that the flow developed in the ALB corresponds to a laminar flow. This allows us to use the model presented in the subsequent chapter for obtaining the bearing properties, disregarding the inclusion of any temporal fluid inertia or turbulent approach. If one estimates, based on the available information at hand, a maximum reduction of the viscosity to a 0.0075 Pa·s for an oil temperature of approximately 70°C, the angular speed for the onset of the

<table>
<thead>
<tr>
<th></th>
<th>$\omega$ [rad/s]</th>
<th>$\mu$ [Pa·s]</th>
<th>Re</th>
<th>$\text{Re}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum (1000 rpm)</td>
<td>104.7</td>
<td>0.023</td>
<td>32</td>
<td>0.027</td>
</tr>
<tr>
<td>Maximum (4000 rpm)</td>
<td>418.9</td>
<td>0.012</td>
<td>250</td>
<td>0.210</td>
</tr>
</tbody>
</table>
transition regime (Re=1000) is approximately 9000 rpm, which is beyond the rotor speed limitation.

### 2.3.1 Influence on the Frequency Response Functions

The Figures 2.11 and 2.12 show the influence of the oil temperature on the FRF shapes. The test rig is loaded with only one disc and the results are presented in the vertical direction only. In the case of the passive lubrication shown in 2.11, i.e. when the high pressure unit is off, a significant difference in amplitudes is seen when the oil is at an ambient temperature of 25°C approx. and when it is heated around 28°C. Above this temperature no significant differences are seen. In the case of hybrid lubrication shown in Figure 2.12 a larger variation is seen, especially for the first resonant zone. In general, the temperature control in the test rig is challenging and influences the system behaviour. To avoid as much as possible the influence of the temperature on the results, all experiments were carried out once the thermal equilibrium was completely set. Finally, Figure 2.13
Figure 2.12: Oil temperature influence. One disc configuration. Hybrid lubrication regime at 1000 rpm and 90 bar,

shows the effect of an increased shaft angular speed on the system response when there is no disc. The gyroscopic effect can be recognized on the first bending natural frequency of around 210 Hz as the angular speed increases.

2.4 Previous Works on the Test Rig

The test rig has been already used to conduct projects aimed at designing controllers for the ALB, as summarized in Table 2.7. In [39], the authors were devoted to defining a reliable linearised mathematical model of the flexible rotor-bearing system which can be used to design observers. The passive and active lubrication were treated separately from a theoretical and experimental point of view, respectively. And the theoretical full dynamic coefficients were obtained by assuming rigid pads and an isothermal approach. They realized early on that the dynamic of the ALB and ball bearing housing as well as sensors pedestal were not completely rigid. This was partly overcome by introducing new degrees-of-freedom related to the ALB and updating the model by fitting
Figure 2.13: Rotor speed influence on the system dynamic response. No disc configuration. Passive Lubrication at 1000 rpm.

curves to the experimental FRFs, with the aid of a self-developed frequency domain method. New tuned parameters for modelling the ALB and ball bearing housing were included in the model. Due to the model updating process, the final mathematical model, hence the observer, is restricted to the operational conditions for which it was obtained, i.e. 1000 rpm of rotational speed and 60 bar of oil supply pressure.

In [40], the authors have extended the study under similar operational conditions, but slightly increasing the high pressure supply to 70 bar; they experimentally recognized an increase in the active force due to an oil supply pressure increase. They have used the same mathematical model already defined and fitted in [39] to design controllers using the model-based control theory. Concepts such as observability, controllability, pseudo modal reduction, optimal deterministic and stochastic observers as well as optimal controllers were introduced. The force produced by the active lubrication was approached experimentally but extending the actuator characterization with the intention of elucidating its dependency on the journal eccentricity as well as on the oil injection pressure. Classical and optimal controllers were designed, implemented and experimentally tested in the test rig with different degrees of success due to, mainly, the discrepancies between the mathematical model and the test rig dynamics.
Within classical controllers, proportional controllers were used to counteract the unbalance whilst the integral controllers were used to affect the journal position within the bearing. Optimal controllers were focused on controlling individual and several modes of the system at the time, but in both cases experiencing spillover problems, i.e. undesired excitation of the unconsidered modes by the modal reduction. It was found that the spillover was more pronounced experimentally rather than theoretically.

In [41], an improvement to the test rig was made by installing horizontal stiffeners to the ALB housing with the intention of restricting its horizontal movement and therefore regarding it as a rigid element, avoiding any additional dynamics in the investigation. This leads to redefining the model by eliminating the extra DOFs previously considered in the mathematical model as done by [39]. The actuator model also had to be updated by obtaining the servovalve quasi-static gains again. Their dependency on operational parameters, such as the eccentricity and supply pressure, were not further studied and it was decided to work with a high pressure supply. Regarding the control design, only model-based control was considered. Optimal LQG regulators were designed. A pseudo modal reduction and the separation of complex states were also needed in order to implement a light discrete version of the controller in real time hardware. Opposed conclusions were obtained on the observability and controllability of the reduced system when compared to the previous work [40], probably due to the new test-rig setup. Finally, during the experimental implementation of the controller a spillover problem appeared around 60 Hz and justified on the un-modelled dynamics disregarded by the model reduction. The main discrepancies on the results were explained by the lack of accuracy of the current rotor-bearing model.

Despite the important advances in control design for the test rig developed from these projects, some improvements were still pending in order to match theory with experiment, as already pointed out by all previous authors. Assuming that the dynamics of the rotor-bearing system is well modelled, and to make the test rig elements more rigid, the ball bearing housing and sensor pedestals were stiffened by installing stiffeners and by making it more massive, respectively. With this, better results were expected.

Table 2.7: Summary of previous works carried out on the test rig.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Year</th>
<th>Passive Approach</th>
<th>Active Approach</th>
<th>Pad Modelling</th>
<th>TPJB Housing</th>
<th>BB Housing</th>
<th>Operational Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[39]</td>
<td>2009</td>
<td>Theoretical</td>
<td>Experimental</td>
<td>Rigid pad</td>
<td>Flexible</td>
<td>Flexible</td>
<td>1000rpm/60bar</td>
</tr>
<tr>
<td>[40]</td>
<td>2010</td>
<td>Theoretical</td>
<td>Experimental</td>
<td>Rigid pad</td>
<td>Flexible</td>
<td>Flexible</td>
<td>1000rpm/70bar</td>
</tr>
<tr>
<td>[41]</td>
<td>2011</td>
<td>Theoretical</td>
<td>experimental</td>
<td>Rigid Pad</td>
<td>Rigid</td>
<td>Flexible</td>
<td>1000rpm/70bar</td>
</tr>
</tbody>
</table>
The tilting-pad bearing was invented independently, and almost simultaneously, by Anthony Michell and Albert Kingsbury\(^1\) at the end of the first decade of the twentieth century. The first designs, which were focused on thrusting rotors, were rapidly implemented in centrifugal pumps, turbines, generators and marine applications. Doubtless, one of its major characteristics is its superior stability property when compared among fluid-film bearings [49]. This great merit, obtained due to the angular adaptive capability of the pads, allowed the bearing to face a variety of bearing loads and speeds, simultaneously reducing the wear and power consumption when compared with cylindrical ones. However, this was not recognized until the beginning of the second half of the century [14]. Since then, a great amount of research has been carried out aiming at a better understanding, modelling and designing of this type of bearing.

3.1 The Conventional TPJB Modelling

The cornerstone work on tilting-pad journal bearings was seamlessly developed by Lund [50] in his landmark paper of 1964. His work provided the first method

to calculate the stiffness and damping coefficients for a TPJB by assuming a synchronous movement of the pads and journal at the shaft angular speed. This approach allowed us to obtain the eight synchronously reduced dynamic coefficients, which can be plugged into any standard rotor-bearing model for its subsequent analysis. Although they are only appropriated for calculating the synchronous unbalance response of the rotor and not for stability analysis, as later clarified by Lund [51], they have been widely used in the industry, becoming an industrial standard [52]. Numerical approaches for obtaining bearing dynamic coefficients based on perturbation methods were also presented later in Lund and Thomsen [53] and Klit and Lund [54], solved by the finite difference and finite elements (variational approach), respectively. A step forward in the modelling of TPJBs was made by Allaire et al. [55], who introduced an approach for calculating the full bearing dynamic coefficients by including the pad’s degrees of freedom in the perturbation analysis, i.e. reflecting the effect of the journal perturbation on the pads and also perturbing the pad’s pitch equilibrium. This approach does not assume any predefined movement of the pads (condensing frequency) so that full dynamic properties matrices, which include the journal and pad DOFs, are obtained. The major advantage of this method is that it enables the analysis of the system stability considering the pad’s contribution.

Many authors have been devoted to incorporating further effects in the bearing modelling which dictate the static and dynamic properties of the bearing. Four approaches can be identified, namely: 1) the isothermal or isoviscous approach, 2) the elastohydrodynamic (EHD) approach, 3) the thermohydrodynamic (THD) approach and 4) the elastothermohydrodynamic (ETHD) approach. The EHD approach includes the effect of the pad flexibility in the determination of the bearing properties. This approach is particularly useful when the bearing is subdued to large bearing loads, i.e. when the pad deformation significantly influences the bearing equilibrium position. The THD approach focuses on including mainly the thermal contribution in the viscosity variation over the pad surface by means of the energy equation, but also in the heat transfer from/to the pad and surroundings, as well as in the thermal deformation induced in the pad. Evidently, the ETHD approach combines both, as well as relevant effects such as the pad pivoting. A mechanical pivot introduces an undamped connection between the pad and ground, which limits the effective bearing stiffness and damping. Deepening the contribution of each author is outside the scope of the thesis; however, for the sake of completeness the following can be cited: Ettles [56, 57], Fillon et al. [58], Brockwell and Dmochowski [59], Fillon et al. [60], Bouchoule et al. [61], Earles et al. [62, 63], Desbordes et al. [64], Kim et al. [65, 66], Suh and Palazzolo [67], among others. Dimond et al. [68] provides a review on TPJB theory and Dimond et al. [69] on experimental identification methods.
3.2 The Active TPJB Modelling

The modelling of the active counterpart of TPJBs to determining the bearing thermal, static and dynamic properties follows the ETHD approach and further incorporates the pressurized radial fluid injection, the pivot flexibility, the hydraulic and pipes dynamics as well as different heating sources into the formulation. The modelling is based on an isothermal model based on the modified version of the Reynolds Equation (MRE), firstly introduced in Santos [15] and thoroughly presented in Santos and Russo [19]. In the MRE the radial oil injection system was simply included through a second order differential equation representing the servovalve dynamics. The model was subsequently expanded in Santos and Nicoletti [20] to incorporate the temperature, hence viscosity, variation across the fluid-film utilizing a modified version of the energy equation which accounted for the radial oil injection of high pressurized oil. Such a THD model, which includes thermal effects, was formulated considering an adiabatic process, i.e. without transferring heat between the fluid-film and surroundings. Instead of moving the subsequent investigation towards an ETHD modelling, the research steered to incorporate the pad compliance, leading to the EHD approach.

The elastohydrodynamic modelling of ALBs was covered in Haugaard and Santos [27]. Further details are found in [70]. The inclusion of the pad flexibility into the bearing modelling is addressed through a finite element approach of the pad which is considered linearly elastic, isotropic and homogeneous. While the fluid domain is modelled through the MRE, the key equations for the pad domain were set through the principle of virtual work and discretised by means of 20-node second order Serendipity finite elements. Such a discretisation allowed the gentle coupling of the 3D-solid and 2D-fluid domains through the use of one of the element faces, leading to a straightforward way to face the fluid-structure interaction between the pad and the lubricant. The main contribution of such a work was to establish a neat way of including and evaluating the influence of the solid flexibility in the modelling. The influence of the lower pad’s modeshapes in the bearing static and dynamic properties was assessed under different conditions of speed and load. Additionally, the sensitivity of the results to relevant design parameters, such as the preload factor was also addressed. It was seen that for the case of hybrid lubrication the effect of the hydrostatic injection in the bearing dynamic properties cancel out the pad flexibility at high load condition. The use of the complete damping and stiffness matrices instead of the (synchronously) reduced ones for stability analysis was repeatedly stressed throughout the contributions [27–29, 70]. The over-prediction of stable regions of proportional and derivative gains was identified as a possible failure source if

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2Bending, skewing and flapping modes with natural frequencies of the order of kHz.
a rigid pad model is used in detriment of the flexible one [29]. The importance and benefit of placing the injection orifices far from the pivot line was also recognized, where the hydrodynamic pressure build up is weaker. One common thing studied under both approaches, i.e. THD and EHD, was the assessment of the bearing performance when the pad featured multi-orifices. Different orifices arrangements were studied in [21, 28, 29], leading to as a main conclusion the bearing enhancement, dynamically as well as thermally, when the pad rendered some orifices towards the leading edge, and emphasized under the condition of light load.

It was not until the work of Cerda [71] that both approaches were unified in the ETHD approach. Among the relevant effects included are the pipelines and pivot dynamics. The importance of the pivot dynamics into the conventional TPJB modelling is a very well known problem, as it can be corroborated by the large amount of literature on the subject, such as [72–75], among others. In short, what the pivot flexibility does to the bearing properties is to reduce the stiffness and damping properties. Another relevant dynamic contribution recognized when modelling the active TPJB is the pipelines dynamics due to the unavoidable entrained air in the hydraulic system; more precisely, the slender pipes connecting the servo valve outflow ports with their corresponding injectors. In this case, the oil film acoustic natural frequency might play an important role by delaying the flow going from the servo valve ports to the journal-pad clearance. This delay entails an undesired phase lag between the control action and the applied active force, as shown in [35].

In Cerda and Santos [33] the stability of a flexible rotor, an industrial gas compressor, was studied when the machine was undergoing three different lubrication regimes. Two developed with a conventional TPJB and one when featuring an active TPJB. The significant effect of temperature was recognized and the first bending mode on the bearing characteristics when featuring a TPJB without injection orifice. Certainly, these conclusions are also valid for active TPJB. Furthermore, the benefit of featuring a combined hydrodynamic and hydrostatic lubrication was highlighted as a means of postponing the onset speed of instability and thus extending the stable operation range of the machine. Two important aspects are gathered when improving the bearing characteristics through the hybrid lubrication: one is the placement of the orifice, which is towards the leading edge and the other one is the injection pattern. Although not specified by the authors, it is most likely that the injection of the high pressurized oil was set on all pads simultaneously; however, this configuration is hard to obtain due to normal pairing of two opposite pads with one servo valve, even more for a 5 pad bearing such as the one under study. The results obtained for the hybrid lubrication by means of a rigid-pad isothermal model were

\[^3\text{According to [76] it ranges between 6\%-20\%}.\]
extended in [34] to account for the pad deformation due to load and thermal expansion and at the same time, to account for the temperature build up in the fluid-film as well as for the heat transfer among fluid-film, pad and shaft. Once the model was validated, it was applied to the same flexible system aforesaid, reinforcing the previous conclusions obtained. Satisfactory results under the hybrid lubrication were again obtained with only one injection orifice placed toward the leading edge of the pad. Additionally, it was recognized that an increase to the injection pressure does not necessarily imply an improvement in the rotor response since the damping ratios of the different modes are affected differently. For instance, while the damping ratio defining the stability onset speed may increase, allowing the stability range to be extend, the damping ratio participating in the unbalance system response might decrease, obtaining a larger resonant peak. Again, the injection pattern was not explicitly defined but it is most likely that it was assumed to be a simultaneous injection from all pads. Within the framework of this research, this issue was experimentally addressed in [P1] in which different injection patterns are studied, such as from only bottom or top pads. It is also covered in [P5] under a model-based point of view. Furthermore, the plausible positioning of the injection nozzle is also studied in [P4], in which similar conclusions regarding the dynamic behaviour improvement were obtained when the injection nozzle was placed towards the leading edge compared with the pivoting line. In Cerda et al. [32] the thermal simplifications assumed to correctly model the heat transfer process between the oil film and pad surface were addressed. In Varela et al. [30] the ETHD model for controllable TPJBs was statically validated against experimental results obtained with the small test rig. Subsequently, the dynamic validation of the controllable TPJB properties, i.e. bearing stiffness and damping, was carried out in [31] using a single-pad bearing configuration. Therein, the results were presented in a condensed way, in which the control law was included as a part of the bearing properties. Table 3.1 summarized the main contributions.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Model</th>
<th>Numerical Approach</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Santos and Russo</td>
<td>Isothermal</td>
<td>Finite Differences</td>
<td>Multi-orifice</td>
</tr>
<tr>
<td>Santos and Nicoletti</td>
<td>THD</td>
<td>Finite Differences</td>
<td>Multi-orifice</td>
</tr>
<tr>
<td>Haugaard and Santos</td>
<td>EHD</td>
<td>Finite Elements</td>
<td>Static validation</td>
</tr>
<tr>
<td>Varela et al.</td>
<td>TEHD</td>
<td>Finite Elements</td>
<td>Single pad dynamic validation</td>
</tr>
<tr>
<td>Varela and Santos</td>
<td>TEHD</td>
<td>Finite Elements</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Main contributions to the controllable TPJB modelling.
3.3 The Controllable Bearing Model

Since the aim of this research is to design model-based controllers for attenuating the lateral vibration of machines with flexible rotors, it is out of the scope to deepen or to extend the current bearing model. Furthermore, this model has been rigorously static and dynamic validated in a series of works [30, 31]. Therefore, this is used here to calculate the bearing dynamic properties used for modelling the entire rotor-bearing system. Nonetheless, some relevant issues are briefly discussed. In the following, the key equations and their main assumption are summarized. These well-known equations are exhaustively covered in [70, 71] and the reader is advised to refer to those for a complete review. The summarized equations here, follow the notation used in reference [71] where the reference coordinate system are those of Figure 3.1 a). The equations are presented for an active TPJB with two pads and one servovalve, as shown in Figure 3.1 b).

3.3.1 Key Equations

- Lubrication theory and dynamics:

1. The Modified Reynolds Equation in pads i=A,B:

   \[ \frac{\partial}{\partial \hat{x}} \left( \frac{h_i^3}{\mu} \frac{\partial p_i}{\partial \hat{x}} \right) + \frac{\partial}{\partial \hat{z}} \left( \frac{h_i^3}{\mu} \frac{\partial p_i}{\partial \hat{z}} \right) = 6U \frac{\partial h_i}{\partial \hat{x}} + 12 \frac{\partial h_i}{\partial t} + 12V_{inj}^i \quad (3.1a) \]
with:

\[ V_{inj}^i = \begin{cases} \frac{p_i - P_{A,B}}{4\mu_0} \left( \frac{d_i^2}{4} - r_i^2 \right) & r_i^2 \leq \frac{d_i^2}{4} \\ 0 & r_i^2 > \frac{d_i^2}{4} \end{cases} \]  \tag{3.1b}

where:

\[ r_i^2 = (\hat{x} - \hat{x}_0)^2 + (\hat{z} - \hat{z}_0)^2 \]  \tag{3.1c}

2. Hagen-Poiseuille injection flow in pads i=A,B:

\[ q_{inj}^i = \int_{S_0} V_{inj}^i dS \]  \tag{3.2}

- **Hydraulics:**

1. Flow in servovalve ports A and B:

\[ q_{A,B} = q_{leak} \pm q_{x,v} \mp K_{pq}(P_A - P_B) \]  \tag{3.3}

2. Servovalve spool-driven flow:

\[ \ddot{q}_{x,v} + 2\xi \omega_v \dot{q}_{x,v} + \omega_v^2 q_{x,v} = R_v \omega_v^2 u \]  \tag{3.4}

3. Servovalve amplifier dynamics:

\[ \tau_a \dot{i} + i = k_a v \]  \tag{3.5}

- **Thermal effects:**

1. Energy equation:

\[
\begin{aligned}
&h \frac{\partial^2 T}{\partial x^2} + h \frac{\partial^2 T}{\partial z^2} + \left( \frac{h^3}{12\mu_0} \frac{\partial p}{\partial \hat{x}} - \frac{\Omega R h}{2\alpha} \right) \frac{\partial T}{\partial \hat{x}} + \frac{h^3}{12\mu_0} \frac{\partial p}{\partial \hat{z}} \frac{\partial T}{\partial \hat{z}} + (\Omega R)^2 \frac{\mu}{h\kappa} + \frac{S_{oil}}{\kappa} = \\
&- \frac{h^3}{12\mu_0} \left( \left( \frac{\partial p}{\partial \hat{x}} \right)^2 + \left( \frac{\partial p}{\partial \hat{z}} \right)^2 \right) - \left[ T_{inj} - T \right] \left( \frac{1}{I_0} + \frac{V_{inj}}{\alpha} \right) + \frac{V_{inj}}{\kappa} \left( p + \frac{4\mu}{3h} V_{inj} \right) \Bigg|_{S_0}
\end{aligned}
]  \tag{3.6}

2. Viscosity law:

\[ \mu(T) = \mu_{ref} e^{\beta(T-T_{ref})} \]  \tag{3.7}

- **Elastic effects (Pseudo-Modal Reduction):**

\[
\begin{aligned}
\ddot{V}_{M}^{T} \ddot{V}_{q} + \ddot{V}_{D}^{T} \ddot{V}_{q} + \ddot{V}_{K}^{T} \ddot{V}_{q} = \ddot{V}_{F}^{T} \\
\dot{M}_{q} + \dot{D}_{q} + \dddot{K}_{q} = \dddot{f}
\end{aligned}
]  \tag{3.8}

\[ \dddot{V}_{F}^{T} \]  \tag{3.9}
3.3.2 On the Boundary Conditions of the MRE

The solutions of the MRE and the Energy equation imply defining adequate boundary conditions within the injection area $S_0$. The injection velocity profile is approximated by a quadratic function derived by assuming a fully developed Hagen-Poiseuille laminar flow in the injector. Such a condition plus the null velocity in the radial direction of the rest of the fluid-film area entail the definition of a continuous piecewise function whose derivative is discontinuous in the limit of the injector. Likewise, an assumption is made for the injected flow temperature in order to solve the energy equation. In such a case, a constant temperature is assumed valid in the injector area, whereas the rest of the fluid-film surface features a different temperature, which depends on the selected approach, commonly the pad temperature is assumed for the fluid-film, see [71]. In this case, discontinuous piecewise functions are used for the temperature boundary condition and its spatial derivative. Such conditions might pose numerical problems for the resolution of the set of equations linked to the ETHD controllable bearing model, especially when the hydrostatic effect predominates over the hydrodynamic one. Such conditions entail discontinuous piecewise function for the pressure and temperature distributions. Aiming at avoiding such numerical instabilities when obtaining the system equilibrium position, defined by the fluid-film pressure and temperature distributions, the use of a smoothed injection velocity profile is suggested.

3.3.2.1 Current approach for $V_{inj}$

The current approach for the injection velocity profile is based on the Hagen-Poiseuille fully developed laminar flow for the injector, as originally proposed by Santos and Russo [19]. The injection velocity profile is expressed in terms of a weighting function $G(\hat{x}, \hat{z})$ as:

$$V_{inj} = \frac{\Delta P}{4\mu_0 l_0} \begin{cases} G(\hat{x}, \hat{z}) & (\hat{x} - \hat{x}_0)^2 + (\hat{z} - \hat{z}_0)^2 \leq \frac{d_0^2}{4} \\ 0 & (\hat{x} - \hat{x}_0)^2 + (\hat{z} - \hat{z}_0)^2 > \frac{d_0^2}{4} \end{cases} \quad (3.10)$$

with:

$$G(\hat{x}, \hat{z}) = \left[ \frac{d_0^2}{4} - [(\hat{x} - \hat{x}_0)^2 + (\hat{z} - \hat{z}_0)^2] \right] \quad (3.11)$$

where in Equation (3.11) $\Delta P = p(\hat{x}, \hat{z}) - P_{inj}$. Its main characteristics can be summarized as:

a) It is a piecewise function, null outside of the injection area $S_0$. 

3.3 The Controllable Bearing Model

b) Its spatial derivative is a discontinuous function, hence it also turns the spatial derivative of the pressure discontinuous.

3.3.2.2 Proposed Approach of $V_{inj}^*$

In an attempt to overcome numerical instabilities due to the limitations of the previous approach, a Gaussian function for $V_{inj}$ is proposed. A two dimensional Gaussian function centred at the injector position $(\hat{x}_0, \hat{z}_0)$ and with equal variance $\sigma$ in both directions $\hat{x}$ and $\hat{z}$ is utilized. Similarly, to weight the amplitudes of both functions, the Gaussian is pre-multiplied by the constant $2\sigma^2$, in which $\sigma$ is to be determined. Thus, the injection velocity is redefined as:

$$V_{inj}^* = \frac{\Delta P}{4\mu_0 l_0} \tilde{G}(\hat{x}, \hat{z})$$

with:

$$\tilde{G}(\hat{x}, \hat{z}) = 2\sigma^2 e^{-\frac{1}{2\sigma^2}[(\hat{x}-\hat{x}_0)^2+(\hat{z}-\hat{z}_0)^2]}$$

Equation (3.12) is valid in the whole fluid domain and $\sigma$ in Equation (3.13) is obtained by equalling the inlet flowrate of both approaches, which produces:

$$q_{inj} = \int V_{inj}(\hat{x}, \hat{z})dA = \int V_{inj}^*(\hat{x}, \hat{z})dA$$

$$\frac{\pi}{2} \frac{\Delta P}{4\mu_0 l_0} r_0^3 = \pi \frac{\Delta P}{4\mu_0 l_0} 4\sigma^4$$

obtaining:

$$\sigma = \frac{r_0}{2\hat{x}}$$

As a result, the suggested injection velocity profile can be written as:

$$V_{inj}^* = \frac{\Delta P}{4\mu_0 l_0} \frac{r_0^2}{\sqrt{2}} e^{-\frac{1}{r_0^2}[(\hat{x}-\hat{x}_0)^2+(\hat{z}-\hat{z}_0)^2]}$$

The characteristics of such an approach are:

- Continuous function valid on the whole pad area $S$.
- Its derivative is a continuous function, hence it avoids numerical problems due to discontinuities.
What is obtained is an MRE which is valid in the whole pad domain with the variance defined in terms of the injection area radius as $\sigma = r_0/2^{0.75}$. Figure 3.2 and Table 3.2 compare the profiles and main flow quantities for a standard pad, while Figure 3.3 overlaps the profile cross sections, along the circumferential direction $\hat{x}$ at the mid of the length span, i.e. $\hat{z} = 0$. By respecting the injected flowrate, the correction of the weighting function $G(\hat{x}, \hat{z})$ lessens the maximum velocity and produces a larger area in which the fluid features radial velocity at the pad surface compared to the injection area. According to calculations, the maximum and mean velocities are reduced by a factor of $\sqrt{2}/2$ (about 29.3% less) when the area is increased by a factor of $\sqrt{2}$ (about 41% more). Figure 3.3 includes all the main inflow quantities for the studied pad, when subjected to a difference in pressure of about 5 bar$^4$. Although these changes do not seem to be significant, their benefits and drawbacks are to be studied with regard to numerical instabilities, especially for lubrication regimes featuring large hydrostatic injection. The first attempts were made in [77]. The suggested approach to approximate the injection velocity profile $V_{inj}^*$, also affects the Energy Equation utilized to determine the temperature distribution of the oil film. Therefore, its effect on the Energy equation should be also studied if the Gaussian weighting function $G(\hat{x}, \hat{z})$ is assumed.

### Table 3.2: Flow quantities comparative

<table>
<thead>
<tr>
<th>Approach</th>
<th>$V_{max}$</th>
<th>$\bar{V}$</th>
<th>$q_{inj}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hagen-Poiseuille ($V_{inj}$)</td>
<td>$\frac{\Delta P r_0^2}{4 \mu_0 l_0}$</td>
<td>$\frac{\Delta P r_0^2}{8 \mu_0 l_0} \hat{z}$</td>
<td>$\Delta P \frac{r_0^4}{2 \mu_0 l_0}$</td>
</tr>
<tr>
<td>Gaussian ($V_{inj}^*$)</td>
<td>$\sqrt{2} \frac{\Delta P r_0^2}{4 \mu_0 l_0}$</td>
<td>$\sqrt{2} \frac{\Delta P r_0^2}{8 \mu_0 l_0} \hat{z}$</td>
<td>$\Delta P \frac{r_0^4}{2 \mu_0 l_0}$</td>
</tr>
</tbody>
</table>

$^\dagger$ Calculated for a 95% of confidence level. I.e. $r_0^* = 2 \sigma = \sqrt{2} r_0$.

$A^* = \pi r_0^*^2 = \sqrt{2} \pi r_0^2$.

### 3.4 Theoretical Calculation of ALB Coefficients

$^4\Delta P = \bar{p}(\hat{x}, \hat{z}) - P_{inj}$. With $\bar{p}$ the averaged pressure in the injection orifice.
3.4 Theoretical Calculation of ALB Coefficients

(a) $V_{in,j}$ profile obtained with Hagen-Poiseuille. (b) $V_{in,j}^*$ profile smoothed with Gaussian function.

**Figure 3.2:** Different velocity profiles for $V_{in,j}$. Zoom close to the injection area of the pad. Injector length and diameter of: $l_0=20\text{ mm}$ and $d_0=3\text{ mm}$.

DOF in the bearing force coefficient matrices. Therefore, the nozzle module allows us to calculate coefficients only for the passive and hybrid lubrication regimes, while the valve one puts no restrictions on all three of them. Depending on what conditions are available for the calculation, i.e. injection pressures or flows, it is more appropriate to utilize one module over the other. Precise measurements of these values on each pad are difficult to obtain, leading to using their best estimates based on previous knowledge. Based on Figure 3.4 and Table 3.3 some guidelines are provided for the calculations.

**Figure 3.3:** Comparison of velocity profiles. Cross section at $\hat{z} = 0$. for the same inflow rate $q_{in,j}$. 

![Velocity profiles comparison](image)
### Table 3.3: Parameter values for ALB force coefficients calculation with the Fortran code.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Nozzle</th>
<th>Valve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive</td>
<td>$P_{inj} = {0 0 0 0}$</td>
<td>$q_0 = -{q_0, q_0, q_0, q_0}$</td>
</tr>
<tr>
<td>Hybrid, Leakage</td>
<td>$P_{inj} = {P_{inj_1}, P_{inj_2}, P_{inj_3}, P_{inj_4}}$</td>
<td>$q_0 = {q_{leak_{1}}, q_{leak_{2}}, q_{leak_{3}}, q_{leak_{4}}}$</td>
</tr>
<tr>
<td>Hybrid, Upward</td>
<td>$P_{inj} = {0 0 P_{inj_3}, P_{inj_4}}$</td>
<td>$q_0 = {-q_0, q_0, q_0, q_0}$</td>
</tr>
<tr>
<td>Hybrid, Downward</td>
<td>$P_{inj} = {P_{inj_1}, P_{inj_2}, 0 0}$</td>
<td>$q_0 = {q_0, q_0, q_0, -q_0, -q_0}$</td>
</tr>
</tbody>
</table>

- For the passive lubrication regime the injection system is turned off, therefore the bearing force coefficients can be calculated by prescribing the reservoir pressure, normally the atmospheric or zero gauge pressure, as the injection pressure in the inlet boundary of the nozzle. This zero pressure entails a negative or backward flow toward the reservoir. Measures of this backward flow can be utilized with the valve module.

- For the hybrid lubrication regime, there are three relevant cases. In the leakage case, the leakage flow of both servovalves is known, then this can be utilized in the valve module for the calculations. Knowing the injection pressure of each pad is more complicated and thus, using the nozzle module is unsuitable.

- In an upward or downward injection scheme, the prescription of the injection pressure in the bottom or upper pads allows us to estimate the bearing force coefficients by using the nozzle module. The measure of these quantities is in practice difficult. This has led to some discrepancies when comparing calculated against experimental results [P3]. Better results might be obtained if pad flow measurements are accessible. Further works should address this problem.

Evidently, for the active lubrication regime, this discussion is not needed since the valve module is the one that accounts for the servovalve dynamics. An accurate characterization of the servovalve will produce the best results. This means obtaining the servovalve parameters $\{\omega_v, \xi_v, R_v\}$ for characterizing the spool-driven flow, the servovalve leakage flow $q_{leak_v}$, and the pressure driven flow constant $K_{pq_v}$ to jointly determine the servovalve flow.

### 3.4.1 On the Normalization of the ALB Force Coefficients

The pseudo-modal reduction implemented in the ETHD Fortran code to include the pad flexibility considers a Euclidean normalization of the pad modeshapes,
hence the resulting bearing force coefficients are expressed in terms of modal coordinates. In order to express them in terms of physical coordinates so that they can be used in the beam finite element formulation of the rotor, the following procedure is carried out. Taking advantage of the inertia matrix of the bearing, which is known in physical units, two pseudo-modal reductions are compared. The equation of motion for the N DOFs of the pad finite elements discretization can be reduced to only n DOFs by constructing the pseudo-modal matrix $\tilde{V}$ containing only the n modes of interest such as the journal ones, the pad tilt and vertical displacement.

1. In the Fortran code the modeshapes are normalized in such a way that the square root of sum of squared values equals one. Hence, the N DOFs $x$ can be written in terms of the n nodal DOFs $q$ as $x = \tilde{V}q$. By replacing this in the equation of motion and left-multiplying by the transpose, it is obtained:

$$\tilde{V}^T \tilde{M} \tilde{V} \ddot{q} + \tilde{V}^T \tilde{D} \dot{\tilde{V}} \ddot{q} + \tilde{V}^T \tilde{K} \tilde{V} q = \tilde{V}^T F$$ (3.18)

$$\tilde{M} \dddot{q} + \tilde{D} \dddot{q} + \tilde{K} q = \tilde{f}$$ (3.19)

2. Let us normalize the modeshapes in such a way that the obtained modal coordinates $q'$ are expressed in physical units. Therefore, the N DOFs $x$ can be written in terms of the n modal DOFs $q'$ as $x = Vq'$. Subsequently, it is obtained:

$$V^T \tilde{M} \dddot{q}' + V^T \tilde{D} \dddot{q}' + V^T \tilde{K} V q' = V^T F$$ (3.20)

$$\tilde{M} \dddot{q}' + \tilde{D} \dddot{q}' + \tilde{K} q' = f$$ (3.21)
Both pseudo-modal matrices \( \tilde{V} \) and \( V \) form the basis of the same subspace, therefore the eigenvectors of the matrices are related to each other through scalars \( A_i \), hence \( \tilde{V} = VA \) or:

\[
\begin{bmatrix}
\tilde{V}_1^1 & \cdots & \tilde{V}_1^n \\
\vdots & & \vdots \\
\tilde{V}_m^1 & \cdots & \tilde{V}_m^n
\end{bmatrix}_{m \times n} =
\begin{bmatrix}
V_1^1 & \cdots & V_1^n \\
\vdots & & \vdots \\
V_m^1 & \cdots & V_m^n
\end{bmatrix}_{m \times n}
\begin{bmatrix}
A_1 & \cdots & 0 \\
\vdots & & \vdots \\
0 & \cdots & A_n
\end{bmatrix}_{n \times n}
\]

(3.22)

The scalar matrix \( A \) is obtained via comparison of the inertia matrices \( M \) and \( \tilde{M} \), since both are known. \( \tilde{M} \) is a code outcome and \( M \) is built with the masses and inertia of the journal and pads. Applying Equation (3.22) and the mass matrices definitions, it can be stated:

\[
\tilde{M} = \tilde{V}^T M \tilde{V} = A^T \tilde{V}^T M \tilde{V} A
\]

(3.23)

\[
\tilde{M} = A^T MA
\]

(3.24)

Since all matrices in (3.24) are diagonal, then the scalar values can be obtained as:

\[
A_i = \sqrt{\tilde{M}_i/M_i} \quad i = 1 \ldots n
\]

(3.25)

Finally, the mass, stiffness and damping matrices in physical units can be obtained as:

\[
M = (A^T)^{-1} \tilde{M} A^{-1}; \quad K = (A^T)^{-1} \tilde{K} A^{-1}; \quad D = (A^T)^{-1} \tilde{D} A^{-1}
\]

(3.26)

To illustrate the matrix changes, let us refer to the contribution \([P4]\). Therein, three DOFs were taken into account: the journal vertical displacement \([\text{m}]\), the pad tilt angle \([\text{rad}]\) and the pad vertical movement \([\text{m}]\). For the case of an angular velocity of 1650 rpm and a bearing load of 1400 N the outcome and corrected matrices for pad C are:

\[
M = \begin{bmatrix}
27 & 0 & 0 \\
0 & 6.59 \cdot 10^{-4} & 0 \\
0 & 0 & 0.81
\end{bmatrix}, \quad \tilde{M} = \begin{bmatrix}
27 & 0 & 0 \\
0 & 1.02 \cdot 10^{-4} & 0 \\
0 & 0 & 1.08 \cdot 10^{-4}
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
2654 & -32 & 2899 \\
-32 & 1.2 & -35 \cdot 10^2 \\
2899 & -35 & 3238
\end{bmatrix}, \quad \tilde{D} = \begin{bmatrix}
2654 & -12.4 & 33.5 \\
-12.4 & 018 & -0.16 \\
33.5 & -0.16 & 0.43
\end{bmatrix} \cdot 10^2
\]

\[
K = \begin{bmatrix}
30145 & -1851 & 32922 \\
-1851 & 36 & 64 \\
32922 & -2021 & 83664
\end{bmatrix} \cdot 10^3, \quad \tilde{K} = \begin{bmatrix}
30145 & -727 & 380 \\
-727 & 6 & 0.3 \\
380 & -9 & 11
\end{bmatrix} \cdot 10^3
\]

(3.27)
3.4 Theoretical Calculation of ALB Coefficients

**Figure 3.5:** Stiffness and damping force coefficients for the ALB with different pads obtained under the studied operational condition. Bearing load of 1400 N. Single pad configuration and flexible pivot.

By condensing away the pad DOFs, the bearing force coefficients can be expressed only in terms of the journal one, allowing the comparison of both set of matrices in frequency domain. Figure 3.5 depicts a continuous line for the different pad designs of contribution [P4], the coefficients obtained from matrices $\mathbf{M} \mathbf{D} \mathbf{K}$ in physical units, whereas the coefficients obtained through $\tilde{\mathbf{M}} \tilde{\mathbf{D}} \tilde{\mathbf{K}}$ in modal coordinates are identified by circle marks for pad C.

Figures 3.6 and 3.7 show the theoretical bearing force coefficients obtained for the system under the first two configurations of Table 3.4 and utilized in the contributions [P5] and [P6]. These configuration are also rendered in Figure 3.8. In the plots, the values have also been condensed: continuous lines represent the values in physical coordinates while discrete marks represent the values in modal ones. In the calculation of the bearing coefficients a pivot stiffness of $4 \cdot 10^8$ N/m has been considered for all pads, independent of the bearing static load. In the figures, no significant differences are observed among the different loads and lubrication regimes. The frequency dependence of the bearing
dynamic properties can be seen. Direct stiffness increases with the frequency while the direct damping reduces. Cross bearing force stiffness and damping coefficients are close to null.

**Table 3.4:** Static bearing loads for the different rotor-bearing configurations used in [P5] and [P6].

<table>
<thead>
<tr>
<th>Test-rig configuration</th>
<th>Details</th>
<th>Static bearing load (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No disc configuration</td>
<td>shaft only</td>
<td>400</td>
</tr>
<tr>
<td>One disc configuration</td>
<td>shaft+80 mm disc</td>
<td>880</td>
</tr>
<tr>
<td>Two discs configuration</td>
<td>shaft+80 and 100 mm discs</td>
<td>1440</td>
</tr>
</tbody>
</table>
3.4 Theoretical Calculation of ALB Coefficients

Figure 3.7: Theoretical bearing force coefficients under 880 N of static loading. One disc configuration.

Figure 3.8: Test-rig configurations.

3.5 Results [P3]: Experimental Identification of ALB Force Coefficients

Publication [P3] deals with the experimental identification of ALB force coefficients. The work goal was to show the modification of the bearing dynamic properties via the active lubrication and not to validate their theoretical calculation. To appreciate easily the variations on the bearing coefficients, a light-load condition was established by using the rotor without discs and at the same time, applying an upward vertical load with the active magnetic bearing. This loading condition may closely resemble the vertical rotor supported by TPJBs. Two main outcomes are highlighted:

1. The hybrid lubrication regimes can increase the bearing stiffness asymmetry and reduce the cross-coupling coefficients, when compared to the passive
3.5 Results [P3]: Experimental Identification of ALB Force Coefficients

![Graphs of stiffness and damping for different controllers](image)

**Figure 3.10:** Identified dynamic coefficients for the ALB. (a) and (b): ALB under feedback-controlled lubrication regime, control law #1 (dashed lines (--)), \( k_p = -30 \text{kV/m} \) & \( k_d = 20 \text{Vs/m} \). (c) and (d): ALB under feedback-controlled lubrication regime, control law #2 (dotted lines (··)), \( k_p_1 = -30 \text{kV/m} \) & \( k_p_2 = -30 \text{kV/m} \). Results obtained under the leakage hybrid case are superimposed with solid lines (••) as a benchmark.

2. The feedback-controlled lubrication based on PD control laws, clearly modifies the ALB dynamic properties. However, these laws must be carefully chosen in order to produce improvements in the whole rotor-bearing system behaviour. Figure 3.10 depicts two control laws, one of them increases the cross-coupling coefficients while the other one does not.

One of the striking issues of this study was the significant bearing cross-coupling coefficients obtained under the studied conditions. It seems that they cannot be neglected under light-load conditions. This claim was also pointed out later by [78], in which the authors established that they can be of the same order of magnitude as the direct ones for vertical machines. Doubtlessly, more research is needed on the TPJBs behaviour under conditions of light loading. The challenging aspect is related to the bearing dynamic coefficients identification in flexible rotors. A great number of publications deal with the experimental
identification of bearing force coefficients [79, 80]. Most of these contributions are advocated to obtain the eight linearized bearing force coefficients in rigid rotor apparatus such as [81, 82]. With regard to flexible rotors, few publications were found [83, 84]. The main problem among them is the large amount of information required to obtain the coefficients due to the shaft flexibility. Wang and Maslen [83] proposed an approach for flexible shafts which requires few measurement points. This procedure was utilized here. Details can be found in [83] or summarized in the contribution [P3]. The bearing dynamic properties, as in most of the approaches, are obtained as the real and imaginary parts of the complex bearing impedance function $H_b(i\omega)$ as:

$$[K_b] = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} = \Re\{H_b(i\omega)\} \quad \quad [D_b] = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix} = \frac{\Im\{H_b(i\omega)\}}{\omega}$$

(3.28)

Uncertainty in the identified coefficients is estimated according to the guidelines given by Moffat [85] and [86]. These guidelines consider both uncertainties in the model as well as in the measurements. Uncertainty in calculating the FRFs was disregarded due to the large number of averages utilized, which according to [87] can be estimated through the standard deviation ($\sigma_{FRF}$) in terms of the coherence function ($\gamma$) and averages ($N_{FRF}$) as:

$$\sigma_{FRF} = \frac{\sqrt{1 - \gamma^2}}{|\gamma|\sqrt{2N_{FRF}}}$$

(3.29)

### 3.5.1 Additional Results on ALB Force Coefficients Identification

Additional identifying of bearing force coefficients under larger static bearing loads were attempted with poor results due to the increasing coupling between the rotor-bearing and foundation dynamics when adding additional discs. The identification method requires an accurate model and since the foundation dynamics is not included, questionable results might be yielded. Nonetheless, in addition to those results presented in the contribution [P3], it was possible to obtain results for the rotor without discs, i.e., with 400 N of static load. Figures 3.11, 3.12 and 3.13 show the eight linearized bearing force coefficients at different angular speeds, 1000 rpm, 2000 rpm and 3000 rpm, respectively. Deterministic noise in the results is due to residual vibration at the rotor angular speed. Unlike the results presented in [P3], the cross-coupling stiffness coefficients are significantly smaller than the direct ones, which resembles more closely the behaviour of tilting pad bearings, supporting horizontal machines. Furthermore, a slight increase in the stiffness direct coefficients with the frequency is
3.5 Results [P3]: Experimental Identification of ALB Force Coefficients 49

Figure 3.11: Identified bearing force coefficients at 1000 rpm. No disc configuration. Bearing static loading of 400 N.

Figure 3.12: Identified bearing force coefficients at 2000 rpm. No disc configuration. Bearing static loading of 400 N.

observed. No big differences are found among the different angular velocities tested besides a slight increase in uncertainty. Finally, damping decreases with the frequency, as expected.
Figure 3.13: Identified bearing force coefficients at 3000 rpm. No disc configuration. Bearing static loading of 400 N.
Chapter 4

The Flexible Rotor - ALB System Modelling

4.1 Flexible Rotor Finite Element Model

The modelling of rotating machinery for analysing its dynamic behaviour is a well established tool with a significant amount of literature covering the topic [88–91]. It is based on the finite element method in which the continuous solid domain of the shaft is discretized in small beam elements in which all governing equations are satisfied. In it, the contributions from machine elements such as: impellers, rotors, couplings, bushes and bearings are introduced at nodes linking two shaft elements. The most cited references on rotor modelling through finite element methods date from the mid-1970s and correspond to Nelson and McVaugh [92] and Nelson [93] advocated by the Euler-Bernoulli and Timoshenko approaches, respectively.

In this work, the mass, stiffness and damping matrices are defined as given in [92] and included in Appendix E for the sake of completeness. Figure 4.1 shows a scheme of the discretization utilized for modelling the shaft, also detailed in Table 4.1. As usual, nodes are placed at relevant positions along the shaft such as every sensor station, disc and bearing position so that all input and output of the system can be monitored. Additional nodes are introduced
Figure 4.1: Finite element discretization scheme.

Table 4.1: 26 finite elements discretization.

