Wave Interaction with Porous Coastal Structures

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$$(EI\nu'')'' = q - \rho A\ddot{v}$$

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Wave Interaction with Porous Coastal Structures

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March 30, 2014
Preface

The present thesis *Wave interaction with porous coastal structures* is submitted as one of the requirements for obtaining the degree of Ph.D. from the Technical University of Denmark. The work was performed at the Department of Mechanical Engineering, Section for Fluid Mechanics, Coastal and Maritime Engineering, under the main supervision of Professor Erik Damgaard Christensen and co-supervisor Professor B. Mutlu Sumer.

The external stay was conducted at University of Texas at San Antonio (UTSA) under the guidance of Assistant Professor Xiaofeng Liu (currently at Penn State University). The help and supervision during this stay, and the continuous collaboration after the return to Denmark, is much appreciated and acknowledged.

A thanks goes to my colleges in our group for any help and discussions during my first more then three years in the group. A special thanks goes to Ph.D. Niels Gjøl Jacobsen, Ph.D. Bo Terp Paulsen, and Ph.D.-student Sina Saremi for many long discussions, sometimes also of relevance for our field of research.

Several M.Sc.-students completed their thesis under guidance of the author and the supervisors. Of relevance for the work presented in the present Ph.D.-thesis, a thanks should be given to Mr. Martin Vistisen who worked on the experimental investigations presented in Chapter 2, and Mr. Morten Ibsen who worked on the numerical modelling presented in Chapter 4.

During the development of the Immersed Boundary Method presented in Chapter 5 some initial tests were performed with the armour unit Xbloc® by Delta Marine Consultants, Netherlands. Drawings and other general information in this relation was provided by Ph.D. Markus Muttray. Although the simulations have not made its way into the present thesis the help and information provided is greatly acknowledged.

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Bjarne Jensen
30th of March 2014
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ABSTRACT

Porous breakwater structures are widely used as protection against waves for ports and harbours as well as for general coastal protection. The structures differ depending on the exact purpose e.g. harbour protection, detached breakwaters, groins, submerged breakwaters etc. Typical types of structures are rubble mound breakwaters and berm breakwaters where common structural elements are core material, filter layers and armour layers. The armour stones serves as the main protection of the filter and core material against wave action. Therefore the armour stones must maintain stable when exposed to waves. The general design methods are based on a long tradition of experimental investigations in scale models. This has resulted in empirical design formulas which in combination with physical model tests during the design phase constitutes the typical approach to breakwater design. Numerical models are also applied as part of investigating and designing breakwaters. The models can provide more detailed information on some topics, such as pressure attenuation through the porous core material, while it is more difficult to simulate the direct destabilisation and movements of individual stones. The present study seeks to extend the methods currently applied to gain more insight into the physical processes involved with armour layer destabilisation. Both experimental and numerical methods are treated.

In Chapter 2 the flow and turbulence around armour layer stones as well as the shear stresses are investigated based on physical experiments. A detailed methodology was applied which takes a different approach than normally seen for breakwater experiments. The physical processes related to generation of turbulence were separated by means of a series of experiments with increasing complexity. Hereby the contribution to generation of turbulence, and destabilizing shear stresses, from the wave breaking, the armour layer, and the porous core was singled out. In Chapter 3 a similar detailed approach was taken towards experimental investigation of the pressure induced forces in the filter layers below the main armour stones. Here it was shown how pressure gradient in the filter layer can contribute to the destabilization of the structure.

In Chapter 4 the numerical approach towards breakwater investigations was treated in terms of resistance type porosity models solved with the Navier-Stokes equations. The method was based on adding the effect of the porous media via the Darcy-Forchheimer equation to the momentum equation. This is a well know method that has been applied for
several decades. A detailed derivation was presented of the volume averaged Reynolds averaged Navier-Stokes (VARANS) equations that forms the basis for the model. With this derivation it was possible to show the origin of the resistance terms which are eventually modelled with the Darcy-Forchheimer equation. The model was calibrated by including several calibration cases that has not previously been applied. Hereby several flow regimes were included giving a better understanding and applicability of the calibrated coefficients. In Chapter 5 the porosity model was extended to be coupled with an immersed boundary method (IBM). This method provides a simple mean of including complex geometries in the numerical model without the need for complicated mesh generation. Hereby parts of the structure, such as the armour layer, can with relative ease be resolved directly. This can provide results such as flow and turbulence around the armour stones and the direct forces on the stones for evaluation of stability. An example was shown with the simulation of a rock toe structure on a rubble mound breakwater. The stones in the toe structure were resolved directly in the model while the rest of the breakwater was included with the porosity model.

In Chapter 6 both experimental and numerical topics are included. The physical experiments includes the first results from the experiments on flow and turbulence around armour stones. These results are presented in greater details in Chapter 2. The numerical simulations includes the flow around the idealised armour layer in terms of spheres which are compared to the measurements from the experiments.
Bølgebrydere er en udbredt konstruktionstype for beskyttelse af havne og terminaler så vel som for generel kystbeskyttelse. Konstruktionerne er opbygget forskelligt afhængig af deres specifikke formål. Typiske bølgebryder-konstruktioner er rubble mound og berm bølgebrydere hvor de gennemgående elementer er kernemateriale, filterlag og ydre dæksten. De ydre dæksten tjener til formål at beskytte filter af kerne materiale mod bølgerne. Det er derfor vigtigt at dækstenene forbliver stabile når de udsættes for bølgepåvirkning.


I Kapitel 2 undersøges strømnings og turbulens samt forskydningsspændinger omkring stenene i de ydre beskyttelseslag ved brug af fysiske eksperimenter. De fysiske processer relateret til generering af turbulens blev blev bestyret ved en serie af eksperimenter med stigende kompleksitet. Hermed var det muligt at vise bidraget til generering af turbulens, og destabiliserende forskydningsspændinger fra bølgebrydning, dækstenene og det porøse kernemateriale. I Kapitel 3 er en lignende tilgang benyttet ved eksperimentel undersøgelse af trykinducerde kræfter i filterlaget under de ydre dæksten. Her blev det vist, hvordan trykgradienter i filterlaget kan bidrage til destabilisering af konstruktionen.

metode kaldet immersed boundary method (IBM). IBM modellen gør det muligt at opløse komplicerede geometrier direkte i den numeriske model uden nødvendigheden at et kompliceret beregningsnet. Hermed kan dele af konstruktionen, f.eks. de ydre dæksten, beskrives direkte i modellen. Det kan give resultater som strømninger og turbulens omkring stenene og en direkte simulering af de kræfter som påvirker de enkelte sten. Tå-beskyttelsen på en bølgebryder blev vist som eksempel, hvor de enkelte sten i tåen er opløst direkte, men resten af bølgebryderen af beskrevet med porøsitetsmodellen.

I Kapitel 6 er både de eksperimentelle og numeriske emner berørt. De første resultater fra de fysiske eksperimenter ang. strømnings og turbulens omkring dækstenene er vist. Disse resultater er vist i større detaljeringgrad i Kapitel 2. De numeriske beregninger vise strømningen omkring de idealiserede dæksten og sammenligner med målingerne fra de fysiske eksperimenter.
CHAPTER 1
INTRODUCTION

The breakwater structure forms a key element in ports and harbour engineering and coastal protection. It is applied to ensure safe and functioning harbours and as a mean to protect coastlines against erosion. The very first harbours were traditionally located in fjords and rivers. Hereby they were sheltered against the most severe wave conditions. This was also the case for the large European ports in the Medieval in e.g. London and Rotterdam. Later port developments in e.g. New York and New Orleans were also naturally protected due to their placement in rivers. During the industrialisation the development went towards larger vessels with a bigger draft. The increasing globally trade continuously increased the need for more transportation by sea and hereby bigger container vessels. The increasing activity within the oil and gas industry during the 70’s and 80’s demanded larger builds of crude oil and LNG carriers. In order to accommodate this the port development needed to follow the same trend. They went towards the coast where some of the biggest breakwater structures were constructed at up to 30-50 m of water depth. Examples of this are the Ciervana breakwater at the port of Bilbao, Spain which extends down to about 30 m and the breakwater at port of Sines, Portugal with a depth of about 50 m.

During several decades the design of breakwaters has been studied experimentally, resulting in design formulas for the various parts of the structure. Although this work has been on-going for several decades the area is still not fully investigated; new and updated formulas are continuously published which are incorporating new parameters and parameter ranges. As described later in this chapter the amount of governing parameters is large which complicates the task of covering the appropriate parameter space in design formulas.