<table>
<thead>
<tr>
<th>#</th>
<th>$L_s$ [mm]</th>
<th>$D_s$ [mm]</th>
<th>#</th>
<th>$L_s$ [mm]</th>
<th>$D_s$ [mm]</th>
<th>#</th>
<th>$L_s$ [mm]</th>
<th>$D_s$ [mm]</th>
<th>#</th>
<th>$L_s$ [mm]</th>
<th>$D_s$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26</td>
<td>40</td>
<td>8</td>
<td>100</td>
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<td>22</td>
<td>10</td>
<td>65</td>
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<tr>
<td>2</td>
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<td>100</td>
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<td>10</td>
<td>65</td>
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<td>3</td>
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<td>50</td>
<td>10</td>
<td>90</td>
<td>90</td>
<td>17</td>
<td>5</td>
<td>90</td>
<td>24</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>11.5</td>
<td>50</td>
<td>11</td>
<td>75</td>
<td>90</td>
<td>18</td>
<td>40</td>
<td>90</td>
<td>25</td>
<td>7.5</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
<td>70</td>
<td>12</td>
<td>75</td>
<td>90</td>
<td>19</td>
<td>40</td>
<td>70</td>
<td>26</td>
<td>7.5</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>40</td>
<td>90</td>
<td>13</td>
<td>45</td>
<td>90</td>
<td>20</td>
<td>50</td>
<td>70</td>
<td>26</td>
<td>7.5</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>90</td>
<td>14</td>
<td>60</td>
<td>99.78</td>
<td>21</td>
<td>45</td>
<td>70</td>
<td>26</td>
<td>7.5</td>
<td>40</td>
</tr>
</tbody>
</table>

at every change in diameter to refine the discretization. Geometrical and inertial properties of the main rigid discs are presented in Table 4.2. Nodes 19 and 21 are utilized to include the contribution of the discs, bearing in mind

Table 4.2: Properties of rigid bodies of the rotor.

<table>
<thead>
<tr>
<th>Mass</th>
<th>$W_r$ [m]</th>
<th>$\phi_{in,r}$ [m]</th>
<th>$\phi_{out,r}$ [m]</th>
<th>Weight [kg]</th>
<th>$I_{Pr}$ [kg · m²]</th>
<th>$I_{Tr}$ [kg · m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disc 80 mm</td>
<td>0.080</td>
<td>0.070</td>
<td>0.290</td>
<td>37.30</td>
<td>0.286</td>
<td>0.417</td>
</tr>
<tr>
<td>Disc 100 mm</td>
<td>0.100</td>
<td>0.070</td>
<td>0.290</td>
<td>45.98</td>
<td>0.407</td>
<td>0.518</td>
</tr>
<tr>
<td>Sleeve</td>
<td>0.099</td>
<td>0.070</td>
<td>0.098</td>
<td>3.08</td>
<td>1.26</td>
<td>1.26</td>
</tr>
<tr>
<td>Excitation Bearing</td>
<td>0.070</td>
<td>0.140</td>
<td>6.48</td>
<td>0.40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
that three configurations can be featured by hanging none, one and two discs. These affect the static bearing load and increase the gyroscopic effect in the system. The actuator positions correspond to node 15, where the ALB is placed. Including the actuating capabilities in the bearing might be considered as not optimal since the active force will be always applied close to a system node. However, one strategy to overcome this difficulty consists of turning the ALB softer in closed-loop. Displacement sensors are placed in nodes, 7, 10, 13, 17 and 23. As explained in Appendix C only probes from 1 to 4 are considered for control purposes. The rest of them were utilized to complement measurements. Displacement sensors 1 and 2 are well placed for measuring the displacement yielded by the lower system modeshapes. Although studies based on the controllability and observability can be carried out to determine the best placement of the actuators and sensors [94, 95], in this work the layout previously described is kept for the reasons already exposed.

Figure 4.2 provides a convergence study of the first three natural frequencies of the shaft in a free-free support condition. The estimated values are contrasted against experimental ones provided in reference [37]. Two approaches were
utilized, i.e. Euler-Bernoulli and Timoshenko. It is noted that convergence is obtained for a discretization above 19 elements for both approaches. Therefore, the discretization utilized in this work suffices the requirements of convergence of the finite element analysis. Since there are no large differences when using any of the approaches, meaning that the shear effects are negligible for this shaft – as expected –, the shaft is modelled with the Euler-Bernoulli approach. Under this approach errors less than -1%, 3% and 1% are found for the three first natural frequencies, respectively.

4.2 Inclusion of the ALB in the Flexible Rotor-Bearing System Model

Bearing dynamic properties for ALBs are covered in detail in chapter 3 and the way these properties, in the form of stiffness and damping force coefficients,
are included in the rotor modelling is explained in contribution [P5] and also reviewed in chapter 6. Figure 4.3 and 4.4 complement this by showing the structure of the bearing force coefficient matrices in terms of the degrees-of-freedom introduced in Table 4.3, which is taken from [P5]. In the abscissa and ordinate are represented the number of the degrees-of-freedom. In blue are depicted the dofs associated with the rotor (mechanical associated with shaft elements), whereas the red ones are linked to the ALB (mechanical associated with the pad and hydraulic ones). Two features can be highlighted: 1) the matrices are sparse, and 2) the link between the hydraulic and mechanical systems is made only in the stiffness matrix, as can be deduced by comparing both figures 4.3 and 4.4. Specifically, the link is made among the rows from 109 to 120 and columns 121 and 121, which correspond to the pads and hydraulic dofs. As discussed in contribution [P5], since the bearing dynamic properties already include the hydraulic contribution from the servo valve via the spool-driven flow dof, there is no need to characterise the active force, as previously done in [40, 41].
4.3 Servovalve Transconductance Amplifier

The dynamic of the servovalve amplifier, MOOG NE 118-205-009, can be described as a first-order system for which the output current \( i(t) \) is driven in terms of an input voltage \( v(t) \) as:

\[
L_a i'(t) + R_a i(t) = v(t)
\]

\[
\tau_a i'(t) + i(t) = kav(t)
\]

where the time constant \( \tau_s \) and gain \( k \) are defined in terms of the amplifier resistance \( R \) and inductance \( L \):

\[
\tau_a = L_a R_a^{-1}; \quad k_a = R_a^{-1}
\]

4.3.0.1 Amplifier parameters identification

Assuming a first order equation for the servovalve amplifiers of Equation (4.1b), the frequency response function reads:

\[
FRF(\omega j) = k_a \frac{1}{\tau_a \omega j + 1}
\]

By knowing the experimental FRFs and separating real and complex terms, the amplifier parameters can be fitted as:

\[
R_a = \Re\{FRF^{-1}\}; \quad L_a = \frac{1}{\omega} \Im\{FRF^{-1}\}
\]

Figure 4.5 shows the FRFs of the servovalve 1 amplifier. These functions were obtained with the rotor-bearing system loaded with one disc at two different angular speeds. Two supply pressures of the hydraulic unit were tested.

**Table 4.3:** Dofs contribution from each subsystem to the whole rotor-active TPJB system. Generalized vector and its total contribution is also included.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Dof vector</th>
<th># dofs</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>rotor:</td>
<td>( x_r = {v_1 w_1 \gamma_{v_1} \ldots, v_n w_n \gamma_{v_n}, \gamma_{w_{v_n}}}^T )</td>
<td>4n_r ( 4 \cdot 27 )</td>
<td>108</td>
</tr>
<tr>
<td>pads:</td>
<td>( s_p = {\theta_1 \ldots \theta_n, \beta_1 \ldots \beta_n, \eta_1 \ldots \eta_n}^T )</td>
<td>( i_p n_p ) ( 3 \cdot 4 )</td>
<td>12</td>
</tr>
<tr>
<td>servovalves:</td>
<td>( q_v = {q_{v_1} q_{v_2}}^T )</td>
<td>( n_v )</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>122</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2N</td>
<td>244</td>
<td></td>
</tr>
</tbody>
</table>
No differences at all were found among these conditions. The results differ only between the two servovalve amplifiers in terms of gains. For servovalve 1, the amplifier gain is around 5 mA/V while for servovalve 2 it is only 4 mA/V. Phase drops from 180 to below 140 for both servovalves above 400 Hz. Table 4.4 summarizes the identified parameters of the amplifiers by fitting first order system parameters. Some facts that can be drawn from this identification are:

1. Time response is equal for both amplifiers, in the order of 0.357 ms or 2800 Hz.

2. The gain of the servovalve amplifier 1 is larger than for the other one. This implies that for moving the spools of both servovalves equally, servovalve 2 requires 33% more control voltage, or energy.

These parameters can be utilized in case the servovalve amplifier dynamics is included in the modelling. In this case, only the phase lag was compensated by
assuming a counter phase response between the servovalve current and amplifier control voltage. This phase lag was assumed constant and equal to 180 degrees, so that only a sign compensation is made when assuming that the servovalve is driven by a control voltage signal rather than a current one.

4.4 Theoretical Modelling Results

The following figures show some theoretical results for the system. Figure 4.6 depicts the modeshapes of the test rig when is loaded with two discs. The rigid modes do not vary significantly from those presented in [P5]. The first bending modes reduce to slightly below 100 Hz for this configuration. The second bending mode is around 355 Hz. All these modeshapes are obtained for an angular velocity of 1000 rpm. The mode associated with the servovalve dynamics have not been plotted.

Figures 4.7, 4.8 and 4.9 portray the Campbell diagrams and Stability maps when loaded with none, one and two discs. In all Campbell diagrams the straight line at 177 Hz corresponds to the servovalve damped natural frequency. It is also drawn from the diagrams that the 1X of the angular velocity never crosses, hence excites the natural frequency associated with the rigid mode. The effect of adding more discs to the system is also clearly seen, which is to separate the forward and backward natural frequencies of the bending modes. This effect seems to be more pronounced when the system is loaded with one disc rather than when loaded with two. It can be deduced from the last Campbell diagram that the only way to cross a critical speed excited by the 1X is when the system is loaded with two discs, and such a critical speed is around 4700 rpm. Finally, and with regard to the stability map, at least for the studied cases in the presented rotor velocity span, there are no instability problems.

<table>
<thead>
<tr>
<th>Table 4.4: Identified parameters of Servovalve amplifiers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_a , (\Omega)$</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Servovalve 1</td>
</tr>
<tr>
<td>Servovalve 2</td>
</tr>
</tbody>
</table>
Figure 4.6: Modeshapes of the test rig with the rotor hanging two discs. Two discs configuration.
Campbell Diagram

Damped Natural Frequency, $\omega_d$ (Hz)

Shaft Angular Speed, $\Omega$ (rpm)

Stability Map

Damping Ratio, $\xi$

Shaft Angular Speed, $\Omega$ (rpm)

Figure 4.7: Campbell diagram and stability map for the rotor-bearing system without discs. Hybrid lubrication at 100 bar. No disc configuration.
Figure 4.8: Campbell diagram and stability map for the rotor-bearing system with one disc. Hybrid lubrication at 100bar. One disc configuration.
Figure 4.9: Campbell diagram and stability map for the rotor-bearing system with two discs. Hybrid lubrication at 100bar. Two discs configuration.
Chapter 5

Results [P1] & [P2]: Model-Free Controllers

5.1 Model-Free Approach for System Characterization

The model-free approach, of an experimental nature, can be used as an alternative to model-based control when a theoretical model of the system is not available or a quick implementation is required. Modelling of physical phenomena is a challenging task. It requires an understanding of the physics behind it, and at the same time, to describe it simply. Precise models lead to complicated mathematical models which commonly do not have analytical solutions. On the other hand, very simple models commonly do not describe the phenomena precisely and reliably enough. Therefore, a balance between these two approaches is normally required. A model-free approach is based on the experimental system characterization to design and develop the controllers. Characterization of the system can be carried out statically, quasi-statically or dynamically, providing the information of the system as calibration functions or more commonly as FRFs. The controllers obtained through this approach are limited to classical proportional-integral-derivative controllers. Two works [P1] and [P2] within this subject have been produced in connection with this project. The first one explores the advantages of using integral controllers whilst the second one deals
with proportional-derivative controllers. Both aim at reducing the lateral vibrations of flexible rotors supported by ALBs. In what follows, the main findings of the two papers are reviewed.

5.2 Integral Control for Changing the Journal Equilibrium Position [P1]

The first studied strategy seeks to explore the possibilities of modifying the bearing properties through non-linearities by affecting the journal equilibrium position. It is well established that bearing force coefficients change with the eccentricity and attitude angle defined by the Somerfeld number [12, 96]. Such an equilibrium modification is only in terms of bearing-journal geometry, since the thermal modification of it, due to the controller action by itself, can be disregarded. The proper controller for this purpose is the integral controller, since it affects the steady-state response of a system, or in other words the journal equilibrium position. Figure 5.1 depicts the idea of a change in the vertical equilibrium position for a plain circumferential bearing. For a defined operational condition, and at a time $t_0$, the journal features the eccentricity point $y_0$. Due to the controller action, it is moved to $y_1$ at $t_1$ to attain the set reference $R_1$. How close the journal can get to the new position depends strictly on how much the journal can be moved inside the bearing at the current operational condition. This is elucidated by characterizing the system at each operational condition required. The main assumption considered is that the system equilibrium position is changed through a quasi-static process so that the inertia forces can be neglected. This allows us to simplify the model and
5.2 Integral Control for Changing the Journal Equilibrium Position

Figure 5.2: Journal center map for the different injection cases and operational conditions. △: case #1. ◊: case #2, reference position for other cases. ○: case #3. □: case #4. The counter clockwise quadrants used in the experimental results are included. Change in the journal equilibrium position.

disregard the system dynamics.

Earlier studies in active lubrication have already dealt with the integral controller, implementing it to bring back the journal position to the bearing center, such as in Nicoletti and Santos [23]. Nonetheless, the idea of using the bearing center as a set point implies a deterioration of the bearing properties, such as damping and the stability margin. Another approach is to set a reference different from the bearing center so that the bearing eccentricity is increased or reduced in order to boost its characteristics.

5.2.1 System Characterization

Figures 5.2 and 5.3 depict the journal center map and calibration curve for the system, respectively, as a quasi-static characterization of it. The characterization is carried out by applying staircase function of the control signal $u$ to the
servovalve within its working range. The increments are small enough to avoid large dynamic system response and back and forth to account for the hysteresis. From Figure 5.2 the following conclusions can be drawn: 1) the larger the pressure, the bigger the journal area covered, and 2) the larger the shaft angular speed, the smaller the journal area covered. This provides us with insight into how much the journal can be moved inside the bearing, hence how much the bearing properties could be affected. Figure 5.3 shows the (quasi-static) calibration curves of the system. Since the pad arrangement is a load-between-pads one, the orthogonal direction of the servovalves are shifted 45 degrees from the horizontal and vertical ones, where the sensors are placed, yielding a MIMO system to be studied. Again, the larger the pressure, the more the journal can be moved for a fixed angular speed. As the angular speed increases, the journal movements become more restricted. It draws attention to the change in direction of the linearised gain (red lines) for servovalve 1 in the horizontal direction with the increased speed. It is also striking that the larger movement in the vertical direction is also achieved by using servovalve 1.

Figure 5.3: Quasi-static system calibration curves for the different operational conditions. In rows, the system response in x-y plane. In columns, the servovalve control signals 1 and 2. Solid lines: linearized gains for cases #2 and #4.
Figure 5.4: Experimental results for case #2 - closed-loop system response and control signals when the shaft is moved in the vertical direction with a sine ramp function. Left column: reference signal to the down (-). Right column: reference signal to the up (+).

5.2.2 Integral Control Performance

Figure 5.4 shows the closed-loop system response and control signals when the shaft is moved in the vertical direction following a sine ramp function, i.e., upward and downward movements. It can be seen that the downward movement is the one with best results from a control viewpoint. The system tracks the reference very well and large downward movements can be imposed. Oppositely, the upward movement does not perform adequately and a smaller set point must be defined. However, as will be seen later, this produces the best results in the whole system dynamic response. When dealing with integral controllers, there are three main parameters to control:
The integral gain $k_i$. The integral gain defines how fast the system reacts to the new position reference. In reference [P1] a criteria is established for synthesizing this gain for a coupled MIMO system. The main objective is to
**Figure 5.6:** Dynamic characterization of the TITO system via servovalves at position \( \vartheta_a \) (red dashed line) and \( \vartheta_b \) (black solid line) for the operational condition of case \#1(c). **A:** \( P_{x1}(j\omega) \), **B:** \( P_{x2}(j\omega) \), **C:** \( P_{y1}(j\omega) \), **D:** \( P_{y2}(j\omega) \). Cases \#2(c) and \#3(c) are also included in gray colors for position \( \vartheta_b \).

keep the overshooting to a minimal in order to avoid possible surface impacts between pad and journal.

The ramp time \( t_r \) is the parameter which defines how low the quasi-static process is to be carried out. The lower the ramp time, the more static the process is considered to be.

Figure 5.5 presents one of the main results obtained in contribution [P1]. Benchmarking the different results obtained with injection schemes against the leakage case (black solid line), it can be seen that a reduction on the system response is obtained when an upward injection scheme (blue dash-dotted line) is established. Additionally, [P1] also presents how, under defined conditions, the integral controllers can be utilized for matching journal center equilibria under passive and hybrid regimes when the ALB is utilized as a calibrated shaker for creating “non-invasive fluid film perturbation forces”.

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**5.2 Integral Control for Changing the Journal Equilibrium Position [P1]**
Results [P1] & [P2]: Model-Free Controllers

Figure 5.7: Stabilizing gains area $D_k$ defined by the D-decomposition method for the MIMO system. A: case #1(c), B: case #2(c), C: case #3(c) and D: common stabilizing area from the intersection of A, B and C areas.

5.3 Proportional-Derivative Control for Affecting the Bearing Properties Around the Equilibrium [P2]

Contribution [P2] deals with the implementation of a proportional derivative controller for reducing the lateral vibration of the system around the equilibrium position of the journal. In this case, a dynamic characterization of the whole mechanical-hydraulic system was needed. Figure 5.6 depicts such a characterization, considering as input and output, the servo valve control voltage and lateral shaft displacement, respectively. The challenge in synthesizing the controller gains was to find a set of gains which can be used in different operational conditions. Following that purpose, a common stabilizing area was found, as presented in Figure 5.7. With the found gains, reductions up to 30% were imposed for the resonant zone related to the first bending modeshape of the system. Figure 5.8 shows these results in the horizontal direction. In the experimental campaigns, the system was loaded with only one disc.
5.3 Proportional-Derivative Control for Affecting the Bearing Properties Around the Equilibrium [P2]

Figure 5.8: Experimental FRFs comparison in the horizontal direction for case #1, 1000 rpm. (-) Passive lubrication. (-) Hybrid lubrication, leakage case with 50 bar. (--) Active lubrication with PD controller #1, 50 bar. (-) Hybrid lubrication, leakage case with 70 bar. (--) Active lubrication with PD controller #1, 70 bar. (--) Active lubrication with PD controller #1, 90 bar. (-) Active lubrication with PD controller #1, 100 bar.
Chapter 6

Results [P4], [P5] and [P6]: Model-Based Controllers

The simplest approach to the system being controlled, under a model-based design perspective, is as a Linear Time Invariant (LTI) system. It does mean that the equation that represents its dynamics must be defined by a linear combination of their independent variables and the constants used for such a linear combination must not vary in time. Most of the systems are governed by non-linear phenomena, and for the dynamic system at hand, the non-linearities arise mostly from the physics governing the fluid film forces. However, a linearised representation of its behaviour can always be obtained around an equilibrium point by means of Taylor expansion. Therefore, the active TPJBs problem is a typical regulation problem, as the journal must remain in the vicinity of the specified equilibrium position defined by the eccentricity and attitude angle within the bearing clearance. In the following, the main findings associated with the model-based control design approach are summarized.
6.1 On the Stability, Controllability and Observability of ALBs [P4]

In contribution [P4], the system fundamentals under the model-based control design approach were studied in the small test rig for different pad layouts, i.e. the stability, controllability and observability. Two main goals were set: 1) to determine the pad layout, in terms of pivoting and injector offset design, which features the best control characteristics, and 2) to study the dynamics of the ALBs by identifying the degrees-of-freedom which dominate the bearing dynamics. The studied pad layouts are presented in Figure 6.1. These pad configurations were obtained varying either the pivot or injection nozzle position so that an offset between them is created. For analysing this system, a simple model of three mechanical degrees-of-freedom plus one hydraulic one is formulated. Among the mechanical ones, the journal vertical movement, the pad tilt and the pad vertical movement (due to the pivot flexibility) are defined, while for the hydraulic subsystem a degree-of-freedom related to the active force is established. The approach for modelling the hydraulic subsystem is a basic one, in which the effect of the hydrostatic force is separated from the effect of the hydrodynamic one. This approach has been used in previous works such as in [39–41] and differs from the one presented in contributions [P5] and [P6] in which both effects are no longer separated.

Figure 6.2 presents the block diagram for the system considering rigid pivot for simplicity, i.e. the vertical movement of the pad is disregarded. It can be seen that all states are controllable, but an additional mechanism is added to access the θ-state when the offset between the pivot and injector is introduced,
6.1 On the Stability, Controllability and Observability of ALBs [P4] 75

Figure 6.2: Block diagram of the rigid rotor-single pad system.

i.e. $\Delta \Theta_p^n$.

The model is validated for the pad with the nozzle and injector aligned in the centre of the pad surface, the pad C. For this single pad-journal system the theoretical and experimental FRFs are compared in Figure 6.3. In general terms, the model matches the experimental information very well for frequencies above 70 Hz for a wide range of loads. However, more research is necessary in order to adjust the theoretical model at low frequencies, especially for low loading conditions. As a main outcome it can be corroborated that the pivot flexibility, responsible for a vertical pad movement, highly affects the ALB dynamics. With a rigid pivot all system modes remain either overdamped or highly damped, at least in the range of load and angular velocity studied. For these
conditions the Sommerfeld number varies between 0.07 and 0.470 approx. Since there will always be some flexibility introduced by the pivot, a coupled mode $\xi - \eta$ in which the journal and pad vibrate oppositely, with almost the same amplitude, will dominate the bearing dynamics. This dominant mode can be controlled and observed. Generally, observability and controllability decrease with an increased load. Additionally, and as elucidated through the degrees of observability, all modes are better observed by measuring the pad tilt angle rather than the journal displacement. However, this is more challenging. Regarding the pad designs, it is seen that a general improvement from a control standpoint is obtained by shifting the injector towards the leading edge or by slightly moving the pivot towards the training pad edge. Both cases increase the wedge effect between the pad and journal surfaces.
6.2 The Modelling and The Hybrid Lubrication Regimes [P5]

Contribution [P5] focuses mainly on the modelling of a flexible rotor supported by ALBs. The main idea behind it is to provide the designer with reliable models that can be used to design model-based controllers. The main challenge in modelling this sort of system relies on the way in which the mechanical and hydraulic subsystems are coupled. Up to this point, most of the approaches have been based on decoupling the active force from the rest of the ALB dynamics. This is a practical approach, since it allows us to experimentally characterize the bearing active force through mainly static procedures. With the maturity of today’s ALB modelling, a more integrated approach can be formulated, which takes into account the hydraulic and mechanical subsystem as a whole. Assuming a linear behaviour for the rotor, the equation of motion subjected to the non-linear fluid film force \( f_\text{p} \) reads:

\[
M \ddot{x}'(t) - \Omega G \dot{x}'(t) + K \dot{x}'(t) = w + f_\text{p}(x'', \dot{x}'', q_v, u, t) + f_{\text{ext}}(t) \tag{6.1}
\]

The fluid film non-linear force is mainly dominated by the servovalve dynamics which defines the spool-driven flows as:

\[
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix} \left\{ \begin{array}{c}
\Delta \dot{q}_{v1}(t) \\
\Delta \dot{q}_{v2}(t) \\
\end{array} \right\} + \left[ \begin{array}{cc}
2\xi_{v1}\omega_{v1} & 0 \\
0 & 2\xi_{v2}\omega_{v2} \\
\end{array} \right] \left\{ \begin{array}{c}
\Delta q_{v1}(t) \\
\Delta q_{v2}(t) \\
\end{array} \right\}
\]

\[
+ \left[ \begin{array}{cc}
\omega_{v1}^2 & 0 \\
0 & \omega_{v2}^2 \\
\end{array} \right] \left\{ \begin{array}{c}
\Delta q_{v1}(t) \\
\Delta q_{v2}(t) \\
\end{array} \right\} = \left[ \begin{array}{cc}
\omega_{v1}^2 R_{v1} & 0 \\
0 & \omega_{v2}^2 R_{v2} \\
\end{array} \right] \left\{ \begin{array}{c}
\Delta u_1(t) \\
\Delta u_2(t) \\
\end{array} \right\} 
\]

(6.2)

By Taylor expanding both subsystems, they can be coupled through the following linear system of equations:

\[
\begin{bmatrix}
M & 0 \\
0 & I \\
\end{bmatrix} \left\{ \begin{array}{c}
\Delta \ddot{x}' \\
\Delta \ddot{q}_v \\
\end{array} \right\} + \left( \begin{array}{cc}
-\Omega G & 0 \\
0 & 0 \\
\end{array} \right) \left\{ \begin{array}{c}
\Delta \dot{x}' \\
\Delta \dot{q}_v \\
\end{array} \right\} + \left( \begin{array}{cc}
-\frac{\partial f_p}{\partial x'} & 0 \\
0 & 2\xi_v\omega_v \\
\end{array} \right) \left\{ \begin{array}{c}
\Delta x' \\
\Delta q_v \\
\end{array} \right\} = \left[ \begin{array}{c}
0 \\
\omega^2 R_v \\
\end{array} \right] \Delta u + \left[ \begin{array}{c}
f_{\text{ext}} \\
0 \\
\end{array} \right] 
\]

(6.3)

In Equation 6.3 the bearing dynamic properties are expressed as:

\[
K|_{\Pi_0} = \begin{bmatrix}
\frac{\partial f_p}{\partial x'} & \frac{\partial f_p}{\partial q_v} \\
0 & \omega_v^2 \\
\end{bmatrix}; \\
D|_{\Pi_0} = \begin{bmatrix}
\frac{\partial f_p}{\partial x'} & 0 \\
0 & 2\xi_v\omega_v \\
\end{bmatrix}
\]
the journal and bearing DOFs plus the parameters of the servovalves. In the case of the bearing stiffness, the coefficients are defined similarly, but an additional term is added. This term, \( \frac{\partial F}{\partial q} \), describes how the fluid film forces vary with small variations of the servovalve spool-driven flow, explicitly defining the link between the mechanical and hydraulic subsystems. Considering this, for each lubrication regime, the equilibrium \( (\Pi_0) \) and the equation of motion can be defined as:

- The passive lubrication \( (p) \): in this case the hydraulic unit is off and there is no control signal \( u = 0 \), nor variable spool-driven flow \( q_v \), thus:

\[
\Pi_0^p = Kx_0^p - w_0 - f_p(x_0^p, 0, q_v^p, 0) = 0
\]
\[
M\ddot{x} + \left( D|\Pi_0^p - \Omega G \right) \dot{x} + \left( K + K|\Pi_0^p \right) x' = f_{ext}
\]  

(6.5a)

(6.5b)

- The hybrid lubrication \( (h) \): in this case the hydraulic unit is on, therefore the equilibrium point changes due to the injection of a flow function of a constant control signal \( u_0 \), thus:

\[
\Pi_0^h = Kx_0^h - w_0 - f_p(x_0^h, 0, q_v^h, u_0) = 0
\]
\[
M\ddot{x} + \left( D|\Pi_0^h - \Omega G \right) \dot{x} + \left( K + K|\Pi_0^h \right) x' = f_{ext}
\]  

(6.6a)

(6.6b)

A common case is when the spool is centred and the leakage flow is injected, hence \( u_0 = 0 \) and \( q_v^h = q_{\text{leak}} \). This is referred to as the leakage case.

- The active lubrication \( (a) \): in this case the position is regarded the same as in the previous one, hence \( \Pi_0^a = \Pi_0^h \); but in addition, there is a controllable fluid-film force exerted on the system due to the spool-driven flow variations \( q_v \) driven by a control signal \( u \), therefore:

\[
\Pi_0^a = \Pi_0^h
\]
\[
M\ddot{x} + \left( D|\Pi_0^a - \Omega G \right) \dot{x} + \left( K + K|\Pi_0^a \right) x = Wu + f_{ext}
\]  

(6.7a)

(6.7b)

With the model defined for each case of lubrication, then utilizing standard model-based tools, an integral control is designed to assist the hybrid lubrication in changing the equilibrium position of the system. Experimental results were obtained for the hybrid lubrication. Figure 6.4 depicts for the system without discs the variations of the FRF amplitude when the hybrid lubrication regime is developed. In this case, the resonant zone due to the first bending mode is around 210 Hz and the hybrid regime is developed injecting only the leakage.
flow. It is seen that a reduction of about 30% is achieved when 90 bar of pressure is utilized. A slightly larger reduction is obtained when an upward injection is featured, as presented in Figure 6.5. As a general outcome of this contribution [P5], it can be said that: 1) the modelling for developing model-based controllers was established, and 2) standard model-based tools (LQR regulator) were used to assist the hybrid lubrication that produces similar results to [P1], since both produced a reduction of lateral vibrations when lifting up the journal.

6.3 The Feedback-Controlled Lubrication Regimes [P6]

With the model of the flexible rotor - ALB system at hand, the model-based control design theory can be used for synthesizing controllers to develop the active or feedback-controlled lubrication regime with the ALB. The ultimate goal of the active lubrication is to improve the damping properties of the bearing and thus, to reduce the lateral vibration and extend the stability range of
Results [P4], [P5] and [P6]: Model-Based Controllers

Figure 6.5: Experimental FRFs in the vertical direction. Closed-loop hybrid lubrication for the upward and downward schemes. Rotor without disc configuration. 4000 rpm.

The rotating machine. This is possible because the bearing properties highly influence the behaviour of the whole rotor-bearing system. Therefore, by modifying the ALB dynamic properties via control, the system lateral vibrations can be reduced. The main advantage of utilizing the model-based approach over the model-free one, lies in the fact that the ALB can be designed as an integrated mechatronic element at the early bearing design stage for the specified machine. Therefore, there is no need for on-site tuning, as normally happens with classical approaches. The contribution [P6] is completely advocated to this goal. Therein, optimal control laws are designed, implemented and tested in the test rig. These controllers are state-feedback controllers with full order stochastic observers. More specifically a Linear Quadratic Gaussian (LQG) regulator, which is compounded by the Linear Quadratic Regulator (LQR) and the Kalman Filter.

Figure 6.6 shows the block diagram of the system in the closed-loop, comprised of the rotor-bearing system plus the observer and the controller, for which the
The governing equations can be summarized as:

\[
\begin{align*}
\dot{\hat{q}}_i &= \begin{bmatrix}
\hat{A}^r & \hat{B}^r K_i & -\hat{B}^r K \\
-\hat{C}^r & 0 & 0 \\
L\hat{C}^r & \hat{B}^r K_i & \hat{A}^r - L\hat{C}^r - \hat{B}^r K_i
\end{bmatrix} \begin{bmatrix}
\hat{q} \\
\hat{\dot{q}}_i \\
\hat{\dot{q}}
\end{bmatrix} + \begin{bmatrix}
0 \\
I_r \\
0
\end{bmatrix} r + \begin{bmatrix}
\hat{B}^r_v \\
0 \\
0
\end{bmatrix} v_1 \\
Y &= \hat{C}^r q + v_2
\end{align*}
\]  

(6.8a)  

(6.8b)

In obtaining the above system of equations, the original state-space formulation has been:

**Pseudo-Modal Reduced.** The standard approach for facing the control design in rotor-bearing systems is through a pseudo-modal reduction where only the lower modes lying in the frequency span of interest are kept for the analysis [97, 98]. This eases the implementation and avoids computational burden by reducing a large number of DOFs of the original mechatronic rotodynamic system to only a few, which describes only the modes kept. However, this is achieved at the expense of spillover problems if not designed carefully. The spillover problem is produced by the so-called “residual” modes which have been left out of the analysis. Two types of spillover can be identified [99]: the observation and control spillover. To avoid the onset of them, low pass digital filters were implemented for filtering the system outputs and control signals if required.

**Figure 6.6:** Closed-loop system block diagram. LQR controller gains \( K \) and \( K_i \). Kalman filter with observer gain \( L \).
Complex States Truncation. The resulting system matrices and vectors after being modal reduced become complex valued. Therefore, a separation of complex states is needed to have a real valued system that can be implemented in a computer, capable of processing only real numbers. In order to represent the original system properly, the system is partitioned into real and imaginary parts as proposed in [100, 101] with what is called an “almost diagonal form”. With this form, each eigenvalue of the Jordan representation is replaced by a $2 \times 2$ block matrix containing the real and imaginary parts of the eigenvalue. At the same time, the input and output matrices are separated into real and imaginary parts. Finally, in order to downscale the system to the original size, the rows and columns of the system related to one of the conjugated eigenvalues are removed, and either the input [40, 41, 100] or output [102] matrix is multiplied by a factor of two. By checking the derivation of the transfer function from the state-space realization, it can be recognized that the same representation can be obtained by multiplying both matrices by $\sqrt{2}$.

6.3.1 On the Observer Design - The Kalman Filter

A full order Kalman Filter [103] was designed as the stochastic observer to estimate the unknown states in the system. The design of the observer followed mainly the recommendations given in [104]. The steady-state observer gain $L$ is obtained upon the solution of the time independent Riccati equation and the uncorrelated measurement and process noise variance matrices $V_2$ and $V_1$, respectively. The first one is readily obtained by characterizing statistically the sensors, as presented in Appendix C.1.1.1, while the second one is more challenging. In the last case, the standard fashion of propagating the noise through the disturbance input matrix $B_v$, provided a known covariance of the system disturbances, did not work properly. Instead, the noise is imposed to the states directly through an identity input matrix $I$ [97]. This identity input matrix is also reduced in order to express the state noise in terms of modal coordinates.

6.3.2 On the Controller Design - The Linear Quadratic Regulator (LQR)

The design of the controller followed the recommendation presented in [104] for a full state-feedback controller with integral action. In this case, the system output $Y$ is used to reconstruct the entire modal state vector $\hat{q}$. The LQR controller is an optimal control in terms of energy balance between states and
6.4 Discussion on the Main Results of the Project

The feedback-controlled lubrication based on LQG regulators was featured by the ALB when the rotor was hanging none, one and two discs. The first two cases did not produce important reductions in the lateral vibration of the rotor due to the fact that, the first bending frequency of the system was too closed to the bandwidth limit of the actuator. Better and satisfactory results were obtained with the rotor hanging two discs. In this configuration, the static bearing load was increased to approx. 1440 N and the first bending frequency was slightly below 100 Hz. Figure 6.7 presents the results for the system under this configuration running at a speed of 1000 rpm. The radial injection system was pressurized with a supply pressure of 100 bar. When comparing the active lubrication based on the LQG regulator with the passive case as reference, a peak reduction of almost 60% is achieved. Even better results are obtained with the PID controller synthesized on-site. However, these results were achieved for the dynamics which the controller was designed for; it means for frequencies above 70 Hz. Although a new equilibrium is imposed when running faster, good results were also obtained with the same controller, but instead running at 2000 rpm, as reported in [P6].

Figure 6.8 shows, under the gray shadow area, what happens at low frequencies. That the amplification around 45 Hz and how this varies by changing the supply pressure are clearly seen. The larger the pressure, the more pronounced the peak. The same effects can be obtained by fixing the pressure and varying a controller master gain. Mortensen [41] reported the same problem in the system. Two attempts were made in order to tackle this problem. Both considered a lumped parameter model for including the contribution either from the bearing pedestals or from an acoustic resonance in the hydraulic pipes. This last was assumed due to a significant entrain of air in the pipelines expected, and due

control signals obtained by minimizing the steady-state cost function:

\[ J = \int_{0}^{\infty} (\hat{q}^T Q\hat{q} + u^T Ru) \, dt \]  \hspace{1cm} (6.9)

However, a significant amount of simulation was necessary in finding the right weighting coefficients for the control \( R \) and states \( Q \) matrices. Generally speaking, the rigid and first bending movement of the rotor required similar weighting coefficients, even though the rigid one is an already very well damped mode. This was unexpected. As closure, it is worth clarifying that, although in contribution [P6] all the design was presented in continuous time, the implementation was carried out in discrete time with the help of adequate Matlab functions and Simulink blocks.
to some unusual behaviour of servo valve 1 observed, which might be matching the resonance. Unfortunately, both attempts did not solve the problem and the problem remains.

It was also realized that this effect, due to some unknown dynamics, becomes stronger with the rotor supporting a larger number of discs. Therefore, additional tests were carried out without discs, with bushings, and with one and two discs. From a control standpoint, the same observer as [P6] was utilized, and only the controller was adapted to the different configurations. Figure 6.9 presents the results for the configuration without discs. As aforementioned, the reduction of lateral vibration was not as significant at the resonant zone. It is worth noting that the same but weaker issue is obtained with the model-based controllers. It also occurs at a higher frequency, around 70 Hz.

Figures 6.10(a) and 6.10(b) depict results under active lubrication in the horizontal and vertical direction respectively when the system is loaded with only
6.4 Discussion on the Main Results of the Project

Figure 6.8: Two discs configuration. Experimental FRF in the Horizontal Direction. 1000 rpm. Input at node 26 and output at node 23. Effect of the supply pressure on the performance of the LQG regulator. 12, 20, 30, 50 and 100 bar.

bushings. For bushings we refer to the small cylinders used to keep the discs in position. Owing to their low weight, the natural frequencies slightly reduce. This can be ascertained with the first bending frequency, which is slightly reduced below 200 Hz. Again, the unknown dynamics dominates the system response in closed-loop with model based controllers, being a little bit stronger than the previous case, especially in the vertical direction. The direction in which all dynamics seem to be more coupled.

After a significant number of experimental campaigns, it is highly believed that the main source of the unknown dynamics affecting the performance of the model-based controllers, is linked to the foundation of the test rig. This ironcast T-slot plate, sufficiently slender, requires a proper modelling in order to catch its dynamics, so that the model-based controllers take them into account. The dynamics seems to be mainly dominated by torsional plate movements, as
Figure 6.9: Frequency response functions of the system without discs at 1000 rpm. 90 bar of supply pressure. Horizontal direction.

(a) Horizontal FRFs.  
(b) Vertical FRFs.

Figure 6.10: Frequency response functions of the system in configuration 2 at 1000 rpm.

reported in Appendix F and also in [105].
The research objective of designing and implementing model-based controllers for flexible rotors supported by actively-lubricated bearings (ALBs) was achieved. To reach that goal, a significant amount of research was needed. Based on the findings of each paper contribution, it can be concluded that:

[P1] The integral controllers can be used to modify the geometrical equilibrium or bearing centre position. For doing so, it is necessary to characterize the system quasi-statically, for instance, under a model-free approach. From a control standpoint, it is easier to move the journal downward with larger setpoints and smaller steady-state errors when compared to any other position. From a system response point of view, a reduced system response at resonances can be obtained by injecting upward from the bottom pads so that the journal is lifted up. This is also seen as a softening of the vertical bearing direct stiffness due to further addition of lubricant between the journal and bottom pads. Integral controllers find applicability in ALBs when used as calibrated shakers.

[P2] The proportional-derivative controllers can be utilized to reduce the lateral vibration of flexible rotors supported by ALBs. The approach can also be model-free with a complete dynamic characterization of the hydraulic and
mechanical systems. By using appropriated synthesizing methods, a single controller can be implemented under different operational conditions. It is seen that the effectiveness of the controller can be significantly improved as the oil pressure of the hydraulic unit is increased.

[P3] The bearing dynamic properties are significantly modified under the hybrid and active lubrication regimes when compared against the passive one. This modification is clearer in bearings under light-load condition. A softening effect is observed when an upward injection scheme, of the hybrid regime, is implemented. It is also shown that different PD control laws produce different effects on the direct and cross-coupling force coefficients, therefore not all control laws are likely to improve the whole rotor-bearing system performance.

[P4] From a control basics point of view, a rocker-pivoted pad configuration with an offset between the injector and the pivot line offers an ALB with increased controllability when compared to a centrally installed injector and pivot configuration. Best results are obtained by slightly moving the pivot line towards the trailing edge. On the other hand, observability can be significantly improved by measuring the pad tilt angle. From a system response standpoint, modes involving either only journal or pad movement are overdamped. The mode which dominates the single-pad ALB dynamic is an out of phase combined translational movement of the journal and pad. This is a direct consequence of the pivot flexibility, which greatly affects the bearing dynamics and performance.

[P5] The joint modelling of the mechanical and hydraulic systems under the different lubrication regimes, namely: passive, hybrid and active is established. For the cases of the passive and hybrid lubrication regimes, the dynamics of the ALB, hence affecting the entire rotor-bearing system, is determined only by the journal and pad dofs, which describe the bearing linear behaviour around their respective equilibria. In these cases, no dynamic from the hydraulic system is included. In the active lubrication case, the same equilibrium of the hybrid regime is established, but in addition, a dof describing the spool-driven flow from the hydraulic unit is included. Within the hybrid lubrication, the advantage of an upward injection scheme against the leakage one is shown again. But this time, aided by an integral controller synthesized using standard model-based control tools. Reductions of 30% in the resonant zone were achieved.

[P6] The active lubrication, specifically the state feedback-controlled lubrication based on LQG regulators, succeeded in reducing the shaft lateral vibration. This is achieved for the dynamics which the controller was designed for. A reduction of about 60% in the resonance amplitude was achieved. Nonetheless, experimental results suggest that additional effort must be addressed
in order to improve the whole system response in closed-loop at low frequencies. Although it is highly likely that the unmodelled dynamics is related to the foundation T-slot plate, it is also recommended to further investigate the pipes acoustic dynamics that may take place along the oil feeding lines.

7.1 Future Aspects

• With regard to the ALB modelling:

1. To carry out a study on the advantages of using the Gaussian profile for the oil injection velocity in the MRE and Energy equation. Especially when high injection pressures are developed, so that large differences in the hydrodynamic and hydrostatic profiles are imposed.

2. To validate the ALB model for arrangements with a larger number of pads, in order to decrease the source of uncertainty when modelling the rotor-bearing system with larger ALBs. So far it has only been done statically and dynamically for a single-pad system.

• With regard to the foundation:

1. Fundamentally, to find and install the test-rig on a rigid foundation, so that its dynamics does not influence the one of the test-rig under study. Another option is to install a rigid frame to support the T-slot plate as suggested by many manufacturers. In this way, all the effort can be targeted to the complete understanding of the flexible rotor-active bearing system behaviour. Resulting in improved modelling of the system, hence better model-based controllers.

2. An alternative approach to solve the issue with the foundation is to include its dynamics in the modelling, so that model-based controllers are designed for the whole system, i.e. the flexible rotor-bearing-foundation system. Although the complexity of the system under study is evidently increased, the current boundary conditions of the foundation, close to a free-free condition after being supported on a gummy bed, makes its inclusion easier. A first approach on this issue has suggested utilizing modal reduced models of the foundation that can be coupled to the rotor-bearing system, for instance, at the bearing nodes.

• With regard to the test-rig improvements:

1. It is recommended to upgrade, or at least re-characterize, both servovalves in order to obtain a better match between theory and experiment in relation to the hydraulic forces. The first option seems to be more
Conclusions and Future Aspects

appropriate since new servovalves can be completely characterized in the small test-rig, which is now completely instrumented for such a purpose.

2. New proximity probes will also be needed in the near future. Alongside this, the inclusion of analog anti-aliasing filters, at least for the displacement sensors used with controllers, is highly recommended.

3. Although it is not strictly needed, an upgrade of the hydraulic system is also recommended. It will avoid cavitation problems of the oil system which affect the repetitiveness of experiments. An oil cooling system is also preferable.

• With all improvements done, run and perform experiments with new model-based controllers at higher speeds, closer to real operational conditions. If there is no success, then more advanced control techniques might need to be implemented, for instance, based on the $H_{\infty}$ control approach.
APPENDIX A

Publications

A.1 [P1]: Exploring Integral Controllers in Actively-Lubricated Tilting-Pad Journal Bearings
Exploring integral controllers in actively-lubricated tilting-pad journal bearings

Jorge G Salazar¹,² and Ilmar F Santos¹

Abstract
Active tilting-pad journal bearings with radial oil injection combine good stability properties of conventional tilting-pad journal bearings with the capability of improving their dynamic properties even more by control techniques. The main contribution of this work is the experimental investigation of integral controllers for feedback-controlled lubrication with the aim of: a) presetting the static journal center and consequently exploring the changes of bearing dynamic properties; b) obtaining an integral controller capable of re-positioning the static journal eccentricity for matching equilibria under conventional hydrodynamic and feedback-controlled lubrication regimes. A novel application is proposed, that tries to build "non-invasive perturbation forces" and uses the active fluid film forces of the bearing as a "calibrated shaker".

Keywords
Tilting-pad journal bearings, feedback-controlled lubrication, integral controller, active lubrication, bearing dynamics

Date received: 24 July 2014; accepted: 7 January 2015

Introduction
Conventional tilting-pad journal bearings (TPJ Bs) are usually chosen when machines supported by cylindrical journal bearings such as pumps, turbines, or compressors experience instability problems or need to have their operational range extended. Such dynamic performance improvement is achieved because of their superior stability properties, which are due to their almost absent cross-coupling terms.¹ However, the more demanding operational conditions and continuous safety requirements have demanded that high-performance turbomachinery operates with enhanced performance; that is, larger stability ranges and lower vibration levels. One proposed way to achieve these requirements is by adapting control systems that aim to modify the bearing dynamic properties. With such control systems, the shaft lateral movements are measured and then used as feedback control signals. These signals, together with suitable controller gains, define the control signals governing the actuators acting on the system. Such actuators can be magnetic,² piezoelectric³,⁴ or hydraulic.⁵ Two types of hydraulic actuators applied to TPJ Bs were compared by Santos: the hydraulic chamber system and the radial oil injection system. Santos concluded that the latter is more appropriate for this type of bearing. Santos and Russo⁶ presented the theoretical study supporting the fundamentals of TPJ Bs with electronic radial oil injection. Such injection involves introducing the lubricant directly into the bearing clearance to modify the pressure distribution, thereby altering the bearing dynamic properties. This design of TPJ Bs (referred to hereafter as the active TPJB) has been extensively studied by Santos et al.⁷–¹¹ with the aim of developing theoretical control strategies and determining the feasibility of applying this design to industrial machinery. These control strategies have been developed based on the elastohydrodynamic approach¹²–¹⁴ for modeling the active TPJB dynamics. The elastothermohydrodynamic approach¹⁵–¹⁸ is the latest, state-of-the-art model, which includes the

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effects of pivot flexibility, pad elastic deformation, oil film and pad temperature, heat transfer with the shaft and surrounding, fluid film pressure field, as well as the hydraulic control system dynamics. Different lubrication regimes can be featured depending on whether the control laws are used and on the type of controllers used.