Although there have been a long development of design methods there do exists examples on breakwater failures. In Figure 1.1 the round head and outer part of the western breakwater at Hornbaek harbour in north Sealand, Denmark is shown. The breakwater is made of natural stones (sea stone) up to about 1 t. Damages occurred during a storm in
December 2013 where approximately one meter of the breakwater crest was pushed into the harbour.

Figure 1.1: Damaged breakwater in Hornbaek harbour, Denmark.

Figure 1.2 shows the breakwater at the east coast of the island of Terceira in the Azores. The breakwater is exposed to Atlantic storms. The breakwater was constructed in 1963 and the armour layer was made of 17 t tetrapods. Already during construction the breakwater was damaged and several repairs were conducted during the following three decades. The structure continued to experience failures and in 2004 a new construction phase was initiated with the armour based on CORE-LOC units up to 31 t.

The breakwater at Sines, Portugal is shown in Figure 1.3. Here a 2 km long breakwater on water depths up to 50 m protects the harbour. The main armour layer was constructed by 42 t Dolosse units. In February 1978 a storm destroyed a large part of the 10,000 Dolosse units. Hereby the seaward side of the breakwater was eroded and the super structure was undermined and consequently destroyed. In December 1978 and February 1979 the breakwater was again exposed to severe storms which destroyed the 5,000 Dolosse units that were placed to repair the first damage. According to Burcharth (1987) the wave conditions during the storms should not have caused significant damages to the breakwater.

In the view of the long tradition of experimental investigations following the same methodology it is interesting to raise the question whether we can apply a different approach in order to gain more insight into the failure mechanisms? Not as a replacement for the current experimental work but as an additional mean of describing the physical processes. Also, it is worth considering the role of numerical modelling both in relation to understanding the physical mechanisms and in the design process. These questions are addressed further in Section 1.4. First, a brief overview is given in relation to the geometrical and hydraulic aspects of breakwaters and destabilizing forces. Also the governing
parameters are summarised together with examples of the design formulas typically used for breakwater armour layers.

Figure 1.2: Collapsed breakwater in the Azores.

Figure 1.3: Collapsed breakwater at Sines harbour, Portugal.
1.1 Wave interaction with breakwaters

Breakwaters are constructed with different geometrical characteristics and choice of materials depending on the purpose of the actual structure. Examples of different types and use of breakwaters are compiled in e.g. Jensen (1984), Bruun (1989), and Goda (2010).

Traditionally rubble mound breakwaters and berm breakwaters are seen in relation to ports and harbours but also as a mean for general coastal protection. For example as detached breakwaters parallel to the shoreline as emerged or submerged structures, or as groins placed perpendicular to the shoreline. Breakwaters and similar structures as a coast and shoreline protection are described in Mangor (2004).

As the breakwater structures are applied to provide protection against wave action in a large variety of situations and conditions the layout and design of these structures are not identical from structure to structure. However, there are several common elements which may be found in most of the breakwater structures. Figure 1.4 shows the cross section of a typical rubble mound breakwater.

![Diagram of a typical rubble mound breakwater](after Burcharth (1993))

The inner part is denoted the core which is often made out of quarry run or gravel. This material is not stable towards wave impact itself and must therefore be further protected. This is done by applying an outer armour layer which is the primary protection of the structure against wave action on the sea side. On the rear side an armour layer may be applied as well as protection against overtopping and wave disturbance. The armour layer is made out of rocks of a sufficient size to remain stable against wave attacks. Alternatives to natural rocks are concrete elements such as the Accropod, Tetrapod, Dolos, and Xbloc. To provide a stable foundation for the armour layer units and to ensure that the core material is not washed out through the armour layer, one or several filter layers are placed between the core and the armour. A toe berm may be included to provide a stable foundation for the armour layer.

These typical elements may all be found to some extend and in different combinations in the various porous breakwater structures. A traditional rubble mound breakwater may be designed similar to the shown example while other types e.g. a detached submerged...
offshore breakwater may consist only of larger armour layer rocks. Common for them all is that the stability of the complete structure is closely linked to the capability of the outer armour layer to remain stable and intact when exposed to waves.

1.1.1 Flow sequences

When the porous breakwater is subjected to waves the flow cycle is divided into a number of sequences given as:

1. Wave approach and initial run-up. Possible wave breaking and initiation of inflow towards the core.
2. Run-up and wave breaking.
3. Maximum run-up and inflow. Outflow initiated at the bottom part of the structure.
4. Run-down.
5. Lowest run-down water level and outflow from the core.