The controller design is a challenging task. Well-tuned gains must be used in order to achieve the desired modification of the TPJB behavior. Essentially there are two approaches for facing the controller design, the model-free\textsuperscript{19,20} and the model-based\textsuperscript{21–23}. The model-free approach does not require a theoretical model and it can be conducted in the frequency domain, by time series data or by considering more sophisticated methods, such as the fuzzy logic\textsuperscript{24,25} or the neural network\textsuperscript{25} methods. On the other hand, the model-based approach is used for optimal control design and requires state-space formulation to develop the controllers. However, the model-free design can be very useful when there is only experimental information about the system available, for instance, the frequency response functions (FRFs) Keel and Bhattacharyya\textsuperscript{26} and Li et al.\textsuperscript{20}. Among others studied the use of the model-free approach for synthesizing proportional-integral-derivative (PID) controller gains, which focused on the use of the bode frequency domain, by time series data or by considering more sophisticated methods, such as the fuzzy logic\textsuperscript{24,25} or the neural network\textsuperscript{25} methods. Buttini and Nicoletti\textsuperscript{27,28} used this model-free approach based on the D-decomposition method for designing controllers aimed at controlling, by means of magnetic actuators, the lateral vibration of a flexible rotor supported by ball bearings.

Recalling that the bearing properties are heavily dependent on the journal position, the main original contribution of this work is a model-free design and implementation of an integral controller (I-controller) for developing the feedback-controlled lubrication with the aim of: a) statically presetting the journal position relative to the bearing surface, i.e. controlling the static journal eccentricity and attitude angle, and consequently exploring the changes of bearing dynamic properties; b) obtaining an I-controller capable of repositioning the static journal eccentricity for matching the equilibrium under conventional hydrodynamic and feedback-controlled lubrication regimes. A novel application is targeted, trying to build “non-invasive perturbation forces” and using the active fluid film forces of the bearing as a “calibrated shaker” to perform dynamic testing in rotating machines. A quasi-static process is followed to change the position of the journal within the bearing once the thermal and static steady-state equilibria of the rotor-bearing system are reached for a given operational condition. The quasi-static system calibration curves are obtained by experimental means to design the controller under the model-free approach. Three different controller gains are synthesized and simulated and the one with the best performance is experimentally tested.

**Experimental facilities**

### The flexible rotor-bearing test rig

Figure 1 shows a picture of the test rig and its main components. The flexible rotor-bearing test rig resembles a large overhung centrifugal compressor supported by an active TPJB. It consists basically of an overhung inertia disc mounted on a 70 mm in diameter and 1150 mm in length flexible shaft supported by two bearings; a ball bearing on its driven side and the active TPJB on its non-driven side with a total weight of 87 kg. The test rig is belt-driven by a 4 hp AC motor connected through a flexible coupling. By means of a frequency driver, the test rig is enabled to run up to 7000 r/min. The first critical bending speed is experimentally identified around 130 Hz and a stable behavior is predicted within the operating speed range. The test rig is also provided with an excitation bearing at its non-driven side, which consists basically of a ball bearing whose internal ring is solidary to the shaft whereas the external one is fixed to a housing. By means of stingers and a force transducer, an electromagnetic shaker can be connected to the housing to carry out experimental dynamic tests. The framed picture of Figure 1 depicts a standard setup of the excitation bearing with the shaker. Low and high pressure supply units sharing the same reservoir are used to provide the lubricant for the low and high pressure lubrication systems.

The test rig was designed and built with the goal of studying the dynamic behavior of the flexible rotor when supported by an active TPJB featuring different lubrication regimes. The operational conditions defining the regimes change the bearing dynamic properties due to their dependency on the Sommerfeld number as well as on the supplied injection pressure, which strongly affect the behavior of the whole system. Different controllers can be digitally implemented and tested to control the lubrication regime.

### The active TPJB (+ servovalves)

Figure 2 depicts a scheme of the active TPJB and Table 1 summarizes its main design characteristics. It comprises four tilting pads in a load-between-pads (LBP) configuration. The pads are made of bronze and have an injection nozzle 3.3 mm in inner diameter placed in the middle of the pad surface, see Figure 2. The active bearing is able to operate under three different lubrication regimes: i) the conventional or passive lubrication regime; ii) the hybrid or adjustable lubrication regime, i.e. the high pressure supply unit is enabled and the servovalves are able to inject a constant lubricant flow into the bearing clearance. This leads to a bearing dynamic properties modification by superimposing the hydrostatic effect onto the hydrodynamic one; iii) the feedback-controlled or active lubrication regime, which is obtained when feedback signals, for instance the shaft lateral
movements, are used to build control laws to dynamically regulate the lubricant flow and vary the hydrostatic effect in time.

The adjustable and feedback-controlled lubrication regimes are developed by two “four way, spool valve configuration” servovalves MOOG™ E760-912 installed orthogonally at 45° over the journal housing. Each servovalve connects a pair of pads with the high pressure supply unit. Servovalve 1 connects to pads 2 and 4 while servovalve 2 with pads 1 and 3. Due to the LBP configuration, the orthogonal directions in which the servovalves act are also shifted 45° from the horizontal direction as it can be noticed in Figure 2. High response servovalves are dynamically characterized by a natural frequency of 350 Hz and a damping ratio of 0.73 (for a standard pressure of 210 bar), which provide a wide linear frequency response. The servovalves driven by digital controllers running in the field programmable gate array (FPGA) hardware play the role of system actuators. Depending on the control signals the servovalves inject lubricant flow directly into the bearing clearance through the nozzles with a pressure in the range of 0–100 bar. This radial electronic oil injection configuration driven by the control signal voltage enables the active TPJB to modify the pressure profile of each pad, to subsequently exert controlled forces over the rotor system. Such controllable forces can modify the journal position and therefore the bearing dynamic properties.

**The instrumentation (digital controller + sensors)**

A dSPACE™ data acquisition system is used, managed via MATLAB-Simulink™ software. This system is able to run an embedded digital controller in its FPGA hardware. The flexible rotor-bearing system is monitored by eddy-current displacement sensors Vibro-Meter™ TQ102 placed orthogonally in pairs in the horizontal and the vertical direction at two different positions of the shaft, as shown in Figure 1, items 2 and 5. The proximity probe signal components are used to determine the shaft center position (DC) in the bearing and the dynamic response (AC) of the flexible rotor around the steady-state equilibrium position. Figure 2 includes in the scheme the two displacement sensors mounted at the bearing mounting ring (Figure 1, item 5) for monitoring the journal. The measurements obtained by the pair of displacement sensors can be considered as the response of the shaft within the bearing, since there is no axial misalignment between the shaft and pads inside the bearing and the shaft moves as a rigid body inside it. Figure 2 also provides the used reference frames. An orthogonal reference frame “x-y” for describing the system response and an auxiliary reference frame “1-2”, aligned with pad mounting/acting directions, shifted by −45° from the horizontal direction. To carry out dynamic testing, a Bruel&Kjaer™ 8200 force cell transducer is placed between the shaker...
and the excitation bearing to measure the excitation force.

The system characterization

(actuators + the flexible rotor-bearing system + sensors)

It is not always possible to obtain simplified mathematical models that can be used to design model-based controllers since models can reach very high complexity by including several effects when trying to precisely describe the system behavior. Instead a model-free approach can be used to design the controller using experimental information. To develop I-controllers based on a model-free approach, a system characterization is required. The system is considered as the flexible rotor-bearing system plus servovalves and sensors.

Model-free approach

The system can be characterized either statically or dynamically. As the control strategy adopted in this work is aimed at modifying the journal position through a quasi-static process, only the static characterization is considered. A quasi-static process is understood as a long time process, compared with the system dynamics, avoiding inertial and damping forces.

The system in an open-loop configuration can be described by the matrix \( P(S) \), where the vector of servovalves control signals \( U = [u_1(t) u_2(t)]^T \) and the system displacements in both orthogonal directions \( Y = [x(t) y(t)]^T \) are considered as the inputs and outputs of the system, respectively, hence \( Y(t) = P(S)U(t) \).

Figure 3 depicts the system block diagram in an open-loop, as well as in a closed-loop configuration. Since each servovalve affects the system in both directions, as seen in Figure 2, the system corresponds to a two-inputs two-outputs (TITO) system.

The system is characterized by obtaining the linearized gains from the quasi-static system calibration curves. Such gains define the \( P(S) \) matrix. In addition, the journal center map, i.e. the area covered by the journal position as a function of the servovalves input signal, is also presented. Table 2 summarizes the operational conditions used for the quasi-static system characterization.

Table 1. Active TPJB design characteristics.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journal radius ((R))</td>
<td>49.89 mm</td>
<td>Injection orifice diameter ((d_0))</td>
<td>3.3 mm</td>
</tr>
<tr>
<td>Number of pads ((n_p))</td>
<td>4</td>
<td>Injection orifice length ((L_0))</td>
<td>21 mm</td>
</tr>
<tr>
<td>Pad inner radius ((R_i))</td>
<td>50 mm</td>
<td>Pad material</td>
<td>Bronze</td>
</tr>
<tr>
<td>Pad aperture angle ((\alpha_0))</td>
<td>69°</td>
<td>Nominal radial clearance</td>
<td>110 ( \mu )m</td>
</tr>
<tr>
<td>Pads configuration</td>
<td>LBP</td>
<td>Assembly radial clearance</td>
<td>83 ( \mu )m</td>
</tr>
<tr>
<td>Pad pivot offset</td>
<td>0.5</td>
<td>Bearing applied load</td>
<td>845 N</td>
</tr>
<tr>
<td>Pivot design</td>
<td>Rocker</td>
<td>Lubrication oil type</td>
<td>ISO</td>
</tr>
<tr>
<td>Pad width ((L))</td>
<td>100 mm</td>
<td>Pad material</td>
<td>Bronze</td>
</tr>
<tr>
<td>Pad thickness ((t))</td>
<td>14 mm</td>
<td>Pad aperture angle ((\alpha_0))</td>
<td>69°</td>
</tr>
</tbody>
</table>

Table 2. Operational conditions used for the quasi-static system characterization.

<table>
<thead>
<tr>
<th>Case</th>
<th>Rotational speed ((\Omega)) ((r/min))</th>
<th>Injection pressure ((P_{n0})) ((bar))</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>1000</td>
<td>20</td>
</tr>
<tr>
<td>#2</td>
<td>1000</td>
<td>60</td>
</tr>
<tr>
<td>#3</td>
<td>4000</td>
<td>20</td>
</tr>
<tr>
<td>#4</td>
<td>4000</td>
<td>60</td>
</tr>
</tbody>
</table>
Table 3. Full range injection conditions of the servovalves (±2 V).\(^a\)

<table>
<thead>
<tr>
<th>Actuator</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Servovalve 1</td>
<td>+</td>
<td>0</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>0</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Servovalve 2</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>0</td>
<td>−</td>
</tr>
</tbody>
</table>

\(^a\)Cases from A to H labeled counter clockwise in Figure 4.

Figure 4. Journal center map for the different injection cases (Table 3) and operational conditions (Table 2). ∆: case #1. ○: case #2, reference position for other cases. ●: case #3. ■: case #4. The counter clockwise quadrants used in the experimental results are included.

as a function of the servovalves working in their full operational range is mapped. The mapping is carried out by determining the maximum shaft center displacement when the servovalve ports which connect a pair of pads are completely opened, see the two opposite pads 1 and 3 in the auxiliary axis 1 and the two opposite pads 2 and 4 in the auxiliary axis 2 in Figure 2. This mapping is realized for all operational cases defined in Table 2, sending a control signal of ±2 V (±20% of the max. voltage) for obtaining full range valve operations. Table 3 summarizes all possible injection combinations, eight cases identified counter clockwise from A to H.

Figure 4 depicts the area where the journal can be moved by means of the hybrid lubrication. In cases D and H only servovalve 1 operates injecting from pad 2 or 4, respectively, whereas in cases B and F only servovalve 2 is in action injecting from pad 3 or 1, respectively. In the injection cases A, C, E, and G, both servovalves operate simultaneously. If both servovalves were completely identical, these cases would lead to journal movements along x and y directions. It can be noticed that the covered area reduces when the shaft angular velocity increases, showing an increasing effect of the hydrodynamic lubrication in comparison with the hydrostatic one. On the other hand, as the injection pressure increases, the covered area increases, showing the significant effect of the hydrostatic lubrication in comparison to the hydrodynamic one. For case #2 with 60 bar of pressure, the journal center can be moved approximately 25% of the assembly diametrical clearance along the x direction (from point E to A) and 34% along the y direction (from point G to C).

Quasi-static system calibration curves. The main purpose of these curves is to obtain a range in which the system behavior can be linearly characterized. Figure 5 illustrates the static input–output calibration curves for all operational cases. The shaft linear displacements in both directions are plotted as a function of the servovalves input voltages. Such curves are obtained by sending a bidirectional staircase step signal, with increments of 0.05 V within the range of 0–4 V, to each servovalve independently, i.e. while the other servovalve is kept closed. Although closed, the servovalve still feeds the system with the leakage flow typical of an underlapped servovalve. This condition is set only for characterizing the servovalves since under normal operational condition, both servovalves operate at the same time, managed by the controller. It can be noticed from these graphs that:

- The higher the injection pressure (\(P_{ij}\)), the more the journal position can be moved due to the hydrostatic effect.
- The higher the rotational speed (\(\Omega\)), the less the journal position can be moved due to the hydrodynamic effect.

It can also be noticed that the system response in the horizontal direction cannot be approximated by a linear curve when the high pressure supply unit is feeding lubricant at 20 bar, because it is not enough pressure to move the shaft. As a consequence, a higher pressure must be used in order to obtain a significant system response. Taking this into account, only the cases with 60 bar in Table 2 are considered hereafter. Thus, the TITO system characterization is described by the four static gains which describe the interaction between the input and output variables as follows:

\[
P(t) = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} \tag{1}
\]

Equation (1) is a constant gain matrix whose term \(P_{pq}\) describes the system response in the p direction due to the action of servovalve q, henceforth monotonically numbered. It is time independent, i.e. \(P(t) = P\) and the system response in an open-loop system configuration can be obtained as
The linearized gains calculated from the calibration curves (colored in red in Figure 5) for cases #2 and #4 are as follows:

\[
P_{1000 \text{ rpm}} = \begin{bmatrix} 11.75 & 31.56 \\ -48 & 23.76 \end{bmatrix} \text{[\text{\mu m/ V}]} \quad (2a)
\]

\[
P_{4000 \text{ rpm}} = \begin{bmatrix} -4.30 & 14.22 \\ -17.32 & 5.06 \end{bmatrix} \text{[\text{\mu m/ V}]} \quad (2b)
\]

These \( P \) matrices of equations (2) can be used for the control design since they represent the system in an open-loop as shown in Figure 3.

**Integral controller design**

With the system characterized, a controller can be designed to move the journal to another position different from the one determined by the thermal and static steady-state equilibria, aimed at exploring how the system dynamic response is affected. The PID controllers are the most commonly used controllers for industrial applications due to their simple implementation. Among the PID controllers, the I-controller is the most appropriate for the purpose of this work because it fulfills the following performance specification:

- The steady-state error \( E(t) \) between the system response \( Y(t) \) and the reference \( R(t) \) must be almost zero to ensure that the new journal position is reached.
- In order to obtain a quasi-static process the rise and settling time must be larger compared to the system dynamics to obtain a slow change of the journal position, hence a controller based on the proportional action should be avoided.
- Overshoot values should be avoided or kept small enough to prevent surface contact when the setpoint is too large. This requirement is consistent with the previous specification; otherwise a derivative action should be implemented.

**The integral controller for TITO systems**

The use of I-controllers and generally PID controllers in TITO systems control is much more challenging when compared with single-input single-output (SISO) systems due to the pairing problem. Such a problem is faced due to the loop interactions, and consequently it must be decided which input should control each output. The more complete and
challenging PID controller is the one which has a dedicated regulator for each combination of output–input variables. Bristol\textsuperscript{29} suggested, based on their interaction measurement, simpler PID controller matrices which consider fewer regulators.

The simplest one and implemented in this work is a skew-symmetric matrix which requires the determination of only one set of the regulator gains; for the case of I-controller, it means one single value for the integral gain $k_i$. With such an approach the servovalve control signals are driven by the sum and subtraction of both error signals. The structure is defined as follows:

$$C(S) = \begin{bmatrix} k_i & k_i \\ -k_i & k_i \end{bmatrix}$$

(3)

If the I-controller transfer function $C(S)$ is added to the system as shown in Figure 3, the closed-loop transfer function of the system $G(S)$ which relates the reference signal with the system response output signal becomes:

$$G(S) = \frac{Y(S)}{R(S)} = [1 + P(S)C(S)]^{-1}[P(S)C(S)]$$

(4)

**Integral controller gain synthesis**

The I-controller gain $k_i$ can be synthesized from the stability viewpoint. Starting from equation (4) it can be demonstrated that the terms of the closed-loop transfer function $G(S)$ with the I-controllers $C(S)$ are defined as follows:

$$G(1, 1) = \frac{P_{12}k_iS + 2|P|k_i^2}{S^2 + Ak_iS + 2|P|k_i^2}$$

$$G(1, 2) = \frac{P_{21}k_iS}{S^2 + Ak_iS + 2|P|k_i^2}$$

$$G(2, 1) = \frac{P_{21}k_iS}{S^2 + Ak_iS + 2|P|k_i^2}$$

$$G(2, 2) = \frac{P_{12}k_iS + 2|P|k_i^2}{S^2 + Ak_iS + 2|P|k_i^2}$$

(5)

where $|P|$ denotes the determinant of the constant gains matrix $P(S)$, $P_{ij}$ denotes the addition (+) or subtraction (−) arithmetical operations of the terms $P_{ij}$ and $P_{j2}$ and $A$ is a constant defined by the system gains as $A = P_{12}^2 + P_{44}^2$.

The system stability is analyzed by means of the characteristic polynomial $\Pi(S)$ of the closed-loop transfer function denominator of equation (5). The system poles are determined by setting the equation $\Pi(S) = 0$, hence the denominator of equation (5) can be rewritten as follows:

$$\Pi(S) = S^2 + Ak_iS + 2|P|k_i^2 = 0$$

(6)

Comparing equation (6) with the second-order polynomial $S^2 + 2\xi\omega S + \omega^2 = 0$ which is characterized by the parameters $\omega$ and $\xi$, and for which the system poles are determined as $S = -\xi\omega \pm j\omega\sqrt{1 - \xi^2}$, the following analogy can be set:

$$2\xi\omega = Ak_i; \quad \omega^2 = 2|P|k_i^2$$

(7)

To ensure stability, the real part of the system poles must be negatively defined, hence $2\xi\omega > 0$. Moreover, the natural frequency must be a real valued number, then $\xi^2 > 0$. Considering these restrictions and the fact that the constant $A$ is negatively defined, $A < 0$, whereas the determinant of $P$ is positively defined, $|P| > 0$ (see example values in Table 5), the following can be stated:

- Since the constant $A$ is negatively defined, the integral gain must be negative as well to guarantee the stability, i.e. $k_i < 0$.
- The absolute value of $k_i$ can be defined in terms of the closed-loop system natural frequency as follows:

$$||k_i|| = \frac{||\omega||}{\sqrt{2|P|}}$$

(8)

- The system damping ratio $\xi$ is independent of the $k_i$ value. In fact, using equation (7) it can be demonstrated that it is defined by $\xi = -A/\sqrt{8|P|}$ for the integral gain $k_i < 0$.

**Simulation of the I-controller**

Simulations are carried out with the aim of synthesizing different gain values for the I-controller based on the control strategy previously developed. To evaluate their performance, the system response was obtained by implementing the closed-loop system model in the Laplace domain with the aid of Simulink. The system response can be obtained from equation (4) by multiplying the closed-loop system transfer function $G(S)$ by the reference vector $R(S)$, i.e. $Y(S) = G(S)R(S)$, where the terms of the matrix $G$ are defined by equation (5). Two reference functions different from the unitary step were used to establish a setpoint value of 10 $\mu$m in both directions at the same time, i.e. $R_0 = [10 \ 10]^{T} \mu$m, a linear and a sine ramp function. These functions are considered to be more appropriate for changing the shaft center position by tracking a reference toward a setpoint quasi-statically. The ramp time $T_{ramp}$ for which the reference
The integral gain values \( k_i \) for case #2 are compared. It can be noticed that:

- The larger the integral gain \( k_i \), the closer the system response to the reference signal and the lower the error.
- The sine ramp function seems to be more appropriate when compared with the linear ramp for changing the journal position due to the smoother transition to the setpoint obtaining almost no overshooting for the higher integral gain values (the system response to the linear ramp function is omitted for the sake of brevity).

### Experimental results

Three fundamental experimental investigations are carried out:

i. **I-controller performance**: The controller performance is evaluated when the journal is positioned at four different setpoints in the four quadrants (I), (II), (III), and (IV) illustrated in Figure 4. A sine ramp function is used as reference and only the simulated gains \( k_i \) with best performance are used to build the I-controller, i.e. \( k_i \). A summary of these gains and the relevant constant for their calculation are summarized in Table 5 for both operational conditions under study. It is worth mentioning that one deals with LBP configuration, conventional hydrodynamic lubrication, and clockwise rotational speed \( \Omega \) (see Figure 2); it means that the journal center moves towards quadrant (III) and (IV) due to a rotor weight load. A very small (almost insignificant) attitude angle can be detected towards quadrants (III), as expected by the hydrodynamic lubrication theory for TPJB.

ii. **System lateral dynamic response at preset journal positions**: Once the journal reaches its static equilibrium position set by means of the I-controller action within the four bearing quadrants, the rotor-bearing system lateral response is experimentally evaluated in the frequency domain up to 200 Hz. The system is excited by a chirp signal applied with an electromagnetic shaker attached to the auxiliary excitation bearing, see the framed picture of Figure 1.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>( k_i ) (mV/( \mu )m/s)</th>
<th>( k_b ) (mV/( \mu )m/s)</th>
<th>( k_c ) (mV/( \mu )m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case #2</td>
<td>-10.5</td>
<td>-31.5</td>
<td>-52.5</td>
</tr>
<tr>
<td>Case #4</td>
<td>-29.6</td>
<td>-88.8</td>
<td>-148</td>
</tr>
</tbody>
</table>

**Table 5. Summary of the used values for integral gain calculation and the dynamic parameters for cases #2 and #4.**

| Case   | \( |P| \) | \( k_i \) (mV/\( \mu \)m/s) | \( \omega_0 \) (Hz) | \( \xi \) |
|--------|------|--------------------------|-------------------|--------|
| Case #2| -44.05 | 1793.8                  | -52.5             | 0.5    | 0.37 |
| Case #4| -30.78 | 224.6                   | -148              | 0.5    | 0.73 |

**Figure 6.** Simulation–system response \( Y(t) \) to the sine ramp function reference \( R(t) \) with I-controller. \( R_0 = [10 \, 10] \mu \)m. \( U(t) \): control signals.

![Figure 6](image-url)
iii. Non-invasive fluid film perturbation forces aided by I-controllers: additionally, the I-controller can be used for re-positioning the static journal eccentricity and matching the equilibria under conventional hydrodynamics and feedback-controllable lubrication regimes. By simultaneously matching the journal equilibria and varying the injection pressure, the lateral dynamics of the rotor-bearing system in the frequency domain is investigated. In that way, the bearing can be used as a calibrated shaker without affecting the original rotor-bearing system equilibrium position.

It is important to recall that, unlike the servovalve characterization, during these investigations none of the servovalves are enforced to keep closed. Moreover, both are switched on and completely managed by the I-controller through the control signals $u_1(t)$ and $u_2(t)$ in order to move the journal within the plane “x-y” (TITO system). Thereby, setpoints can be carefully defined as references for the I-controller taking into account the limitations of the servovalves (journal center map) under the operational condition imposed.

The I-controller performance

The I-controllers are digitally implemented with the integral gains $k_i$, summarized in Table 5. The journal center is moved from the steady-state equilibrium to the up (blue colored), down (red colored), left (cyan colored) and right (magenta colored) directions. The performance of the I-controllers is evaluated at the four different setpoint values achieved by the sine ramp function with a ramp time of $T_{ramp} = 25$ s as reference.

Figures 7 and 8 depict the results obtained for case #2 when the shaft moves horizontally and vertically, due to the I-controller actuation, respectively. Figures 9 and 10 illustrate the results obtained for case #4. To facilitate the reading of the large amount of information from the set, subfigures (a) and (d) depict the journal movements in the horizontal direction with subfigures (b) and (e) in the vertical direction. Subfigures (c) and (f) depict the control signals governing the servovalves.

From Figure 7, it can be noticed that in general, the system response is capable of tracking the reference signal towards the setpoint of 20 $\mu$m to the left and to the right directions, i.e. $R_0 = [-20 \text{ to } 0] \mu$m and $R_0 = [0 \text{ to } 20] \mu$m, but with some small inconveniences.

On the other hand, when the reference signal is set to the right, the shaft cannot move further than 15 $\mu$m because of the servovalve 2 control signal getting saturated (see Figure 7f). On the other hand, when the reference signal is set to the left, the shaft can reach the setpoint but with some oscillations during the track of the reference signal. These ripples which also appear in the system vertical response can be reduced by changing the gain value of the I-controller. When the journal moves towards the down direction, from Figure 8(b), the journal center is able to track the reference signal very well, achieving the down position with a setpoint of $30 \mu$m, i.e. $R_0 = [0 \text{ to } 30] \mu$m. In this case, small errors during the ramp time and almost no steady-state error can be seen. The opposite occurs when the reference signal is set to move the journal to the upper position with a

![Figure 7](image1.png)

**Figure 7.** Experimental results for case #2—closed-loop system response and control signals when the shaft is moved in the horizontal direction with a sine ramp function. Left column: reference signal to the left (−). Right column: reference signal to the right (+).

![Figure 8](image2.png)

**Figure 8.** Experimental results for case #2—closed-loop system response and control signals when the shaft is moved in the vertical direction with a sine ramp function. Left column: reference signal to the down (−). Right column: reference signal to the up (+).
The phenomena already seen when the shaft is moved in the horizontal direction appear again; there are some oscillations during the reference tracking and the system does not reach its setpoint value due to, in this case, the control signal saturation of servovalve 1, see Figure 8(f).

Figures 9 and 10 depict the results obtained for the operational case #4 with the updated gain value of $k_i$ from Table 5. The main difference from the previous results is that since the operational condition has changed, the servovalve responses are not the same; the shaft center cannot be moved as much as in case #2. For the horizontal cases as well as for the up position of the vertical case, a setpoint of 7 μm was used, i.e. $R_0 = [0 7] \text{μm}$. For the down position, a setpoint of 15 μm was used, i.e. $R_0 = [0 15] \text{μm}$. In general, it can be noticed that the system response is able to track the reference signal with different degrees of success in the steady-state error. The best performance is obtained when the journal is moved to the right position and a poor performance is obtained when the shaft is moved to the up position. It can be seen that the ripples are not present for this operational condition and the reference signal is tracked better when the journal is moved to the down position. Regarding the cross-coupling effects, it can be seen from both operational cases, that there is a significant influence in the cross-system response when the shaft is moved in the opposite orthogonal direction. This effect is clearer in Figure 10, when the journal is moved in the vertical direction (Figure 10b and c) and a significant response in the horizontal direction (Figure 10a and d) is obtained too.

System lateral dynamic response at preset journal positions

Since the I-controller is capable of changing the journal position, the effect on the system lateral dynamic response can be explored. Changing the journal position in a controlled manner is similar to the effect featured with the adjustable lubrication regime, for which the injected lubricant flow is obtained with the servovalves completely open, but in this case the flow is commanded by the control signals aimed at reaching the setpoint. By doing so, the active injection system is acting mainly on the static force equilibrium rather than on the thermal equilibrium, hence its thermal effects are less significant. Such steady-state equilibrium corresponds to the one reached with the high pressure hydraulic unit turned on, hereafter referred to as the leakage flow case. The system lateral dynamic responses were obtained when the journal position was changed to the four different positions previously tested. The system dynamic response when the test rig is running under passive lubrication regime and when it is running with the high pressure hydraulic unit enabled and feeding with the leakage flow, are considered as benchmarks.

Figure 11 depicts the FRFs obtained at these new journal positions. These FRFs have been obtained by means of a chirp signal applied with the electromagnetic shaker placed at the excitation bearing (see Figure 1 item 1) when the test rig was subjected to the operational condition of case #2. In Figure 12, the preset journal positions and their corresponding setpoints are plotted. The journal center map defined in Figure 4 for the same operational condition is also...
presented. By turning the high pressure hydraulic unit on, a new journal steady-state equilibrium is reached due to the leakage flow into the bearing gap. Significant changes in the system dynamic response are consequently detected. This claim can be proven through the increase of the FRFs amplitudes, i.e. passive hydrodynamic lubrication (green solid line) and hydrodynamic plus hydrostatic coming from the leakage flow (black thick solid line). Using the leakage case black FRF as reference, one can notice that there are two setpoints in which an increase of the system lateral responses are obtained and two in which they are reduced. An increase in FRF amplitudes correspond to the right and down setpoint positions. These positions correspond to the borders of quadrant (IV) defined previously, where the shaft is moved in the same direction as the applied load and counter to the direction of the small attitude angle achieved by the hydrodynamic equilibrium position. In the two additional cases, corresponding to the up and left setpoint positions, the borders of quadrant (II), where the shaft is moved counter to the load direction and in the same direction as the small attitude angle, a decrease of system response is observed. This can be seen by analysing the FRFs with blue and cyan dash-dotted lines. It is important to highlight that such vibration amplitude reduction is obtained when compared with the leakage case (black thick solid line), not with the passive case (green solid line).

Figure 11. Experimental FRFs—comparison of system lateral dynamic responses at preset journal positions. Operational conditions of case #2 (test performed after thermal equilibrium condition is reached). Direct vertical FRFs obtained through a chirp signal. Solid line: passive case. Thick solid line: leakage case. Dashed lines: down and right cases. Dash-dotted lines: left and up cases.

Figure 12. Preset journal positions—reached journal positions ("O" marker) and setpoints ("X" marker). Operational condition of case #2. Movements along the x and y directions between setpoints represent 24% and 27% of the assembly diametrical clearance, respectively.

Non-invasive fluid film perturbation forces aided by I-controllers

The servovalve input signal \( u_i(t) \) and the supplied injection pressure \( P_{inj} \) are the two main parameters responsible for the dynamic modification of the pad pressure distribution, generating active fluid film forces. Such forces can be used either to control or to excite the rotor-bearing system. The feasibility of using the active lubrication as a "calibrated shaker" would be extremely useful towards "in-situ" rotor dynamic testing, identification of machine parameters, and aiding fault diagnose procedures. The idea of using controllable fluid film bearings as a calibrated shaker has been mentioned in Santos’s work \(^{33}\) and investigated experimentally in Santos and Varela’s work, \(^{24}\) but not yet fully exploited. In Santos and Varela’s \(^{34}\) work, the maximum values of the two main input parameters are experimentally found and suggested, having the focus on building non-invasive perturbation forces. With “non-invasive force” we have in mind “small perturbation forces” built by the controllable radial oil injection which do not significantly alter the fluid film dynamics, i.e. the stiffness and damping properties of the film, but still perceptible and measurable, allowing for measurements of FRFs with good coherence. From Santos and Varela’s \(^{34}\) work one additional challenge is identified: by turning on the servovalves, the journal equilibrium position changes due to the leakage flow into the bearing gap, and consequently also the rotor-bearing system dynamics. This experimental study is devoted to overcoming this problem and, via I-controller, bringing the journal center equilibria in both
lubrication regimes (hydrodynamic and hybrid) to the same setpoint.

Figure 13 depicts the system lateral response at location 2 of Figure 1 as a function of the excitation frequency. Amplitude, phase, and coherence functions are presented when the injection pressure is set to 12 bar, a low pressure in order to avoid invasive perturbation forces. The green thin curve illustrates the dynamic behavior under passive lubrication, which corresponds to the dynamic behavior of the original rotor-bearing system (the main reference curve). The black thick curve shows the system dynamic behavior obtained with the high pressure hydraulic unit turned on (leakage case). The curve in a red dashed line illustrates the result obtained when the I-controller is used, matching the journal equilibria. It can be noticed from the shaft center position table (journal equilibria) adjoined to the graph in Figure 13 that the I-controller succeeded in returning the journal position to the original one. In the leakage case with injection pressure of 12 bar, the system dynamic behavior is significantly altered when compared to the passive case. Activating the I-controller, the FRF resembles pretty well the dynamic response of the original passive case, especially for the frequency range up to 150 Hz.

Conclusions

Based on the experimental results obtained, the following conclusions can be addressed:

- The I-controller can be used to change the journal position within the linear range of the system through a quasi-static process. It was experimentally realized that the system response can track better the reference signal when the journal position is moved to quadrant (IV) of the bearing gap. If the journal position is moved to quadrant (II) some oscillations during the reference signal tracking are noticed. It was observed that these oscillations can be diminished when the integral gain $k_i$ is changed.
- For this special design of the active TPJB, the system dynamic amplitudes cannot be significantly reduced compared to the passive lubrication case, by only changing the shaft center position to a pre-defined equilibrium via I-controllers. However, if the lubrication regime with the leakage flow is used as a benchmark, some reduction of the dynamic response can be obtained when the shaft is moved to quadrant (II). It is important to stress that such conclusions are true for TPJB with one single nozzle machined in the pad middle and without any kind of pocket (shallow nor deep).
- The I-controller can successfully bring the journal position back to its original one defined by the passive lubrication. By simultaneously choosing low pressure values (in this experimental study, up to 12 bar), no significant changes in rotor-bearing system dynamics are noticed. It leads to the conclusion that it is possible to generate “non-invasive fluid film perturbation forces” and use the actively-lubricated fluid film bearing as a “calibrated shaker”, by choosing correct injection pressures and amplitude signal for the servovalves, and simultaneously using a properly-designed I-controller to match the journal center equilibria under both lubrication regimes.

Conflict of interest

None declared.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

References


\( k \) integral gain (mV/\( \mu \text{m} \text{s} \))

\( P_j \) open-loop system constant gain (\( \mu \text{m}/\text{V} \))

\( P_{\text{oil}} \) oil injection pressure (bar)

\( P_{\text{p}i} \) system gain matrix elements (\( \mu \text{m}/\text{V} \))

\( P_{\text{p}q} \) constant = \( P_j \pm P_k \) (\( \mu \text{m}/\text{V} \))

\( P(S) \) open-loop system transfer function (\( \mu \text{m}/\text{V} \))

\( P(t) \) system gain matrix (\( \mu \text{m}/\text{V} \))

\( R_\text{o} \) reference setpoints = \([r_{ox} \quad r_{oy}]^T \) (\( \mu \text{m} \))

\( R(S), R(t) \) reference signals = \([r_x(t) \quad r_y(t)]^T \) (\( \mu \text{m} \))

\( S \) Laplace variable

\( T_{\text{ramp}} \) time of the ramp function (s)

\( U(S), U(t) \) control signals = \([u_1(t) \quad u_2(t)]^T \) (\( \mu \text{m} \))

\( Y(S), Y(t) \) system response = \([x(t) \quad y(t)]^T \) (\( \mu \text{m} \))

\( \xi \) second order system damping ratio

\( \Pi(S) \) characteristic polynomial of the transfer function

\( \omega \) second order system natural frequency (Hz)

\( \Omega \) shaft angular velocity (r/min)
A.2 [P2]: Feedback-Controlled Lubrication for Reducing the Lateral Vibration of Flexible Rotors Supported by Tilting-Pad Journal Bearings
Feedback-controlled lubrication for reducing the lateral vibration of flexible rotors supported by tilting-pad journal bearings

Jorge G Salazar¹,² and Ilmar F Santos¹

Abstract
The feedback-controlled lubrication regime, based on a model-free designed proportional–derivative controller, is experimentally investigated in a flexible rotor mounted on an actively-lubricated tilting-pad journal bearing. With such a lubrication regime, both the resulting pressure distribution over the pads and hence the bearing dynamic properties are dynamically modified. The control strategy is focused on reducing the lateral vibrations of the system around its operational equilibrium within a wide frequency range. To synthesize the proportional–derivative controller gains, an objective function is optimized in the stabilizing gain domain and then chosen from a subdomain imposed by servovalve restrictions. This work demonstrates enhancements of the dynamic response of flexible rotor-bearing systems supported by an active tilting-pad journal bearing by means of the feedback-controlled lubrication regime featured via proportional–derivative controllers.

Keywords
Feedback-controlled lubrication, active tilting-pad journal bearing, active bearing design, multi-input multi-output systems, vibration reduction

Introduction
For several decades, fluid film journal bearings have been used as a standardized machine element that can be easily specified for a new machine. Most of the earlier drawbacks, such as poor energy efficiency, high bearing wear, and instability problems have been satisfactorily addressed with the current designs. Nevertheless, new requirements are continuously emerging and nowadays higher efficiency, more severe operational conditions of velocities and loads, lower vibration levels as well as the capability of adjusting their dynamics for a maintenance-free operation are currently requested.¹ In such a framework, tilting-pad journal bearings (TPJBs)—a well-known type of fluid film bearing with superior stability properties—have naturally evolved to mechatronic machine elements by means of the addition of control systems for responding to these new requirements. Different types of actuators have been studied,³–⁶ nonetheless the one presented in this work is based on the hydraulic type. TPJBs with electronic radial oil injection or actively-lubricated TPJBs or simply active TPJBs⁷,⁸ are provided with servovalves, which inject high pressurized lubricant into the bearing clearance through nozzles placed in orifices commonly drilled in the middle of the pad surface. For such mechatronic bearings, control signals govern the servovalves, regulating the lubricant injection flow and hence the pressure distribution composed by the hydrodynamic and hydrostatic effects. As a result, the dynamic properties of the bearings are modified. Mathematical models for active TPJBs follow the modeling of conventional TPJBs and have evolved from the elastohydrodynamic approach⁹ to the current thermoelastohydrodynamic approach,¹⁰,¹¹ accounting for the pivot flexibility, the heat transfer to the solid and fluid surroundings, as well as for the servovalves and pipe dynamics. Depending on whether control laws are used or not, different

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lubrication regimes can be featured. The used control strategy also defines the developed lubrication regime.

Within the active TPJBs framework, earlier theoretical studies\textsuperscript{12–14} have been focused on including the control laws in the model for a rigid rotor-active TPJB test bench; however, few studies have been focused on flexible rotor systems. There are roughly two ways of reducing the lateral vibrations of a rotor-bearing system, the first one by changing its equilibrium position, and the second by affecting the bearing properties around the equilibrium. Previous work\textsuperscript{15} on the same flexible rotor system focused on exploring the reduction on the system lateral response by changing the journal equilibrium position via an integral controller. The second way of reducing the system lateral vibrations can be reached by implementing the feedback-controlled lubrication regime based on proportional–derivative (PD) controllers. It has been shown theoretically that these controller gains act directly on the bearing stiffness and damping parameters.\textsuperscript{16} Thus, the lateral vibrations can be controlled around the shaft operational equilibrium position.

The main original contributions of this work are the design and implementation of a PD controller to feature the feedback-controlled lubrication regime with the aim of reducing the lateral vibration of a flexible rotor-active TPJB test bench. To synthesize the PD controller gains an objective function is optimized in the stabilizing gain domain and then the set of the stabilizing PD-gains are chosen from a subdomain imposed by the servovalve restrictions. The D-decomposition approach,\textsuperscript{17–19} previously used with experimentally described single-input single-output (SISO) systems,\textsuperscript{20,21} is now extended to determine the stabilizing gain domain within the controller parameter space for an experimentally characterized multi-input multi-output (MIMO) system. The dynamic characterization of the whole system (servovalves + mechanical system + sensors) required for designing the controller is provided by means of a model-free characterization in the form of measured frequency response functions (FRFs) obtained by excitation via the servovalves.

**Experimental facilities**

*The flexible rotor-bearing test stand*

The test stand, which is comparable to an industrial machine, is an 87 kg overhung rotor mounted on a 1150 mm flexible shaft supported by the active TPJB and by a ball bearing at its driven side. The test stand is belt-driven by means of a layshaft flexibly coupled to the shaft at its driven side. The power is supplied by a 4 hp AC motor which through a frequency driver enables the test stand to run up to 7000 r/min. The test stand allows for studying the dynamic behavior of flexible rotor-bearing systems when different feedback-controlled lubrication regimes depending-on-the-adopted-control-law are developed with the active TPJB. Two ways of exciting the test stand are currently enabled to carry out modal parameter identification. The first one, by means of an excitation bearing placed at the free end and connected to an electromagnetic shaker through stingers and a force transducer to apply unidirectional forces. The second one, an active magnetic bearing (AMB) placed next to the active TPJB, toward the driven end side, which can exert forces over the rotor in all possible directions without any physical link to it. Figure 1 shows an overview scheme of the test stand with its primary parts and the experimental instrumentation setup used for this work. The used reference frame is also included in the scheme.

*The controllable journal bearing*

The controllable/active bearing is a TPJB with four bronze pads in a load-between-pads (LBP) configuration. The pads are rocker pivoted in the circumferential middle of the pad, i.e. with an offset of 0.5. Further design parameters are included in Table 1. The active or controllable feature of the bearing is
developed by an electronic radial oil injection system as proposed by Santos and Russo. This injection system superimposes a hydrostatic pressure over the hydrodynamic pressure distribution by injecting pressurized oil between the journal-pad clearance through a nozzle placed in the middle of the pad. The high pressure oil flow is controlled by two high-frequency response servovalves, each one is coupled to a pair-wise of counter pads. The lubricant is supplied for the conventional and for the active lubrications by a low (max. 2 bar) and a high (max. 100 bar) pumping unit, respectively. Figure 2 depicts a scheme of the radial oil injection system overlapped to a picture of the active bearing. Proximity probes used for monitoring and feeding back controllers are also included (position 7a in Figure 1). The active bearing is capable of operating under three different lubrication regimes, namely: (a) the conventional or hydrodynamic lubrication regime, also called “passive”, (b) the hybrid or adjustable lubrication regime, which is a combination of the hydrodynamic case with an hydrostatic effect added by the injection system, and (c) the feedback-controlled or active lubrication regime whose hydrostatic contribution to the pressure field is controlled by well-tuned control gains. Commonly, the controller is digitally implemented in an FPGA or similar hardware with real-time processing capabilities.

**PD controller design**

When the control strategy is focused on diminishing the vibration around the equilibrium position the most suitable controllers for such a purpose are the PD controllers. This is because their gains act over the instant error and its time derivative (related with the system position \( Y(t) \) and velocity \( _Y \dot{Y}(t) \)) instead of acting upon the cumulative error (related to the mean position value \( Y(t) \)) as is the case with the integral controllers, which are more suitable for affecting the system equilibrium position. In order to properly

### Table 1. Conventional and controllable design parameters of the controllable tilting-pad journal bearing or active TPJB.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conventional design</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Journal radius ((R))</td>
<td>49.89</td>
<td>mm</td>
</tr>
<tr>
<td>Pad inner radius ((R_i))</td>
<td>50</td>
<td>mm</td>
</tr>
<tr>
<td>Pad aperture angle ((\alpha_p))</td>
<td>69</td>
<td>degrees</td>
</tr>
<tr>
<td>Pad width ((L))</td>
<td>100</td>
<td>mm</td>
</tr>
<tr>
<td>Pad thickness ((t))</td>
<td>14</td>
<td>mm</td>
</tr>
<tr>
<td>Nominal radial clearance</td>
<td>110</td>
<td>(\mu)m</td>
</tr>
<tr>
<td>Assembly radial clearance</td>
<td>83</td>
<td>(\mu)m</td>
</tr>
<tr>
<td>Bearing applied load</td>
<td>890</td>
<td>N</td>
</tr>
<tr>
<td>Lubrication type/oil</td>
<td>Oil mist / ISO VG22</td>
<td>–</td>
</tr>
<tr>
<td>Sprinkler number</td>
<td>4</td>
<td>–</td>
</tr>
<tr>
<td>Oil flow (at 1 bar)</td>
<td>1 L/min</td>
<td></td>
</tr>
<tr>
<td><strong>Controllable design</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Servovalve type</td>
<td>MOOG E760-912</td>
<td>–</td>
</tr>
<tr>
<td>Servovalve configuration</td>
<td>4 way, spool valve</td>
<td>–</td>
</tr>
<tr>
<td>Injection orifice diameter</td>
<td>3.3 mm</td>
<td></td>
</tr>
<tr>
<td>Injection orifice length</td>
<td>21 mm</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 2](image-url)
design the PD controller, a reliable dynamic plant characterization in a wide frequency range is needed instead of the quasi-static calibration curves used, as in the case of integral controllers. Owing to changes in the system dynamics with the operational conditions, produced mainly by the gyroscopic effect and by the nonlinear bearing properties, different operational conditions are taken into account. Table 2 summarizes the operational conditions considered for the plant characterization and the controller design, hereafter referred to as the cases from #1(a) to #3(c).

Model-free plant characterization

The plant dynamics can be described either by theoretical or experimental models. Working with theoretical models is not a straightforward task due to the complexity that they normally reach for describing their behavior more precisely. On the other hand, since the goal is to demonstrate the effectiveness of the feedback-controlled lubrication, the usage of experimental models is sufficient for the work purpose and of high importance for industrial applications. In this case, all uncertainties yielded by theoretical models can be disregarded since the experimental models take them already into account. The experimental plant dynamic characterization is carried out by means of the measured FRFs obtained via the servovalves and defined between the system input and output variables, i.e. in this case between the servovalve control signals and the lateral movements of the shaft; for instance, at the bearing housing position or at the shaft free end (positions 7a and 7b in Figure 1).

Therefore, the plant characterization includes not only the mechanical system dynamics but also the servovalves and proximity probe dynamics. Since the four pads of the bearing are in a LBP configuration, i.e. the orthogonal directions used for describing the applied load and also the measurements are shifted 45° related to the servovalves' orthogonal directions, the bearing can be considered as a two-input two-output (TITO) system for the control design aim. Contrarily, if the bearing design were considered as a two-input two-output (TITO) system for the controller design purpose, the bearing can be considered as a TITO system for the controller design purpose.

Table 2. Operational conditions used for the model-free dynamic plant characterization and the PD controller design.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \Omega ) (r/min)</th>
<th>( P_{\text{sup}} ) (bar)</th>
<th>( P_{\text{sup}} ) (bar)</th>
<th>( P_{\text{sup}} ) (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>1000</td>
<td>30</td>
<td>50</td>
<td>70</td>
</tr>
<tr>
<td>#2</td>
<td>2500</td>
<td>30</td>
<td>50</td>
<td>70</td>
</tr>
<tr>
<td>#3</td>
<td>4000</td>
<td>30</td>
<td>50</td>
<td>70</td>
</tr>
</tbody>
</table>

\( \Omega \): shaft angular velocity; \( P_{\text{sup}} \): oil supply pressure of the radial injection unit.

The closed-loop TITO plant configuration

Figure 4 depicts the plant block diagram in open (solid line) and closed-loop (dashed line) configurations in which all variables are defined in the
frequency domain instead of the Laplace domain to work with the experimental data, i.e.

\[ S = j! \]

where \( j \) denotes the complex unity. \( R(j\omega) \) stands for the reference signal, \( E(j\omega) \) for the error, \( U(j\omega) \) for the control signal, \( D(j\omega) \) for the system disturbance, \( Y(j\omega) \) for the system output, \( P(j\omega) \) the plant response, \( C(j\omega) \) PD controller.

**Figure 3.** Dynamic characterization of the TITO system via servovalves at position \( G_a \) (red dashed line) and \( G_b \) (black solid line) of Figure 1 for the operational condition of case #1(c). A: \( P_x(j\omega) \); B: \( P_y(j\omega) \); C: \( P_x(j\omega) \); D: \( P_y(j\omega) \). Cases #2(c) and #3(c) are also included in gray colors for position \( G_b \).

**Figure 4.** Open (solid line) and closed-loop (dashed line) plant block diagrams. \( R(j\omega) \): reference signal; \( E(j\omega) \): error; \( U(j\omega) \): control signal; \( D(j\omega) \): system disturbance; \( Y(j\omega) \): system output; \( P(j\omega) \): the plant response; \( C(j\omega) \): PD controller.

Table 2. Hence the closed-loop transfer function \( G(j\omega) \) of the plant is defined by:

\[
G(j\omega) = \frac{Y(j\omega)}{D(j\omega)} = [I + P(j\omega)C(j\omega)]^{-1}[P(j\omega)] \tag{1}
\]

where \( I \) denotes the identity matrix. Two matrices define the closed-loop transfer function of the plant defined in equation (1), i.e. the open-loop plant transfer function \( P(j\omega) \) which is completely known and experimentally defined for each operational condition.
This adopted TITO structure is defined as:

\[ \mathbf{P}(\omega_0) = \begin{bmatrix} P_{x1}(\omega_0) & P_{x2}(\omega_0) \\ P_{y1}(\omega_0) & P_{y2}(\omega_0) \end{bmatrix} \] (2)

and the PD controller transfer function which must be designed. Different structures can be adopted for the PD controller transfer function matrix \( \mathbf{C}(\omega_0) \) when dealing with the TITO system, commonly referred to as the pairing problem. These structures are based on the loop interactions and assessed in terms of their interaction measure.\(^5\) For instance, a dedicated PD controller for each matrix element of \( \mathbf{C}(\omega_0) \) requires synthesizing eight controller gains. Contrarily, a decentralized matrix, i.e. each control signal \( u(t) \) is commanded for only one error signal \( e_i(t) \), would require the synthesis of four controller gains. A simpler form, is to work with the same PD controller for all matrix elements of \( \mathbf{C}(\omega_0) \), for which only one set of controller gains must be determined. This adopted TITO structure is defined as:

\[ \mathbf{C}(\omega_0) = \begin{bmatrix} \mathbf{C}_{P1}(\omega_0) & \mathbf{C}_{P2}(\omega_0) \\ -\mathbf{C}_{P1}(\omega_0) & \mathbf{C}_{P2}(\omega_0) \end{bmatrix} \] (3)

where the term \( \mathbf{C}_{P}(\omega_0) \) is the standard SISO transfer function for the PD controller defined in the frequency domain by:

\[ \mathbf{C}_{P}(\omega_0) = \frac{u(\omega_0)}{e(\omega_0)} = k_p + j_0k_d \] (4)

isolating the PD controller complex gain of equation (4), equation (3) can be rewritten conveniently as:

\[ \mathbf{C}(\omega_0) = (k_p + j_0k_d) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \] (5)

with such a structure, the servovalve control signal \( u_i(t) \) is managed by the sum of the errors \( e_i(t) + e_j(t) \) and the servovalve control signal \( u_j(t) \) is managed by the subtraction of the error signals \( e_i(t) - e_j(t) \). This is expressed in the frequency domain as:

\[ \mathbf{U}(\omega_0) = \mathbf{C}_{P}(\omega_0) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \mathbf{E}(\omega_0) \] (6)

\[ \begin{bmatrix} u_1(\omega_0) \\ u_2(\omega_0) \end{bmatrix} = \mathbf{C}_{P}(\omega_0) \begin{bmatrix} e_1(\omega_0) + e_2(\omega_0) \\ e_1(\omega_0) - e_2(\omega_0) \end{bmatrix} \] (7)

**Setting up the reference signal \( \mathbf{R}(t) \) and the control law \( \mathbf{U}(t) \)**

Once the PD controller structure is defined the next step is to define the error signals governing the PD controller by setting the reference signals. In the time domain the error signals are defined by \( \mathbf{E}(t) = \mathbf{R}(t) - \mathbf{Y}(t) \), i.e.:

\[ \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} - \begin{bmatrix} x_0 + x(t) \\ y_0 + y(t) \end{bmatrix} \] (8)

where the subscripts define the orthogonal directions and the constants \( x_0 \) and \( y_0 \) are the equilibrium position given by the mean value of the displacement sensor signals for the current operational condition. When dealing with an integral controller, the reference signal is set as the new desired equilibrium position, that is \( (x_0, y_0) \); but when dealing with PD controllers and since the control strategy has the goal of reducing the lateral vibration around the operational equilibrium position \( (x_0, y_0) \), the reference signal can be set as the same operation equilibrium point, \( \mathbf{R}(t) = \mathbf{Y}_0 = [x_0 \ y_0]^T \), then the error at each orthogonal direction can be defined by the vibration around the operational equilibrium point as:

\[ \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} = \begin{bmatrix} x_0 - x_0 + x(t) \\ y_0 - y_0 + y(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \] (9)

or expressed in frequency domain as:

\[ \mathbf{E}(\omega_0) = -\mathbf{Y}(\omega_0) \] (10)

Substituting equation (10) into equation (6) leads to the following control law for the servovalves:

\[ \mathbf{U}(\omega_0) = -\mathbf{C}_{P}(\omega_0) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x(\omega_0) \\ y(\omega_0) \end{bmatrix} \] (11)

Expanding equation (11), the control law is written as:

\[ \begin{bmatrix} u_1(\omega_0) \\ u_2(\omega_0) \end{bmatrix} = -(k_p + j_0k_d) \begin{bmatrix} x(\omega_0) + x(\omega_0) \\ y(\omega_0) - x(\omega_0) \end{bmatrix} \] (12)

or in the time domain as:

\[ \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = -k_p(\dot{x}(t) + \dot{x}(t)) - k_d(y(t) + x(t)) \]
\[ u_2(t) = -k_d(\dot{y}(t) - \dot{x}(t)) - k_p(y(t) - x(t)) \] (13)

**Controller gain synthesis**

To synthesize the PD controller gains \( k_p \) and \( k_d \) of equation (12) an optimization problem is formulated. In such a case, an objective function is pursued to be maximized/minimized within the stabilizing domain of the controller gains, defined in the parameter space \( k_p \times k_d \). To define the objective function, the control strategy of suppressing the vibration...
amplitudes is kept in mind for the whole considered frequency range. For defining the stabilizing gain domain, the D-decomposition method \(^{17,19}\) is used. Then, some restrictions related to the servovalve control signals are applied.

The objective function \(I(k_p, k_d)\): The aim of using a PD controller is to reduce the lateral vibration of the system so that the amplitude of the closed-loop system response \(G(j\omega)\) is lower than the amplitude of the open-loop system response \(P(j\omega)\) if possible in the whole frequency range defined as \(D_{\omega} = \{\omega \in \mathbb{R} | 0 < \omega < \omega_{\text{max}}\}\). If the phase margin specification is neglected, then the following function regarding the gain margin can be defined for each frequency \(\omega\):

\[
\mathcal{F}(j\omega, k_p, k_d) = |P(j\omega)| - |G(j\omega, k_p, k_d)| : \forall \omega \in D_{\omega}
\]

where \(|\cdot|\) stands for the absolute value. Therefore, if the area under the curve is used as an index to assess the performance of the controller in the whole frequency domain \(D_{\omega}\), then the following objective function can be defined:

\[
I(k_p, k_d) = \int_{\omega_{\text{min}}}^{\omega_{\text{max}}} \mathcal{F}(j\omega, k_p, k_d) d\omega
\]

(15)

The pair of controller gains \((k_p^*, k_d^*)\) which maximize the index of equation (16) is defined by:

\[
I^* = \max\{I(k_p, k_d)\}; \quad k_p^*, k_d^* \in D_k
\]

(17)

The stabilizing gain domain \(D_k\): Different methods for determining the stabilizing gain domain can be found in literatures.\(^{24–27}\) However, most of them are aimed at synthesizing the gains of decentralized controllers for theoretical or experimental SISO system models. In Buttini and Nicoletti,\(^{20,21}\) the D-decomposition method, developed by Neimark and summarized in Gryazina et al.,\(^{17–19}\) divides the controller parameter space \((k_p \times k_d)\) into root invariant regions for which the number of stable and unstable poles remains constant. For experimental MIMO plant representations, the stability in such areas can be assessed by the generalized (MIMO) Nyquist stability criterion.\(^{25}\) For MIMO systems the stability invariant region boundaries are defined by equaling the determinant of the inverse of the sensitivity function to zero,\(^{18,19}\) i.e.:

\[
det(\mathbf{I} + C(j\omega)P(j\omega)) = 0
\]

(18)

Considering the plant characterization \(P(j\omega)\) of equation (2) and the PD controller \(C(j\omega)\) defined by equation (3) and the identity matrix \(I\), equation (18) can be reduced to:

\[
det(P(j\omega))C_{PD}^2 + (p_{11} - p_{22} + p_{11} + p_{22})C_{PD} = 0
\]

(19)

Equation (19) is a second-order complex equation with one complex variable \(C_{PD}\). Rearranging equation (19) in real and complex terms, the following two equations can be obtained:

\[
a_k k_p^2 - \omega^2 a_k k_d^2 - 2a_k a_k k_d + b_k k_p - b_k a_k k_d = -1
\]

\[
a_k k_p^2 - \omega^2 a_k k_d^2 - 2a_k a_k k_d + b_k k_p - b_k a_k k_d = 0
\]

(20)

where the terms \(a_k, b_k, a_k\), and \(b_k\) are defined by:

\[
a_k = \Re\{a\} = \Re\{\det(P(j\omega))\}
\]

\[
a_k = \Im\{a\} = \Im\{\det(P(j\omega))\}
\]

\[
b_k = \Re\{b\} = \Re\{\det(p_{11} - p_{22} + p_{11} + p_{22})\}
\]

\[
b_k = \Im\{b\} = \Im\{\det(p_{11} - p_{22} + p_{11} + p_{22})\}
\]

(21)

Equation (20) is a system of two equations with two variables — the controller gains \(k_p\) and \(k_d\) — which can be solved numerically and taking into account that the solution should not be the singular solution and that the controller gains \(k_p\) and \(k_d\) must be real values.

The restricted gain subdomain \(D_k\): A restricted gain subdomain can be defined due to the maximum allowed values of the control signals \(U_i(t)\) which in turn limit the PD gains values too. The control signals can be limited by the D/A converter (± 10 V), by the linear range of the servovalves (± 0.5 V over their offset values) or by the full servovalve range (± 2 V). They can also be limited, for instance, to restrict the system response at the active TPJB as a fraction of the assembly clearance, to avoid surface contact between the shaft and the bearing. In this work, due to some uncertainties on the servovalves’ linear range, since there are no spool position feedback signals, the control signals are restricted to their full operational range, limiting the values of the PD controller gains \(k_p\) and \(k_d\).

**Synthesized PD controller gains:** Figure 5 depicts the stabilizing domains in the parameter space found by means of the D-decomposition method for the cases \#1(c), \#2(c), and \#3(c) of Table 2 and identified by A, B, and C, respectively. For obtaining these areas,
equation (19) was used with the data provided from the experimental plant characterization of Figure 3. Results are very sensitive to the experimental data quality, which are improved as the pressure of the high pressure supply unit is increased. With the higher pressure of 70 bar considered (cases (c) in Table 2), the information contained between 20 and 140 Hz has provided acceptable values of all coherence functions and has been used for the calculations. In Figure 5, the black area denoted by \( D \) represents the intersection of all areas and it is considered the common stabilizing gain domain \( D_k \) for determining the pair of gain values \((k_p, k_d)\), which can be indistinctly used for all operational conditions. For analyzing the stability of the closed-loop system, the Nyquist stability criterion for MIMO system was employed as stated in Skogestad and Postlethwaite.\(^{28}\) Three pairs of controller gains \((k_p, k_d)\) were randomly selected and assessed for stability. The first one inside the black \( D \) area, the second one at the boundary, and the third out of the area. Since the plant is properly defined, i.e. it has no unstable poles, no encirclements of the origin in the Nyquist plot are obtained in the closed-loop configuration, or a pass through the origin, and hence the \( D \) area is stable.

Figure 6 shows the objective function \( I(k_p, k_d) \) defined in equation (16) normalized by its maximum absolute value under the operational conditions #1(c) within the common stabilizing domain \( D_k \). All other cases have been omitted for the sake of briefness. Since the objective function \( I(k_p, k_d) \) is defined based on the open and closed-loop transfer function matrices (see equation (16)) it is also defined as a
matrix and it yields different values for the different input/output relationships. The subplots A, B, C, and D show the different relationship between the servovalve control signals and the system response in the same form as defined previously for Figure 3. In general, it can be noticed that better values of the objective function are obtained with higher values of the gains. However, the servovalve control signal ranges restrict the area from which the PD controller gains can be selected and the subdomain $D_k$ must be determined. A simple experimental test was carried out to determine the restricted subdomain. The proportional $k_p$ and derivative $k_d$ gains were tuned one at a time until the servovalves reached their saturation limits of $\pm 2$ V over their offset values ($2.5$ V and $2.05$ V for the servovalves 1 and 2, respectively). Limit values of $k_p = 0.018$ V/$\mu$m and $k_d = 3 \times 10^{-5}$ Vs/$\mu$m were determined defining a restricted rectangle from which the gains can be selected. This rectangle of the subdomain $D_k$ is included in the plot of the objective function in Figure 6.

Table 3 summarizes three different PD controllers selected from the subdomain $D_k$ to be digitally

![Figure 7](image_url)
implemented in the FPGA hardware and experimentally tested. The PD controller #1 was randomly chosen. The other two controllers were obtained by increasing one gain at a time to see their effect on the system response; i.e. for the PD controller #2 the proportional gain was increased from 0.006 to 0.01 V/μm and for the PD controller #3 the derivative gain was increased from $2 \cdot 10^{-5}$ to $3 \cdot 10^{-5}$ Vs/μm.

Experimental results

Experimental procedure

To implement the controllers of Table 3 the shaft displacements were obtained from the proximity probes at point 7b in Figure 1, whereas the velocities were integrated from the accelerometer placed at the excitation bearing, at point 8 in Figure 1, close to the proximity probes. This approach was adopted to avoid unsuitable velocity signals that can affect the control signals due to the amplified noise coming from the numerical time derivative of the displacement signals and due to the phase lags obtained by the use of low pass filters, commonly used to avoid noise. A system identification test aided by the electromagnetic shaker was carried out to characterize the dynamic behavior of the flexible rotor-bearing system at each orthogonal direction for all considered operational conditions of Table 2, except for cases #1(a), #2(a), and #3(a) with the lowest pressure. The tests were performed by sweeping a sinusoidal signal in the frequency range of interest applied by the shaker at position 10 and measured at position 7b of Figure 1 respectively. The experimental procedure was as follows:

- The test rig is run under conventional lubrication for each angular velocity until the steady-state (s-s) is

Figure 8. Experimental FRFs comparison in the vertical direction for case #2 of Table 2, 2500 r/min. (——) Passive lubrication. (——) Hybrid lubrication, leakage case with 50 bar. (——) Active lubrication with PD controller #1, 50 bar. (——) Hybrid lubrication, leakage case with 70 bar. (——) Active lubrication with PD controller #1, 70 bar. (——) Active lubrication with PD controller #1, 90 bar.
reached. Then, the system identification test is performed in both orthogonal directions. These benchmark records are identified as the passive cases.

- The test rig is run under hybrid lubrication for each angular velocity and pressure considered until the s-s condition is reached. Then, the system identification test is performed in both orthogonal directions. These records are identified as the leakage case.
- Once the s-s is reached for each operational case and after the leakage cases, the different PD controllers are tested with the test rig running under feedback-controlled lubrication. Then, for all controllers the system identification test is performed. These records are identified as the active cases with the controllers of Table 3.

**Experimental FRFs**

Figure 7 shows the experimental FRFs obtained in the horizontal direction for case #1 of Table 2, i.e. 1000 r/min. The dynamic response of the system is depicted in green under passive or conventional lubrication as a benchmark. In gray and black are the responses under hybrid lubrication for 50 and 70 bar respectively, identified as the leakage case while red and dark red depict the system response under active or feedback-controlled lubrication with the PD controller #1 of Table 3 for the same pressures. Results for the remaining PD controllers have been omitted since they do not present improvements compared to PD controller #1. From the graph, it can be noticed that the system response is effectively reduced on the resonant zone when the PD controller #1 is used. This reduction is more evident as the pressure of the supply unit is increased. To make the pressure effect more evident, blue and dark blue responses have also been included under active lubrication with 90 bar and 100 bar respectively. A reduction of about 30% can be obtained when the peak amplitudes of the active case with 100 bar is compared against the passive one. Figure 8 shows the experimental FRFs obtained in the vertical direction for case #2 of Table 2, i.e. 2500 r/min. The same color description has been used in this figure. Unlike the horizontal direction, in the vertical one there is a second resonant zone around 165 Hz. The PD controller #1 in the vertical direction seems to be acting mainly over the first resonant zone, the same as for the horizontal case, for which it can be stated that a reduction of the system response is obtained below 120 Hz. The results for case #3 have not been reported because they are highly affected by noise, obtaining a poor coherence.

**Conclusions**

Based on the experimental results obtained in this paper, the following conclusions can be summarized regarding the feedback-controlled lubrication applied to flexible rotor-bearing systems:

- The feedback-controlled or active lubrication based on PD controllers can be used to reduce the lateral vibration of a flexible-rotor bearing system supported by an active TPJB, improving its dynamic performance.
- The effectiveness of the vibration reductions around the equilibrium position, strongly depend on the machine operational conditions as well as on the orthogonal directions in which these reductions are sought. It has been noticed that the effectiveness of the PD controller on reducing the lateral vibration can be significantly improved as the pressure set on the high pressure supply unit is increased.
- A simple PD controller can be used for such a goal and their proportional and derivative gains can be synthesized optimizing an objective function within the stabilizing gain domain or from a narrowed restricted subdomain due to the experimental control signal restrictions. A model-free approach of the plant dynamics based on the measured FRFs via servovalves provides the required plant information for the controller design aim. Such a procedure could be very attractive and useful for industrial applications of the feedback-controlled lubrication.

**Conflict of interest**

None declared.

**Funding**

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

**References**


Appendix

Notation

\( C_{(j_0)} \) controller transfer function (v/m)
\( C_{yp}(j_0) \) SISO PD controller transfer function (v/m)
\( D_k \) stabilizing gain domain
\( \bar{D}_k \) restricted gain subdomain
\( D_{(j_0)} \) system disturbances (V)
\( e_{x,y} \) system error in the \( x, y \) direction (m)
\( E(j_0), E(t) \) system error = \( [e_x(t), e_y(t)]^T \) (m)
\( \mathcal{F}(j_0, k_p, k_d) \) gain margin function
\( G_{(j_0)} \) closed-loop plant transfer function (m/V)

\( I \) identity matrix
\( I(k_p, k_d) \) objective function
\( j \) complex unity, \( \sqrt{-1} \)
\( k_d \) derivative gain (Vs/m)
\( k_p \) proportional gain (V/m)
\( P_{sup} \) oil supply pressure of the radial injection unit (bar)

\( P_{(j_0)} \), \( P_{(j_0), P_{(j_0)} \}} \) open-loop plant transfer function (m/v)
\( P_{yp}(j_0), P_{yp}(t) \) plant gain matrix elements,
\( i = x, y, j = 1, 2 \) (m/v)
\( q_{h}(t) \) high-pressure oil flow 1,2 (m/s)
\( q_{l}(t) \) low-pressure oil flow (m/s)
\( q_{o}(t) \) return oil flow (m/s)
\( R(j_0), R(t) \) reference signals = \( [r_x(t), r_y(t)]^T \) (m)
\( S \) Laplace variable
\( u_{g}(t) \) control signal of servovalve 1,2 (V)
\( U_{(j_0)}, U(t) \) control signals = \( [u_1(t), u_2(t)]^T \) (V)
\( x(t) \) shaft horizontal displacement (m)
\( y(t) \) shaft vertical displacement (m)
\( Y_{(j_0)}, Y(t) \) system response = \( [x(t), y(t)]^T \) (m)
\( Y(t) \) system velocity = \( [\dot{x}(t), \dot{y}(t)]^T \) (m/s)
\( \bar{Y}(t) = Y_0 \) system equilibrium position = \( [x_0, y_0]^T \) (m)
\( \omega \) excitation frequency (Hz)
\( \Omega \) shaft angular velocity (r/min)
A.3 [P3]: Experimental Identification of Dynamic Coefficients of Lightly-Loaded Tilting-Pad Bearings Under Several Lubrication Regimes
Experimental identification of dynamic coefficients of lightly loaded tilting-pad bearings under several lubrication regimes

Jorge G Salazar\textsuperscript{1,2} and Ilmar F Santos\textsuperscript{1}

Abstract
This paper presents the identified dynamic coefficients of a lightly loaded actively lubricated bearing under three lubrication regimes: passive, hybrid and feedback-controlled. The goal is to experimentally demonstrate the feasibility of modifying the bearing dynamic properties via active lubrication. Dominated by the latest two regimes, the bearing properties become adjustable or controllable due to the injection of either a constant or variable pressurized oil flow. Such a flow is regulated by a hydraulic control system composed of (a) a high-pressure oil supply unit, (b) servovalves, (c) radial injection nozzles, (d) displacement sensors and (e) well-tuned digital controllers. A scaled-down industrial rotor featuring active lubrication, composed of a flexible rotor supported by a four-rocker load-between-pads tilting-pad bearing under light load condition, is used for this objective. The experimental identification is performed by means of measured frequency response functions and a rotor finite element model. Predicted coefficients are also provided for benchmarking. Comparing results between the different regimes, presented along with their expanded uncertainty, provides the experimental evidence of the bearing properties modification via active lubrication.

Keywords
Tilting-pad journal bearings, lightly loaded bearing, active lubrication, dynamic force coefficients, frequency-domain identification

Introduction
Tilting-pad journal bearings (TPJBs) have experienced widespread usage to the point of becoming a standard machine element when designing high-speed turbomachinery. This is due to their distinguishing stability characteristics among fluid film bearings,\textsuperscript{1} which strongly influence the dynamic characteristics of the entire rotor-bearing system. Such characteristics are predefined at an early design stage when selecting machines for specific process lines. However, increasing demands of plant production still require faster machines with enhanced load-carrying capacity and dynamic stability able to adapt themselves to the new requirements. One way to fulfill these requirements is by modifying the bearing properties according to such demands. Nevertheless, the dynamic properties of a standard TPJB are completely determined by its Sommerfeld number,\textsuperscript{2} and there is no way of significantly “on-line” changing such properties. In order to provide “in-situ”, “on-line” and “on-demand” capabilities of adaptation, standard TPJBs have been re-designed and transformed into a mechatronic machine element.

Santos\textsuperscript{3} proposed two different design solutions for TPJBs with controllable characteristics based on hydraulic actuators: (1) the hydraulic chamber system and (2) the hydraulic radial oil injection system. The present work is focused on the second design solution also known as actively lubricated bearing\textsuperscript{4} (ALB). This system injects pressurized oil into the bearing clearance through radial nozzles usually placed at the midspan of the pad surface.

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Servovalves, commanded by well-defined control laws, control the pressurized oil flow injection, resulting in a modification of the oil film pressure field and thereby of the “controllable forces” exerted on the rotor. Most of the early theoretical studies on ALBs considered isoviscous hydrodynamic models, focusing on the development of control strategies for rigid as well as flexible rotor applications. Multibody dynamics and finite element methods are often used to describe the behaviour of rigid and flexible rotating elements, i.e. discs and shaft. Their dynamics are linked to the bearing dynamics through force coefficients of stiffness and damping. In the case of ALBs, the dynamics of hydraulic components and of the feedback control system are additionally included in the modelling. In this multiphysics modelling approach, experimental identification of bearing force coefficients is key to ensuring model accuracy.

The bearing coefficients are experimentally identified in terms of the journal degrees-of-freedom (DOFs) since they are of crucial importance and normally the only ones easily accessible via eddy-current displacement sensors. Such DOFs are normally called “master” DOFs, DOFs such as the pad tilting,\(^{10}\) pad bending\(^{11}\) and pad pivot flexibilities\(^ {10,12-16}\) are not directly measured, but strongly influence the dynamic behaviour of such force coefficients, leading to a frequency dependency. Furthermore, in the case of ALBs, the DOFs related to servovalve dynamics, pressure–flow relationship and feedback control make the frequency dependency of such force coefficients even stronger. The DOFs different from those of the journal are normally called “slaves” DOFs. To make the theoretical force coefficients comparable to those experimentally obtained, a dynamic condensation of the “slaves” DOFs is necessary.

By using the identification methods based on the frequency domain, the bearing dynamic characteristics – represented by complex impedance functions – are determined in a broad frequency range aided by multifrequency excitations. One of the most used methods is the KCM model\(^ {7-20}\) introduced for hydrostatic bearings by Rouvas and Childs\(^ {21}\) and mostly applied to “floating bearing-fixed shaft” setups after Glienicke.\(^ {22}\) The KCM approach experimentally addresses the frequency dependency by introducing a set of mass coefficients which account for bearing stiffening or softening. However, its application is meant for rigid rotors. To cope with flexible rotors, like the one in this work, Arumugam et al.\(^ {23}\) and Wang and Maslen\(^ {24}\) proposed approaches based on “fixed bearing-free shaft” configurations. The limitation of such approaches arises when dealing with systems with a large number of DOFs. This limitation can be more easily overcome by introducing selector matrices,\(^ {25}\) which allow for the selection of a few DOFs related to excitation and measurement points.

Two main publications related to the identification of dynamic coefficients of controllable fluid film bearings are found in the literature. Santos\(^ {13}\) investigated a pair of tilting-pads controlled by hydraulic chambers and the frequency dependency of stiffness and damping coefficients is theoretically as well as experimentally studied. Therein, a simple hydrodynamic (isothermal) model with rigid pads supported on flexible membranes is explored. Conversely, in Cerda and Santos,\(^ {26}\) the stiffness and damping coefficients for a single tilting-pad under several lubrication regimes are theoretically and experimentally researched. Therein, a complex elastothermohydrodynamic (ETHD) model for a single-pad ALB is used. The experimental work is carried out using a simple test-rig, but with pads fully instrumented.

In this framework, the main goals of this paper are:

(a) to experimentally demonstrate the feasibility of modifying the dynamic force coefficients of full ALB composed of four pads-controlled pairwise by two servovalves. Lightly loaded conditions are used with the aim of resembling certain radial compressor configurations and some applications to vertical turbomachinery\(^ {25}\) in which TPJBs are prone to instabilities due to, among others, the lack of damping. Such instabilities, led by low static loads, have been profusely reported for instance by Olsson,\(^ {27}\) White and Chan,\(^ {28}\) Flack and Zuck\(^ {29}\) and Lie et al.\(^ {30}\) among others.

(b) to build an experimental database for validation of the ETHD model applied to ALBs under different operation conditions, which will be useful and available for other authors interested in the dynamic behaviour of TPJB under several lubrication regimes and the frequency dependency of its force coefficients. Due to the frequency-dependence nature of the bearing force coefficients, the experimental identification procedure is carried out in the frequency domain aided by the approach presented by Wang and Maslen,\(^ {24}\) taking advantage of the finite element model to include the shaft flexibility and to compute the expanded uncertainty as proposed by Moffat.\(^ {31}\)

The flexible rotor – ALB test-rig

The test-rig is depicted in Figure 1(a). It comprises an approximately 50 kg and 1150 mm long shaft \(\oplus\) supported by an ALB \(\oplus\) and rigidly supported by a ball bearing \(\oplus\) at its driven end. It is flexibly driven by a layshaft which in turn is belt-driven by a 4 hp AC motor \(\oplus\) provided with a frequency converter to run up to 7000 r/min. An active magnetic bearing \(\oplus\) is currently mounted between bearings to exert vertical loads up to 1900 N.\(^ {32}\) An excitation bearing \(\oplus\) is placed at the free end to carry-out model parameter identification by means of an electromagnetic shaker. The ALB is a TPJB with four bronze pads in a load-between-pads (LBP) configuration. The pads are
rocker-pivoted in the circumferential middle of the pad, i.e. with an offset of 0.5. The controllable or active feature of the bearing is developed by a hydraulic radial oil injection system as proposed by Santos. This injection system adds a hydrostatic pressure to the hydrodynamic pressure distribution by injecting pressurized oil between the journal and pad clearance through a nozzle placed in the middle of the pad surface. The pressurized oil flow is controlled by two high-frequency response servovalves installed orthogonally at 45°, aligned with the “1–2” reference frame, each one coupled to a pairwise of counter pads. The lubricant is supplied by a low pressure (max. 2 bar) and high pressure (max. 100 bar) pumping units for the passive and active lubrication cases, respectively. Figure 1(b) depicts a scheme of the radial oil injection control system overlapped to the ALB with its main parts, the high pressure supply unit, the servovalves, proximity sensors, and the digital controller (FPGA). Low pressure and return pumping units as well as the fixed orthogonal reference frames “x–y” and “1–2” are also included.

Table 1. Conventional and controllable design parameters of the actively lubricated bearing (ALB).

<table>
<thead>
<tr>
<th>Conventional design parameters</th>
<th>Value</th>
<th>Units</th>
<th>Controllable design parameters</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journal radius (R)</td>
<td>49.89</td>
<td>mm</td>
<td>Servovalve type</td>
<td>MOOG E760-912</td>
<td>–</td>
</tr>
<tr>
<td>Pad inner radius (R_p)</td>
<td>50</td>
<td>mm</td>
<td>Servovalve configuration</td>
<td>four-way, spool valve</td>
<td>–</td>
</tr>
<tr>
<td>Pad aperture angle (α_p)</td>
<td>69</td>
<td>°</td>
<td>Cut-off frequency (210 bar)</td>
<td>350</td>
<td>Hz</td>
</tr>
<tr>
<td>Pad width (L)</td>
<td>100</td>
<td>mm</td>
<td>Damping (210 bar)</td>
<td>0.7</td>
<td>–</td>
</tr>
<tr>
<td>Pad thickness (t)</td>
<td>14</td>
<td>mm</td>
<td>Control flow (210 bar)</td>
<td>19.2</td>
<td>L/min</td>
</tr>
<tr>
<td>Nominal radial clearance (C_p)</td>
<td>110</td>
<td>μm</td>
<td>Cut-off frequency (100 bar)</td>
<td>260</td>
<td>Hz</td>
</tr>
<tr>
<td>Assembly radial clearance (C_b)</td>
<td>83</td>
<td>μm</td>
<td>Injection orifice diameter (d_0)</td>
<td>3.3</td>
<td>mm</td>
</tr>
<tr>
<td>Lubrication oil type</td>
<td>ISO VG22</td>
<td>–</td>
<td>Injection orifice length (L_0)</td>
<td>21</td>
<td>mm</td>
</tr>
<tr>
<td>Nominal flow (2 bar)</td>
<td>1.4</td>
<td>L/min</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. Flexible rotor – ALB test-rig. (a) picture of the test-rig and its main components: ⊙ the excitation bearing, ⊠ the flexible shaft, ⊡ the actively lubricated bearing (ALB), ⊢ servovalves, ⊣ the active magnetic bearing, ⊤ the ball bearing, ⊥ the AC motor with frequency driver. (b) A scheme of the radial oil injection control system overlapped to the ALB with its main parts, the high pressure supply unit, the servovalves ⊢ and ⊣, proximity sensors ⊤ and the digital controller (FPGA). Low pressure and return pumping units as well as the fixed orthogonal reference frames “x–y” and “1–2” are also included.

The three lubrication regimes

The ALB is capable of operating under three different lubrication regimes, namely:

i. the passive regime which gives the hydrodynamic backup support in terms of the load-carrying capacity in case of a hydraulic injection system failure;

ii. the hybrid lubrication regime which is a combination of the passive case with a hydrostatic effect developed by the hydraulic injection system. The larger the pressure in the high pressure unit, the more pronounced the hydrostatic effect obtained. Since the bearing is a four pad arrangement in an LBP configuration and governed by two servovalves shifted 45°, the journal can be moved within the plane “x–y”.

However, since the servovalves’ dynamic properties are slightly different, the journal cannot be strictly moved upward or downward without the aid of an
When the servovalve spool is kept centred there is still some hydrostatic effect detectable as a consequence of leakage flow through servovalve ports. The servovalves used are of an underlapped type (Note: If the width of the land is smaller than the port in the valve sleeve, the valve is said to have an open center or to be underlapped). This case is referred to throughout the paper as “leakage case”. When the lubricant is injected from the two bottom pads, leading to vertical upward lift forces, it is referred to as upward case, see Figure 2(a). When it is injected from the two upper pads, it is referred to as downward case, see Figure 2(b).

iii. **the feedback-controlled lubrication regime** (active lubrication) in which the hydrostatic effect is dynamically modified by the servovalves and well-tuned digital controllers. Different classical or modern control strategies can be developed aided by model-free or model-based approaches. For simplicity, the ones utilized in this work are based on a proportional-derivative (PD) controller.

**ALB modelling**

The state-of-the-art regarding ALB modelling requires the inclusion of several effects apart from the well-known hydrodynamic oil film pressure build-up to achieve an acceptable level of accuracy, i.e. thermal effects related to the oil film temperature build-up, heat transfer among fluid film, bearing pads and surroundings, flexibility associated with compliant pivot and pads due to the exerted loads. Additionally, for coping with the controllable features, it is necessary to also include servovalve and pipe flow dynamics. An exhaustive revision of all the involved equations can be found in Cerda and Santos, in which the bearing dynamic properties have been validated using a single pad system. Figure 3 shows the predicted coefficients for a full ALB under the operational conditions tested, i.e. 3000 rpm, for an almost null applied load (light-load condition) and 80 bar of supply pressure for the injection system. To incorporate the pivot stiffness in calculations, a value of $2 \times 10^7$ N/m has been considered based on the experimental results obtained in a test-rig with a similar pad design. Bearing force coefficients are depicted in Figure 3(a) and (b) for the ALB operating under passive and hybrid lubrication regimes. Due to the bearing symmetry the direct- and cross-coupling coefficients are, respectively, equal in both directions, for passive and hybrid cases. Furthermore, the cross-coupling coefficients are negligible compared to the direct ones, despite the lightly loading condition imposed. In the frequency range studied, theory predicts almost constant direct stiffness coefficients in the order of $10^7$ N/m and low damping for the passive case (solid line). Almost negligible changes can be observed for the leakage case (dash-dot line). Contrarily, in the upward (dotted line) and downward (dashed line) cases both the stiffness and damping force coefficients significantly increase (about 2.3 times at low frequencies for the stiffness direct coefficients) and slight differences between them can be seen. Such an increase is more significant for the damping coefficients at lower frequencies. The difference between upward and downward cases is also more prominent at lower frequencies.

Results under the feedback-controlled lubrication regime are reported in Figure 3(c) and (d). The control law simulated is reported in Table 2 and it corresponds to the PD-controller #1. The controller

**Figure 2.** Injection cases for the hybrid lubrication. (a) upward injection through the two bottom pads, resulting in a lifting force. (b) downward injection through the two upper pads, resulting in a loading force.
imposes large modification of the cross-coupling coefficients (dotted and dash-dot lines) rather than the direct coefficients (solid and dashed lines). It does not necessarily mean an improvement in rotor-bearing system behaviour, it just illustrates the stronger dependency of the bearing force coefficients on the control law implemented.

**Identification of ALB force coefficients**

A mathematical model capable of representing the relevant rotor-bearing system dynamics must be formulated as a first step. If the shaft is modeled as rigid, then the approach presented by Arumugam et al.\textsuperscript{23} can be used since a reduced number of DOFs are utilized, leading to the eight linearized oil film bearing coefficients by comparing directly the experimental FRFs against the theoretical ones. Examples of its application to cylindrical and TPJBs can be found in the same reference,\textsuperscript{23} and to air foil journal bearings and polymer-faced TPJBs in Larsen et al.\textsuperscript{36} and Simmons et al.,\textsuperscript{37} respectively.

If shaft flexibility cannot be neglected, the shaft model can be built using the finite element method.\textsuperscript{38} Due to the substantially large number of DOFs, it becomes unfeasible to obtain experimental input/output relationships of every single DOF to build an identification procedure. This difficulty can be overcome by applying the method presented by Wang and Maslen,\textsuperscript{24} which allows the inclusion of the shaft flexibility and to identify unknown dynamics from a reduced number of input/output relationships. For the sake of completeness, the method is summarized in the following section. Nevertheless, the reader

![Figure 3](image)

**Figure 3.** Predicted ALB coefficients by the ETHD approach. (a) and (b) coefficients under passive and hybrid lubrication regimes. Solid line (–): passive lubrication. Dash-dot line (–.): hybrid lubrication, leakage case. Dotted line (–): hybrid lubrication, upward injection. Dashed line (– -): hybrid lubrication, downward injection. (c) and (d): coefficients under feedback-controlled lubrication. PD-controller #1 of Table 2. Dashed line (– -): kxx and dxx. Dotted line (–): kxy and dxy. Dash-dot line (–.): kyx and dyx. Solid line (–): kyy and dyy.

**Table 2.** Main parameters for the lubrication regimes featured with the ALB.

<table>
<thead>
<tr>
<th>Lubrication</th>
<th>Injection</th>
<th>PD-controller #1</th>
<th>P-controller #2</th>
<th>k_p (kV/m)</th>
<th>k_d (Vs/m)</th>
<th>k_p1/k_p2 (kV/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Hybrid</td>
<td>85</td>
<td>Leakage</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Hybrid</td>
<td>85</td>
<td>Upward(15 μm)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Hybrid</td>
<td>85</td>
<td>Downward(30 μm)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Active #1</td>
<td>85</td>
<td>–</td>
<td>–30</td>
<td>+20</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Active #2</td>
<td>85</td>
<td>–</td>
<td>+30/−30</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
is advised to refer to literature for a comprehensive presentation of the method.

**Finite element model of the shaft and rotor**

The flexible shaft, depicted in Figure 4, is discretized using 21 finite elements. The model accounts for inertia and flexibility of the shaft. Damping is neglected. The relevant nodes for the identification procedure are highlighted with black dots, i.e. point (B) for the ball bearing, point (R) for the active magnetic bearing rotor, point (S) for the shaft center of mass, point (T) for the placement of the ALB, point (P) for the placement of the sensor and point (E) for the excitation bearing. Main distances are also depicted and defined as \( l_{ij} \), for which the subscripts stand for the distance between points \( i \) and \( j \). Rigid discs for the active magnetic bearing rotor \( M^R \) and the excitation bearing \( M^E \) are placed in their respective nodes, taking into account lumped mass and inertia. The shaft mass \( M^S \) is distributed along its nodes. The mean values of these parameters are given in Table 3 along with their uncertainty information.

**Linking theoretical and experimental FRFs**

The procedure is aimed at obtaining the equivalent bearing complex impedance function \([H_l(i\omega)]\) by means of the measured frequency response functions \([FRF(i\omega)]\) and an equivalent mathematical model of the test setup. The test setup can be dynamically described around an equilibrium position by the well-known equation of motion

\[
[M][\ddot{q}] + ([D] - \Omega[G])\dot{q} + ([K] + [K_0])q = [f]
\]

where \([M]\) stands for the generalized inertia matrix, \([G]\) for the gyroscopic matrix, \([K]\) for the stiffness matrix and \([f]\) and \([q]\) represent the generalized external force and displacement coordinate, respectively. \(\Omega\) denotes the angular velocity. The contribution from the ALB in terms of stiffness \([K_0]([\Omega])\) and damping \([D_0]([\omega])\) is to be determined. Under the assumption of a linear system and considering a harmonic excitation \(f(t)\) of frequency \(\omega\), the generalized coordinate \(q(t)\) and its time derivatives are dominated by a harmonic response at the same frequency, hence they can be written using complex notation as

\[
[f(t)] = [f_0]e^{i\omega t}; \quad [q(t)] = [q_0]e^{i\omega t}; \quad [\dot{q}(t)] = -\omega^2 [q_0]e^{i\omega t}
\]

**Figure 4.** Schematic of the flexible rotor and ALB test-rig depicting the finite element discretization. The inertial reference frame – \(xyz\) – is included. Note that the bearing cross-coupling force coefficients have been omitted for simplicity.

**Table 3.** Model parameters considered for the uncertainty evaluation.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mean value</th>
<th>Source</th>
<th>Error limits</th>
<th>Standard uncertainty</th>
<th>Parameters</th>
<th>Mean value</th>
<th>Source</th>
<th>Error limits</th>
<th>Standard uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M^R)</td>
<td>49.55 kg</td>
<td>CAD file</td>
<td>±1%</td>
<td>±0.2528 kg</td>
<td>(I_p)</td>
<td>1.0015 m</td>
<td>Measured</td>
<td>±0.005 m</td>
<td>0.0029 m</td>
</tr>
<tr>
<td>(M^B)</td>
<td>6.48 kg</td>
<td>Measured</td>
<td>±0.020 kg</td>
<td>±0.0115 kg</td>
<td>(I_{RE})</td>
<td>1.0790 m</td>
<td>CAD file</td>
<td>±1%</td>
<td>0.00551 m</td>
</tr>
<tr>
<td>(M^E)</td>
<td>1.112 kg</td>
<td>Measured</td>
<td>±0.0020 kg</td>
<td>±0.0112 kg</td>
<td>(r^3)</td>
<td>A</td>
<td>CAD file</td>
<td>±1%</td>
<td>0.01A/1.96</td>
</tr>
<tr>
<td>(I_{BR})</td>
<td>0.4850 m</td>
<td>Measured</td>
<td>±0.005 m</td>
<td>±0.0029 m</td>
<td>(r^3, E)</td>
<td>A</td>
<td>Estimated</td>
<td>±3%</td>
<td>0.03A/1.96</td>
</tr>
<tr>
<td>(I_{BT})</td>
<td>0.7315 m</td>
<td>CAD file</td>
<td>±1%</td>
<td>±0.0037 m</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Combining equation (2) and equation (1) leads to
\[
-\omega^2[M] + i\omega[D_0(\omega)] - \Omega[G]) + [K] + [K_s(\omega)]^{-1} [q_b] = [q_b]
\]
\[
|[Z_s] + [H_s(\omega)]|^{-1} = [FRF(\omega)]
\]

Equation (3) states the relationship between the system dynamic stiffness matrix \([Z_s]\) from the finite element model, the unknown bearing impedance function \([H_s(\omega)] = [K_s(\omega)] + i\omega[D_0(\omega)]\) and the matrix \([FRF(\omega)]\) containing the input/output relationships for every DOF in the model. Experimentally, it is not possible to determine every component of this matrix, hence some additional techniques must be applied. By introducing the usage of selector matrices \([S_T]\) to deal only with several excited/sampled locations, the bearing impedance function \([H_s(\omega)]\) can be determined by
\[
[H_s(\omega)] = [-[A_{TT}] + [A_{TE}]][A_{TE}]
\]
\[
- [FRF^*(\omega)]^{-1} [A_{PT}]^{-1}
\]
\[
[A_{TE}] = [S_T]^T[Z_s]^{-1}[S_T];
[A_{PT}] = [S_T]^T[Z_s]^{-1}[S_T]
\]
\[
[A_{TT}] = [S_T]^T[Z_s]^{-1}[S_T]
\]
where \([S_T]\) stands for the selector matrix associated with the excited DOF (point E), \([S_T]\) stands for the measured degrees of freedom (point P) and \([S_T]\) stands for the bearing degrees of freedom (point T). The required experimental data are significantly reduced from N DOFs to 2 DOFs, since the main requirement to apply the method is that the number of excitations and sensors match the dimension of the unmodeled bearing dynamics. Hence the reduced measured FRFs matrix \([FRF^*(\omega)]\) only contains the transfer functions between the excitation point (E) and the response at the measurement point (P). From equation (4a) the bearing dynamic properties are obtained as the real and imaginary parts of the bearing impedance which reads
\[
[K_s] = \begin{bmatrix}
K_{sx} & K_{sy} \\
K_{sx} & K_{sy}
\end{bmatrix} = \Re([H_s(\omega)])
\]
\[
[D_0] = \begin{bmatrix}
D_{sx} \\
D_{sy}
\end{bmatrix} = \Im([H_s(\omega)]/\omega)
\]

Experimental procedure

The bearing dynamic coefficients are identified in the frequency range 15–130 Hz. A reference operational condition is used: angular velocity of 3000 r/min and almost null load on the ALB. Such a load condition is realized aided by the magnetic bearing. As already mentioned, such a condition has been selected with the goal of mimicking a rotor under light-load condition. Although magnetic bearings introduce negative stiffness in open-loop configuration, its magnitude is approximately 100 times smaller than the ones to be identified, hence its contribution to the whole system dynamics can be neglected without loss of stringency. This can be corroborated either experimentally or theoretically. A supply pressure of 85 bar has been used for the active control system. Such a relative high pressure is set in order to make the hydrostatic effect more notorious in the bearing and to keep the cut-off frequency of the servovalves out of the frequency range of study (260 Hz with approximately 100 bar). It is also slightly higher than the pressure used in the modelling, trying to account for measurement errors and pressure losses in the hydraulic system. For all lubrication regime cases, care is taken to guarantee that the test-rig reaches thermal and geometric steady-state equilibria.