During these sequences a number of flow stages are present. In Andersen et al. (2011) the run-up flow was divided into the flow stages as shown in Figure 1.5. Here the flow is seen to change considerably in space and time during the run-up phase. At the lower part of the structure the thickness of the run-up wedge may be several times the roughness of the armour layer. Here the flow has similarities with a rough bottom channel flow. At the upper part of the structure the run-up wedge thickness is less than the roughness which may resemble flow around obstacles as described in Andersen et al. (2011).

![Flow regimes during run-up](Figure 1.5: Flow regimes during run-up. After Andersen et al. (2011)).

Following the same methodology as for the run-up phase the flow stages for the run-down phase are shown in Figure 1.6. Again, the flow is dominated by the upper part of the structure where the wedge thickness is small compared to the roughness and the lower part with a relatively larger wedge thickness. In addition to this a formation of a breaking bore or hydraulic jump can be seen during the run-down as described in Pedersen and Gjevik (1983).
For both the run-up and run-down an additional flow stage can be defined in terms of the porous flow in the core of the structure.

The different flow stages shows the complexity related to evaluating the flow interaction with the structure. Depending on the flow stage different physical processes governs the forces, and thereby the destabilization of the structure.

### 1.1.2 Hydraulic forces on the armour layer

The flow sequences outlined in Section 1.1.1 will give rise to a hydraulic loading acting on the armour stones. The external forces acting on the outer armour layer are mainly governed by the wave induced loading during the run-up and run-down stage. The individual stones will experience a resulting force which decomposes into a drag force acting parallel to the underlying filter or core material and a lift force acting perpendicular to this. The wave induced forces may differ according to the type of wave action. A surging wave may introduce a flow over the armour layer very much governed by run-up and run-down while a plunging wave breaking on the breakwater slope may give raise to maximum loads during short impact periods.

The flow in and out of the porous core may be important for the stability of the outer armour layer. Especially the flow out of the core may contribute to a force acting on the armour stones away from the core. In Hald (1998) this process was exemplified during run-up and run-down. It is described how the out going flow and the associated forces are largest during the run-down where the lowering of the surface elevation inside the core is delayed compared to the surface elevation outside the core causing an outward directed flow. In addition to the forces directly from the flow around the armour stones another destabilizing effect can be generated due to the delay of the flow inside the filter and core material. This delay may cause inward and outward directed pressure gradients in the outer layers of the core or filter. The outward directed pressure gradient will cause a suction effect which will act as a destabilizing force.

Including the forces described above as destabilizing, a force balance may be set-up for a
single armour stone where the stabilizing forces are introduced as the gravitational force and reaction forces from adjacent armour stones. Figure 1.7 shows a schematic representation of the force balance during run-down.

![Diagram](image_url)

**Figure 1.7: Schematic force balance for an armour layer stone. After Hald (1998).**

The forces which will act as destabilizing are given as the drag force, $F_D$, and the lift force, $F_L$. The stabilizing force will be the gravitational force, $F_g$, and also the reaction forces from adjacent stones may act as stabilizing. The flow from inside the core and outwards will add to the drag and lift force depending on the orientation of the outward directed flow. Both the drag and lift force are composed of the contribution from the different flow processes such as boundary layer flow above the armour layer, flow around the individual stones, wave breaking processes, and flow from the porous core. Results from the traditional physical experiments gives the combined effect of these processes in terms of an observation of whether the stones are moving are remains in place.

### 1.1.3 Breakwater failure modes

In Figure 1.4 some typical structural parts of a breakwater were shown. As a results of the very different scales and materials also the type of failures will differ throughout the structure. The failure of breakwater structures are divided into failure modes depending on the type of failure, see e.g. Abbott and Price (1994) and Burchar and Liu (1995). Some examples are instabilities and movements of armour stones, slip failures where larger parts of the armour, filter and/or core material slides downward the slope, settlement of the core and subsoil, and sea bed scour at the toe structure. These failures are summarised in Figure 1.8.

For the present study the relevant failures are mainly direct instabilities and movements of
armour stones and destabilization of the filter layer due to e.g. suction mechanisms. However, the approach presented in this work towards detailed experimental investigations and the numerical modelling concept can be applied for investigations of other failure modes as well.