To ensure only vertical journal movements under the hybrid lubrication regime, i.e. upward and downward cases, an I-controller with gain of \(k_i = -30 \text{kV/(ms)}\) is used. The maximum vertical displacement from the equilibrium position is \(+15 \mu\text{m}\) upward and \(-30 \mu\text{m}\) downward. Two types of control laws are implemented and presented below based on a PD-controller and a P-controller. Their gain values are summarized in Table 2 together with the main parameters of all lubrication regimes used. The experimental procedure to synthesize the proportional \(k_p\) and derivative \(k_d\) gains for the feedback-controlled lubrication regime is thoroughly explained by Salazar and Santos.

Feedback-controlled lubrication control laws

Control law #1: If the simplest control law is considered, only a pair of control gains \((k_p, k_d)\) must be determined. Despite such an advantage, an important drawback is obtained by using the same control gains to govern both servovalves. Their dynamics cannot be independently managed. The servovalve control signals \((u_1, u_2)\) can be obtained in terms of the measured displacement as
\[
\begin{bmatrix}
\alpha_1 \\
\alpha_2
\end{bmatrix} = -k_p \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix} \begin{bmatrix}
x_P \\
y_P
\end{bmatrix} - k_d \begin{bmatrix}
1 & 1 \\
-1 & 1
\end{bmatrix} \begin{bmatrix}
x_P \\
y_P
\end{bmatrix}
\]
\[
(6)
\]
rotor-bearing system dynamics, it illustrates the capability of influencing the bearing force coefficients, which is one of the work goals.

Control law #2: A more aggressive control law can be chosen for modifying the direct coefficients. However, additional gains for the controller must be determined. If the following proportional control law is used, then only a pair of proportional gains \( k_{p1}, k_{p2} \) for the P-controller must be synthesized. This control law #2 allows us to command the servovalves as

\[
\left\{ u_1 \right\} = - \begin{bmatrix} k_{p1} & k_{p2} \\ rk_{p1} & -k_{p2} \end{bmatrix} \left\{ x_p \right\}
\]

where \( r \) is a constant identified in the experimental gain matrix, which relates the control signals with the active forces. It is shown in appendix 1, under the same simplifying assumption adopted, that each gain \( k_p \) modifies the direct stiffness coefficients, and that the cross-stiffness coefficients stay unaltered. Since the derivative gains \( k_d \) are disregarded, the bearing damping properties are not affected. This control law can be beneficial for the rotor-bearing system, since it can be used to increase the asymmetry of the bearing direct force coefficients while not affecting the cross-coupling ones.

Uncertainty analysis

To estimate the interval (normally, with 95% of confidence) on which the results are thought to lie, the total uncertainty of the identified coefficients is calculated as proposed by Moffat10 following the ISO GUM10 recommendations. It is considered that measurement random uncertainties in the FRFs are not influencing the results, and this work only accounts for measurement and modelling systematic uncertainties. A large amount of FRFs averages is considered to minimize their standard deviation and to work with their mean values as their best estimates. Referring to equation (1), the modelling uncertainties arise from the determination of the system dynamic stiffness \([Z_a]\) and from the length of the finite elements.

Table 3 summarizes all parameters considered for the evaluation of the uncertainty along with their mean values, sources, error limits and standard uncertainties. Uncertainties in the inertia are evaluated considering the uncertainties in their respective radius of mean value \( J \). For the measured parameters, their error limits are obtained from their instruments, while for the case of parameters obtained from CAD files their errors are estimated to be lower than 1%. In the case of estimated radius, a bigger error of 3% is considered. A normal distribution with a 95% confidence interval level is considered for all parameters, except for the measured ones, for which a uniform distribution with 100% confidence interval level is considered. Regarding the transducers, the load-cell manufacturer informs linearity of ±1% at maximum load, while for the proximity probe a linearity of ±3% is estimated based on a similar transducer. For both sources of uncertainty, a normal distribution with 95% confidence level is taken into account. Finally, a normal distribution with a confidence level of 95% is considered to calculate the expanded uncertainty of results.

The parameters which strongly contribute to the total uncertainty are: the transducer sensitivities and lengths, particularly the displacement sensor sensitivity and the distances \( l_{ar} \text{ and } l_{at} \). In general, acceptable values of the expanded uncertainties are obtained which are to be presented simultaneously with the results.

Rotor-bearing system response: Experimental FRFs

Insight into the overall system behaviour can be obtained by analysing the experimental FRFs under several lubrication regimes. Moreover, such FRFs are fundamental for bearing parameter identification. Four representative cases are presented: the passive and the hybrid (leakage case) lubrication regimes in Figure 5 and two feedback-control lubrication regimes in Figure 6. These FRFs are obtained with the aid of an electromagnetic shaker by sweeping a linear bidirectional chirp excitation from 15 to 130 Hz with a target time of 45 s for 10 min. All signals are simultaneously sampled at a frequency of 6000 Hz. The FRFs are calculated by means of the H1 estimator, 20,000 samples, an overlap of 80% and flattop windowing. Shaft displacements were recorded with an 8 mV/µm eddy-current inductive sensors with 2-mm linear range, ranging between 30 and 50 µm.

The used test setup configuration entails that the dynamic forces are exerted in the excitation bearing (point E), and the system response is measured in the proximity probe location (point P). If the passive FRFs, presented in Figure 5(a), are used as reference, it is noted that the rotor lateral vibration amplitudes are drastically reduced when the hybrid lubrication is activated, Figure 5(b), leading to significant damping improvement in the frequency range analysed. Comparing the FRFs of Figure 6 with ones of Figure 5, it can be clearly noticed that depending on the adopted control law the rotor amplitude can be either reduced or increased. It is clear that the control law #1, Figure 6(a), generates a resonant zone around 60 Hz, whereas the control law #2, Figure 6(b), produces a further reduction of the amplitude below 100 Hz. The implications of these FRFs in terms of bearing force coefficients are presented next.

Bearing force coefficients – Passive and leakage cases

Figure 7 shows the identified force coefficients for the ALB under passive and hybrid lubrication regimes in the frequency range studied. Such results are obtained
Generally speaking, the stiffness coefficients for the passive case (dashed lines) can be considered almost constant in the whole frequency range used. With regard to the expanded uncertainty, low values are obtained with the largest one at about $\pm 20\%$ for the $K_{yy}$ term. Comparing with theoretical coefficients, good agreement for the direct stiffness coefficients is found. However, large discrepancies for cross-coupling stiffness coefficients are shown, for which the model predicts almost null cross-coupling coefficients. The cross-coupling stiffness coefficients are negative and have the same order of magnitude, i.e., $10^7$, when compared to the direct ones. The strong cross-coupling effect is fundamentally detected only under light-load conditions. To further investigate it, two additional experiments are carried out and reported in appendix 2. Firstly, to eliminate the hypothesis of...
starving lubrication and consequently generating a cross-coupling effect followed by reducing damping coefficients, several supply pressure and flow conditions are tested. The cross-coupling coefficients are not significantly affected for any of these cases. Secondly, 900 N downward load is applied to the shaft via AMB. For such a new loading condition, the cross-coupling coefficients are reduced. Other authors, such as Childs and Carter and Rodriguez and Childs, also report cross-coupling coefficients within the same order of magnitude as the direct ones, although those experiments do not correspond to the same TPJB design and load conditions.

Back to Figure 7(b), it can be seen that the damping coefficients weakly depend on the frequency for the passive case. Due to light load condition, their values are small, i.e. in the order of $10^4$, especially for frequencies over 40 Hz. The experimental results are little affected by the expanded uncertainty. Comparison of the experimental damping coefficients against the predicted ones shows poor agreement, which set some room for modelling improvements. It is important to underline here that, the work goal is to demonstrate experimentally the modification of the bearing force coefficients due to active lubrication and not to, from any point of view, validate the theoretical model.

In Figure 7(c) and (d), the bearing dynamic coefficients for the leakage case are illustrated using solid lines. Comparing the passive and the leakage lubrication cases, it becomes evident that the vertical direct stiffness coefficient $K_{xy}$ significantly increases its value to about $7\cdot10^7$ N/m above 50 Hz, while the values of the horizontal direct stiffness coefficient $K_{xx}$ remain almost the same, i.e. the bearing becomes more asymmetric. For $K_{xx}$, a stronger frequency dependency is seen after 50 Hz. Comparing the passive and the leakage lubrication cases, it can be seen that the cross-coupling coefficients are significantly reduced for the hybrid case. The expanded uncertainty becomes more important as the frequency increases and it is far more important for the vertical direct stiffness coefficient $K_{yy}$, with a magnitude of about $\pm17\%$. In the case of the damping coefficients, Figure 7(d), it is noticed that the bearing becomes more damped, especially in the vertical direction, i.e. increased value of $D_{yy}$. Moreover, all damping coefficients significantly diminish from $10^5$ to $10^4$ over the frequency span of 60 Hz. In the horizontal direction, at low frequencies, the damping coefficients $D_{xx}$ and $D_{xy}$ are significantly smaller than those in the vertical direction. Over 60 Hz, $D_{yy}$ provides the largest damping to the system, while the cross-coupling coefficients $D_{yx}$ and $D_{xy}$ are negative. Considering the damping coefficients, the expanded uncertainty is more significant for the direct coefficient $D_{yy}$ at low frequencies where it reaches $\pm37\%$.

**Bearing force coefficients – Hybrid cases**

Figure 8 depicts comparative plots of the identified dynamic properties of the ALB for all hybrid lubrication regimes, i.e., for the leakage, upward and downward injection cases. The leakage case, plotted in solid lines, is used as a benchmark, while the upward and downward cases are plotted with dotted and dashed lines, respectively. Although it can be argued that all differences among the cases lie within

---

**Figure 7.** Identified dynamic coefficients for the ALB. (a) and (b): ALB under passive lubrication regime (dashed lines (- -)). (c) and (d): ALB under hybrid lubrication regime, the leakage case (solid lines (–)).
the confidence limits, it is clear that the vertical direct stiffness coefficient $K_{yy}$ is reduced when the shaft is moved upward, closer to the bearing center, which turns the ALB softer in the vertical direction. In the downward injection case, the cross-coupling coefficients are significantly reduced for frequencies higher than 50 Hz and are simultaneously less influenced by uncertainties. Regarding the damping coefficients, there are no significant differences between the different hybrid cases and they behave similarly within the uncertainty bounds. Hence, it can be stated that the damping is hardly affected by the mentioned changes under these different hybrid lubrication conditions. Comparison against theoretical results of Figure 3(b) shows fair agreement in the sense that theoretical as well as experimental stiffness and damping coefficients increase their values when compared against the passive case. However, comparing upward against leakage case, a softening effect in the vertical direction can be observed. Such an experimental finding though is hardly predicted by the theoretical model. Furthermore, among the three hybrid lubrication cases, the leakage case shows the largest discrepancies between simulations and experiments, see Figure 7(c) and (d). Further efforts towards theoretical modelling improvements are necessary.

**Bearing force coefficients – Feedback-controlled cases**

Figure 9 shows selected results for the identified coefficients under the feedback-controlled lubrication regimes in the frequency range of 60–130 Hz. Figure 9(a) and (b) shows the results obtained with the active lubrication defined by the control law #1, i.e. a PD-controller and Figure 9(c) and (d) the coefficients obtained using the control law #2, a P-controller. Gains of both control laws are summarized in Table 2. As a benchmark, the coefficients obtained for the leakage case are added with thin solid lines. Using the PD-controller #1 (dashed lines), the direct stiffness coefficients (Figure 9(a)) have a constant behaviour over the frequency range investigated, suppressing the frequency dependency of $K_{xx}$ seen under the leakage case. The cross-coupling coefficients, as aforementioned, are strongly affected by the control law adopted. Indeed, this particular behaviour is also theoretically reproduced as seen in Figure 3(c) and (d). Between 60 and 80 Hz large
uncertainties can be detected for both direct coefficients with maximum confidence limit of ∆±37%. For the cross-coupling coefficient $K_{xy}$ smaller uncertainties are found in the whole frequency span analysed. The same behaviour is observed for the damping coefficients, i.e. the direct damping coefficients vary significantly when compared to the one obtained in the leakage case and the cross-coupling damping coefficients are the ones mostly affected by the controller. Large uncertainty of about 40% is seen for the cross-coupling damping coefficients, especially between 60 and 80 Hz. The cross-coupling stiffness coefficients are less affected by the controller #2 (dotted lines (---)) compared to controller #1, see Figure 9(c). The direct stiffness coefficients are the most affected by the control law #2, leading to a reduction of the direct stiffness coefficients, as it was pursued. Damping coefficients, see Figure 9(d), are also modified by the control law, even though the featured controller is a P-controller.

**Conclusions**

The contribution of this paper is mainly experimental in nature. It should be re-emphasized that the work goal was to show the modification of the bearing dynamic properties via the active lubrication and it should not be seen, under any case, as an attempt to validate the theoretical bearing coefficients. Having said that and keeping in mind the light-load condition imposed on a bearing supporting a “flexible” rotor, the comparison between theoretical and experimental results shows a fair agreement for a reduced number of cases only. This suggests the need for further improvements of the multiphysics modelling of the ALB and of the identification modelling as well. In the first case, the way how all different regimes are modelled should be further investigated and in the second case, the extension of the identification modelling to account for further dynamics, such as, for instance, foundation and hydraulic dynamics shall be included if needed. Currently, both approaches are being researched.

Again, heading towards the research goal and in the light of the experimental investigations followed by comprehensive uncertainty analysis, it can be concluded that:

- The development of the hybrid or feedback-controlled lubrication regimes clearly modify the rotor-bearing system properties as a whole, significantly reducing the rotor vibration amplitudes. This can be clearly seen from the FRF's used to identify the bearing coefficients under several lubrication conditions.
- The stiffness coefficients identified under passive lubrication show weak frequency dependency in the whole range of study. Considering the direct stiffness coefficients, good agreement between simulated and experimental results is found. Nevertheless, it is not the case for the cross-coupling stiffness coefficients.
The hybrid lubrication regimes can increase the bearing stiffness asymmetry and significantly contribute to reducing the cross-coupling coefficients identified under light-load conditions. Furthermore, under these lubrication regimes, the ALB becomes more damped.

The hybrid lubrication regimes also allow us to modify the direct stiffness coefficients by changing the journal equilibrium position aided by I-controllers. A softening of the vertical direct stiffness coefficient—in comparison to the leakage case—can be observed, due to the shaft lifting to a more centred equilibrium position.

The feedback-controlled lubrication clearly modifies the ALB dynamic properties and can be developed using classical PD controllers. By properly choosing the control law and gains, beneficial modification of the rotor-bearing system dynamic properties can be achieved. It was shown that different control laws can produce different effects on the direct and cross-coupling force coefficients.

Acknowledgments

The authors would like to express their deepest acknowledgments to Mr. Alejandro Cerda Varela for his valuable support during the experimental tests and fruitful discussions on the matter.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

References


**Appendix**

**Notation**

A mean value of radius (m)
[C] controller transfer function matrix

[D] identified bearing damping
[D] bearing damping matrix
[d] theoretical bearing damping
[f] generalized external force vector
[FRF] theoretical frequency response functions
[FRF] measured frequency response functions

[G] system gyroscopic matrix
[H] bearing complex impedance function
i complex unity,
K system stiffness matrix
K bearing stiffness matrix
K identified bearing stiffness
K theoretical bearing stiffness
K proportional gain matrix
K derivative gain matrix
K proportional gains (V/m)
Ij distance between points “i” and “j” (m)
M shaft mass (kg)
M active magnetic bearing rotor mass (kg)
M excitation bearing mass (kg)
P sup oil supply pressure of the radial injection unit (bar)
(q) (q) (q) generalized displacement, velocity and acceleration coordinate vectors
q q q high pressure oil flow 1,2 (m/s)
q q q low pressure oil flow 1,2 (m/s)
r constant of the servovalve gain “W /”
r high pressure oil flow 1,2 (m/s)
W high pressure oil flow 1,2 (m/s)
W low pressure oil flow 1,2 (m/s)
W return oil flow (m/s)
W constant of the servovalve gain “W /”
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Appendix 1. Effect of Control Law on Bearing Force Coefficients

The matrix $[W]$ defines the relationship between the control signals $[u]$ and active fluid film forces $[f]$, taking into account the dynamics of amplifiers, servovalves and pipelines. Normally, the matrix $[W]$ is frequency dependent. Nevertheless, using short pipelines, high-response servovalves and high values of pressurization such a frequency dependency can be neglected in the frequency range studied. Therefore, the matrix $[W]$ can be considered constant and defined by $W(kN/V)$ and $r$ as

$$[W] = \begin{bmatrix} W_{x1} & W_{y1} \\ W_{x2} & W_{y2} \\ \end{bmatrix} = \begin{bmatrix} W & W \\ -rW & W \\ \end{bmatrix}$$

Equation (9) shows that the variation in the system dynamic stiffness is $[\Delta Z] = [W][C]$. Depending on the DOFs fed back to the controller, $[Z]$ is differently affected by the control law. If the lateral displacements of the journal at point $T$ are fed back to the controller, then the change in $[Z]$ will be influenced solely by the journal movements, i.e. $[Z_{TT}] = [Z_{TO}] + [\Delta Z]$. If other DOFs are used for building up the control law, for instance the shaft extremity (point $P$), then nodes $T$ and $O$ are coupled via the controller and the lateral movements of node $P$ will influence $[Z]$ and thereby the bearing force coefficients, i.e. $[Z_{TO}] = [Z_{TO}] + [\Delta Z]$.

By applying a dynamic condensation to keep only the ALB DOFs, the reduced stiffness and damping matrices of the ALB can be defined as follows

$$K = \Re \left( \begin{bmatrix} Z_{TT} + (\omega^2 M_{TO} - \dot{Z}_{TO}) \\ (\omega^2 M_{OO} - Z_{OO})^{-1} (\omega^2 M_{OT} - Z_{OT}) \end{bmatrix} \right)$$

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$$[\Delta Z] = [W][C]$$

Equation (11) shows that the variation in the system dynamic stiffness is $[\Delta Z] = [W][C]$. Depending on the DOFs fed back to the controller, $[Z]$ is differently affected by the control law. If the lateral displacements of the journal at point $T$ are fed back to the controller, then the change in $[Z]$ will be influenced solely by the journal movements, i.e. $[Z_{TT}] = [Z_{TO}] + [\Delta Z]$. If other DOFs are used for building up the control law, for instance the shaft extremity (point $P$), then nodes $T$ and $O$ are coupled via the controller and the lateral movements of node $P$ will influence $[Z]$ and thereby the bearing force coefficients, i.e. $[Z_{TO}] = [Z_{TO}] + [\Delta Z]$.

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It becomes evident that by using the control law #2 the direct force coefficients are the only ones affected, and the cross-coupling coefficients remain unchanged.

Appendix 2. Bearing force coefficients – Influence of lubricant feeding and loading

Figures 10 and 11 report the changes in the bearing stiffness coefficients due to lubricant supply pressure (Flooding lubrication between pads) and loading condition under the passive lubrication regime. Figure 10 clarifies that an increase in the lubricant supply pressure does not significantly affect the stiffness coefficients, eliminating the hypothesis of starving lubrication conditions contributing to a high level of cross-coupling coefficients.

Figure 11 shows the bearing stiffness coefficients under two different load conditions, i.e. (a) lightly loaded and (b) downward loaded via AMB. It is evident that the loading significantly affects the order of the cross-coupling coefficients compared to the direct ones.

Figure 10. Identified dynamic stiffness for the ALB under passive lubrication regime. Different feeding pressures. Dotted lines (-·): 0.50 bar. Dashed lines (- -): 1.50 bar. Solid lines (-): 2.40 bar.

Figure 11. Identified dynamic stiffness for the ALB under passive lubrication regime. Different loading conditions. Solid lines (- -): 900 N downward loaded. Dashed lines (- · -): Light-load condition.
A.4 [P4]: On the Controllability and Observability of Actively Lubricated Journal Bearings With Pads Featuring Different Nozzle-Pivot Offsets
On the Controllability and Observability of Actively Lubricated Journal Bearings With Pads Featuring Different Nozzle-Pivot Configurations

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The fundamental properties of an actively lubricated bearing (ALB) from a control viewpoint are investigated, i.e., the stability, controllability, and observability. The ALB involves the addition of an oil injection system to the standard tilting-pad journal bearing (TPJB) to introduce constantly and/or actively high pressurized oil into the rotor-pad gap through, commonly, a single radial nozzle. For the work goal, a four degrees-of-freedom (DOF’s) ALB system linking the mechanical with the hydraulic dynamics is presented and studied, comprising: (i) the vertical journal movement, (ii) the pad tilt angle, (iii) the vertical pad movement—due to the pivot flexibility, and (iv) the controllable force as the hydraulic DOF. The test rig consists of a rigid rotor supported by a single rocker-pivoted rigid pad. A thorough parametric study is carried out by investigating the effects of: (a) nozzle-pivot offset, (b) pivot flexibility, and (c) bearing loading on these control basics in order to determine the pad with the best control characteristics. Different nozzle-pivot offsets can be set by varying the positioning of either the injection nozzle or the pivot line. The influence of the pivot compliance on the bearing dynamics is assessed by benchmarking the results obtained with the flexible pivot against the rigid pivot. Three different bearing loads are studied. According to the results, the proposed configurations, especially the offset-pivot pad with slight offsets, improve the bearing control characteristics by introducing an extra mechanism to access the system states. The loading condition modifies the stability, controllability, and observability, while the pivot flexibility highly affects the ALB dynamics. [DOI: 10.1115/1.4033053]

Keywords: tilting-pad journal bearing, active lubrication, actively lubricated bearings, model-based control design, stability, controllability and observability
2 Insight Into the Active Lubrication Principles

The aim of this work is to contribute to the state-of-the-art in ALBs modeling from a control viewpoint, by investigating the system stability, controllability, and observability of a simplified ALB system. This is achieved through an in-depth analysis in which both theoretical and experimental assessments of a rigid rotor-single rigid pad arrangement have been carried out. The assessment of these model-based control basics aims to setup the baseline knowledge for analyzing more complex rotor-bearing systems, such as flexible rotors supported by multipad bearings. Additionally, a radial oil injection system, which is comprised of the same reservoir, a high-pressure pumping unit, a four-port servovalve, hydraulic pipelines, and pressurized oil injection nozzles, is added with the aim of turning the bearing controllable so that: (i) the bearing properties can be modified and (ii) controllable forces can be applied. The four-port servovalve is connected to two counter-face pads in order to apply alternating (two pads) or fluctuating (only one pad) controllable forces in the pad direction. Each pad is provided with an injection nozzle which is responsible for injecting the extra pressurized lubricant into the bearing clearance. This modifies the fluid film thickness and therefore the pressure distribution.

The pad with a single centered radial nozzle has already been produced and thoroughly studied and tested in Refs. [6,7], and [13]. Normally for this pad, the nozzle and pivot positions are coincident. Figure 2 shows a schematic of the pad-injector cross section with its main components. Other configurations can be developed, for instance, by shifting the nozzle or pivot position such as the ones proposed in Sec. 3. Other authors have studied pads with multi-ornices in order to improve the bearing thermal [14,15] and dynamic [16,17] performances. However, these kinds of pads have not been widely produced; hence, they are not considered in this study. With the inclusion of the radial oil injection system, the bearing is capable of undergoing the following three different types of lubrication regimes.

2.1 Passive Lubrication Regime. The conventional lubrication regime in which the hydrodynamic effect provides the load carrying capacity of the bearing and its dynamic properties. For control purposes and since there is not a controller, this regime is also referred to as passive lubrication.

2.2 Hybrid Lubrication Regime. In this regime, the hydrostatic effect is activated. Depending on the servovalve’s spool position, the high pressurized oil is either injected through the upper or bottom pad. The spool position is driven by a control signal which if set to constant a permanent lubricant injection can be obtained by either of the pads. Even when there is no control signal governing the servovalve and the spool is centered, a small leakage flow is still injected due to the tolerances of the underlapped design. In general terms, the higher the pressure, the more pronounced the hydrostatic effect in the bearing. This yields a modification of the journal equilibrium position as a consequence of the extra constant hydrostatic force applied. By using the

![Fig. 1 Scheme of a simplified ALB consisting of: a passive lubrication system, low pressure pump + sprinklers; an active lubrication system, high pressure pump + servovalve + injection nozzles. Both lube systems are connected to the same reservoir.](image1)

![Fig. 2 Sketched view of the pad-injector section.](image2)
journal center position as a feedback signal, this regime can be aided to attain or to maintain a predefined equilibrium by integral controllers [18].

2.3 Active Lubrication Regime. If the control signal driving the servovalve is defined by a control law, then the whole or part of the hydrostatic force exerted over the journal can be actively controlled. By using the journal position as a feedback signal, control laws based on proportional-derivative controllers for instance can be implemented with the aim of varying the hydrodynamic force in response to the journal movement [19]. This control effect is commonly interpreted as a bearing properties modification around the equilibrium position. However, these variable controllable forces are treated here as system excitations. For control purposes, this regime is named active lubrication.

3 Pads Under Study: Nozzle-Pivot Offsets

Figures 3(a)–3(c) show the schematics of the different pads to be studied. All of them are symmetric along the pad length and they only differ in the position of the injector and the pivot lines, defined by their offset values \( \Theta_{\text{nozzle}} \) and \( \Theta_{\text{pivot}} \), respectively. These offsets are defined as the ratio of the angle from the leading edge to the nozzle and pivot locations, respectively, to the entire pad arc length \( \varphi_p \). The main design goal is to contribute to the pad design purposes, it is assumed that the bearing force and moment can be considered part of the system in the left-hand side of the equation of motion imposed by an operational condition. In addition to the Sommerfeld number, such a condition is also defined, in the case of the ALBs, by the pressurized oil injection flow \( q_{\text{inj}} \). For control design purposes, it is assumed that the bearing force and moment defined in Eq. (1) can be mathematically separated into two terms: the first one contains the hydrodynamic force plus the constant part of the hydrostatic force, termed \( f^p \), and the second one includes the variable part of the controllable hydrostatic force, termed \( f^s \). The \( f^p \) force can be described by means of linear coefficients proportional to displacements and velocities; therefore, they can be considered part of the system in the left-hand side of the equation of motion. On the contrary, the \( f^s \) is kept as a system excitation at the right-hand side. This approach leads to the linearized equation of motion which is written as

\[
\begin{bmatrix}
    m_0 & 0 & 0 \\
    J_p & 0 & 0 \\
    0 & m_p & 0
\end{bmatrix} \begin{bmatrix}
    \dot{z} \\
    \dot{\varphi} \\
    \dot{\eta}
\end{bmatrix} + \begin{bmatrix}
    D_{zz} & D_{z\varphi} & D_{z\eta} \\
    D_{\varphi z} & D_{\varphi\varphi} & D_{\varphi\eta} \\
    D_{\eta z} & D_{\eta\varphi} & D_{\eta\eta}
\end{bmatrix} \begin{bmatrix}
    \dot{z} \\
    \dot{\varphi} \\
    \dot{\eta}
\end{bmatrix} = \begin{bmatrix}
    f^p \\
    f^\tau \\
    f^\eta
\end{bmatrix}
\]

where \( K_{ij} \) and \( D_{ij} \) stand for the stiffness and damping force coefficients between the corresponding \( i \)th and \( j \)th DOFs, respectively. It is important to stress that the linearized bearing force coefficients are only obtained considering the hydrodynamic and the constant hydrostatic forces. The variable force \( f^s \), which is exerted over the journal and the pad, produces a moment \( \tau^s \), which is dynamically dependent on the control signal \( u \). Such a dependency can be described in the frequency domain by the relationship \( f^s = H_f(\omega)u \). Here, the “calibration function” \( H_f(\omega) \) includes all

4 Theoretical Investigation

The governing equations for ALB and the basics for the system assessment from a control viewpoint are presented.

4.1 The Rigid Rotor—Single Pad System Dynamics. Figure 4 shows a mechanical model of the system to be studied: the rigid rotor-single rigid pad system. The rotor is restricted to move only in a vertical direction while the pad is allowed to tilt and translate. The deformation of the pad as well as the movement of the rotor in a horizontal direction is not considered in order to keep the analysis simple. The pivot deformation is included in the analysis due to its significant influence on the TPJB dynamics [20–24], leading to a vertical displacement of the pad. Under these assumptions, three mechanical DOFs, i.e., the vertical journal movement \( z \), the pad tilt angle \( \varphi \), and the pad displacement \( \eta \) (due to the pivot deflection), describe the behavior of the system governed by the nonlinear equations of motion. Such equations are obtained by balancing both forces and moments over the rotor and the pad, respectively.

\[
\begin{align*}
\Sigma F_z: & \quad m_0 \ddot{z} = f(p(z, \varphi, q_{\text{inj}}, t)) \\
\Sigma M_{\varphi}: & \quad J_p \ddot{\varphi} = \tau(p(z, \varphi, q_{\text{inj}}, t)) \\
\Sigma F_\eta: & \quad m_p \ddot{\eta} = -f(p(z, \varphi, q_{\text{inj}}, t))
\end{align*}
\]

Fig. 3 Different tilting pads with pressurized oil injection nozzle: (a) offset-nozzle pad, \( \Theta_{\text{nozzle}} = 0.5 \), \( \Theta_{\text{pivot}} = 0.5 \), (b) centered-nozzle pad, \( \Theta_{\text{nozzle}} = \Theta_{\text{pivot}} = 0.5 \), (c) offset-pivot pad, \( \Theta_{\text{nozzle}} = 0.5 \), \( \Theta_{\text{pivot}} = 0.5 \), and (d) comparison of an offset-nozzle pad with a centered-nozzle pad
A (8) can be rewritten as

\[ a \text{ normally written in the standard state-space} \]

Further, the latest findings [15] emphasize the importance of considering the frequency range of interest. This is not desired when applying the control algorithm to the servovalve response when its natural frequency is within the range of the control signal. This moment can be mathematically described as

\[ f^c + 2\zeta_H \omega_n f^c + \omega_n^2 f^c = \omega_n^2 \kappa_H u \]  

Here, \( \omega_n \) and \( \zeta_H \) stand for the hydraulic natural frequency and damping ratio, respectively, whereas \( \kappa_H \) constant depends on the journal eccentricity. Two ways to determine Eq. (4) arise. The first one implies a theoretical treatment where the pressure profile, due to a control signal \( u \), is isolated from the pressure distribution. Then, it is integrated to obtain the controllable force \( f^c \). The other one implies an experimental determination of this force, where both the control signal \( u \) and the controllable force \( f^c \) are registered, before fitting a 1DOF model. In this work, the second way is utilized based on the work presented in Ref. [13]. By combining Eq. (4) with Eq. (2), the controllable force \( f^c \), originally at the right-hand side, is now accounted for as an extra system state. The system of equations is hence expanded to a 4DOFs system as

\[
\begin{bmatrix}
 m_0 & 0 & 0 & 0 \\
 0 & J_p & 0 & 0 \\
 0 & m_p & \eta & 0 \\
 0 & 0 & 1 & f^c \\
 \end{bmatrix} \begin{bmatrix}
 \dot{x} \\
 \dot{\phi} \\
 \dot{\eta} \\
 \dot{f} \\
\end{bmatrix} + \begin{bmatrix}
 D_{1c} & D_{10} & D_{00} & 0 \\
 D_{2c} & D_{20} & D_{00} & 0 \\
 D_{3c} & D_{30} & D_{00} & 0 \\
 0 & 0 & 0 & 2\zeta_H \omega_H \\
\end{bmatrix} \begin{bmatrix}
 x \\
 \phi \\
 \eta \\
 f^c \\
\end{bmatrix} = \begin{bmatrix}
 0 \\
 0 \\
 0 \\
\end{bmatrix} u 
\]

(5)

The link between the mechanical system and the hydraulic one, mainly dictated by the servovalve dynamics, is produced in the fourth column of the stiffness matrix in Eq. (5). Using \( M, D \), and \( K \) as notation for the mass, damping, and stiffness matrices, respectively, the expanded equation of motion can be rewritten as

\[ M \ddot{x} + D \dot{x} + K x = w u \]  

(6)

with \( x = \{ \xi \ \theta \ \eta \ f^c \}^T \) the generalized displacement vector, \( w = \{ 0 \ 0 \ \omega_n^2 \kappa_H \}^T \) stands for the hydraulic gain vector, and \( u \) is the control signal.

4.3 System Control Properties. A reduction to a first-order state-space formulation of the rotor-pad system is set. In doing so, the state-space variables are defined as: \( x_1 = x \) and \( x_2 = f^c \). The dynamics can be mathematically described as

\[ f^c + 2\zeta_H \omega_n f^c + \omega_n^2 f^c = \omega_n^2 \kappa_H u \]  

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\[
\begin{bmatrix}
 m_0 & 0 & 0 & 0 \\
 0 & J_p & 0 & 0 \\
 0 & m_p & \eta & 0 \\
 0 & 0 & 1 & f^c \\
\end{bmatrix} \begin{bmatrix}
 \dot{x} \\
 \dot{\phi} \\
 \dot{\eta} \\
 \dot{f} \\
\end{bmatrix} + \begin{bmatrix}
 D_{1c} & D_{10} & D_{00} & 0 \\
 D_{2c} & D_{20} & D_{00} & 0 \\
 D_{3c} & D_{30} & D_{00} & 0 \\
 0 & 0 & 0 & 2\zeta_H \omega_H \\
\end{bmatrix} \begin{bmatrix}
 x \\
 \phi \\
 \eta \\
 f^c \\
\end{bmatrix} = \begin{bmatrix}
 0 \\
 0 \\
 0 \\
\end{bmatrix} u 
\]

(5)

The link between the mechanical system and the hydraulic one, mainly dictated by the servovalve dynamics, is produced in the fourth column of the stiffness matrix in Eq. (5). Using \( M, D \), and \( K \) as notation for the mass, damping, and stiffness matrices, respectively, the expanded equation of motion can be rewritten as

\[ M \ddot{x} + D \dot{x} + K x = w u \]  

(6)

with \( x = \{ \xi \ \theta \ \eta \ f^c \}^T \) the generalized displacement vector, \( w = \{ 0 \ 0 \ \omega_n^2 \kappa_H \}^T \) stands for the hydraulic gain vector, and \( u \) is the control signal.

4.3 System Control Properties. A reduction to a first-order state-space formulation of the rotor-pad system is set. In doing so, the state-space variables are defined as: \( x_1 = x \) and \( x_2 = f^c \). Hence, the following system of equations is obtained:

\[
\begin{bmatrix}
 x_1 \\
 x_2 \\
\end{bmatrix} = \begin{bmatrix}
 0 & 1 & 0 & 0 \\
 -M^{-1} K & -M^{-1} D & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
 X_1 \\
 X_2 \\
\end{bmatrix} + \begin{bmatrix}
 0 \\
 0 & 0 & 0 & 0 \\
\end{bmatrix} u 
\]

(7)

where the matrices of Eq. (8) are readily identified from Eq. (7) with \( A \) being the state matrix, \( B \) is the input matrix, and \( C \) is the output matrix.

Transactions of the ASME

Table 1 Offsets for the different pad layouts to be investigated expressed as a fraction of the pad arc

<table>
<thead>
<tr>
<th>Offset-nozzle pads</th>
<th>Centered-nozzle pad</th>
<th>Offset-pivot pads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ωnozzle</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Ωpivot</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>ΔΦ</td>
<td>-0.2</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

*The pad #C is the only one produced and tested so far.*
corresponding output matrix. The m-dimensional output vector \( \mathbf{Y} \) is obtained by multiplying the output matrix \( \mathbf{C} \) by the state-space vector \( \mathbf{X} = [\xi \ \theta \ \eta \ \mathbf{f}']^\top \). The \( m \times n \) output matrix \( \mathbf{C} \) is a binary matrix containing ones in the corresponding measured states. Expanding Eqs. (8a) and (8b), the complete state-space system formulation reads

\[
\begin{bmatrix}
\xi \\
\theta \\
\eta \\
\mathbf{f}'
\end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -\varepsilon_{g}^{h} & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\xi \\
\theta \\
\eta \\
\mathbf{f}'
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
\mathbf{u}
\end{bmatrix}
\tag{9a}
\]

It is clear from Eq. (9) that the system has a single-input, the control signal \( \mathbf{u} \). The number of outputs depends on the structure of the output matrix \( \mathbf{C} \), which is defined based on the sensors’ arrangement. Due to restricted accessibility in the industry, commonly only the journal displacement \( \xi \) is measured through displacement sensors. In this case, a single-input single-output system is obtained and the elements of the output matrix \( \mathbf{C} \) are defined as: \( C_{\xi} = 1, C_{\theta} = C_{\eta} = C_{\mathbf{f}'} = 0 \). The measurement of other variables is more difficult, since it requires a more instrumented machine. Nonetheless, the authors of Ref. [13] have recently been able to measure the controllable force \( \mathbf{f}' \) acting directly over the rotor and the authors of Ref. [25] have measured the pad tilt angle \( \theta \) in order to obtain rotor-pad transfer functions. These variables could also be hypothetically measured, leading to a single-input multiple-outputs system. Figure 5 suggests an arrangement of two displacement sensors looking perpendicularly at the pad back surface. With this configuration, both the pad tilt angle \( \theta \) and the pad vertical movement \( \eta \) can be obtained. Finally, case C shows a way of obtaining the active forces \( \mathbf{f}' \) by means of a pressure transducer connected to the injection orifice. Other means might be set, for instance by placing a dynamic load cell in the back of the pad or by directly measuring the force over the rotor as presented in Ref. [13], provided that no other forces are acting simultaneously.

It is worth mentioning that, in this work, the experimental analysis carried out only considers as output the journal displacement \( \xi \), which in most cases is the only DOI available. On the other hand, the theoretical analysis is performed by further including all the mechanical DOFs at the same time, i.e., \( C_{\xi} = C_{\theta} = C_{\eta} = C_{\mathbf{f}'} = 1 \). This approach allows us to analyze the system observability with the three mechanical DOFs output, obtaining the same results as if they were analyzed separately. The hydraulic DOI has not been considered in order to simplify the analysis for two main reasons: (1) the dynamic variation of the controllable force \( \mathbf{f}' \) with the control signal \( \mathbf{u} \) is already known through Eqs. (4) and (2) the system observability through \( y_{3} = f' \) is mainly constrained to the mode of the hydraulic system as it was checked preliminarily.

4.3.1 System Stability and Diagonal Form

The major requirement of system stability is assessed by the real part of its eigenvalues \( \lambda \), which must be lower than zero, i.e., \( \Re(\lambda) < 0 \). Most of the bearings are inherently stable; others such as active magnetic bearings require a closed-loop to stabilize them [26]. Recall that the eigenvalues \( \lambda \), and its respective right \( \phi \), and left \( \psi \), eigenvectors are obtained by solving the standard eigenvalue problem of the following form:

\[
J_{\mathbf{N}} \mathbf{N}_{\omega} = \mathbf{N}_{\omega} \mathbf{N}_{\omega} \mathbf{B}
\]

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MAY 2017, Vol. 139 / 031702-5

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The right and left modal matrices can be constructed columnwise by the eigenvectors as \( \Phi = \{ \phi_1, \phi_2, \ldots, \phi_k \} \) and \( \Psi = \{ \psi_1, \psi_2, \ldots, \psi_k \} \), respectively. Likewise, \( \Phi \) is the diagonalized form of the matrices \( \Phi A \Phi^{-1} \) and \( \Phi B \Phi^{-1} \). The right and left modal matrices can be constructed columnwise by the eigenvectors as \( \Phi = \{ \phi_1, \phi_2, \ldots, \phi_k \} \) and \( \Psi = \{ \psi_1, \psi_2, \ldots, \psi_k \} \), respectively. Likewise, \( \Phi \) is the diagonalized form of the matrices \( \Phi A \Phi^{-1} \) and \( \Phi B \Phi^{-1} \).

The aforementioned test of full controllability and observability states are stable, the system is said to be stabilizable/detectable. If all uncontrollable/unobservable states are observable, the system can be controlled/observed. If all uncontrollable/unobservable terms are zero, then the system can be controlled/observed.

\[
\begin{align*}
A\phi_i &= \lambda_i \phi_i \\
A^T\psi_j &= \lambda_j \psi_j
\end{align*}
\] (10a)

(10b)

In Eq. (13a), the index \( \cos(z_{ij}) \) measures the controllability of the \( i \)-th mode from the \( j \)-th input and in Eq. (13b) the index \( \cos(\beta_j) \) measures the observability of the \( i \)-th mode in the \( j \)-th output. These quantities form the cos\( A \) and cos\( B \) matrices, respectively. Although these measures are affected by the mode normalization and the state-space realization [27], unlike the invariant binary assessment of controllability and observability, they are comparable to each other and shed some light on the modal controllability and observability. Gross measurements of controllability and observability are also proposed in Ref. [29]. In this case, the gross measurement of controllability, of all modes due to the \( i \)-th input in the system, is evaluated as the norm of the \( i \)-th column of the cos\( A \) matrix. On the other hand, the observability of all modes through the \( j \)-th output is evaluated as the norm of the \( j \)-th row of the cos\( B \) matrix. These gross measures can also be premultiplied by the norm of the \( b_j \) or \( c_k \) vectors in order to have them taken into account in the calculations. The indices of Eq. (13) might be conveniently calculated in different coordinates. When calculated in Jordan realization, this allows us to determine by direct examination of the state input \( b_j \) and output \( c_k \) matrices, which states are uncontrollable/unobservable, since zeroes will be placed in its corresponding term. In balanced coordinates [31], for instance, the relative importance of different modes is weighted in aiding to define the optimal position of allocated sensors and actuators [30,32].

5 Theoretical Results

The stiffness and damping force coefficients obtained through the ETHD model and validated in Ref. [7] are recalculated and used here. Specifically, they are obtained when the test rig undergoes hybrid lubrication at 1650 rpm with a constant oil inflow. The supply pressure is set to 5 bar by a control signal of 0.1 V, approximately. The same three loading conditions are also replicated here, i.e., 700 N, 1400 N, and 2800 N. These operational and loading conditions are used throughout this work. The calibration curves for the hydraulic system \( H_i(s) \), obtained in Ref. [13], close to the studied operational conditions are also utilized. Three different values of \( k_h \) have been considered to account for its variation with the eccentricity set by the operational conditions and the different bearing loads. The pipeline dynamics has been neglected since it is assumed that there is a low flow of entrained air in the pipelines; therefore, its main dynamic contribution is expected to be at higher frequencies. Thus, the analysis can be kept at a manageable level.

The results are presented in the main text for an intermediate bearing load of 1400 N taking into account the flexible pivot. Results under other load conditions are included in Appendices unless otherwise specified. The system of equations is included in Appendix B for all pads under 1400 N. Other loads have not been included. Additionally, bearing force coefficients for 1400 N and the calibration function fitted for the hydraulic system under all loading conditions are included in Appendix C.

5.1 Stability Results for the Different Pads, Table 2 summarizes the eigenvalues \( \xi_j \), damped frequencies \( \omega_{ij} \), and damping ratios \( \zeta_j \) of the system, considering the pivot flexibility in the different pads studied. Modes (eigenvalues and eigenvectors) are named according to the DOFs that significantly participate and characterize the dynamic response, i.e., \( \xi_{j-pnl} \), \( \theta_{j-pad} \), \( f'_{j-hyd} \), and \( \zeta_{j-cpld} \). The respective mode shapes are presented in Table 3 for all pads, while the eigenvectors of overdamped modes have been excluded. In the journal mode shape \( \zeta_{j-pnl} \), neither the pivot nor the hydraulic \( f' \) DOFs have any influence on the response. In the coupled mode shape \( \zeta_{j-cpld} \), the pad vibrates vertically in a counter phase with a slightly smaller amplitude than the journal,

\[
\cos(z_{ij}) = \frac{\| b_j \|}{\| \phi_j \|} \| \psi_i \| \| b_j \| \}
\] (13a)
Table 2: Eigenvalues $\lambda$, damped frequencies $\omega_d$, and damping ratios $\zeta$ of the system under studied operational conditions

<table>
<thead>
<tr>
<th>Pad #N2</th>
<th>Pad #N1</th>
<th>Pad #C</th>
<th>Pad #P1</th>
<th>Pad #P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda (s^{-1})$</td>
<td>$\omega_d (Hz)$</td>
<td>$\zeta$</td>
<td>$\lambda (s^{-1})$</td>
<td>$\omega_d (Hz)$</td>
</tr>
<tr>
<td>$\theta$-pad</td>
<td>-653,830.8</td>
<td>0</td>
<td>1</td>
<td>-577,809.9</td>
</tr>
<tr>
<td>$\zeta$-pad</td>
<td>-124.5 + 147.0i</td>
<td>23.4</td>
<td>0.65</td>
<td>-142.1 + 155.8i</td>
</tr>
<tr>
<td>$f'$-hyd</td>
<td>-636.2 + 826.5i</td>
<td>131.5</td>
<td>0.61</td>
<td>-636.2 + 826.5i</td>
</tr>
<tr>
<td>$\zeta$-cpled</td>
<td>-177.5 + 1179.5i</td>
<td>187.7</td>
<td>0.15</td>
<td>-190.3 + 1172.9i</td>
</tr>
<tr>
<td>$\zeta$-hyd</td>
<td>177.5</td>
<td>1179.5i</td>
<td>187.7</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Applied bearing static load of 1400 N.

Table 3: Underdamped right mode shapes $\phi_j$ of the system under studied operational conditions

<table>
<thead>
<tr>
<th>Pad #N2</th>
<th>Pad #N1</th>
<th>Pad #C</th>
<th>Pad #P1</th>
<th>Pad #P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_j$</td>
<td>$\phi_j$-hyd</td>
<td>$\phi_j$-cpled</td>
<td>$\phi_j$</td>
<td>$\phi_j$-hyd</td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.0 $\times$ 10^{-16}</td>
<td>1.0 $\times$ 10^{-16}</td>
<td>1</td>
<td>0 $\times$ 10^{-16}</td>
</tr>
<tr>
<td>$\theta$</td>
<td>22.6 $\times$ 10^{-16}</td>
<td>4.4 $\times$ 10^{-16}</td>
<td>3.4 $\times$ 10^{-16}</td>
<td>26.4 $\times$ 10^{-16}</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.0 $\times$ 10^{-16}</td>
<td>0.9 $\times$ 10^{-16}</td>
<td>0.9 $\times$ 10^{-16}</td>
<td>0.9 $\times$ 10^{-16}</td>
</tr>
<tr>
<td>$f'$</td>
<td>0.0 $\times$ 10^{-16}</td>
<td>1.9 $\times$ 10^{-16}</td>
<td>1.9 $\times$ 10^{-16}</td>
<td>1.9 $\times$ 10^{-16}</td>
</tr>
</tbody>
</table>

Applied bearing static load of 1400 N.
whereas the \( f^\text{p} \) DOF does not participate in the response. For the hydraulic mode shape \( f^\text{hyd} \), the hydraulic DOF features the largest response, while other DOFs react in a similar way as in the \( \xi \)-cpled mode shape. Tables with modal information for loading conditions different from 1400 N are included in Appendix D. It is noted that for every pad analyzed, all modes are stable regardless of the loading condition studied, i.e., \( R(\lambda_i) < 0 \). The two fundamental modes correspond to the ones identified as \( \theta \)-pad and \( \xi \)-nal modes. The \( \theta \)-pad mode is overdamped regardless of the pad type. This behavior does not change if the load is increased; however, when the load is reduced it might become underdamped for the pad \#P2 (see Table 9 in Appendix D). The mode related to the hydraulic unit, the \( f^\text{hyd} \), does not vary with the pads or with the load, as expected. A general outcome is that the damping ratios \( \zeta \) related to the mechanical system—excluding the hydraulic dynamics—increase as one goes from pad layouts \#N to \#P in Table 1. This indicates that the offset-pivot pad configurations \#P1 and \#P2 have more damped modes than the offset-nozzle pads \#N1 and \#N2 and the centered-nozzle pad \#C, which becomes overdamped in most cases. With regard to the damped natural frequencies \( \omega_n \), an increase and a reduction of the frequency linked to the \( \xi \)-cpled mode are observed when comparing the offset-nozzle pads or the offset-pivot pads against the centered-nozzle pad, respectively. On the contrary, for the \( \xi \)-nal mode all pads differ from the centered-nozzle pad exhibit slightly larger frequencies. This is highly influenced by the loading condition, especially at light loads, as corroborated in Tables 9 and 10 of Appendix D. Interestingly, at 700 N, all modes become underdamped with the pad \#P2, despite the \( \xi \)-nal and the \( \theta \)-pad modes already behaving as overdamped modes with the \( \xi \)-pad modes already behaving as overdamped modes. This behavior does not change if the load is increased with the loads or with the loads. The rest of the modes are significantly less controllable, the coupled \( \eta \)-cpled become more damped for the offset-pivot pads, obtaining a larger value for pad \#P2. Finally, it is clearly seen that the eigenvalues related to the hydraulic mode \( f^\text{hyd} \) do not change with the different pads and although they are placed more to the left than the others, they fall into the frequency span of interest. Again, in order to prevent the hydraulic and pivot dynamics, a high response servovalve and stiff pivot should preferably be used.

### 5.2 Controllability and Observability of the Different Pad Designs

Table 3 summarizes the binary assessment of the controllability and observability for the different pads under the three loading conditions in its original form. The controllability \( M_c \) and observability \( M_o \) matrices are not included for the sake of brevity. None of the matrices are full rank, meaning that the system is stabilizable/detectable but not fully state controllable/observable for any of the pads. In terms of controllability, it can be said that the system has less controllable modes as the load is increased. This characteristic seems to be triggered at low loads for the offset-nozzle pad \#N2 and does not significantly affect the offset-pivot pad \#P2 in the loading range studied. It is also seen that the controllability is increased to four for the offset-pivot pad \#P2 at light load conditions since all modes become underdamped (see Table 9 in Appendix D). With regard to the observability, this is hardly affected by the parameters studied. For the offset-nozzle pads and the centered-nozzle pad, it does not change with the load condition. For the offset-pivot pads it varies with the load, especially for pad \#P2.

How controllable/observable the modes are is elucidated by the degree of controllability and observability with formula (13). The results of the degree of controllability are shown in Table 5, for all pads, under a loading condition of 1400 N. The larger degree of controllability is associated with the \( f^\text{hyd} \) mode, which does not vary either with the pads or with the loads. The rest of the modes are significantly less controllable, the coupled \( \eta \)-cpled

---

### Table 4: Binary assessment of controllability and observability of pad designs under different loading conditions: 700 N, 1400 N, and 2800 N

<table>
<thead>
<tr>
<th>Bearing load</th>
<th>Pad #N2</th>
<th>Pad #N1</th>
<th>Pad #C</th>
<th>Pad #P1</th>
<th>Pad #P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>700 N</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1400 N</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2800 N</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bearing load</th>
<th>Pad #N2</th>
<th>Pad #N1</th>
<th>Pad #C</th>
<th>Pad #P1</th>
<th>Pad #P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>700 N</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1400 N</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2800 N</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

---

### Table 5: Degrees of modal controllability for the different pads under 1400 of bearing load

<table>
<thead>
<tr>
<th>Mode</th>
<th>Pad #N2</th>
<th>Pad #N1</th>
<th>Pad #C</th>
<th>Pad #P1</th>
<th>Pad #P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )-pad</td>
<td>1.8 x 10^{-14}</td>
<td>2.0 x 10^{-14}</td>
<td>2.2 x 10^{-14}</td>
<td>5.4 x 10^{-14}</td>
<td>1.4 x 10^{-13}</td>
</tr>
<tr>
<td>( \xi )-nal</td>
<td>4.6 x 10^{-15}</td>
<td>4.3 x 10^{-16}</td>
<td>9.6 x 10^{-16}</td>
<td>1.8 x 10^{-16}</td>
<td>6.6 x 10^{-17}</td>
</tr>
<tr>
<td>Coupled modes</td>
<td>7.7 x 10^{-12}</td>
<td>6.2 x 10^{-12}</td>
<td>5.4 x 10^{-12}</td>
<td>2.6 x 10^{-12}</td>
<td>3.5 x 10^{-11}</td>
</tr>
</tbody>
</table>

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Transactions of the ASME
presents the results of the degree of observability at outputs. However, the offset-pivot pads are, comparatively, noticed that the centered-nozzle pad and the larger value is obtained for the offset-nozzle pads compared with the output 1 is larger for the offset-nozzle pads compared with the centered-nozzle pad and the larger value is obtained for the offset-pivot pad #P1. A significant increase is obtained if output 2 is considered, i.e., \( y_2 = \theta \), the pad tilt angle. This means that the measurement of the pad tilt angle \( \theta \) would significantly improve the gross observability of all pads, by a factor of ten, approximately. Furthermore, from output 3 the system is even more weakly observable than it is from output 1. These characteristics do not vary significantly with a larger bearing load, i.e., 2800 N. For a

The hydraulic mode has not been analyzed since it hinders the analysis due to its larger value of modal controllability. On the other hand, a different situation is obtained from the gross measurement of observability of all modes at the different outputs. If the journal displacement is only considered, \( y_1 = \xi \) (see Fig. 5(a)) then the gross measurement of observability of all modes at output 1 is larger for the offset-nozzle pads compared with the centered-nozzle pad and the larger value is obtained for the offset-pivot pad #P1. A significant increase is obtained if output 2 is considered, i.e., \( y_2 = \theta \), the pad tilt angle. This means that the measurement of the pad tilt angle \( \theta \) would significantly improve the gross observability of all pads, by a factor of ten, approximately. Furthermore, from output 3 the system is even more weakly observable than it is from output 1. These characteristics do not vary significantly with a larger bearing load, i.e., 2800 N. For a

Table 6 Degrees of modal observability for the different pads under 1400 N of bearing load

<table>
<thead>
<tr>
<th>Mode</th>
<th>Output ( \theta )-pad</th>
<th>Output ( \xi )-junal</th>
<th>Output ( \psi )-pooled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pad #N2</td>
<td>2.8 \times 10^{-9}</td>
<td>2.2 \times 10^{-9}</td>
<td>2.3 \times 10^{-7}</td>
</tr>
<tr>
<td>Pad #N1</td>
<td>2.9 \times 10^{-9}</td>
<td>2.7 \times 10^{-9}</td>
<td>2.3 \times 10^{-7}</td>
</tr>
<tr>
<td>Pad #C</td>
<td>3.1 \times 10^{-9}</td>
<td>3.3 \times 10^{-9}</td>
<td>2.9 \times 10^{-7}</td>
</tr>
</tbody>
</table>

Pivot damping ratio (\( \xi_{\text{p}}, \psi_{\text{p}} \)) 0.61
Injection nozzle radius 3 mm
Injection nozzle length 20 mm
Injection nozzle offset 5 mm
Constant injection pressure 5 bar

Including the levered arm weight.

Table 7 Gross measurement of controllability from input \( u \) and observability at outputs \( y_1, y_2 \), and \( y_3 \)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Offset-nozzle pads</th>
<th>Centered-nozzle pad</th>
<th>Offset-pivot pads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pad #N2</td>
<td>5.4 \times 10^{-3}</td>
<td>5.5 \times 10^{-3}</td>
<td>1.4 \times 10^{-2}</td>
</tr>
<tr>
<td>Pad #N1</td>
<td>4.6 \times 10^{-4}</td>
<td>3.8 \times 10^{-4}</td>
<td>1.5 \times 10^{-4}</td>
</tr>
<tr>
<td>Pad #C</td>
<td>7.4 \times 10^{-3}</td>
<td>6.8 \times 10^{-3}</td>
<td>4.0 \times 10^{-3}</td>
</tr>
<tr>
<td>Pad #P1</td>
<td>3.0 \times 10^{-4}</td>
<td>2.6 \times 10^{-5}</td>
<td>6.6 \times 10^{-3}</td>
</tr>
</tbody>
</table>

Bear load of 1400 N:

\( \xi = \xi_{\text{p}}, \psi = \psi_{\text{p}} \)

Table 8 Conventional and controllable design parameters of the investigated system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional design</td>
<td>49.23 mm</td>
<td>( R_b )</td>
</tr>
<tr>
<td>Journal radius</td>
<td>49.62 mm</td>
<td>( R_b )</td>
</tr>
<tr>
<td>Pad axial length (( L_{ax} ))</td>
<td>100 mm</td>
<td>( L_{ax} )</td>
</tr>
<tr>
<td>Pad arc length (( \alpha ))</td>
<td>69 deg</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>Pad thickness</td>
<td>12 mm</td>
<td>( t )</td>
</tr>
<tr>
<td>Pad mass (( m_{\text{pad}} ))</td>
<td>8000 kg</td>
<td>( m_{\text{pad}} )</td>
</tr>
<tr>
<td>Pad inertia (( J_{\text{pad}} ))</td>
<td>6.6 \times 10^{-4} kg( \cdot )m(^2)</td>
<td>( J_{\text{pad}} )</td>
</tr>
<tr>
<td>Rotor equivalent mass (( m_{\text{rot}} ))</td>
<td>27 kg</td>
<td>( m_{\text{rot}} )</td>
</tr>
<tr>
<td>Pivot design</td>
<td>Rocker</td>
<td>( \theta )</td>
</tr>
<tr>
<td>Pivot offset</td>
<td>According to Table 1</td>
<td>( \delta )</td>
</tr>
<tr>
<td>Pivot stiffness</td>
<td>3 – 5 \times 10^{6} N/m</td>
<td>( k_{\text{p}} )</td>
</tr>
<tr>
<td>Pivot damping</td>
<td>0 – 8 \times 10^{5} Ns/m</td>
<td>( c_{\text{p}} )</td>
</tr>
<tr>
<td>Journal material</td>
<td>Steel</td>
<td>( J_{\text{pad}} )</td>
</tr>
<tr>
<td>Journal angular speed (( \omega ))</td>
<td>1650 rpm</td>
<td>( \omega )</td>
</tr>
<tr>
<td>Applied load</td>
<td>700–4000–2800 N</td>
<td>( L_{\text{app}} )</td>
</tr>
<tr>
<td>Load angle</td>
<td>On pad</td>
<td>( \delta_{\text{load}} )</td>
</tr>
<tr>
<td>Lubricant type</td>
<td>ISO VG22</td>
<td>( \text{Lubricant type} )</td>
</tr>
<tr>
<td>Controllable design</td>
<td>Four-ways, two-stage spool</td>
<td>( \text{Controllable design} )</td>
</tr>
<tr>
<td>Servovalve type</td>
<td>Four-ways, two-stage spool</td>
<td>( \text{Servovalve type} )</td>
</tr>
<tr>
<td>Servovalve cutoff frequency (( \omega_{\text{cut}} ))</td>
<td>166 Hz</td>
<td>( \omega_{\text{cut}} )</td>
</tr>
<tr>
<td>Servovalve damping ratio (( \xi_{\text{p}}, \psi_{\text{p}} ))</td>
<td>0.61</td>
<td>( \xi_{\text{p}}, \psi_{\text{p}} )</td>
</tr>
<tr>
<td>Injection nozzle radius 3 mm</td>
<td>Injection nozzle length 20 mm</td>
<td>Injection nozzle offset According to Table 1</td>
</tr>
</tbody>
</table>

Fig. 7 ALB test rig with centered-nozzle pad. \( \xi \): servovalve, \( \psi \): AC motor, \( \theta \): electromagnetic shaker, \( \theta \): rotot, \( \delta \): levered arm, \( \xi \): load cell, and \( \psi \): displacement sensor (not seen).
lightly loaded condition of 700 N, the gross measurements of observability, from outputs 1 and 2, are slightly increased.

6 Experimental Validation for Centered-Nozzle Pad

The ALB test rig has been thoroughly presented in a number of publications [6,7,13]. It is a simple rigid rotor with two opposite pads fully instrumented with the aim of studying the fundamentals of active lubrication. Figure 7 shows a picture of it, highlighting its main components. Table 8 summarizes its main standard and controllable design parameters. A single bottom pad configuration...
of the test rig with the centered-nozzle pad presented in Table 1 is used.

The results concerning stability and controllability and observability are experimentally validated for the already built pad #C. This validation is carried out by comparing predicted against experimental frequency response functions (FRFs), in order to assess the validity of the dynamic model linearization. Results are obtained once the system is in force and thermal steady-state equilibria, imposed by the angular speed and injection pressure of the hydraulic system. Uncertainty in experimental FRFs is estimated as Type A according to ISO GUM [13]. The mean value is used as the best estimate and its standard deviation is employed to calculate the expanded uncertainty. The standard deviation is obtained with a 95% confidence level and 700DOF (blocks).

The experimental FRFs are obtained by sweeping a chirp signal from 10Hz to 300Hz. The dynamic force is applied through an electromagnetic shaker connected to the levered arm which supports the rotor. This implies the definition of an external force vector \( f_e = (f_x \ 0 \ 0 \ 0) \) and null control signal \( u = 0 \) in Eq. (5). Hanning windowing and a sampling frequency of 2000 Hz are utilized. The system response is recorded by means of a displacement sensor looking at the levered arm. Figure 8 shows a comparison of the theoretical against experimental FRFs obtained in m/N for the three different loading conditions. Generally speaking, the two main experimental trends, yielded by the change in the loading condition, are well represented by the theoretical FRFs. The first one is an increase of the damped natural frequency \( \omega_d \) with the load. This leads to a shift toward the higher frequencies of the resonant zone, identified as around 150 Hz under 700 N. This resonance, linked to the coupled mode, is very well predicted by the model. The second one is an important reduction of the system response at low frequencies due to the increased load. Nonetheless, the predicted amplitudes below 50 Hz are larger than the ones obtained experimentally, especially for 700 N. The uncertainty confidence levels obtained for the FRFs are high due to the large amount of averages utilized, leading to narrow plotted uncertainties.

7 Frequency Response Functions for Offset-Nozzle Pads and Offset-Pivot Pads

Once the model is dynamically validated for the centered-nozzle pad, the results can be extended for the proposed pads. Figure 9 presents the FRFs of the outputs as a function of the control signal. The figure includes all pads under the intermediate loading conditions. The comparison between the theoretical and experimental FRFs shows a good agreement, especially for the centered-nozzle pad. The FRFs for the offset-nozzle pad and offset-pivot pad are also presented, and the differences in the response are attributed to the different geometries and loading conditions. The uncertainty levels obtained for these FRFs are also shown, highlighting the variation between the theoretical and experimental results.

Fig. 12 Calibration function for the hydraulic system under the studied operational conditions. Controllable force \( f_c \) as a function of control signal \( u \). \( \bigcirc \) pad #N2, \( \triangle \) pad #N1, \( \square \) pad #C, \( \bigtriangleup \) pad #P1, \( \bigtriangledown \) pad #P2. Controllable force \( f_c \) as a function of control signal \( u \). \( \bigcirc \) pad #N2, \( \triangle \) pad #N1, \( \square \) pad #C, \( \bigtriangleup \) pad #P1, \( \bigtriangledown \) pad #P2. Solid line (–): flexible pivot and dashed line (- -): rigid pivot.

Fig. 13 Theoretical FRFs. Bearing applied load of 700 N. \( \bigcirc \) pad #N2, \( \bigtriangleup \) pad #N1, \( \square \) pad #C, \( \bigtriangledown \) pad #P1, \( \bigtriangledown \) pad #P2. Solid line (–): flexible pivot and dashed line (- -): rigid pivot.

Fig. 14 Theoretical FRFs. Bearing applied load of 2800 N. \( \bigcirc \) pad #N2, \( \bigtriangleup \) pad #N1, \( \square \) pad #C, \( \bigtriangledown \) pad #P1, \( \bigtriangledown \) pad #P2. Solid line (–): flexible pivot and dashed line (- -): rigid pivot.
bearing load of 1400 N. Solid and dashed lines depict the system responses considering flexible and rigid pivot, respectively. Other load conditions are included in Appendix F. Only the DOFs related to the mechanical system are presented in the FRFs. Such FRFs are presented in decibel to make the results comparable within the same scale. It can be seen from these FRF plots that dynamic responses are dominated by the coupled mode $\eta$-cpled, producing a resonant zone between 150 Hz and 200 Hz. The system response of the offset-nozzle pads is slightly larger than the reference centered-nozzle pad. This is clearer in a linear scale ($m/\text{V}$). The opposite occurs for the offset-nozzle pads, and larger differences are obtained for the offset-nozzle pad #P2, for which the response of the pad tilt angle $\theta$ at low frequencies is larger compared to the rest of the pads. This characteristic might be considered as a disadvantage for this pad since it significantly affects the steady-state equilibrium of the pad. The behavior of the offset-pivot pad #P2 is even worse under light load (see Fig. 13 in Appendix F). Under such a condition, all modes become under-damped and participate in the system dynamic response (see Table 9 in Appendix D). Under a larger bearing load, i.e., 2800 N, the journal and pad translational responses are closer to each other for all pads. The pad tilt angle $\theta$ remains larger at low frequencies for pad #P2 compared to the rest of the pads.

8 Conclusions

The theoretical investigation sheds some light on which of the studied pads gather the best control characteristics and which of the design parameters influence its behavior the most. It also contributes to determining which of the modes dominate the system dynamic response. The following is concluded from this theoretical investigation, experimentally validated by the existing centered-nozzle pad:

- The difference in offsets between the injection nozzle and pivot lines introduces an additional mechanism for accessing and controlling the tilt angle of the pad featuring the injection nozzle.
- The degree of modal controllability shows that, without considering the hydraulic mode, the most controllable mode is the coupled mode $\eta$-cpled. Furthermore, the gross measurement of modal controllability shows that the offset-pivot pads have better controllability than the centered-nozzle pad and offset-nozzle pads. These measures decrease with an increased load.
- The degree of modal observability shows that the most observable mode is also the coupled mode, $\eta$-cpled. The gross measurement of observability shows that by measuring the journal displacement, an improvement in observability can be obtained with the offset-nozzle pads and with the offset-pivot pad #P1. Observability also reduces with an increased load. Additionally, a substantial improvement in the state observability can be gained by measuring the pad tilt angle $\theta$ as output.
- The offset-nozzle pads slightly improve the performance of pads with the injection nozzle from a control viewpoint. However, the best overall control characteristics are found in the offset-pivot pad #P1 with a slight offset. One fact must be kept in mind: the offset should not be too large in order to avoid undesired pad behavior, i.e., $|\Delta \theta| \leq [0.1]$.
- The ALB system dynamics is strongly influenced by the pivot flexibility and the system hydraulic dynamics. The pivot flexibility introduces the counter face coupled mode $\eta$-cpled which dominates the system response. To avoid their dynamic effects, the rigid pivot pad and high response servovalve must be targeted.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>$j$th column vector of the input matrix</td>
</tr>
<tr>
<td>$B$</td>
<td>transformed state-space input matrix</td>
</tr>
<tr>
<td>$C$</td>
<td>state-space output matrix</td>
</tr>
<tr>
<td>$c$</td>
<td>$k$th row vector of the output matrix</td>
</tr>
<tr>
<td>$C_i$</td>
<td>transformed state-space output matrix</td>
</tr>
<tr>
<td>$\cos A$</td>
<td>controllability degree matrix</td>
</tr>
<tr>
<td>$\cos B$</td>
<td>observability degree matrix</td>
</tr>
<tr>
<td>$\cos (\beta_i)$</td>
<td>controllability degree index</td>
</tr>
<tr>
<td>$D$</td>
<td>system damping matrix</td>
</tr>
<tr>
<td>$D_i$</td>
<td>$i$th damping force coefficient</td>
</tr>
<tr>
<td>$f$</td>
<td>fluid-film force (N)</td>
</tr>
<tr>
<td>$f_e$</td>
<td>external generalized force vector</td>
</tr>
<tr>
<td>$f^c$</td>
<td>variable hydrostatic forces (N)</td>
</tr>
<tr>
<td>$f^p$</td>
<td>hydrodynamic + constant hydrostatic forces (N)</td>
</tr>
<tr>
<td>$H(u)$</td>
<td>transfer function between $u$ and $f^c$</td>
</tr>
<tr>
<td>$f_p$</td>
<td>pad mass moment of inertia (kg/m²)</td>
</tr>
<tr>
<td>$K$</td>
<td>system stiffness</td>
</tr>
<tr>
<td>$K_{ij}$</td>
<td>$i$th stiffness force coefficient</td>
</tr>
<tr>
<td>$L$</td>
<td>pad axial length (m)</td>
</tr>
<tr>
<td>$M$</td>
<td>system mass matrix</td>
</tr>
<tr>
<td>$M_i$</td>
<td>system controllability matrix</td>
</tr>
<tr>
<td>$M_L$</td>
<td>system observability matrix</td>
</tr>
<tr>
<td>$m_p$</td>
<td>pad mass (kg)</td>
</tr>
<tr>
<td>$m_o$</td>
<td>rotor equivalent mass (kg)</td>
</tr>
<tr>
<td>$q_{inj}(t)$</td>
<td>injected flow (m³/s)</td>
</tr>
<tr>
<td>$R_p$</td>
<td>pad inner radius (m)</td>
</tr>
<tr>
<td>$r$</td>
<td>time (s)</td>
</tr>
<tr>
<td>$u(t)$</td>
<td>servovalve control signal (V)</td>
</tr>
<tr>
<td>$w$</td>
<td>hydraulic gain vector</td>
</tr>
<tr>
<td>$X_1$, $X_2$, $X_3$</td>
<td>state-space displacement, velocity, and general vector</td>
</tr>
<tr>
<td>$Y$</td>
<td>state output vector</td>
</tr>
<tr>
<td>$Z$</td>
<td>transformed modal coordinates</td>
</tr>
<tr>
<td>$z_i$</td>
<td>pad arc length (rad)</td>
</tr>
<tr>
<td>$\Delta \theta^o$</td>
<td>offset difference</td>
</tr>
<tr>
<td>$\zeta_f$</td>
<td>hydraulic damping ratio</td>
</tr>
<tr>
<td>$\delta_f(t)$, $\delta_f$</td>
<td>$i$th system eigenvalue</td>
</tr>
<tr>
<td>$\ddot{\theta}(t)$</td>
<td>pad tilt angular displacement, velocity, acceleration</td>
</tr>
<tr>
<td>$\Theta_i$</td>
<td>nozzle/pivot offset. Ratio of the angle from the leading edge to the nozzle/pivot location to the entire pad arc length</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>hydraulic proportional constant (N/s)</td>
</tr>
<tr>
<td>$A$</td>
<td>eigenvalue diagonal matrix</td>
</tr>
<tr>
<td>$a_i$</td>
<td>$i$th eigenvalue</td>
</tr>
<tr>
<td>$a_{j,i}$</td>
<td>$j$th row vector of the output matrix</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>fluid-film moment (N m)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$i$-system right eigenvector</td>
</tr>
<tr>
<td>$\psi$</td>
<td>left modal matrix</td>
</tr>
<tr>
<td>$\omega_{nf}$</td>
<td>hydraulic natural frequency (Hz)</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>rotor angular speed (rpm)</td>
</tr>
</tbody>
</table>

Appendix A: Pressure Profiles and Their Centroids

Appendix B: System of Equations for the ALB Under 1400 N of Bearing Load: Different Pads

B.1 Offset-Nozzle Pad System of Equations, Pad #N2.
B.3 Centered-Nozzle Pad System of Equations, Pad #C.

\[
\begin{bmatrix}
27 & 0 & 0 & 0 \\
0 & 6.6 \times 10^{-4} & 0 & 0 \\
0 & 0 & 0.81 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
+ \begin{bmatrix}
3.78 \times 10^5 & -3817 & 4.12 \times 10^5 & 0 \\
-3812 & 138.8 & -4163 & 0 \\
4.12 \times 10^5 & -4169 & 4.58 \times 10^5 & 0 \\
0 & 0 & 0 & 1.27 \times 10^3 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
= \begin{bmatrix}
3.33 \times 10^3 & -2.85 \times 10^6 & 3.64 \times 10^3 & -1 \\
4095 & 4.20 \times 10^4 & 4469 & -1.2 \times 10^{-2} \\
3.64 \times 10^3 & -3.11 \times 10^6 & 8.75 \times 10^3 & 1 \\
0 & 0 & 0 & 1.09 \times 10^6 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
12.7 \times 10^3 \\
\end{bmatrix}
\]  

(B1)

B.2 Offset-Nozzle Pad System of Equations, Pad #N1.

\[
\begin{bmatrix}
27 & 0 & 0 & 0 \\
0 & 6.6 \times 10^{-4} & 0 & 0 \\
0 & 0 & 0.81 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
+ \begin{bmatrix}
3.24 \times 10^5 & -3605 & 3.54 \times 10^5 & 0 \\
-3601 & 134.5 & -3933 & 0 \\
3.54 \times 10^5 & -3938 & 3.94 \times 10^5 & 0 \\
0 & 0 & 0 & 1.27 \times 10^3 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
= \begin{bmatrix}
3.18 \times 10^3 & -2.37 \times 10^6 & 3.47 \times 10^3 & -1 \\
3.60 \times 10^3 & 4.01 \times 10^4 & 3.93 \times 10^4 & -6.01 \times 10^{-3} \\
3.47 \times 10^3 & -2.59 \times 10^6 & 8.57 \times 10^3 & 1 \\
0 & 0 & 0 & 1.09 \times 10^6 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
12.7 \times 10^3 \\
\end{bmatrix}
\]  

(B2)

B.3 Centered-Nozzle Pad System of Equations, Pad #C.

\[
\begin{bmatrix}
27 & 0 & 0 & 0 \\
0 & 6.6 \times 10^{-4} & 0 & 0 \\
0 & 0 & 0.81 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
+ \begin{bmatrix}
2.65 \times 10^5 & -3159 & 2.90 \times 10^5 & 0 \\
-3157 & 120.1 & -3448 & 0 \\
2.90 \times 10^5 & -3450 & 3.24 \times 10^5 & 0 \\
0 & 0 & 0 & 1.27 \times 10^3 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
= \begin{bmatrix}
3.01 \times 10^3 & -1.85 \times 10^6 & 3.29 \times 10^3 & -1 \\
5.83 \times 10^4 & 3.58 \times 10^4 & 6.37 \times 10^4 & 0 \\
3.29 \times 10^3 & -2.02 \times 10^6 & 8.37 \times 10^3 & 1 \\
0 & 0 & 0 & 1.09 \times 10^6 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
12.7 \times 10^3 \\
\end{bmatrix}
\]  

(B3)

B.4 Pivot-Nozzle Pad System of Equations, Pad #P1.

\[
\begin{bmatrix}
27 & 0 & 0 & 0 \\
0 & 6.6 \times 10^{-4} & 0 & 0 \\
0 & 0 & 0.81 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
+ \begin{bmatrix}
1.27 \times 10^5 & -691.4 & 1.40 \times 10^5 & 0 \\
691.4 & 38.56 & -819.1 & 0 \\
1.40 \times 10^5 & -820.5 & 1.63 \times 10^5 & 0 \\
0 & 0 & 0 & 1.27 \times 10^3 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
= \begin{bmatrix}
2.65 \times 10^7 & -9.99 \times 10^6 & 3.08 \times 10^7 & -1 \\
-1.20 \times 10^8 & 1.73 \times 10^6 & -1.58 \times 10^3 & -6.01 \times 10^{-3} \\
2.94 \times 10^7 & -1.13 \times 10^6 & 8.18 \times 10^7 & 1 \\
0 & 0 & 0 & 1.09 \times 10^6 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\dot{\xi} \\
\eta \\
\dot{\eta} \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
12.7 \times 10^3 \\
\end{bmatrix}
\]  

(B4)
Table 9: Eigenvalues $\lambda$, damped frequencies $\omega_d$, and damping ratios $\zeta$ of the system under studied operational conditions

<table>
<thead>
<tr>
<th>Pad #N2</th>
<th>Pad #N1</th>
<th>Pad #C</th>
<th>Pad #P1</th>
<th>Pad #P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ (s$^{-1}$)</td>
<td>$\omega_d$ (Hz)</td>
<td>$\zeta$</td>
<td>$\lambda$ (s$^{-1}$)</td>
<td>$\omega_d$ (Hz)</td>
</tr>
<tr>
<td>$\theta$-pad</td>
<td>$\zeta$</td>
<td>$\lambda$</td>
<td>$\omega_d$</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>$\zeta$-pad</td>
<td>$\zeta$</td>
<td>$\lambda$</td>
<td>$\omega_d$</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>$\eta$-cpled</td>
<td>$\zeta$</td>
<td>$\lambda$</td>
<td>$\omega_d$</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>Applied bearing static load of 700 N.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10: Eigenvalues $\lambda$, damped frequencies $\omega_d$, and damping ratios $\zeta$ of the system under studied operational conditions

<table>
<thead>
<tr>
<th>Pad #N2</th>
<th>Pad #N1</th>
<th>Pad #C</th>
<th>Pad #P1</th>
<th>Pad #P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ (s$^{-1}$)</td>
<td>$\omega_d$ (Hz)</td>
<td>$\zeta$</td>
<td>$\lambda$ (s$^{-1}$)</td>
<td>$\omega_d$ (Hz)</td>
</tr>
<tr>
<td>$\theta$-pad</td>
<td>$\zeta$</td>
<td>$\lambda$</td>
<td>$\omega_d$</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>$\zeta$-pad</td>
<td>$\zeta$</td>
<td>$\lambda$</td>
<td>$\omega_d$</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>$\eta$-cpled</td>
<td>$\zeta$</td>
<td>$\lambda$</td>
<td>$\omega_d$</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>Applied bearing static load of 2800 N.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 11 Underdamped right mode shapes \( \phi \) of the system under studied operational conditions

<table>
<thead>
<tr>
<th>Pad #2</th>
<th>Pad #1</th>
<th>Pad #3</th>
<th>Pad #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>( f )</td>
<td>( \phi )</td>
<td>( \zeta )</td>
</tr>
<tr>
<td>15.9 Hz</td>
<td>131.5 Hz</td>
<td>166.4 Hz</td>
<td>19.3 Hz</td>
</tr>
<tr>
<td>15.6 Hz</td>
<td>131.5 Hz</td>
<td>166.4 Hz</td>
<td>19.3 Hz</td>
</tr>
<tr>
<td>15.6 Hz</td>
<td>131.5 Hz</td>
<td>166.4 Hz</td>
<td>19.3 Hz</td>
</tr>
<tr>
<td>15.6 Hz</td>
<td>131.5 Hz</td>
<td>166.4 Hz</td>
<td>19.3 Hz</td>
</tr>
</tbody>
</table>

Applied bearing static load of 700N.

### Table 12 Underdamped right mode shapes \( \phi \) of the system under studied operational conditions

<table>
<thead>
<tr>
<th>Pad #2</th>
<th>Pad #1</th>
<th>Pad #3</th>
<th>Pad #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>( f )</td>
<td>( \phi )</td>
<td>( \zeta )</td>
</tr>
<tr>
<td>15.9 Hz</td>
<td>131.5 Hz</td>
<td>166.4 Hz</td>
<td>19.3 Hz</td>
</tr>
<tr>
<td>15.6 Hz</td>
<td>131.5 Hz</td>
<td>166.4 Hz</td>
<td>19.3 Hz</td>
</tr>
<tr>
<td>15.6 Hz</td>
<td>131.5 Hz</td>
<td>166.4 Hz</td>
<td>19.3 Hz</td>
</tr>
<tr>
<td>15.6 Hz</td>
<td>131.5 Hz</td>
<td>166.4 Hz</td>
<td>19.3 Hz</td>
</tr>
</tbody>
</table>

Applied bearing static load of 2300N.
B.5 Pivot-Nozzle Pad System of Equations, Pad #P2.

\[ \begin{bmatrix}
27 & 0 & 0 & 0 & 4.67 \times 10^4 & -158 & 5.31 \times 10^4 & 0 \\
0 & 0.66 \times 10^{-4} & 0 & 0 & -157.3 & 9.62 & -211.0 & 0 \\
0 & 0 & 0.81 & 0 & 5.31 \times 10^{-4} & -211.8 & 6.76 \times 10^6 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1.27 \times 10^7 & 0 \\
\end{bmatrix}
\]

\[ + \begin{bmatrix}
2.37 \times 10^7 & -4.70 \times 10^4 & 2.83 \times 10^7 & -1 \\
-2.12 \times 10^7 & 8302 & -2.68 \times 10^4 & -1.20 \times 10^{-2} \\
2.74 \times 10^7 & -5.58 \times 10^4 & 8.05 \times 10^7 & 1 \\
0 & 0 & 0 & 1.09 \times 10^6 \\
\end{bmatrix}
\begin{bmatrix}
\xi \\
\theta \\
\eta \\
\zeta' \\
\end{bmatrix}
\]

\[ \begin{bmatrix}
\xi \\
\theta \\
\eta \\
\zeta' \\
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

(Appendix C: Bearing Force Coefficients and Hydraulic Calibration Function)

(Appendix D: Eigenvalues and Mode shapes for 700 N and 2800 N)

(Appendix E: Tables of Degree of Modal Observability for 700 N and 2800 N)

(Appendix F: Theoretical FRFs for 700 N and 2800 N)

### Table 13: Degree of Modal Observability for the different pads under different loading conditions (700 N and 2800 N)

<table>
<thead>
<tr>
<th>Output</th>
<th>700N</th>
<th>2800N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pad #N2 y1</td>
<td>1.1 \times 10^{-8}</td>
<td>1.2 \times 10^{-8}</td>
</tr>
<tr>
<td>y2</td>
<td>5.2 \times 10^{-6}</td>
<td>9.3 \times 10^{-5}</td>
</tr>
<tr>
<td>y3</td>
<td>4.1 \times 10^{-7}</td>
<td>1.2 \times 10^{-5}</td>
</tr>
<tr>
<td>Pad #N1 y1</td>
<td>1.2 \times 10^{-8}</td>
<td>1.2 \times 10^{-8}</td>
</tr>
<tr>
<td>y2</td>
<td>6.5 \times 10^{-9}</td>
<td>1.2 \times 10^{-8}</td>
</tr>
<tr>
<td>y3</td>
<td>5.7 \times 10^{-6}</td>
<td>9.3 \times 10^{-5}</td>
</tr>
<tr>
<td>Pad #C y1</td>
<td>1.3 \times 10^{-8}</td>
<td>1.2 \times 10^{-8}</td>
</tr>
<tr>
<td>y2</td>
<td>6.3 \times 10^{-6}</td>
<td>9.4 \times 10^{-5}</td>
</tr>
<tr>
<td>y3</td>
<td>6.6 \times 10^{-7}</td>
<td>1.4 \times 10^{-4}</td>
</tr>
<tr>
<td>Pad #P1 y1</td>
<td>1.0 \times 10^{-7}</td>
<td>1.0 \times 10^{-7}</td>
</tr>
<tr>
<td>y2</td>
<td>2.6 \times 10^{-5}</td>
<td>1.0 \times 10^{-5}</td>
</tr>
<tr>
<td>y3</td>
<td>3.8 \times 10^{-6}</td>
<td>7.3 \times 10^{-6}</td>
</tr>
<tr>
<td>Pad #P2 y1</td>
<td>4.0 \times 10^{-7}</td>
<td>5.6 \times 10^{-7}</td>
</tr>
<tr>
<td>y2</td>
<td>1.4 \times 10^{-4}</td>
<td>5.6 \times 10^{-7}</td>
</tr>
<tr>
<td>y3</td>
<td>1.2 \times 10^{-5}</td>
<td>1.7 \times 10^{-6}</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Mode</th>
<th>Pad #N2</th>
<th>Pad #N1</th>
<th>Pad #C</th>
<th>Pad #P1</th>
<th>Pad #P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta)-pad</td>
<td>(1.3 \times 10^{-13})</td>
<td>(1.3 \times 10^{-13})</td>
<td>(1.4 \times 10^{-13})</td>
<td>(6.2 \times 10^{-13})</td>
<td>(4.0 \times 10^{-12})</td>
</tr>
<tr>
<td>(\delta)-pad</td>
<td>(1.1 \times 10^{-13})</td>
<td>(2.0 \times 10^{-13})</td>
<td>(4.4 \times 10^{-13})</td>
<td>(1.9 \times 10^{-13})</td>
<td>(4.0 \times 10^{-12})</td>
</tr>
<tr>
<td>(\zeta)-pad</td>
<td>(3.4 \times 10^{-11})</td>
<td>(2.1 \times 10^{-11})</td>
<td>(1.3 \times 10^{-11})</td>
<td>(6.4 \times 10^{-14})</td>
<td>(2.2 \times 10^{-10})</td>
</tr>
<tr>
<td>(\zeta)-pad</td>
<td>(3.4 \times 10^{-11})</td>
<td>(2.1 \times 10^{-11})</td>
<td>(1.3 \times 10^{-11})</td>
<td>(6.4 \times 10^{-14})</td>
<td>(2.2 \times 10^{-10})</td>
</tr>
<tr>
<td>(f)-pad</td>
<td>(9.6 \times 10^{-4})</td>
<td>(9.6 \times 10^{-4})</td>
<td>(9.6 \times 10^{-4})</td>
<td>(9.6 \times 10^{-4})</td>
<td>(9.6 \times 10^{-4})</td>
</tr>
<tr>
<td>(f)-pad</td>
<td>(9.6 \times 10^{-4})</td>
<td>(9.6 \times 10^{-4})</td>
<td>(9.6 \times 10^{-4})</td>
<td>(9.6 \times 10^{-4})</td>
<td>(9.6 \times 10^{-4})</td>
</tr>
<tr>
<td>(\phi)-pad</td>
<td>(5.6 \times 10^{-11})</td>
<td>(5.9 \times 10^{-11})</td>
<td>(6.1 \times 10^{-11})</td>
<td>(4.1 \times 10^{-10})</td>
<td>(8.0 \times 10^{-11})</td>
</tr>
<tr>
<td>(\phi)-pad</td>
<td>(5.6 \times 10^{-11})</td>
<td>(5.9 \times 10^{-11})</td>
<td>(6.1 \times 10^{-11})</td>
<td>(4.1 \times 10^{-10})</td>
<td>(8.0 \times 10^{-11})</td>
</tr>
</tbody>
</table>

### References


A.5 [P5]: Active Tilting-Pad Journal Bearings Supporting Flexible Rotors: Part I – The Hybrid Lubrication
Active tilting-pad journal bearings supporting flexible rotors: Part I – The hybrid lubrication

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A R T I C L E   I N F O

Keywords:
Active tilting-pad journal bearings
Adjustable lubrication
Vibration suppression
Rotordynamics

A B S T R A C T

This is part I of a twofold paper series, of theoretical and experimental nature, presenting the design and implementation of model-based controllers meant for assisting the hybrid and developing the feedback-controlled lubrication regimes in active tilting-pad journal bearings (active TPJBs). In part I, the flexible rotor-active TPJB modelling is thoroughly covered by establishing the link between the mechanical and hydraulic systems for all regimes. The hybrid lubrication is herein covered in depth; from a control viewpoint, an integral controller to aid such a regime is designed using model-based standard tools. Results show slight improvement on the system dynamic performance by using the hybrid lubrication instead of the passive one. Further improvements are pursued with the active lubrication in part II.

1. Introduction

The fast development of the mechatronics has allowed a variety of machine elements to be upgraded. Among bearings, active magnetic [1,2], gas (compressible) [3–5] and oil-film (uncompressible) bearings [6,7] can be recognized. Within oil-film bearings, the development has focused on active tilting-pad journal bearings (active TPJBs) because it is an intrinsically stable bearing [8,9] so it can safely run under demanding operational conditions with or without the active feature. Under a mechatronic approach, sensing and actuating capabilities are incorporated into the active bearing design. Suitable actuators for TPJBs are of a hydraulic nature [10], although magnetic or piezoelectric [11,12] can also be implemented. In 1994, Santos [13] compared two types of hydraulic actuators for turning the conventional TPJBs into “active TPJBs”, the chamber and the oil radial injection systems, with the last one as the most appropriated choice [14]. By injecting high pressurized oil through orifices, commonly only one centrally machined at the pad surface, three lubrication regimes can be developed due to the combination of the hydrostatic with the hydrodynamic principles: the conventional (pure hydrodynamic), the hybrid (hydrodynamic plus hydrostatic) and the controllable (hydrodynamic plus variable hydrostatic). In Santos and Russo [15], the isothermal modelling of active TPJBs featuring the radial oil injection system was firstly introduced. Since then, a great amount of research has been carried out mainly in two fundamental branches. The first one involves the modelling of active TPJBs, and the second focuses on the control design for such bearings. When modelling the active TPJBs, all considerations of an elastohydrodynamic (EHD) approach are maintained including also the radial oil injection, hydraulic and pipelines dynamics among the relevant effects. A detailed development of the modelling on active TPJBs can be found in [16–19]. On the other branch, the advances in the field of control design for active TPJBs have been deeply subjugated to the availability of accurate and reliable bearing models. Without such models to precisely predict the bearing properties, the integration of both areas for developing model-based controllers is limited. This limitation has circumscribed the control design to mainly classical PID controllers either via previous experimental system characterization or in-situ tuning. Theactual maturity of the active TPJBs modelling, which allows us to comprehend the physics behind it, can lead to designing model-based controllers at an early stage of the bearing design. This avoids calculating the PID parameters by heuristic means or by tuning them on site after the bearing is manufactured. Works related to control design for active TPJBs can be found in [20–26].

In the industry, the majority of the critical machines feature flexible rotors which make them worth analysing when supported by active TPJBs, in the same way as when they are supported by conventional TPJBs [27–32]. In rotating systems, the bearing dynamic properties heavily influence the whole system dynamics because they provide the main source of energy dissipation through the lubricating fluid film. Besides the damping reduction with the increased angular velocity, this property also reduces with the excitation frequency, which indeed
lowers the damping ratio of the flexible system modes. In this case, the design of model-based controllers for governing the active TPJBs becomes even more relevant and challenging for increasing the damping of such modes, provided proper actuators are available. Only a few publications, such as [24,22,33], deal with the targeted systems from a theoretical and experimental perspective simultaneously. Contrarily, rigid rotor systems are more profusely covered [13,14,34,23,20,21].

In this framework, the main contribution of this work is fundamentally theorectical by presenting the modelling of flexible rotors when supported by active TPJBs. This modelling is based on the standard beam finite element formulation for the rotor and by including the full bearing matrices derived from an ETHD approach for the active TPJBs. This further includes the pad degrees-of-freedom (dofs) – tilt, bending and radial movement – and an extra dof – the servovalve spool-driven flow – which makes the link between the mechanical and hydraulic systems. Special emphasis is given to the modelling of the different lubrication regimes by analysing the influence of the fluid-film forces on the journal equilibrium and the bearing dynamic properties, leading to appropriated models for the passive, hybrid and active lubrication regimes. These models can be used for assisting the journal position changes under the hybrid lubrication regime or for designing model-based state-feedback controllers to develop the active lubrication, as presented next in part II. In this part, the hybrid lubrication is thoroughly covered and an integral controller derived using model-based tools is used to aid the journal equilibrium position changes. This study on flexible rotor-active bearing systems offers a more complete analysis that includes the flexible rotor dynamics and also serves to supplement the shortage identified previously.