![Failure modes for rubble mound breakwater](image)

**Figure 1.8:** Failure modes for rubble mound breakwater. After Burcharth and Liu (1995).

### 1.2 Experimental design methods for armour layers

As described in the previous sections the breakwater structure consists of several structural parts that must all remain stable during wave loading. The forces on the structure are governed by a number of both hydraulic and structural parameters. The structural parameters are given as:

- Armour stone diameter, $D_{n50}$
- Armour stone grading, $D_{85}/D_{15}$
- Armour stone roughness
- Armour stone density, $\rho_s$
- Armour layer thickness
- Angle of the slope, $\gamma$
- Crest height
- Porosity, $n$
- Placement of the armour stones
- Internal friction angle between armour and filter layer

Correspondingly a number of governing hydraulic parameters are given as:
• Wave height, $H$
• Wave period, $T$
• Number of waves, $N$
• Type of wave spectra
• Water depth, $h$
• Water density, $\rho_w$

When incorporating all governing parameters into the design process for e.g. the armour layer it becomes a complicated task. As such there exists no general functional relationship that relates the stability of armour layers to the governing parameters. The design of the armour layer is traditionally based on stability formulas derived from physical model experiments. The formulas are derived with the applicability limited to the conditions and parameters investigated in the experiments. This has lead to a large amount of design formulas. However, they all have the same general form as:

$$\frac{H}{\Delta D_{n50}} = Kf(\cdot)$$  \hspace{1cm} (1.1)

where $H$ is the wave height, $\Delta$ is the relative submerged stone density, $D_{n50}$ is the equivalent cube length, $K$ is an empirical coefficient, and $f(\cdot)$ is a function of various parameters.

One of the most referenced stability formula for armour layers is the stability formula by Hudson (1959):

$$\frac{H}{\Delta D_{n50}} = (K_D \cot \gamma)^{\frac{1}{3}}$$  \hspace{1cm} (1.2)

where $K_D$ is an empirical parameter and $\gamma$ is the slope angle. The formula is rather simple and the stability is expressed only as a function of the slope angle. The effect of all other governing parameters, both structural and hydraulic, must be incorporated via the empirical coefficient.

In van der Meer (1988) another stability formula was presented which takes into account several of the governing parameters. The formula was presented for plunging waves as:

$$\frac{H}{\Delta D_{n50}} = 6.2 \xi_0^{-0.5} P^{0.18} \left( \frac{S}{\sqrt{N_z}} \right)^{0.2}$$  \hspace{1cm} (1.3)

and for surging waves as:

$$\frac{H}{\Delta D_{n50}} = \xi_m P \sqrt{\cot \alpha} P^{-0.13} \left( \frac{S}{\sqrt{N_z}} \right)^{0.2}$$  \hspace{1cm} (1.4)

where $\xi_m$ is the surf similarity parameter, $P$ is the permeability factor, $S$ is the damage level, and $N_z$ is the number of waves.
More recent formulas are shown in e.g. Andersen and Burcharth (2010) and van Gent (2013) where prediction methods for a rubble mound breakwater with a berm is given. Also specific structural parts of the breakwater is treated in this way. This is e.g. van Gent and van der Werf (2014) and Muttray (2013) where specific design formulas for a rock toe is presented. These more recent formulas, also including those of van der Meer (1988), includes the effect of both geometrical parameters such as slope angle and permeability, and hydraulic parameters such as the wave condition in terms of the surf similarity parameter. Several empirical parameters are also introduced which shows the limitations of these formulas in terms of the range of data applied for determining these parameters.

1.3 Numerical approach to breakwater design

Numerical methods have been applied in relation to coastal structures and breakwaters for several decades. In particular solutions of the Navier-Stokes equations have been modified to handle the flow in porous media. The actual porous media can not be resolved directly for large structures as breakwaters and as such the effect of the porosity is added to the flow rather then the actual porous media. This effect, in terms of an added resistance, is commonly included as the extended Darcy-Forchheimer equation:

\[ I = a \rho u_i + b \rho \sqrt{u_i u_i} u_i \tag{1.5} \]

where \( \rho \) is the density of the fluid, \( u_i \) is the Cartesian velocity vector, and \( a \) and \( b \) are resistance coefficients. The resistance coefficients must be determined e.g. based on experiments. Often the formulation presented in Engelund (1954) and van Gent (1995) have been applied.

Some examples of numerical models that used this approach is van Gent et al. (1994), van Gent (1995), Troch and De Rouck (1998), and Liu et al. (1999). In Hsu (2002) the volume averaging of the Navier-Stokes equations (VARANS) was introduced in order to derive the porous media equations. Hereby the extra resistance terms were shown however they still needed to be modelled by the Darcy-Forchheimer equation. This formulation was applied in several models such as Garcia et al. (2004) and Lara et al. (2006). In del Jesus et al. (2012) a new version was introduced which also made use of the volume averaging procedure but with some discrepancies compared to the previous versions.

The resistance-type porosity models have a limitation in the information that can be achieved. It is possible to estimate e.g. the armour layer stability based on velocities near the surface of the porous structure. But this requires the knowledge about force coefficients for the drag and lift forces. The details on the forces on individual parts of the structure can not be simulated. Possible destabilization evaluated directly based on the forces are thus not a possibility. With respect to evaluation of stability these type of models does not provide more information than obtained from physical experiments.
One reason for applying numerical modelling is to get more detailed results which can not be achieved with physical experiments. Therefore it is attractive to be able to resolve the flow in the armour layer and thereby simulating the forces directly on each stone. This becomes a complicated task in terms of the model setup for the computational mesh. A random packing of naturally shaped stones may prove to be nearly impossible to resolve with a good quality mesh. This can also be seen in terms of the previous examples of models with this capability. In Latham et al. (2008), Maeno et al. (2009), and Ren et al. (2013) armour stones have been resolved in the simulation. But to overcome the complexity of mesh generation the stones were included via a discrete element modelling (DEM) method. Here the stones are included as solid objects/particles that are not directly resolved with a body-fitted mesh.

1.4 Outline of current work

The work compiled in this thesis covers a number of topics related to wave interaction with porous breakwaters seen from both an experimental and numerical point of view. The following five main chapters includes five papers each covering a specific topic. Some papers have been published, some are under review or will be submitted shortly. The current status is given on the title page before each chapter.

When reviewing the previous work and design methods as briefly summarized above in relation to an experimental and a numerical approach, some questions arises that have formed the basis for the present work.

- Can we design a simple/generic experimental setup that can single out some of the physical processes involved in armour layer destabilization?
- Is it possible to measure the contribution to e.g. turbulence from the individual structural parts?
- What is the magnitude of pressure gradient induced forces in the filter layers? Do they give a significant contribution to the destabilizing forces?
- Can the resistance coefficients for porosity models be determined taking into account the entire parameter space and several flow regimes?
- Can we extend the current porosity model approach to handle a direct resolution of selected parts of the structure with a simple mesh generating routine?
- Can a numerical model provide more detailed information about the loading process on individual stones?

Regarding the experimental approach for investigating breakwater stability and design in general, the typical methodology is focused on a scale model of a complete breakwater including the different parts such as core, filter and armour layer. The model is build as
a direct scaled copy of the prototype structure. The response of the structure when exposed to waves, e.g. the amount of damages is compiled into empirical design formulas as described in Section 1.2. Another experimental approach is to construct an experimental setup that includes the physical processes however not necessarily as a direct copy of the structure of interest. The experimental model may not look like the real life prototype but the physical processes are reproduced while the design of the experiment enables measurements of the physical processes. In this study this approach has been adopted for experimental investigations of breakwaters. The goal was to design the physical experiments in such a way that more details about the physics could be achieved and with the possibility to distinguish the physical processes from each other.

The first two papers (Chapters 2 and 3) covers the experimental investigations of the physical processes related to destabilizing of breakwater armour layers. Detailed measurements of velocity, turbulence, and pressure were performed. In Chapter 2, the flow, turbulence and shear stresses at the armour layer was described. The experiments included a smooth impermeable bed, a rough impermeable bed, and a porous bed. With these different setup it was possible to single out some of the physical mechanisms related to generation of turbulence and the associated shear stresses. In Chapter 3, detailed measurements of pressure gradients below the armour layer are presented. Here it was shown how the outward directed pressure gradients in the core or filter layers contributes to the destabilizing of the armour layers. Also shear stresses were derived from measurements of turbulence.

For a numerical handling of breakwaters there are examples of model developments during the last three decades. As described in Section 1.3 the most common approach is the resistance type models incorporated in the solution of the Navier-Stokes equations. For a practical engineer this large amount of models may create some confusion as some differences appears in terms of the model formulation. Also calibration of the resistance coefficients have generally been done based on a limited amount of calibration cases. A good understanding of the variations of the parameters for coastal engineering is needed.