2. The test rig facilities

The rotor-bearing test rig, shown in Fig. 1(a), resembles an industrial overhung centrifugal compressor. The driven torque is delivered through a flexible coupling connected to a lasash, which is belt-driven by an AC motor. The rotor is supported by a ball and an active TPJB at the driven and free end respectively. Discs can be overhung at the shaft free end to resemble impellers and to increase the gyroscopic effect. None, one and up to two discs can be suspended. Without discs the rotor behaves as a rigid rotor at frequencies below 150 Hz, whereas any extra disc means analysing it as a flexible one. The system can be excited either through an electromagnetic shaker connected to the excitation bearing at the shaft end or through the active magnetic bearing placed between bearings. The main design and operational characteristics of the test rig are summarized in Table 1.

2.1. The active TPJB and lubrication regimes

The controllable bearing, schematized in Fig. 1(b), is a tilting-pad journal bearing with 4 bronze pads in a load-between-pads configuration. The pad is centrally pivoted with a rocker type pivot. The active or controllable feature of the bearing is rendered by an electronic radial oil injection system as proposed by Santos [15]. This injection system combines a hydrostatic with the hydrodynamic pressure distribution by injecting pressurized oil in the journal-pad clearance through, in this case, a single central nozzle aligned with the pivot line. The high pressure oil flow is injected by two high-frequency response servovalves, where each one couples to a pair of counter pads. Such pad sets aremono-installed. This servovalve-pad configuration enables us to freely exert controllable forces within the bearing plane so that any external force, such as imbalance, can be counteracted. One of the main components of the servovalve is the spool, whose position is driven by an input control signal which if set to positive, connects the supply port with one pad or if set to negative with the counter pad. With a zero input control signal, the servovalve’s spool is centred to keep both ports closed, hence blocking the flow to the pads. However, due to tight fabrication tolerances between spool lands and ports being difficult to produce, the spool lands undercut the ports and a small leakage flow is always injected [39]. Further design parameters are presented in Table 2.

Fig. 2 depicts the layout of the hydraulic units connected to the active TPJB. This includes a flooding system for the conventional lubrication and the electronic radial oil injection system for developing the hybrid and active lubrication when enabled. These regimes, which are mathematically approached in the next section, can be further described as follows:

The Passive Lubrication Regime: provides the main bearing dynamic properties and its load carrying capacity due to the hydrodynamic lubrication. There is no controller implemented. It also acts as backup in case of failure of the radial oil injection system. This regime is utilized for benchmarking. The Hybrid Lubrication Regime: in this regime, which is one of the research objectives of this work, the high pressure unit is turned on and the hydrostatic contribution to the pressure build up is enabled. Depending on the servovalve’s spool position, the high pressurized oil can be permanently injected by different pad combinations, leading to a change of the journal equilibrium position within the “x-y” plane and hence on the bearing dynamic properties. For horizontal machines, injecting from bottom pads has shown to produce a vertical bearing softening [40] and to reduce the system response amplitude [25]. In general terms, the higher the supply pressure the easier it is to change the journal equilibrium position. By using feedback signals, this regime can be aided by integral controllers to attain or maintain a predefined equilibrium, especially when servovalve dynamics are unequal [25].

The Active Lubrication Regime: if the control signals driving the servovalves are defined by a control law, then the hydrostatic fluid-film force exerted over the journal can be actively-controlled. This is achieved by using the system lateral movements or their estimates as feedback signals to synthesize classical or model-based control laws. This was carried out in [26] based on proportional-derivative controllers, which have been also used to demonstrate the bearing properties modification by control laws in [17,19]. Provided a reliable rotor-bearing system model, system states can be estimated so that model-based state-feedback controllers can be designed and implemented.

3. The rotor - active TPJB modelling

The framework for the modelling of flexible rotor supported by active TPJBs when featuring the three lubrication regimes is set. Firstly, the generalized governing equations are obtained upon the linearisation of the equation of motion subjected to the fluid-film nonlinear forces. Then, the modelling of the active TPJB through the ETHD approach is briefly addressed. Full bearing dynamic coefficients, which includes the pads and hydraulic dynamics, are reviewed for the three regimes. Lastly, the coupling of the active TPJB properties with a beam-based finite element model of the rotor is covered. Table 3 summarizes the contribution of dofs from each subsystem, which will be used throughout this section. The rotor subsystem introduces 4 dofs per node, two translational and two rotational. The pads introduce three dofs per pad: the pad tilt, bending and radial translation (due to the pivot flexibility). Finally, two extra dofs are introduced because of the hydraulic system, one per servovalve.

3.1. Linking the rotor-bearing with the hydraulic dynamics for the different lubrication regimes

Fig. 3 presents the mechanical model of the integrated rotor-active
The TPJB system with the servo hydraulic subsystem. The link between them is the fluid-foil force $f_{\text{ff}}(t)$ generated by the hydrodynamic and hydrostatic lubrications. These two effects together are responsible for: 1) the static bearing force – the load carrying capacity- and 2) two dynamics forces, namely: the bearing impedance force and the controllable force produced mainly due to variation of the hydraulic system control signal, affecting the hydrostatic injection of high pressurized oil. Uncoupling these forces is unphysical but mathematically convenient since it allows us to isolate the controllable force which can be experimentally characterized or theoretically calculated as presented in [41]. This approach has been utilized in works such as ([21,22,42]). Another way to link the systems is by incorporating the hydraulic system as a part of the bearing dynamics. This leads to solving the bearing and servovalve dynamic equations together and to extending the amount of dofs defining the bearing dynamic properties by incorporating the speed-driven flow as a new dof. The way in which this link is made along with defining the generalized equation for the lubrication regimes are presented next.

Consider the equation of motion of the rotor subjected to the non-linear fluid-foil forces $f_{\text{ff}}$, the rotor distributed weight $w$ and external dynamic forces $f_{\text{ext}}$:

$$\mathbf{M}(\mathbf{x}) + \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{K}(\mathbf{x}) = \mathbf{w} + \mathbf{f}_{\text{ff}}(\mathbf{x}^*, \mathbf{x}, \mathbf{q}_p, \mathbf{u}, t) + \mathbf{f}_{\text{ext}}(t)$$

where $\mathbf{x} = [y_1, y_2, y_3]^T$ is the extended generalized vector which includes the rotor and pad's dofs, i.e. it does not include the hydraulic dofs. The fluid-foil forces $f_{\text{ff}}$ are a function of the journal translational dofs in which the bearing is connected (node j) and the pad's dofs, i.e. $\mathbf{x}^* = [y_1, y_2, y_3]^T$, as well as its time derivative. It is also a function of the speed-driven flow $\mathbf{q}_p$ and the control signal $\mathbf{u}$. This force is simultaneously applied to the journal dofs and to the pad's dofs as forces and moments. The servovalve injection flow is a non-linear function of the spool position and the load pressure difference, i.e. $Q_p = Q_p(y_1, y_2, y_3)$. This flow links the hydraulic and bearing dynamics through a set of constitutive equations. Upon a linearisation of the servovalve injection flow, the variations of the speed-driven flow $\Delta q_p$ are determined by the second order linearised equation governing the servovalve dynamics:

$$\begin{bmatrix} 1 & 0 & \Delta q_p(1) \\ 0 & 1 & \Delta q_p(1) \\ \Delta q_p(2) & \Delta q_p(2) \\ \Delta q_p(3) & \Delta q_p(3) \end{bmatrix} = \begin{bmatrix} 2H + \alpha_1 & 0 & 0 \\ 0 & 2H + \alpha_1 & 0 \\ \alpha_1 & 0 & \alpha_1 \end{bmatrix} \begin{bmatrix} \Delta q_p(1) \\ \Delta q_p(2) \\ \Delta q_p(3) \end{bmatrix}$$

(2)

The above equation written in matrix form reads:

$$\mathbf{M} \ddot{\mathbf{x}}(t) + 2\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{K}(\mathbf{x}) = \mathbf{w} + \mathbf{f}_{\text{ff}}(\mathbf{x}^*, \mathbf{x}, \mathbf{q}_p, \mathbf{u}, t) + \mathbf{f}_{\text{ext}}(t)$$

(3)

In order to linearise the Eq. (1), it is Taylor expanded by rewriting it as a functional:

$$\Pi = \mathbf{M}(\mathbf{x}) + \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{K}(\mathbf{x}) - \mathbf{w} - \mathbf{f}_{\text{ff}}(\mathbf{x}^*, \mathbf{x}, \mathbf{q}_p, \mathbf{u}, t) - \mathbf{f}_{\text{ext}}(t)$$

and redefining the variables as variations around an equilibrium point:

$$\Pi = \mathbf{M}(\mathbf{x}^*) + \mathbf{C}(\mathbf{x}^*, \dot{\mathbf{x}}^*) + \mathbf{K}(\mathbf{x}^*) - \mathbf{w} - \mathbf{f}_{\text{ff}}(\mathbf{x}^*, \mathbf{x}^*, \mathbf{q}_p, \mathbf{u}, t) - \mathbf{f}_{\text{ext}}(t)$$

(4)
The Taylor series expansion of Eq. (4) becomes:

\[ f'(q, u) = \frac{\partial f}{\partial q} \Delta q + \frac{\partial f}{\partial u} \Delta u + \frac{\partial f}{\partial q} \Delta q + \frac{\partial f}{\partial u} \Delta u \]

The time variable \( t \) has been omitted for simplicity and the weight and external forces are defined as purely static and dynamic loadings respectively. The left-hand side of Eq. (6) is equal to zero by definition, i.e. \( \Pi = 0 \) for all variables. On the right-hand side, the terms higher than second order are disregarded, i.e. \( O(\Delta^2) \approx 0 \). Thus, at the equilibrium point at which \( x_0 = x_0 \), it can be written:

\[ \Pi_0 = Kx_0 - w_0 - f_0(x_0, 0, q_{0b}, u_0) = 0 \]

The Jacobian terms are determined as:

\[ \frac{\partial \Pi}{\partial x'} = M \quad \frac{\partial \Pi}{\partial u'} = K \]  
\[ \frac{\partial \Pi}{\partial q'} = \Omega - \frac{\partial \Pi}{\partial q} \quad \frac{\partial \Pi}{\partial u} = \frac{\partial \Pi}{\partial u_0} \]

where the variation of the fluid-film forces due to the variation of the bearing displacement, velocities and spool-driven flow as well as the parameters from the servovalve dynamics represents the bearing dynamic stiffness and damping properties of an active TPJB for a defined equilibrium \( \Pi_0 \), i.e.:

\[ K_{0} = \begin{bmatrix} \frac{\partial f_{0}}{\partial x} & \frac{\partial f_{0}}{\partial u} \\ \frac{\partial f_{0}}{\partial q} & \frac{\partial f_{0}}{\partial u} \end{bmatrix} \]

These full bearing matrices of Eq. (10) are constant, i.e. they are not frequency dependent, and are obtained through a perturbation analysis.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Dof vector</th>
<th>( \sigma ) dofs</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>rotor</td>
<td>( q ) ( u ) ( v ) ( w )</td>
<td>4 ( n_r )</td>
<td>4 ( n_r )</td>
</tr>
<tr>
<td>pads</td>
<td>( \theta ) ( \beta ) ( \eta ) ( \gamma ) ( \nu ) ( \alpha ) ( \zeta ) ( \rho ) ( \theta )</td>
<td>( n_p )</td>
<td>( n_p )</td>
</tr>
<tr>
<td>servovalves</td>
<td>( q ) ( u ) ( v ) ( w )</td>
<td>( n_v )</td>
<td>( n_v )</td>
</tr>
<tr>
<td>N</td>
<td>122</td>
<td>2( N )</td>
<td>244</td>
</tr>
</tbody>
</table>
of an ETHD modelling for controllable bearing as presented by [19]. These matrices can be directly plugged into Eq. (9), with previous sorting and zero-padding. The link between the rotor-bearing and the hydraulic dynamics is made through the columns of the stiffness matrix \( \mathbf{D}_{\text{hyd}} \), which link the dofs of the servovalve with the rotor-bearing system, and it represents the variation of the fluid-film forces due to the variation of the spool-driven flow. With slight abuse of notation to disregard \( \Delta \), Eq. (9) is rewritten as:

\[
\mathbf{M} \ddot{\mathbf{x}} + (\mathbf{D}_{\text{hyd}} - \Delta \mathbf{GI}) \mathbf{x} + (\mathbf{K} + \mathbf{K}_{\text{p}}) \mathbf{x} = \mathbf{W}_{\text{in}} + \mathbf{f}_{\text{e}}
\]

where \( \mathbf{x} = [x, q]^T \) considers all dofs of the system, see Table 3. And the control input matrix \( W \) only depends on the supply pressure \( P_{\text{in}} \) of the injection system which defines the servovalve natural frequency and input gain. By considering Eq. (7) for the steady-state equilibrium and Eq. (11) for the vibrations around it, it is obtained for:

- The passive lubrication (\( \Pi^0 \)): in this case the hydraulic unit is off and there is no control signal \( u = 0 \), nor variable spool-driven flow \( q \) thus:

\[
\begin{align*}
\mathbf{M} \ddot{\mathbf{x}}^0 + (\mathbf{D}_{\text{hyd}} - \Delta \mathbf{GI}) \mathbf{x}^0 + (\mathbf{K} + \mathbf{K}_{\text{p}}) \mathbf{x}^0 &= \mathbf{W}_{\text{in}} + \mathbf{f}_{\text{e}} \\
\mathbf{P}^0 &= \mathbf{W}_{\text{in}} + \mathbf{f}_{\text{e}}
\end{align*}
\]

- The hybrid lubrication (\( \Pi^h \)): in this case the hydraulic unit is on, therefore the equilibrium point changes due to the injection of a flow function of a constant control signal \( u_0 \), thus:

\[
\begin{align*}
\mathbf{M} \ddot{\mathbf{x}}^h + (\mathbf{D}_{\text{hyd}} - \Delta \mathbf{GI}) \mathbf{x}^h + (\mathbf{K} + \mathbf{K}_{\text{p}}) \mathbf{x}^h &= \mathbf{W}_{\text{in}} + \mathbf{f}_{\text{e}} \\
\mathbf{P}^h &= \mathbf{W}_{\text{in}} + \mathbf{f}_{\text{e}}
\end{align*}
\]

A common case is when the spool is centred and the leakage flow is injected, hence \( u_0 = 0 \) and \( q^h = q_{\text{in}} \). This is referred to as the leakage case.

- The active lubrication (\( \Pi^a \)): in this case the equilibrium position is regarded as the same as in the previous one, hence \( \Pi^a = \Pi^h \); but in addition, there is a controllable fluid-film force exerted on the system due to the spool-driven flow variations \( q \), driven by a control signal \( u \), therefore:

\[
\begin{align*}
\mathbf{M} \ddot{\mathbf{x}}^a + (\mathbf{D}_{\text{hyd}} - \Delta \mathbf{GI}) \mathbf{x}^a + (\mathbf{K} + \mathbf{K}_{\text{p}}) \mathbf{x}^a &= \mathbf{W}_{\text{in}} + \mathbf{f}_{\text{e}} \\
\mathbf{P}^a &= \mathbf{W}_{\text{in}} + \mathbf{f}_{\text{e}}
\end{align*}
\]

Equations (12), (13) and (14) set the mathematical modelling framework for the analysis of the different lubrication regimes developed with the active TPJB. This implies that the bearing dynamic properties \( \mathbf{D}_{\text{hyd}} \) and \( \mathbf{K} \) are determined distinctly for the passive and every injection combination of the hybrid lubrication regime, which indeed determine a new journal equilibrium. In the case of the active lubrication regime, the same bearing coefficients calculated for the hybrid case can be utilized. Naturally, the passive lubrication regime defined by Eq. (12), is used as a benchmark when specifying the advantage or drawbacks of the other regimes. Moreover, the active lubrication regime is the only one which includes the variation of the spool-driven flow \( q \), while for passive and hybrid ones it only influences the equilibrium position through the constant flow \( q_{\text{in}} \). As aforementioned, this work focuses on the hybrid lubrication, Eq. (13), aiming at describing how the change of the journal equilibrium position \( \Pi^h \) and hence the bearing dynamic properties, affect the entire rotor-bearing system response. The active lubrication with model-based controllers, based on Eq. (14), is treated in part II.

3.2. Modelling the active TPJB

Lund [43], Lund and Thomsen [44], Springer [45] and Allaire et al. [46], among others, have provided a way of calculating the frequency dependence synchronously reduced as well as the full bearing force coefficients; the latter by including the pad dofs in the perturbation analysis thus eliminating the frequency dependence. Alongside the computational development these first isothermal models incorporated thermal and compliance effects in a more accurate model for conventional TPJB, the elasto thermo-hydrodynamic (ETHD) model [43,46–50]. Regarding the active TPJBs, the initial isothermal modelling proposed by Santos and Russo [15] has been extended by the subsequent contributions of Santos and Nicoletti [16] (THD) and Santos and Haugaard [17] (EHD) to finally bridge the gap between both approaches in the ETHD formulation with the works of Varela et al. [18,19]. The main ETHD equations are listed below.

- The Modified Reynolds Equation for modelling the fluid-film pressure distribution in active TPJB by incorporating the high pressure oil flow injection as an equivalent perturbation of the velocity field [15].
- The Energy Equation for Active Lubrication: for modelling the local variation of viscosity of the fluid-film as a function of the oil film temperature, also affected by the oil jet injection [16].
- The Pad Equation of Motion: A pseudo modal reduction of the pad finite element formulation to incorporate the pad and pivot compliance in the modelling [17].
- The Servovalve Dynamics: A second order differential equation which relates to high pressure inflow as a function of the servovalve control signal \( q_{\text{in}} \) (already presented in Eq. (3)).

The joint numerical solution of the listed equations, by the finite element method, leads to the calculation of the fluid-film force \( f_{\text{fi}}(t) \) in equilibrium due to fluid-film pressure build up. Subsequently, the perturbation of such a pressure distribution yields the full set of the bearing dynamic force coefficients.

For the passive and hybrid regimes, provided the operational conditions of load and velocity, the bearing force coefficients are calculated either by defining the injection flow \( q_{\text{in}} \) or the injection pressure \( P_{\text{in}} \) on each nozzle, whichever variable is better known. For the passive one the injection pressure must be nil, i.e. \( P_{\text{in}} = 0 \), which entails a negative flow \( q_{\text{in}} < 0 \). In the case of hybrid lubrication, it is simpler to set an injection flow defined by a constant control signal \( q_{\text{in}}(u_0) \). When this control signal is nil, the flow reduces to the leakage flow, i.e. \( q_{\text{in}} = q_{\text{in}} \). In both regimes (\( \Pi^a \)), the bearing force coefficients are \( (2 + j \omega) \times (2 + j \omega) \) matrices, which written as a bearing impedance reads:
The above matrix is defined only in terms of the journal $v_i \omega^2_i$ and pad’s dofs $v_i$. Variation of the spool-driven flow $q_i$ is not included for these regimes. $H_j$ stands for the 2×2 sub-matrix associated with the active TPJB node (dark gray), $H_p$ and $H_n$ are $\times n$ and $n \times 2$ sub-matrices, respectively, which couple the journal dofs with the pad ones (light gray) and $H_p$ is a diagonal sub-matrix associated only with the pad’s dofs. The length of the pad’s dofs vector depends on the number of pads $n_p$ and pad’s modes $\omega^p_i$ being considered in the pseudo modal reduction, i.e. $n_p \omega^p_i$. For a four pad bearing and considering the pad modes under study, 12 pad’s dofs are obtained which lead to a matrix of 14×14. In the case of active lubrication regime (4), the spool-driven flow of the servovalves is introduced and the matrix of Equation (15b) is extended to a $(2 + 2 \cdot n_p + n_v) \times (2 + 2 \cdot n_p + n_v)$ matrix:

$$\mathbf{H}^{(b)}_{115} = \left[ \begin{array}{ccc} H_{ij} & H_{is} & H_{iv} \\ H_{bj} & H_{bs} & H_{bv} \\ 0 & 0 & H_{cv} \end{array} \right] = \cdots$$

where the term $H_{ij}$ (soft gray) accounts for the servovlae’s dynamics and the terms $H_{ij}$ and $H_{ij}$ (dark gray) make the link between the hydraulic and mechanical systems. These full set of bearing force coefficients are to be used in the rotor-bearing model without any further reduction to avoid system under-modelling, and hence stability margin over estimation [31,51].

**3.3. Coupled model of the flexible rotor with active TPJB full matrices**

The model of the rotor is based on a Euler-Bernoulli beam finite element approach for the shaft which accounts for the translational and angular movements of the nodes [52], i.e. 4 dofs per node. Discs, bushes and coupling hubs are incorporated as rigid discs adding mass and inertia at their respective nodes. If no structural damping is considered nor the bearing impedance, then the equation of motion for the rotor around an equilibrium point is written as follows:

$$M \ddot{x} + \left( \begin{array}{c} \mathbf{G}^T \mathbf{x} + \mathbf{K} \mathbf{x} = \mathbf{f} \end{array} \right)$$

where $x$ stands for a $4n_r \times 1$ vector containing only the dofs of the rotor discretization as defined in Table 3. Fig. 4 depicts the discretization utilized highlighting the main nodes. Short elements do not pose numerical problems. The coupling of the linear bearing force coefficient matrices of the active TPJB entails a system dofs expansion. In the case of the passive or hybrid regimes, the generalized vector is augmented to account for the pad dofs as $x' = [x_s \ x_f]$ and for active lubrication it is augmented to also account for the spool-driven flow as $x = [x_s \ x_f \ q_s]$. By zero-padding the system matrices $M'$, $G'$, $K'$ and vector $f'$, Eq. (18) can be augmented as:

$$\mathbf{M} x' = \mathbf{G} x' + \mathbf{K} x' = \mathbf{f}$$

where $x' = x$ in the case of passive or hybrid regimes or $x' = x$ in the active case. The bearing matrices $K'_{ij}$ and $D'_{ij}$ are sorted out in terms of the new augmented state vector $x'$ or $x'$ accordingly and zero-padded as follows:

$$H^{(b)}_{ij} = K^{(b)}_{ij} + j \omega D^{(b)}_{ij}$$

where in Eq. (19) the inner matrix corresponds to the passive and hybrid cases (Eq. (15b)), whereas the outer one does for the active (Eq. (16)). Same procedure is done with the mass and inertia bearing matrix also yielded by the ETHD approach. Adding bearing matrices to the left hand side of Eq. (18), the equation of motion of the rotor-bearing system is stated as:

$$\mathbf{M} \ddot{x} + \left( \begin{array}{c} \mathbf{G}^T \mathbf{x} + \mathbf{K} \mathbf{x} + \mathbf{f} \end{array} \right)$$

**Fig. 4.** 26 shaft elements discretization with 27 nodes and 108 dofs. Main nodes are highlighted.
The above equation is generalized (*) and it is detailed for each lubrication regime in Eqs. (12), (13) and (14).

4. The hybrid lubrication regime

The hybrid lubrication changes the system dynamic properties by injecting high pressurized oil into the bearing gap, which entails a change in the journal equilibrium position. This constant oil can be injected through different injection arrangements as discussed in [25]. Two of these are depicted in Fig. 5 for a four pad bearing. These changes in equilibrium might be aided by integral controllers. Next, the influence of the regime on the system dynamic properties with and without the integral controller is revised and a model-based integral controller is presented, drawn from the work presented in paper II.

The equation of motion under hybrid lubrication (Eq. (13)) is rewritten in state-space formulation. Matrices \( \{A, B, \bar{B}, C\} \) define the state-space formulation for the vector \( \dot{X} = X' \). Furthermore, the system is pseudo-modal reduced and complex states separated to obtain an equivalent state-space formulation \( \{A, \bar{B}, \bar{B}, C\} \) in terms of the modal vector \( q \) of the slowest system modes. These system dynamic properties are influenced by the changes in the journal equilibrium position \( \Pi_0 \) since each equilibrium position defines a unique pair of stiffness and damping properties, i.e. \( K_{\Pi_0} \) and \( D_{\Pi_0} \), for the ALB. Depending on how this equilibrium changing process is carried out regarding whether controllers are utilized or not, eigenvalues are affected differently, thus one can identify:

**The Hybrid Lubrication Regime in Open-Loop.** Under this configuration no control law is implemented. This implies that the bearing \( (K_{\Pi_0} \text{ and } D_{\Pi_0}) \) and thus the rotor-bearing system properties \( \det(A - \lambda) = 0 \) are modified only due to the change in \( \Pi_0 \).

**The Hybrid Lubrication Regime in Closed-Loop.** Under this configuration an integral control law is implemented to aid the journal position changes. In this case, in addition to the modifications already described in the open-loop case \( (K_{\Pi_0} \text{ and } D_{\Pi_0}) \), changes in the system are due to the closed-loop eigenvalues defined by:

\[
\det\left\{A - \lambda \begin{bmatrix} A & BK \\ -C & 0 \end{bmatrix}\right\} = 0
\]

(21)

4.1. A model-based integral control design

Integral controllers do not require any system model to synthesize their gains, relying only on the direct comparison of the system output against a reference for obtaining them. However, some standard approaches of model-based control can be utilized. The integral action, which ensures zero steady-state error, is included by system augmenta-

Fig. 5. Upward and downward injection arrangements for a 4-pad bearing.

Fig. 6. Theoretical modeshapes with their respective damped natural frequency and damping ratio. Hybrid lubrication regime at 1000 rpm and 90 bar of supply pressure.
tion, i.e. by incorporating the time derivative of the integral state vector \( \dot{q}_i \) defined as the error between the desired position and the current one \( q_r = r - \dot{r} \), or \( q_r = r - Cq \). If the control law is defined as \( u = K_iq \), then the closed-loop augmented system can be written as:

\[
\begin{bmatrix}
q \\
\dot{q}_i \\
\dot{r}
\end{bmatrix} =
\begin{bmatrix}
-A & BK & 0 \\
-C & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
q \\
\dot{q}_i \\
\dot{r}
\end{bmatrix} +
\begin{bmatrix}
B \dot{r} \\
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} v
\]

or in a reduced form \( [\dot{\tilde{q}}, \dot{\tilde{r}}, \dot{\tilde{C}}] \) for a state vector \( \tilde{Z} = [\dot{q}, \dot{q}_i, \dot{r}] \).

A linear-quadratic-regulator (LQR), which is an optimal control in terms of energy balance, is chosen since it provides an intuitive way to synthesize the integral gains by weighting only the integral states and the control signals in the steady-state cost function:

\[
J = \int_0^\infty \left( Z^T Q Z + \dot{v}^T R \dot{v} \right) dt
\]

where \( Q \) is the \( n \times n \) state weighting diagonal matrix, which in this case has non-zero weighting constants only at the positions associated with the integral states \( q_i \). \( R \) is a 2x2 matrix to weight the servovalve control signals. Both matrices are obtained via simulations.

5. Model validation

Theoretical results are reported jointly with their experimental counterpart to validate them. This is done for the system without discs configuration and featuring the passive and hybrid lubrication regimes. For both cases an angular velocity of 1000 rpm and a pressure of 90 bar in the hybrid lubrication cases are considered. Results are presented as plots of modeshapes with their damped natural frequency and damping ratio for the non-stationary response and as frequency response functions (FRFs) for the stationary one. In the calculation of the bearing force coefficient matrices \( K_{\Pi h} \) and \( D_{\Pi h} \), presented in Appendix A for the leakage case of the hybrid lubrication regime, a pivot stiffness of \( 4 \times 10^8 \) (N/m) is considered equally in all pads. The bearing inertia matrix \( M \) is also provided. Excitation is introduced to the system through the excitation bearing at node 26, dof 101 and 102, and the system response is obtained at the pedestal of sensors 1 and 2, which corresponds to the node 23, dof 89 and 90. Fig. 6 shows the first modes of the system without any disc featuring hybrid lubrication at 1000 rpm and 90 bar of pressure supply. These modes correspond to the conical and first bending modes, both forward and backward around 10 Hz and 210 Hz respectively. The conical mode in this particular case describes almost a straight line due to the weak gyroscopic effect at such speed. Furthermore, this mode has a damping ratio more than ten times lower than the first bending mode.
times larger than the first bending one, which explains its low influence in the FRFs.

Figs. 7 and 8 report the FRFs under passive lubrication. Fair agreement is obtained with differences in the damped natural frequency and in the damping ratio mainly in the vertical direction. The mismatch in phase is increased at higher frequency due to what is seen as a linear increase of the experimental phase with excitation frequency. Similarly, Figs. 9 and 10 show the FRFs in both directions for the system featuring the hybrid lubrication with 90 bar of supply pressure. In this case, the fitting of FRFs is better in relation to the amplitudes, with better results in the horizontal case. The same trend in the experimental phase is observed. It is considered that the model satisfies the requirement for developing controllers, bearing in mind the identified mismatching.

6. Experimental setup

Experimental campaigns are carried out under two system configurations: by hanging none and one disc (80 mm disc), i.e., 400 N and 880 N of static bearing loading. Three angular speeds are tested: 2000 rpm and 4000 rpm for the system without disc and 1000 rpm with the system with one disc, which leads to an increase of the gyroscopic effect. To develop the hybrid lubrication regime 60 bar and 90 bar are used with no disc configuration and 100 bar with one disc. The experiments start when force and thermal equilibria are established. The system lateral response is obtained by sweeping a bidirectional chirp signal from 20 to 250 Hz in a 10 s linear ramp over 5 min. Other types of excitation signals can be used to reduce the application time required, such as Pseudo Random Binary Sequences (PRBS) or Schroeder Phased Harmonic Signals (SPHS) [53], and maybe tried in the future instead of the swept-sine signals used. The lateral movement is obtained at the pedestal of sensor 1 when exciting at the excitation bearing, see Fig. 4. FRFs are obtained with a Hanning window, 1 Hz of frequency resolution and 50% of overlap.

7. Results under open-loop hybrid lubrication

The rotor without discs is firstly studied. No injection schemes are attempted in open-loop. Instead, the hybrid lubrication regimes obtained by injecting only the leakage flow are tested at two supply pressures. Fig. 11 compares at 2000 rpm the system response under 880 N of static bearing loading. Three angular speeds are tested: 2000 rpm and 4000 rpm for the system without disc and 1000 rpm with the system with one disc, which leads to an increase of the gyroscopic effect. To develop the hybrid lubrication regime 60 bar and 90 bar are used with no disc configuration and 100 bar with one disc. The experiments start when force and thermal equilibria are established. The system lateral response is obtained by sweeping a bidirectional chirp signal from 20 to 250 Hz in a 10 s linear ramp over 5 min. Other types of excitation signals can be used to reduce the application time required, such as Pseudo Random Binary Sequences (PRBS) or Schroeder Phased Harmonic Signals (SPHS) [53], and maybe tried in the future instead of the swept-sine signals used. The lateral movement is obtained at the pedestal of sensor 1 when exciting at the excitation bearing, see Fig. 4. FRFs are obtained with a Hanning window, 1 Hz of frequency resolution and 50% of overlap.

The rotor without discs is firstly studied. No injection schemes are attempted in open-loop. Instead, the hybrid lubrication regimes obtained by injecting only the leakage flow are tested at two supply pressures. Fig. 11 compares at 2000 rpm the system response under
hybrid lubrication regime with 60 and 90 bar against the conventional passive lubrication regime. The effectiveness of injecting constant oil aiming at improving the damping is evident at the resonance around 210 Hz. Moreover, the higher the supply pressure, the more significant the vibration reduction obtained, in this case around 30%. Results under the same conditions, but at 4000 rpm, are depicted in Fig. 14. The vibration reduction obtained, in this case around 30%. Results under closed-loop hybrid lubrication are depicted in Fig. 13 with the expected reduction achieved by the hybrid lubrication. It is also noted that below 200 Hz new dynamics are taking place around 100 Hz for the passive case which is suppressed out with the hybrid lubrication regime.

8. Results under closed-loop hybrid lubrication

The rotor without discs is then studied in closed-loop configuration. The supply pressure is raised up to 180 bar for which the oil temperature reached around 58 °C and the angular velocity is reduced to 1000 rpm. An integral controller synthesized upon the rotor-bearing model is utilized to feature upward and downward injection schemes, as pursued in part II. In this work the modelling of a flexible rotor mounted on active TPJB is mathematically reviewed. Three possible cases of lubrication regimes are investigated: passive, hybrid and active. Special attention is paid to describing the link between the hydraulic and mechanical subsystems for each of the regimes. In the light of the theoretical treatment and experimental results exposed, it can be concluded:

- From a theoretical viewpoint, the validation of the whole rotor-bearing system dynamic response is considered satisfactory under the passive and hybrid lubrication regimes.
- Furthermore, it is established that the bearing force coefficients calculated for the hybrid regime and linearized around the same journal equilibrium point, can be utilized for the active lubrication by further including the servovalve dynamics.
- From an experimental point of view, it is shown under different system configurations and operational conditions that improvement of the system’s lateral dynamic response can be achieved by featuring the hybrid or adjustable lubrication in comparison to the passive regime. Even further reduction can be attained by establishing an upward injection scheme, either with or without integral control. Reductions of about 30% were achieved around the resonant zone with the hybrid lubrication.
- Further vibration reductions might only be possible by adding feedback control to the lubrication regimes, as pursued in part II.

9. Conclusions

In this work the modelling of a flexible rotor mounted on active TPJB is mathematically reviewed. Three possible cases of lubrication regimes are investigated: passive, hybrid and active. Special attention is paid to describing the link between the hydraulic and mechanical subsystems for each of the regimes. In the light of the theoretical treatment and experimental results exposed, it can be concluded:

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- Furthermore, it is established that the bearing force coefficients calculated for the hybrid regime and linearized around the same journal equilibrium point, can be utilized for the active lubrication by further including the servovalve dynamics.
- From an experimental point of view, it is shown under different system configurations and operational conditions that improvement of the system’s lateral dynamic response can be achieved by featuring the hybrid or adjustable lubrication in comparison to the passive regime. Even further reduction can be attained by establishing an upward injection scheme, either with or without integral control. Reductions of about 30% were achieved around the resonant zone with the hybrid lubrication.
- Further vibration reductions might only be possible by adding feedback control to the lubrication regimes, as pursued in part II.

Appendix A. Active TPJB matrices. 1000 rpm and 400 N of static load. 90 bar of supply pressure

\[
\begin{bmatrix}
4.7e7 & 5.1e7 & 9e5 & -1.1e6 & 1.3e6 & 1.7e6 & 2e5 & -4.9e5 & 4.5e5 & -9.2e5 & -9.5e6 & 8.7e6 & 1.8e7 & -1.7e7 \\
-5.1e7 & 4.7e7 & 1.1e6 & 8.8e5 & 1.7e6 & -1.3e6 & 5.1e5 & 2e5 & 9.2e5 & 4.5e5 & -8.6e6 & -9.6e6 & 1.7e7 & 1.9e7 \\
-1.3e5 & 5.9e4 & 9.7e3 & 0 & 0 & 1.5e4 & 0 & 0 & 0 & 4.8e4 & 0 & 0 & 0 \\
-5.8e4 & -1.3e5 & 0 & 9.3e3 & 0 & 0 & 1.5e4 & 0 & 0 & 0 & 4.7e4 & 0 & 0 \\
-2.3e5 & 1.4e5 & 0 & 1.9e4 & 0 & 0 & 2.4e4 & 0 & 0 & 0 & 5.8e4 & 0 & 0 \\
-1.2e5 & 2.3e5 & 0 & 1.5e4 & 0 & 0 & 2.4e4 & 0 & 0 & 0 & 5.8e4 & 0 & 0 \\
6.7e4 & 3.1e5 & 9.4e3 & 0 & 0 & 4.6e5 & 0 & 0 & 0 & -1.6e5 & 0 & 0 & 0 \\
3.1e5 & 6.6e4 & 1.9e4 & 0 & 0 & 4.6e5 & 0 & 0 & 0 & -1.6e5 & 0 & 0 & 0 \\
-4.7e4 & 5.3e4 & 0 & 2.9e4 & 0 & 0 & 4.7e5 & 0 & 0 & 0 & 3.1e5 & 0 & 0 \\
-5.3e5 & -4.8e4 & 0 & 0 & -2.9e4 & 0 & 0 & 4.7e5 & 0 & 0 & 0 & 3.1e5 & 0 & 0 \\
3.7e6 & -2.2e7 & -1.3e6 & 0 & 0 & -4.6e5 & 0 & 0 & 0 & 4.1e8 & 0 & 0 & 0 \\
2.2e7 & 3.3e6 & 0 & -1.3e6 & 0 & 0 & -4.5e5 & 0 & 0 & 0 & 4.1e8 & 0 & -1.6 \\
1.2e6 & 3.7e6 & 0 & 1.9e6 & 0 & 0 & 8.9e5 & 0 & 0 & 0 & 4.2e8 & 0 & 0 \\
3.7e7 & -1.3e6 & 0 & 0 & -1.9e6 & 0 & 0 & 8.9e5 & 0 & 0 & 0 & 4.2e8 & 0 & 0 \\
\end{bmatrix}
\]

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A.6 [P6]: Active Tilting-Pad Journal Bearings Supporting Flexible Rotors: Part II – The Model-Based Feedback-Controlled Lubrication
Active Tilting-Pad Journal Bearings Supporting Flexible Rotors: Part II – The Model-Based Feedback-Controlled Lubrication

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Abstract

This is part II of a twofold paper series dealing with the design and implementation of model-based controllers meant for assisting the hybrid and developing the feedback-controlled lubrication regimes in active tilting pad journal bearings (active TPJBs). In both papers theoretical and experimental analyses are presented with focus on the reduction of rotor lateral vibration. This part is devoted to synthesising model-based LQG optimal controllers (LQR regulator + Kalman Filter) for the feedback-controlled lubrication and is based upon the mathematical model of the rotor-bearing system derived in part I. Results show further suppression of resonant vibrations when using the feedback-controlled or active lubrication, overweighting the reduction already achieved with hybrid lubrication, thus improving the whole machine dynamic performance.

Keywords: Tilting-Pad Journal Bearing, Actively-Lubricated Bearing, Active Vibration Suppression, Rotordynamics

1. Introduction

The control of vibration in rotating machinery has been achieved both passively and actively. Passive elements, such as squeeze-film dampers \[1\] and seal dampers \[2\] introduce dissipative forces to the system counteracting

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destabilizing forces and consequently reducing vibrations. However, without simultaneous sensing and actuating capabilities their potential to adapt and perform adequately in a wide frequency range becomes limited. The active elements overcome this limitation by incorporating actuators and sensors, which in a closed-loop configuration provide adaptability to wider frequency ranges and higher efficiency toward vibration reduction. Most of the active elements for rotating machines are built upon bearings. The control of vibrations is attained by modifying the bearing properties accordingly to the excitation loads and operational conditions. A variety of bearing types (ball bearings, magnetic bearings, compressible and incompressible fluid film bearings) combined with several actuator types (magnetic, piezoelectric, pneumatic, hydraulic) leads to a variety of mechatronic devices or simply active bearings [3, 4, 5, 6, 7, 8, 9]. Herein, an incompressible fluid-film bearing with hydraulic actuator, namely tilting-pad journal bearings with active lubrication [10, 11, 12], is theoretically as well as experimentally investigated. The dynamic characteristics of rotors supported by TPJBs can be controlled either by modifying the journal-pad clearance through pad pushers – piezoelectric [13] or hydraulic [11] – or by direct modification of the fluid-film pressure distribution [14] via the active lubrication. In [15] the authors simulated an active TPJB with piezoelectric pushers controlled by an LQR regulator, while in [10] the author investigated the lateral dynamics of a rigid rotor controlled by pads on hydraulic pushers and introduced the active lubrication principle applied to TPJBs.

Active TPJBs aiming at improving damping properties and the stability margin of rotating machines have been theoretically investigated in [16, 17]. Bearing damping properties in active TPJBs are improved by featuring the feedback-controlled lubrication. Under this lubrication regime, the system response, namely the rotor lateral vibration, is utilized to generate suitable signals to command the servovalves that control the high pressurized oil flow. Synthesising PID controllers seems to be an adequate control design approach once the rotor-bearing system is manufactured. Some model-free approaches [18, 19] or even “on-site” gain tuning can be used to obtain good controllers. The works [20, 21, 22, 23] are focused on PID controllers for active TPJBs. Nonetheless, if the synthesis of the controller is to be considered as a part of the machine design process, i.e. before the whole rotating machine is manufactured, then a model-based approach becomes an attractive control design tool, allowing for an optimization of the whole electro-mechanical system dynamics. The accuracy in describing the dy-
namic behavior of passive TPJBs [24, 25, 26, 27, 28, 29] and active TPJBs connected to hydraulic servosystems [14, 30, 31, 32, 33] makes feasible today’s model-based control design approaches. Regarding the active control of flexible rotors supported by fluid-film bearings, a theoretical study on full and reduced modal state controllers was presented in [34]. In [6] the vibration suppression was theoretically and experimentally studied though using magnetic actuators. Different state-feedback controllers were studied and the system model reduction was based upon retaining dominant modes; no spillover problems were observed. [35] also emphasizes the modal reduction of large rotordynamics systems whilst an LQR controller was designed. Linked with active TPJBs, in [36] a compressor supported by this kind of bearing was theoretically studied. Output feedback control and pole placement methods were used. In all these contributions the need for reducing the size of the rotordynamics model by modal approaches is addressed, which might lead to spillover problems.

The main contribution of this work is to present the design and implementation of a model-based controller for a flexible rotor supported by an actively-lubricated TPJB. The objective of the controllers is to reduce the amplitude of the frequency response at resonance in closed-loop, while the TPJB is actively lubricated. From the theoretical standpoint, the rotor-bearing system modelling based on part I is hereby used. The model is modal-reduced and its states are further complex separated to design the LQG regulator. From an experimental standpoint, comparisons of the system response in closed-loop with LQG against the PID controller are presented. Additional tests on slightly modified system are carried out to check the actuator bandwidth. Finally, a discussion on additional pinpointed dynamics is presented.

2. Test Rig Facilities Recap

The flexible rotor-bearing test rig was introduced in part I of this series of papers, more details can be found therein. It resembles a large overhung centrifugal compressor for which the rotor is supported by an actively-lubricated TPJB as the one shown in Figure 1. The feedback-controlled regime, as defined in part I, is obtained by dynamically controlling the injection of pressurized oil directly into the bearing gap via servovalves. Three rotor configurations can be obtained by hanging different numbers of discs at the free-end, i.e. none, one or two discs. Consequently, three different levels of static bearing loading can be obtained, i.e. 400 N, 880 N or 1440 N,
respectively. This also strengthens the gyroscopic effect due to the addition of the disc inertia and reduces the natural frequencies of the system. For instance, the first bending mode is reduced from 210 Hz to 150 Hz and 95 Hz by augmenting the disc number, respectively. The frequency bandwidth of the servovalves and consequently of the active forces strongly depends on the supply pressure $P_s$. For the current application high response servovalves are used, with a cutoff frequency of 350 Hz for a nominal pressure of 210 bar and 260 Hz at 100 bar, the maximum design pressure for the current system.

3. Linear Model of the Rotor-Bearing System

The modelling of the flexible rotor-bearing system under different lubrication regimes was also presented in part I. The theoretical model is based on a finite element approach, in which each shaft element is represented by two nodes with 4 dofs each and where the active TPJB is included by using the full matrices of the dynamic linear coefficients obtained by an Elasto-Thermo-Hydrodynamic (ETHD) model of the bearing. In the case of the active lubrication regime, the equilibrium position $\Pi_{a0}$ can be regarded as the same as in the hybrid lubrication regime, which is defined by the force and thermal equilibria as well as by the supply pressure of the injection system. Therefore, the linear equation of motion at a defined rotational speed $\Omega$ and a given supply pressure $P_s$ reads:

$$M \ddot{x} + (D|\Pi_s - \Omega G) \dot{x} + (K + K|\Pi_s) x = Wu + f_{ext}$$ (1)
where on the left hand side $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{G}$ stand for the rotor mass, stiffness and gyroscopic matrices and $\mathbf{K}|_{\Pi_0^0}$ and $\mathbf{D}|_{\Pi_0^0}$ stand for the full dynamic coefficient matrices of the actively-lubricated bearing at the equilibrium condition $\Pi_0^0$, which include the servovalves dynamics. The generalized coordinated vector is composed of the rotor and active tilting pad dofs: $\mathbf{x} = \{v_1 w_1 \gamma_{v_1} \gamma_{w_1} \ldots v_n w_n \gamma_{v_n} \gamma_{w_n}, \theta_1 \ldots \theta_{np} \ldots \beta_1 \ldots \beta_{np}, \eta_1 \ldots \eta_{np}, q_{v_1} q_{v_2}\}^T$. On the right hand side, $\mathbf{f}_{\text{ext}}$ stands for the external forces applied to the rotor and $\mathbf{u}$ for the $2 \times 1$ control signal vector containing the driven voltage of each servovalve, i.e. $\mathbf{u} = \{u_1 u_2\}^T$. $\mathbf{W}$ is a sparse input control matrix whose non-zero elements are defined by the servovalve parameters dependent on $P_s$ as:

$$W(i_{q_{v_1}}, 1) = \omega_{v_1}^2 R_{v_1} \quad W(i_{q_{v_2}}, 2) = \omega_{v_2}^2 R_{v_2}$$

Equation (1) can be rewritten as:

$$\mathbf{M}\ddot{\mathbf{x}} + \bar{\mathbf{D}}\dot{\mathbf{x}} + \bar{\mathbf{K}}\mathbf{x} = \mathbf{W}\mathbf{u} + \mathbf{f}_{\text{ext}}$$

(3)

4. Modal-Reduced State-Space Model

In order to design a model-based controller, the system governing equation (3) must be rewritten in a state-space formulation. To reduce computational burden when implemented, a pseudo-modal reduction scheme is chosen which considers only the slowest eigenvalues. As a consequence, a separation of complex states is needed for implementing the controller in real time processors, which does not work with complex numbers.