The first part of the numerical work related to resistance type models is presented in Chapter 4. Here the focus is on the detailed derivation of the VARANS equations making use of the volume averaging theorem which is generally not presented in previous publications. The implemented model was applied for investigating the variation of the resistance parameters covering the entire parameter space. Several calibration cases were introduced in order to cover several flow regimes. In Chapter 5, a numerical method referred to a the immersed boundary method (IBM) is introduced. The method makes it possible to include complex geometries, such as an armour layer, in a computational mesh without the need for complicated mesh generation. The model was implemented and coupled to the porosity model presented in Chapter 4. With this new coupling the hybrid approach was proposed for simulating coastal structures. An example was given where the toe structure of a breakwater was resolved with the immersed boundary model while the main breakwater was modelled by the porosity model.

In Chapter 6 both experimental and numerical topics are included. This chapter presents
the contribution to the International Conference on Coastal Engineering, 2012. The experimental results deals with the first outcome of the experiments on flow and turbulence around the armour layer which are presented in more details in Chapter 2. The numerical part presents a detailed simulation of the flow around the spherical roughness elements applied as a generic armour layer. The results were compared to the measured velocities above and in-between the spheres.

1.4.1 Remarks on future work

In relation to the above mentioned work some remarks are given in the following regarding the current status and future work.

The experimental investigations took a different approach then typically applied for breakwater structures. Some details were shown on the wave interaction with rubble mound breakwaters and in particular the armour layer. The same methodology can be applied to investigate many other geometrical and hydraulic effects. Regarding the results presented in the present thesis some future topics could be effects of irregular waves, variation of slope angle, effect of thickness of armour layer etc. Also other parts of the structure could be of interest. Currently an on-going project seeks to extend the results on pressure induced forces and shear stresses (Chapter 3) to include a toe structure. Here the same methodology is applied for detailed measurements of pressure and flow in the toe.

The hybrid modelling concept with the combination of the porosity model and the immersed boundary model is an attractive way of getting more details from the numerical models than possible from a porosity model only. The possibility to only resolve those details which are of interest with the IBM model gives a more efficient approach. The simplicity in terms of grid generation makes the method useful not only for research purposes but also for more practical engineering tasks. The present work coupled the IBM model with the porosity model including the free surface VOF formulation. An example of the possible extend of the model was shown by simulating a rock toe structure. Future work in this relation will be to further validate the model in terms of detailed force and pressure measurement on both idealised (cylinder, sphere) and real armour stones placed on filter and core material. Also, a planned extension of the present model includes six-degree-of-freedom rigid body motion module for simulation of actual movements of armour stones due to the hydrodynamic forces.

1.5 References


CHAPTER 2

FLOW AND TURBULENCE AT RUBBLE MOUND BREAKWATER ARMOUR LAYERS UNDER SOLITARY WAVE

Submitted for publication as:
Flow and turbulence at rubble mound breakwater armour layers
under solitary wave

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Abstract
This paper presents the results of an experimental investigation of the flow and turbulence at the armour layer of rubble mound breakwaters during wave action. The study was focused on the details of the flow and turbulence in the armour layer and the effect of the porous core on the flow and stability. In order to isolate the processes involved with the flow in the porous core the experiments were conducted with increasing complexity and hereby adding the different physical process to the experiments. Following this methodology, three parallel experiments were performed: 1) a rigid-bed experiment with a smooth sloping bed, 2) a rigid-bed experiment with large roughness elements added to the sloping bed, and 3) a porous bed experiment where the porous core was added below the sloping bed. In this paper the focus is on the details of a single cycle of wave approach, run-up, and run-down. In order to isolate this wave cycle the experiments were performed applying a solitary wave. The individual sources of generation of turbulence were distinguished and the effect of the armour layer and porous core was described in terms of a reduced impact of the run-down process, production of lee wake turbulence, and less transport of turbulence up above the armour layer. The shear stresses were evaluated from the measurements of turbulence and they were associated to the run-up and run-down phases. The Shields parameter, determined from the shear stresses, was found to be reduced by 30 % as a result of the porous core material.