By choosing the state displacement and velocity vector $\mathbf{X} = \{\mathbf{x} \dot{\mathbf{x}}\}^T$, the state-space representation of the LTI system of Equation (3) is written as:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{u} + \mathbf{B}_v\mathbf{v}_1$$

(4a)

$$\mathbf{Y} = \mathbf{C}\mathbf{X} + \mathbf{v}_2$$

(4b)

where for the state equation (4a) the state matrix $\mathbf{A}$, the input matrix $\mathbf{B}$ and the disturbance input matrix $\mathbf{B}_v$ are defined as:

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}\bar{\mathbf{K}} & -\mathbf{M}^{-1}\bar{\mathbf{D}} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{W} \end{bmatrix} \quad \mathbf{B}_v = \begin{bmatrix} 0 \\ \mathbf{M}^{-1} \end{bmatrix}$$

(5)

The external forces acting upon the system are considered as process or state disturbances $\mathbf{v}_1 = f_{\text{ext}}$. In the measurement equation (4b) the output matrix
is defined as $C = \begin{bmatrix} \Pi & 0 \end{bmatrix}$ where the matrix $\Pi$ is a binary matrix with ones in the measured states. $v_2$ stands for the measurement noise.

Stability is assessed by the real part of the eigenvalues $\lambda_i$ which must be less than zero, i.e. $\Re\{\lambda_i\} < 0$. Eigenvalues $\lambda_i$ and the correspondent right $\Phi_i$ and left $\Psi_i$ eigenvectors are obtained by solving the standard eigenvalue problems [37]:

$$A\Phi_i = \lambda_i\Phi_i \quad A^T\Psi_i = \lambda_i\Psi_i$$

(6)
eigenvectors are normalized according to $\Psi_i^T\Phi_i = 1$. Controllability and observability are evaluated by the “degrees” of controllability and observability of the $i^{th}$ mode from the $j^{th}$ input or $k^{th}$ output, respectively, forming the cos $A$ and cos $B$ matrices whose elements are defined as [38]:

$$\cos(\alpha_{ij}) = \frac{|\Psi_i^Tb_j|}{\|\Psi_i\||b_j|} \quad \cos(\beta_{ki}) = \frac{|c_k^i\Phi_i|}{\|c_k\||\Phi_i|}$$

(7)

This approach has been previously used in [39, 40] and is chosen over the standard invariant binary test based on the rank assessment [37] because it allows us to judge separately the controllability and observability of each mode. By taking the norm of each column of the cos $A$ matrix and each row of the cos $B$, gross measurement of controllability and observability are obtained by considering the contribution of all modes in the respective input $u_i$ and outputs $Y_k$.

The system as presented in Equation (4a) is computationally heavy for implementing model-based controllers in real time since it requires a large number of states to be controlled. This is mainly due to the large number of shaft elements necessary to properly describe the rotor dynamics. In addition to that, a significant amount of dofs are added to describe the pad and hydraulic dynamics of the active TPJB. Therefore a system reduction is required. A number of reduction schemes can be used [41], however, the pseudo modal reduction is the most appropriate since only the lowest modes lying in the frequency range of interest are preserved while higher modes are discarded. Nonetheless, the model reduction is obtained at the expense of increasing the odds of producing observer or controller spillover problems [34]. Under this scheme the physical general coordinates are approximated by $n$ modal coordinates such that $\mathbf{X} = \mathbf{P}_r\mathbf{q}$ and $\dot{\mathbf{X}} = \mathbf{P}_r\dot{\mathbf{q}}$, for which the pseudo-modal right and left matrices $\mathbf{P}_r$ and $\mathbf{P}_l$ are constructed column-wise with the selected modes as:

$$\mathbf{P}_r = [\Phi_1 \Phi_2 \Phi_3 \cdots \Phi_n] ; \quad \mathbf{P}_l = [\Psi_1 \Psi_2 \Psi_3 \cdots \Psi_n]$$

(8)
The system of equations (4) is rewritten as:

\[
\dot{q} = A^r q + B^r u + B^r_v v_1 \\
Y = C^r q + v_2
\]  
(9a)  
(9b)

with:

\[
A^r = P_1^T A P_r \\
B^r = P_1^T B \\
B^r_v = P_1^T B_v \\
C^r = C P_r
\]  
(10)

When making the similarity transformation with the pseudo-modal matrices \( P_r \) and \( P_l \), which contain complex valued modes, the resulting system matrices also become complex valued; therefore, a separation of complex states is needed in order to rewrite a real valued system that can be implemented in a computer, capable of processing only real numbers. The system is partitioned into real and imaginary parts as proposed in [39, 42] obtaining a double size 2n system defined by the matrices:

\[
\bar{A}^r = \begin{bmatrix} & & \\
& \Re\{\lambda_i\} & -\Im\{\lambda_i\} \\
& \Im\{\lambda_i\} & \Re\{\lambda_i\} \\
& & \ddots
\end{bmatrix}
\]

\[
\bar{B}^r = \begin{bmatrix} \Re\{\mathcal{B}^r_i\} & \Im\{\mathcal{B}^r_i\} & \cdots \end{bmatrix}^T \\
\bar{B}_v^r = \begin{bmatrix} \Re\{\mathcal{B}_v^r_i\} & \Im\{\mathcal{B}_v^r_i\} & \cdots \end{bmatrix}^T \\
\bar{C}^r = \begin{bmatrix} \Re\{\mathcal{C}^r_i\} & -\Im\{\mathcal{C}^r_i\} & \cdots \end{bmatrix}
\]  
(11)

Provided that the eigenvalues are complex conjugated – no real eigenvalues –, the system can be downsized back to \( n \) states by keeping in the matrices \( \{\bar{A}^r, \bar{B}^r, \bar{B}_v^r, \bar{C}^r\} \) only the rows and columns associated with one of the conjugated eigenvalues and by multiplying by two the input matrices \( \{\bar{B}^r, \bar{B}_v^r\} \). Therefore:

\[
\dot{q} = \bar{A}^r q + \bar{B}^r u + \bar{B}_v^r v_1 \\
Y = \bar{C}^r q + v_2
\]  
(12a)  
(12b)

Equation (12) is the system representation in terms of the selected modal coordinates \( q \). No correction for the steady-state error is made, assuming small contribution of higher modes to the steady state response. The reduced model can be used for designing controllers by influencing only the included system modes.
5. Controller Design

With the availability of an LTI system model, the necessary controllers for developing the active lubrication with the TPJB can be part of the bearing design process. The control objective is to reduce vibrations in resonance zones, or in other words, to achieve FRF spectra that minimise peak responses. A Linear Quadratic Gaussian (LQG) regulator is to be designed by combining an optimal Linear Quadratic Regulator (LQR) with a stochastic optimal observer, the Kalman filter. Provided the system is stabilizable and taking advantage of the separation principle, the observer and controller can be indistinctly designed following the recommendations of [37].

In most practical applications only a few states can be measured, in this case up to four. This poses the need to estimate correctly the system states through observers. Two cornerstone works have paved the way for how observers are designed, the deterministic Luenberger observer [43] and the stochastic Kalman filter [44]. The observer used in the design of the LQG regulator corresponds to a full order Kalman filter. This stochastic observer, which takes into account the measurement $v_2$ and process $v_1$ noises, is the optimal observer for LTI systems provided the existence and propagation of noise. Even if the system does not behave linearly, it is the best estimator that can be designed. Referring to Figure 2 and using the system of equations (12), the defining equations of the observer are:

\[
\dot{\hat{q}} = \bar{A}\hat{q} + \bar{B}u + L(Y - \hat{Y}) \tag{13a}
\]

\[
\hat{Y} = \bar{C}\hat{q} \tag{13b}
\]

where $L$ is the steady-state observer gain obtained in terms of the variance matrix $V_2$, the output matrix $\bar{C}$, and the error covariance matrix, which is the solution of the time independent Riccati equation for the uncorrelated noise variance matrices $V_2$ and $V_1$ assumed for the measurement $v_2$ and process $v_1$ noises, respectively. Obtaining these matrices is the more challenging task in the design of the filter; the approach here utilized will be discussed later.

With the availability of the state estimates $\hat{q}$, a full state-feedback controller can be designed. Full state-feedback means that the system output $Y$ is used to reconstruct the entire “modal” state vector $\hat{q}$ and not only the missing states. The LQR controller is an optimal control in terms of energy balance between states and control signals obtained by minimizing the
steady-state cost function:

\[ J = \int_0^\infty (\hat{q}^T Q \hat{q} + u^T R u) \, dt \]  

(14)

where \( Q \) is a positive semi-definite \( n \times n \) diagonal matrix utilized for weighting the states and \( R \) is a positive definite \( 2 \times 2 \) matrix to weight the servovalve control signals. These matrices are obtained via trial and error tests through simulations until control requirements are met. This approach provides an intuitive way to synthesise the controller gains by weighting only the modal states of interest while control signals are kept within specifications. The control law is defined by:

\[ u = -K \hat{q} \]  

(15)

where the control gain \( K \) is obtained in terms of the control weighting matrix \( R \), the input matrix \( \bar{B}r \) and the solution of the steady-state Riccati equation for \( Q \) and \( R \). The control matrix \( K \) includes only proportional and derivative gains; integral action for keeping the system around the equilibrium can also be included in the calculation by means of system augmentation. In this case the state vector is augmented as \( \bar{q}_{lqr} = \{\hat{q} q_i\}^T \) and the obtained gain matrix is \( K_{lqr} = [K - K_l] \). The integral states are defined as the integral of the output steady-state error and are incorporated via the time derivative equation \( \dot{q}_i = r - \bar{Y} = r - \bar{C}^r \hat{q} \), with \( r \) being the desired position reference.

Figure 2 shows the block diagram of the system in closed-loop, comprised of the rotor-bearing system plus the observer and the controller. The governing equations in closed-loop can be summarized as:

\[
\begin{bmatrix}
\dot{\hat{q}}_i \\
\bar{q}_i \\
\dot{\hat{q}}
\end{bmatrix}
=
\begin{bmatrix}
\bar{A}^r & \bar{B}^r K_i & -\bar{B}^r K_l \\
-\bar{C}^r & 0 & 0 \\
\bar{L} \bar{C}^r & \bar{B}^r K_i & \bar{A}^r - \bar{L} \bar{C}^r - \bar{B}^r K_l
\end{bmatrix}
\begin{bmatrix}
\hat{q} \\
q_i \\
\dot{\hat{q}}
\end{bmatrix}
+
\begin{bmatrix}
0 \\
I_r \\
0
\end{bmatrix}
\begin{bmatrix}
r \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\bar{B}^r \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
\]  

(16a)

\[
Y = \bar{C}^r q + v_2
\]  

(16b)

for which the eigenvalues of the closed-loop system are determined by:

\[
\det \left( \lambda I - \begin{bmatrix}
\bar{A}^r - \bar{B}^r K_l & \bar{B}^r K_i \\
-\bar{C}^r & 0
\end{bmatrix}
\right) = 0
\]  

(17a)

\[
\det \left( \lambda I - (\bar{A}^r - \bar{L} \bar{C}^r) \right) = 0
\]  

(17b)
Equation (17a) and equation (17b) stand for the controller and observer eigenvalue equations respectively.

6. Theoretical Results

The system is firstly studied with the rotor without discs. Under this condition the first bending mode of the rotor-bearing system lies slightly above 200 Hz. The static bearing load is 400 N. The simulated operational conditions are $\Omega$ 1000 rpm and $P_s$ 100 bar. The shaft discretization is reported in part I. In the current case, 26 finite elements ensure convergence of the shaft representation and place nodes at the more significant locations for the study. The rotor contributes 108 dofs whilst the active TPJB has 14 additional ones. There are three dofs per pad (tilt, bending and radial movement) plus one dof per servovalve. The full-order system has a total of 122 dofs, i.e. $4n_r + n_p i_p + n_v$; it totalizes $2N = 244$ states which is reduced to $n = 12$ modal states. The eddy-current sensors are installed at node 23 with sensors 1 and 2 and at node 17 with sensors 3 and 4. Uneven sensors are horizontally oriented while even ones are vertically. The active TPJB is located at node 15. The full order model can be reproduced with the information provided in the paper series. Table 1 summarizes the eigenvalues of the full model up to around 400 Hz and the ones kept in the reduced order model up to around 210 Hz. The full order model is obtained considering the spool of
the servovalve centred, i.e. injecting only the leakage flow. In the leftmost column an indication of the subsystem that the eigenvalues are related to are presented. To fulfill the complex separation approach, slow overdamped modes lying in the studied range are excluded in advance, hence they are not affected by the LQR controller. This is the same as not weighting them in the \( Q \) matrix and it contributes to lowering the reduced-order system size even more, critical for the controller implementation. The lower eigenvalues of around 10 Hz correspond to the rotor-bearing system conical mode which is very well damped. The next ones at 177 Hz are introduced by the servovalve dynamics, which have the larger damping ratio of the whole system. The highlighted frequencies around 210 Hz correspond to the first bending modes of the rotor-bearing system. These modes dominate the FRFs due to their relative low damping ratio. As a reference, higher bending modes are also depicted around 400 Hz for the full model.

Figure 3 shows the modal controllability degrees of the six retained modes in the reduced-order model, sorted as presented in Table 1 along with their conjugate, i.e. twelve states in total. The same sorting is utilized in subsequent figures of observability. It is clear that the included modes are more controllable from input 2 than from input 1 with gross measurements of controllability of 13·10^{-4} and 1·10^{-4}, respectively, i.e. 13 times more controllable from input 2. From Figure 3 b) the most controllable mode corresponds to the servovalve mode. Among the rotor-bearing modes, conical modes are slightly more controllable than the bending ones. Regarding observability,

<table>
<thead>
<tr>
<th>Full model</th>
<th>Reduced model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_i ) (sec(^{-1}))</td>
<td>( \omega_d ) (Hz)</td>
</tr>
<tr>
<td>1) R-B</td>
<td>-55.22±64.09i</td>
</tr>
<tr>
<td>2) R-B</td>
<td>-54.75±64.14i</td>
</tr>
<tr>
<td>3) SV</td>
<td>-1188±1112.2i</td>
</tr>
<tr>
<td>4) SV</td>
<td>-1188±1112.2i</td>
</tr>
<tr>
<td>5) R-B</td>
<td>-66.27±1315.8i</td>
</tr>
<tr>
<td>6) R-B</td>
<td>-66.33±1318.1i</td>
</tr>
<tr>
<td>7) R-B</td>
<td>-236.53±2588.1i</td>
</tr>
<tr>
<td>8) R-B</td>
<td>-351.49±2589.8i</td>
</tr>
</tbody>
</table>

Table 1: Open-loop eigenvalues \( \lambda_i \), damped frequencies \( \omega_d \) and damping ratios \( \zeta \) of the full and reduced order model of the rotor-bearing system. R-B: Rotor-Bearing system. SV: Servovalves, hydraulic system.

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Figure 3: Modal Controllability degrees of the reduced-order model. a) from servovalve 1. b) from servovalve 2.

Figure 4 depicts the degrees of observability from sensors 1, 2, 3 and 4 with gross measurements of observability of $19.1 \cdot 10^{-5}$, $17.9 \cdot 10^{-5}$, $15.4 \cdot 10^{-5}$ and $14.4 \cdot 10^{-5}$, respectively. Clearly the most observable mode corresponds to the conical one. On the other hand, the first bending mode is more observable from sensors 1 and 2 than from 3 and 4. To diminish the computation burden, only sensors 1 and 2 are utilized in the controller design and in the experimental work to observe the bending mode. Servovalve modes are unobservables. A two input - two output plant is established.

The LQG regulator is designed for controlling the first bending mode of the rotor-bearing system in the no disc configuration. This implies that their corresponding modes (1, 2, 5 and 6) are heavily penalized in comparison with the servovalve ones (3 and 4). The utilized $R$ and $Q$ matrices are presented in Equation (18). Some heuristic rules [37] were followed to determine the weights but they were finally determined after intensive trial and error tests. As mentioned, the rotor-bearing modes are highly penalized with the weights in the order of $10^6$, while the modes included by the servovalve modelling are quite less penalized by only 1. The last two weights penalize the integral states. Control signals are weighted in a way that simulated signals are bounded $\pm 1.5$ V to stick to the linear behaviour of servovalves while exciting
For the Kalman filter, the measurement and process noise covariance matrices are needed. The first one is readily obtained by characterizing statistically the sensors, while for the second one different approaches might be used. The first one consists of, provided a known covariance of the system disturbances, propagating the noise to the states through the disturbance input matrix $B_v$. Another form is to impose the noise to the states directly through an identity input matrix $I$ [41]. This identity input matrix also has to be reduced following the same steps as for the input matrix $B_v$ in order to express the state noise in modal coordinates. Both options were simulated, with the last
one obtaining better results. The noise covariance matrices are defined as:

\[ \mathbf{V}_2 = \text{diag}\{ (1.422 \cdot 10^{-7})^2, (3.932 \cdot 10^{-7})^2 \} \] \[ [m^2]; \quad (19) \]

\[ \mathbf{V}_1 = 10^5 \text{diag}\{4, 5, 15, 40, 58475, 17300, 56734, 3482, 1059, 2521, 229, 49, 1453, 5038\}; \]

In Figure 5 the theoretical FRFs of the full and reduced open-loop system as well as the reduced-order system in closed-loop are shown. It is seen that the reduced-order model reconstructs closely the full order one with small discrepancies towards the steady-state response. On the other hand, the vibration reduction in the resonant zone is achieved in both directions with the closed-loop reduced-order system, but at the expense of an increase in the system response at low frequencies. Table 2 summarizes the controller and observer eigenvalues of the closed-loop system. Unlike the pole placement approach in which, provided the system is stabilizable, eigenvalues can be modified at will, under optimal control they are obtained only after simulations. In closed-loop, one of the conical eigenvalues becomes overdamped, while for the other one the damping ratio is slightly increased. The servo-valve eigenvalues remain almost unaltered. While for one of the first bending modes, the damped natural frequency is slightly reduced to around 200 Hz whilst its damping ratio is increased significantly. The last two eigenvalues are linked to integral action; they were obtained relatively slowly meaning

<table>
<thead>
<tr>
<th>LQR regulator</th>
<th>Kalman Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_i (\text{sec}^{-1}) )</td>
<td>( \lambda_i (\text{sec}^{-1}) )</td>
</tr>
<tr>
<td>1) R -B</td>
<td>-316.9</td>
</tr>
<tr>
<td>2) R -B</td>
<td>-92.03</td>
</tr>
<tr>
<td>3) R -B</td>
<td>-69.1±63.4i</td>
</tr>
<tr>
<td>4) SV</td>
<td>-1323±1104i</td>
</tr>
<tr>
<td>5) SV</td>
<td>-1188±1112i</td>
</tr>
<tr>
<td>6) R -B</td>
<td><strong>726.35±1262.8i</strong></td>
</tr>
<tr>
<td>7) R -B</td>
<td><strong>122.87±1317.2i</strong></td>
</tr>
<tr>
<td>8) I</td>
<td>-0.0008</td>
</tr>
<tr>
<td>9) I</td>
<td>-0.0012</td>
</tr>
</tbody>
</table>

Table 2: Closed-loop eigenvalues \( \lambda_i \), damped frequencies \( \omega_d_i \) and damping ratios \( \zeta_i \) of the LQR regulator and Kalman filter observer. R-B: Rotor-Bearing system. I: integral action.
Figure 5: Theoretical FRFs in the Horizontal (a) and c)) and Vertical (b) and d)) direction. Solid line (–) corresponds to the open-loop full order system, i.e. system under hybrid lubrication. Dashed dotted line (-.) corresponds to the reduced-order model response. The dashed line (--) corresponds to the reduced-order system in closed-loop, i.e. featuring the LQG regulator.

that steady-state errors will be slowly compensated. Kalman filter features two fast real eigenvalues at the left half plane which dominates the observer dynamics.

7. Experimental Results

The experiments start when force and thermal equilibria were established for the same system configuration and operational conditions simulated. The implementation of controllers is carried out in discrete time considering a sampling frequency $F_s$ of 3 kHz. Results are presented only in the horizontal direction. The system is excited with an electromechanic shaker connected to the excitation bearing, corresponding to node 26, dofs 101. The system response is obtained at sensor pedestal 1, which corresponds to node 23, dofs 89 of the model. For obtaining the FRFs a chirp signal is swept in the frequency range covering the first bending frequency of the rotor-bearing system. The sweeping time was set to 10 s and the recording time was 5 min. Post processing parameters are: Hanning window, 1 Hz of frequency resolution and 50% of overlap. To avoid the onset of observer or controller spillover problems, low pass digital filters were implemented for filtering the
system outputs and control signals if required.

Figure 6 presents the system output $Y_1$ along with the corresponding estimate $\hat{Y}_1$ produced by the Kalman filter at low and high frequency ranges. Estimates correspond pretty well with the measurement which makes the observer a suitable one with regards the definitions of the covariance matrices $V_1$ and $V_2$, especially regarding the last one.

Figure 7 presents the results for the no disc configuration. In this case the resonance is slightly above 200 Hz and the active lubrication based upon the synthesised LQG controller and a variation of it were tested. This variant, the LQG*, ponders the first bending mode slightly less. No further im-
Figure 7: No disc configuration. Experimental FRF in the horizontal direction. 1000 rpm. 90 bar. Input at node 26 (dof 101) and output at node 23 (dof 89).

Improvements are obtained with the active lubrication and vibrations slightly differing from the hybrid case. This might be because servovalves are acting at their bandwidth limit for the 90 bar of supply pressure used. Under this supply pressure the natural frequency of the hydraulic system is around 250 Hz which, along with a 0.73 of damping factor, implies a decay in the servovalve response above 170 Hz approximately. To delve into this assumption the system is studied with the rotor hanging one disc and with the same supply pressure. Under this configuration the bearing supports a heavier load, 880 N approximately. Alongside, the damped natural frequency of the first bending mode of the rotor-bearing system is reduced to about 150 Hz, which lies within the linear bandwidth of the actuator. Figure 8 shows the results in the horizontal direction for such a configuration and operational condition.
A new LQG controller was synthesised following the same procedure presented previously. In addition, for benchmarking purposes, an on-site tuned PID controller was also synthesised for developing the active lubrication in this configuration. The proportional, derivative and integral matrices were defined as $K_p = 10^3[7 - 7; 7 7]$ (V/m), $K_d = [50 - 40; 40 50]$ (Vs/m) and $K_i = 10^4[1 - 1; 1 1]$ (V/ms), respectively. The tuning of an appropriate PID controller is not trivial and an important amount of trial and error tests are run until obtaining a performance that met the requirements. It is noted that the tendency in the results remains, in the sense that vibrations are reduced with the hybrid lubrication regime when compared against the passive one. But in this case further reductions are obtained with the active lubrication, which performs slightly better with PID control rather than with the LQG controller.
regulator. Nonetheless, reductions are still not so significant.

To further reduce in frequency the resonant zone and ensure a better actuator performance, an additional disc was mounted, obtaining two discs hanging in the rotor. Static bearing load increases up to 1440 N. The supply pressure was slightly increased to 100 bar. Figure 9 depicts the experimental FRFs obtained with both controllers, the same PID controller and a new synthesised LQG regulator. Again, the same synthesising procedure presented in the theoretical section was followed. Additionally, the system response under passive and hybrid lubrication is also included for comparison. Under the hybrid lubrication, the spool is kept centred and only the leakage flow into the bearing clearance occurs. A significant reduction in amplitude is obtained in the resonant zone when the hybrid lubrication is featured in

Figure 9: Two discs configuration. Experimental FRF in the horizontal direction. 1000 rpm. 100 bar. Input at node 26 (dof 101) and output at node 23 (dof 89).
Figure 10: Two discs configuration. Experimental FRF in the horizontal direction. 2000 rpm. 100 bar. Input at node 26 (dof 101) and output at node 23 (dof 89).

comparison with the passive or conventional one. This is due to the injection of high pressurized oil at 100 bar which modifies the bearing stiffness and damping properties. Further reductions are obtained in the resonant zone with the LQG regulator and even more with the PID controller. A fitting of a single degree of freedom curve to the passive and active – with LQG control – cases reveals an increase of the damping ratio of approximately five times, from 0.067 to 0.33, which is half of that predicted. Doubtless, a better performance of the actuator is obtained when two discs are hanged on the rotor than when is only one. As a point of comparison, in Figure 9 a peak reduction of about 70% is obtained with active lubrication based on PID control when compared to the passive one, whereas in Figure 8 this reduction is only of the order of 50%.
Figure 11: Two discs configuration. Experimental FRF in the Horizontal Direction. 1000 rpm. 100 bar. Input at node 26 (dof 101) and output at node 23 (dof 89).

Despite the LQG controllers being designed for an angular speed of 1000 rpm, in this configuration it was also tested at a higher speed. Figure 10 shows such results for 2000 rpm. The PID controller was retuned in order to meet the requirement of reducing vibrations at such velocity. The proportional and derivative gain matrices were updated to $K_p = 10^4[1 - 1; 1 1]$ (V/m) and $K_d = [20 - 10; 10 20]$ (Vs/m). It can be noticed that in general the vibration amplitudes reduce and both controllers manage to reduce the vibration amplitudes around the 90 Hz resonance. The PID controller still performs better than the LQG regulator. In all studied cases, no spillover problems at higher frequencies were observed for the model-based controllers, hence signal filtering was not needed.
8. Discussion

Previous results have achieved the control goal of reducing the vibrations in the frequency range for which the controllers were designed, i.e. around the resonant zone of the first bending frequency of the rotor-bearing system. However, at lower frequencies than 70 Hz, the implementation of the LQG regulators yields an unexpected resonance which only shows up with this sort of controller. Figure 11 depicts the same results as Figure 9 but extended at low frequencies up to 20 Hz (highlighted with light a gray transparent layer). Frequencies below this value are avoided due to the seismic mass behaviour of the shaker. The resonant zone is identified at around 40 Hz only for the LQG controller. It is also confirmed that the active lubrication based on the PID controllers perform well at low frequencies with only a small amplification of rotor vibration amplitudes close to 20 Hz. Figure 12 depicts the voltage control signals commanding the servovalves when performing the tests with the LQG regulator. The rise in amplitude of the servovalves clearly noted in servovalve 1 before 2 seconds corresponds to such a 40 Hz resonance. It is also seen that signals are bounded between ±1 V and that control signal 2 is larger and noisier. Since the performance significantly deteriorates below 70 Hz only for the LQG controller, it is assumed that there are unmodelled
Figure 13: Two discs configuration. Experimental FRF in the Horizontal Direction. 1000 rpm. Input at node 26 (dof 101) and output at node 23 (dof 89). Effect of the supply pressure in the performance of the LQG regulator. 12, 20, 30, 50 and 100 bar.

dynamics. To explore the possible causes, the supply pressure $P_s$ is varied to make the controller less aggressive, even though the controller is originally designed for 100 bar. According to the servovalve specifications, a reduction of the pressure supply implies to lessen the servovalve natural frequency and hence obtain a narrower bandwidth of effectiveness.

Figure 13 shows the effect of reducing the supply pressure $P_s$ in the radial oil injection system. It is noticed that as the pressure reduces, the frequency and amplitude of the resonant zone around 40 Hz also reduces. It is also evident that as the pressure reduces the effectiveness of the controller around 90 Hz (rotor-bearing resonance) deteriorates as expected. It is believed that this is due to a strong interaction between the rotor-bearing system with
torsional movements of the foundation, which indeed are linked through the housing stiffeners and boosted at higher pressures. Given the existence of this unmodelled dynamic present in the LQG regulator, it was not possible to apply the complete control signals to both servovalves for pressure over 50 bar, cases for which control signals were reduced by a master gain to 40%. Figure 14 depicts the same results as in Figure 9 but with a supply pressure of 30 bar, showing that with a less aggressive controller in terms of pressure, a good balance between the two resonant zones around 40 Hz and 90 Hz can be obtained.

Figure 14: Two discs configuration. Experimental FRF in the Horizontal Direction. 1000 rpm. 30 bar of supply pressure. Input at node 26 (dof 101) and output at node 23 (node 89).
9. Conclusions

In this work, the design and implementation of model-based controllers for developing the feedback-controlled lubrication in actively-lubricated TPJBs were presented. The work demonstrates flexible rotor systems supported by the mentioned kind of bearings. The designed controllers are full-state optimal LQG controllers which harness the characteristics of LQR regulators with stochastic Kalman filters. Despite the results being obtained under operational conditions less demanding than those imposed on industrial machinery, results are valuable since they pinpoint latent problems when it comes to real applications. In the light of the simulations and the experimental campaigns, it can be concluded that:

- The current maturity in the modelling of actively-lubricated TPJBs based on the ETHD approach allows us to include full matrices of the bearing dynamic properties in the rotor-bearing modelling to design and develop model based controllers.

- The design of model-based controllers based on full state-feedback requires a model reduction. An appropriate reduction scheme is the pseudo-modal, which preserves only the slowest eigenvalues. A further treatment of complex states is also required in order to implement discrete controllers in processing units. Such a complex separation requires only complex conjugated eigenvalues in the frequency range of interest to be considered. Disregarding in advance real eigenvalues, does not pose a problem when dealing with the LQR regulator since it has the same effect as not weighting them with the modes weighting matrix.

- The LQG controller is a suitable option for developing feedback-controlled lubrication in active TPJBs since it allows us to focus the actuator energy in the modes of interest, such as bending modes of the rotor machine. It can be designed during the machine design process. Caution is advised when selecting the state or process noise, which might be backed up with simulations.

- From an experimental viewpoint, it was shown that the LQG controller performs well for the dynamics it was designed for, i.e., the first bending of the rotor-bearing system. This entails that the controller can be jointly designed with bearings from the first stages of the bearing design and manufacturing.
• Despite the effectiveness shown by the LQG regulator, it appeared obvious that a more accurate model which describes the dynamics of the rotor-bearing system and also the foundation dynamics is needed. The modal-reduced dynamics of the foundation must be incorporated in the modelling, so that bending and especially torsional movements can be incorporated. Further studies will target proper modelling, reducing and including the flexible foundation dynamics.

• Additional reduction of vibrations is obtained with the feedback-controlled lubrication over the hybrid and passive ones. Although PID controllers already deliver good results in a wider frequency range, they are suitable only as a post-production approach. In order to transform the active TPJB into an end-user machine element, efforts toward model-based controllers must be aimed.

Alternatively, and as a future work, robust $H_\infty$ controllers may also be approached under an optimization standpoint, so that discrepancies between theory and experiment can be treated as uncertainty for improved closed-loop stability.


APPENDIX B

Bearing Clearance Measurement

The preload factor $m_p$, defined by Eq. (B.1) in terms of the bearing clearance $c_b$ and pad clearance $c_p$, is one of the most important design parameters of the tilting-pad journal bearings. It strongly influences the dynamic characteristics of the bearing by determining, along with other parameters such as the static bearing load, the number of loaded pads. Generally speaking, low values of preload increase the bearing damping and have almost no effect on the stiffness [106]. Contrarily, higher values of the preload decrease the damping properties of the bearing. Zero preload values are not recommended because they can affect the rotor stability and lead to unloaded pads. Typical values of $m_p$ are between 20% and 60%.

$$m_p = \left(1 - \frac{c_b}{c_p}\right)$$ (B.1)

To determine $m_p$, the bearing clearance must be measured. The assembled bearing clearance $C_b$ can be measured based on the tilting pad bearing configuration. For bearings with an even pads number in an LOP configuration the measurement is trivial. For bearings with an even pads number in an LBP and with an odd pads number in LOP and LBP configuration, the measurement is not straightforward since the shaft sinks between two pads due to their tilting capability. References [107] and [106] give guidance on these two general cases.
Two ways of determining the bearing clearance $c_b$ have been used to determine the bearing preload in a kind of cross check. The first one is by measuring the lift of the shaft, see Figure B.1(b), then by geometrical relationship the clearance is calculated as $\sqrt{2}/2$ the shaft lift. The second one is by measuring directly $c_b$ along the direction of two counter pads, see Figure B.1(a).

![Figure B.1: The assembled bearing clearance measurement.](image)

Using statistical tools the assembled bearing clearance is determined with a mean value of $C_b = 84 \, \mu m$ with a standard deviation of $\sigma = \pm 5 \, \mu m$ for an interval of confidence of 95 %. This leads to a $m_p$ value ranging from [22 28]% with a mean value of 25%.
APPENDIX C

Calibration of the Sensors

C.1 Displacement sensors

Two brands of displacement sensors have been used to register the shaft lateral displacements. However, Pulsotronic sensors have been used only to characterize the system, whereas the Vibrometer sensors have been considered for control purposes due to their superior performance. Although Pulsotronic sensors conveniently include their electronics inside, their variable electric noise and a time delay identified make them inconvenient for compensating shaft run out and for using with controllers. The calibration of the sensors is carried out by reading its output voltage $V$ when facing a target specimen at a known distance $X$. Then, a first order polynomial is adjusted to determine its voltage offset $V_0$ and sensitivity $S$ as:

$$V = V_0 + SX$$  \hspace{1cm} (C.1)

A voltage value $V^*$, approximately in the middle of the measuring range, is determined as a target voltage when installing them. In this way, a gap value close to the mean distance between the probe and the shaft surface is set to ensure a correct utilization of the probe linear range.
C.1.1 Vibrometer Displacement Sensors

Vibrometer type 102 eddy current displacement sensors with IQS 603 demodulator conditioners have been used. Figure C.1 presents the calibration curves of the sensors utilized and identified as sensor 1, sensor 2, sensor 3 and sensor 4. Additionally, the information is also summarized in Table C.1.

![Calibration Curves of Vibrometer Proximity Probes](image-url)

**Figure C.1:** Calibration curves of Vibrometer proximity probes.
Table C.1: Sensitivity, offset and gap values of Vibrometer sensors.

<table>
<thead>
<tr>
<th>Sensor #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity(^\d), S [V/mm]</td>
<td>-7.80</td>
<td>-7.83</td>
<td>-8.22</td>
<td>-7.75</td>
</tr>
<tr>
<td>Offset, (V_0) [V]</td>
<td>0.46</td>
<td>0.57</td>
<td>-0.57</td>
<td>0.36</td>
</tr>
<tr>
<td>Gap, (V^*) [V]</td>
<td>-6.13</td>
<td>-5.99</td>
<td>-5.87</td>
<td>-6.19</td>
</tr>
</tbody>
</table>

\(^\d\) Gains mean value of -7.9 mV/\(\mu\)m. Calibrated against a St 60-2 made specimen.

### C.1.1.1 Noise Quantification

The following figures describe the noise quantification for the Vibrometer proximity probes. The noise is quantified through the standard deviation of a sufficiently long time series for each sensor. These values are used in the design of observers within a model-based control approach as indicatives of the measurement noise. The standard deviation is estimated by fitting a normal distribution curve to the histogram of each time series.
C.1.2 Pulsotronic Displacement Sensors

Pulsotronic proximity inductive sensors KJ2-M8MB40-ANU have also been used, but only to characterize the mechanical system when necessary. They have not been used for control. According to the manufacturer, they measure within 0.5 to 2 mm having an output signal range from 0 to 10 V. The switching frequency corresponds to 400 Hz. Subsequently, the calibration curves of all sensors are presented along with their voltage offset $V_0$, sensitivity $S$ and the gap value $V^*$ set when installed.

<table>
<thead>
<tr>
<th>Sensor #</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity, $S$ [V/mm]</td>
<td>8.41</td>
<td>8.60</td>
<td>8.68</td>
<td>8.43</td>
<td>8.24</td>
<td>8.42</td>
</tr>
<tr>
<td>Offset, $V_0$ [V]</td>
<td>4.29</td>
<td>3.51</td>
<td>3.34</td>
<td>3.42</td>
<td>3.44</td>
<td>3.25</td>
</tr>
<tr>
<td>Gap, $V^*$ [V]</td>
<td>-3.24</td>
<td>-4.20</td>
<td>-4.43</td>
<td>-4.13</td>
<td>-3.96</td>
<td>4.31</td>
</tr>
</tbody>
</table>

$\dagger$ Gains mean value of 8.49 mV/μm. Calibration carried out against a St 60-2 made specimen.
Figure C.3: Calibration curves of Vibrometer proximity probes.
C.2 Load Cells

Load cells are used to calibrate dynamically the ALB and AMB. The used load cells correspond to the type HBM U9B with amplifier AE101. These sets allow static as well as dynamics loads to be measured. Figure C.4 presents the calibration curves of both cells used and identified by the last digits of their serial number, i.e. 025 and 027.

![Load Cells Calibration](image)

**Figure C.4:** Calibration of the load cells.

C.3 Current Sensor for the Servovalve Amplifier

The current sensor Honeywell CSIW6B200M was used for measuring the control current produced by the servovalve amplifier MOOG E122-205. This type of miniature wired open-loop current sensor is fed with 9 V of supply power and has a coil resistance of 4 Ω in order to alter as little as possible the servovalve original circuit (40 Ω in parallel coils configuration). As a drawback, its linear range of ±200 mA is misused. Figure C.5 depicts the sensor calibration curve for the working range of the servovalve, i.e. ±40 mA.
The sensors have been placed in a home-made measurement box. When connecting the measurement box, the I/O switch of the printed circuit board (PCB) indicated in the Figure C.6, must be in position 0 in order to close the loop and measure. Position 1 is normal, no measure is performed.

**Figure C.5**: Honeywell current sensor calibration curve.

**Figure C.6**: PCB Interface. I/O switches for activating the servovalve control current measurement.
The run out is a significant issue when measuring with non-contact inductive proximity probes. The term refers to undesired signals of a mechanical and/or electrical nature, basically understood as noise, which are sensed by the probe and interpreted as physical displacements of the target. Although this problem can be significantly mitigated with capacitive probes, they are only suitable for clean environments, making them inappropriate in the majority of rotodynamic applications. Other solutions mention the usage of less inductive material as a target, such as titanium or aluminium, but they seem non applicable \(^1\). This Appendix explains the way the run out was treated in this project and some main considerations relating to its compensation.

### D.1 Work Principle of Displacement Sensors

The inductive proximity probe works based upon the “eddy current” principle. It means that, a high frequency magnetic field induces a parasitic current in

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the ferromagnetic target. This current is proportional to the distance to the target and any variation of it modulates the magnetic field. With proper electronics (demulator+amplifier), these modulations can be converted into a readable voltage signal. Referring to Figure D.2, and in the absence of any error source, the sensor shall deliver $h_0$ as a constant distance between the probe and the shaft. Since the shaft is spinning, two error sources arise: one of a mechanical nature and another of an electrical nature.

- The mechanical error is due to the roughness of the shaft surface, even though high surface tolerances are specified. Additionally, a perfect flat surface is expensive and almost impossible to obtain.

- The electrical error is due to a non homogeneous material which, in turn, can modify the magnetic field. The insert identified as A in Figure D.1, can affect the magnetic field of the sensor every time it passes in front of the sensor, leading to an electrical error in the signal.

![Figure D.1](image_url)

**Figure D.1:** Run out phenomenon in rotating machinery.

Despite the presence of the run out, the displacement signals can be treated and compensated online for it.
D.2 Compensation of the Run Out

In order to compensate for the run out in the displacement signals, it must be known and available in advance as a function of the shaft angle in look-up tables. Then, by simultaneously sampling the displacement with the shaft angle signals, the run out can be subtracted at each sampling time. This implies knowing the shaft angular position all the time through an encoder. The encoder provides a sufficient number of pulses per revolution, normally ranging from 360 to 2000, which if sampled with a board capable of treating quadrature signals, the shaft angular position can be determined with a high resolution, e.g. an encoder with 500 pulses/rev, as the one used, provides an angular resolution of \( \Delta \theta = 360 \cdot (500 \cdot 4)^{-1} = 0.18 \) deg. The steps for subtracting the run out are summarized as follows:

1. Run the machine at low rotational speed. Low enough to consider that the displacement seen through the proximity probe is only due to the run out and not to vibration inherent in the machine. An angular speed below 10% of the nominal speed can be considered enough. Here, a speed around 60 rev\(^{-1}\) was utilized.

2. Record both the shaft angular position and the displacement signal. The larger the block size the more available the information for generating the run out reference signals. The amount of samples is limited by the capacity of the DAQ board.

3. Save the reference signals in look-up tables and make them available during the acquisition of the displacement signals at nominal rpm.

4. Subtract from the displacement signal the run out reference one at each sampling time obtained based on shaft angular position from the look-up table.

D.3 Run Out Reference Signals

Figure D.2 summarizes the run out reference signals for the ten displacement sensors used in the test rig. The first four correspond to the Vibrometer sensors, whereas number five to ten are Pulsotronic. Each pair of sensors are plotted overlapping each other for comparison purposes, bearing in mind that they are physically delayed 90 degrees or a quarter of a revolution because they are orthogonally installed. Two things can be noticed from these figures: first, there is a scratch on the shaft surface at the position of sensors 1 and 2, close to 180
degrees. Second, there is a significant mismatch between sensors 7 and 8 due to the electric noise, characteristic of Pulsotronic sensors as previously mentioned. In general terms, the run out reference signals must be updated frequently during the experimental campaigns since they change along with the change of temperature in the system influenced by the operational conditions. This phenomenon is explained by the axial expansion of the shaft with the temperature, which changes the original target surface of each probe. The reported ones were obtained at ambient temperature, around 25 degrees.
Figure D.2: Run out reference signals for the proximity probes.
E.1 Shaft Elements Matrices

\[ M_t^s = \frac{\mu l}{420} \begin{bmatrix} 156 & 0 & 0 & 156 & 0 & -22l & 4l^2 \\ 0 & 156 & 0 & -22l & 4l^2 & 22l & 0 \\ 0 & 0 & 13l & 156 & 0 & 54 & 0 \\ 22l & 0 & 0 & 13l & 156 & 0 & 54 \\ 54 & 0 & 0 & 13l & 156 & 0 & -13l \\ 0 & 54 & -13l & 0 & 0 & 156 & 0 \\ 0 & 13l & -3l^2 & 0 & 0 & 22l & 4l^2 \end{bmatrix} \] (E.1)

\[ M_r^s = \frac{\mu r^2}{120l} \begin{bmatrix} 36 & 0 & 0 & -3l & 3l & 0 & 0 & 36 \\ 0 & 36 & 0 & -3l & 3l & 0 & 0 & 36 \\ 0 & 0 & 36 & 0 & 3l & -l^2 & 0 & 0 \\ 36 & 0 & 0 & -l^2 & 0 & 3l & -3l & 4l^2 \end{bmatrix} \] (E.2)
\[
G^s = \frac{\rho l}{15 l} \begin{bmatrix}
0 & 36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3l & 4l^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 36 & 3l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-36 & 0 & 0 & 3l & 36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3l & 0 & l^2 & -3l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3l & -l^2 & 0 & 0 & -3l & 4l^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}; \quad G^s = -G^{sT} \quad (E.3)
\]

\[
K^s = \frac{EI}{(1 + a)l^2} \begin{bmatrix}
12 & 0 & 12 & 0 & 0 & 6l & (4 + a)l^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 6l & (4 + a)l^2 & 0 & 0 & 6l & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-6l & 0 & 0 & 0 & 6l & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-12 & 0 & 0 & 6l & (2 - a)l^2 & 0 & 0 & 0 & 0 & 6l & (4 + a)l^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -12 & -6l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 \\
0 & 6l & (2 - a)l^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 \\
-6l & 0 & 0 & (2 - a)l^2 & 6l & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 \\
\end{bmatrix} \quad (E.4)
\]

E.2 Disc Inertia and Gyroscopic Matrices

\[
M^d = \begin{bmatrix}
m_d & 0 & 0 & 0 \\
0 & m_d & I^d_D & 0 \\
0 & 0 & I^d_D & 0 \\
0 & 0 & 0 & I^d_D \\
\end{bmatrix}; \quad G^d = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -I^d_p & 0 \\
0 & 0 & I^d_p & 0 \\
\end{bmatrix} \quad (E.5)
\]

E.3 Ball Bearing Stiffness Matrix

\[
K^{bb} = \begin{bmatrix}
k_{bb} & k_{bb} & 0 & 0 \\
k_{bb} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix} \quad (E.6)
\]
Appendix F

Baseplate Experimental Modal Analysis

The test-rig is mounted on a T-slot metal plate which is resting on the building floor made of concrete. Figure F.1 summarizes the first three modeshapes of the T-slot plate where the test-rig (not shown) is mounted. These were obtained through an experimental modal analysis, where the structure (plate + test rig) was excited in a fixed point via an electromagnetic shaker and the system response was measured with an accelerometer roving throughout all points. Nine points were defined: points 1, 2 and 3 are at the driven side of the test-rig, points 4, 5 and 6 are aligned with the ALB position and points 7, 8 and 9 are placed at the free-end. The excitation was introduced via point 9. In order to obtain the experimental FRFs, a bidirectional chirp signal was swept in the frequency range of interest.

A visual inspection of the plate allows us to recognize the boundary conditions imposed over it. In this case, the plate is not supported on the floor evenly, producing a sort of a cantilever beam condition, i.e., clamped toward the driven side and free at the non-driven one. This condition produces three relevant modeshapes lying in the studied frequency range which, unfortunately, couple with the rotor-bearing dynamics. The first one is around 20 Hz and it resembles somehow a kind of first bending mode shape of a cantilever beam. The next two at 65 Hz and 130 Hz are torsional in nature with smaller displacements.
However, these modes strongly affect the whole dynamics of the rotor-bearing-foundation system. Similar results were obtained in [105], but with the rotor hanging two discs instead of only one. As a last attempt to decouple the foundation dynamics from the rotor-bearing system one, a bed of gummy was installed underneath the plate. Such a bed is composed of two different types of gummies as shown in Figure F.2 to add stiffness and damping properties at the same time. The objective was not completely achieved. Instead, a freely supported condition was obtained for the plate. Nonetheless, this condition facilitates the future modelling of the plate dynamics.

\begin{figure}[h]
\centering
\begin{subfigure}[b]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure_f1a.png}
\caption{(a) Plate with the measurement points.}
\end{subfigure} \hspace{0.05\textwidth}
\begin{subfigure}[b]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure_f1b.png}
\caption{(b) First plate modeshape.}
\end{subfigure}
\begin{subfigure}[b]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure_f1c.png}
\caption{(c) Second plate modeshape.}
\end{subfigure} \hspace{0.05\textwidth}
\begin{subfigure}[b]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure_f1d.png}
\caption{(d) Third plate modeshape.}
\end{subfigure}
\caption{Figure F.1: (a) Plate with the measurement points. (b) First plate modeshape. (c) Second plate modeshape. (d) Third plate modeshape.}
\end{figure}
Figure F.2: Gummies set installed underneath the T-slot metal plate foundation.
Bibliography