Keywords: Wave-structure interaction, Porous flow, Turbulence, Shear stresses, Breakwater stability, Model scale experiments

1. Introduction

The rubble mound breakwater structure is used within coastal and harbour engineering for providing sheltering against offshore wave action. Often the breakwater structure is designed as a porous structure which allows water to flow through the structure while the wave energy is removed. The internal porous core is made of gravel materials with specific gradations. To protect these rather fine grained internal layers from being eroded by the waves one or several layers of larger stones are placed on top of the core material. These are referred to as cover or armour stones.

It is a common practice to evaluate the stability and general functioning of breakwater structures by means of physical experiments. This can be seen for example in Jensen (1984). The experiments often includes a complete scale model of the breakwater structure including core material, filter layers, and armour stones. The response of the structure is observed as function of the applied wave conditions. With this methodology the design formulas have been derived which are used for practical engineering design. Regarding stability of armour layers this includes e.g. the formulas of Hudson (1959), van der Meer (1987), and the general guidelines given in CIRIA et al. (2007).

Recent examples on such experiments are Comola et al. (2014) where stability of a breakwater roundhead
The run-up and run-down process. Examples are Grilli et al. (1994) where solitary wave breaking induced by a breakwater was investigated, Sumer et al. (2011) who studied flow and sediment transport due to a plunging solitary wave, and Lara et al. (2012) who applied a solitary wave for investigating wave interaction with a breakwater both experimentally and numerically.

It is noted that two processes in ordinary oscillating waves were missing in the present idealized solitary wave case, namely the process controlling the wave setup and the process controlling the water table in the porous core of the structure.

The present experimental work focused on the details of the armour layer flow and the effect of the porous flow in the core in relation to turbulence and stability. In order to isolate the processes involved with the flow in the porous core the experiments were first carried out with a completely impermeable bed, both with and without roughness elements on the bed, and subsequently repeated with a porous bed (porous core material).

The present paper is a detailed continuation of the work presented in Jensen et al. (2012) where some first results were presented. It also follows the same detailed methodology as applied in Jensen et al. (2014) where the focus was on pressure induced forcing in the core material.

The paper is organized as follows: The experimental setup and instrumentation are presented in Section 2. Section 3 presents the applied wave condition. A general description of the run-up and run-down process is given in Section 4. The results in terms of velocities and turbulence are presented in Section 5 and 6. In Section 7 the Shields parameter, the parameter characterising the mobility of the armour stones, is discussed. Finally, conclusions are given in Section 8.

2. Experimental setup

All tests were carried out in the wave flume referred to as Flume No. at the hydraulic laboratory at the Technical University of Denmark (DTU). The flume has a length of 25 m, a width of 0.6 m, and a depth of 0.8 m. The water depth for the present experiments was fixed at 0.4 m. The flume is equipped with a piston-type wave maker in one end. At the general testing area the sides of the flume is made of transparent glass, which enables a visual observation of the experiments as well as Laser Doppler anemometry (LDA) measurements from...
the side. An overview of the entire flume setup is shown in Figure 1.

The front and rear slope of the breakwater were arranged with a slope of 1:1.5. For the rigid bed experiments (smooth and rough bed) the sloping bed was made out of a plastic PVC plate with a thickness at 20 mm. The width corresponded to the width of the flume at 0.6 m. The plate was fixated both at the top and the bottom of the flume to ensure that there was no movements of the bed during wave action. The small gap between the PVC plate and the side and bottom of the flume was sealed with silicone filler to ensure that no water exchange took place between the front and rear side of the slope.

For the rough bed experiments the PVC plate was covered with one layer of spherical plastic elements with a diameter of $d = 38$ mm. The spheres were glued to the bed in a $90^\circ$ arrangement as shown in Figure 2.

For the porous bed experiments the core material was made of the same spherical plastic elements used for the armour layer. A cage was constructed with a perforated steel plate to hold the interior plastic spheres in place. The perforated plate substituted the impermeable PVC plate. The plate had a thickness of 2 mm. The perforations were made as quadratic voids with dimension of 1 cm and a void-to-plate ratio (porosity) of 0.41. This corresponded approximately to the porosity of the core material of $n = 0.4$. The armour layer for the porous bed experiments was identical to that applied for the rough bed experiments as the plastic spheres were glued to the perforated steel plate in the same arrangement.

Two types of measurements were performed: velocity measurements and surface elevation measurements. Measurements of velocities and turbulence were performed with LDA. A DANTEC two-component LDA system was applied in back-scatter mode where the two velocity components, horizontal and vertical, were measured simultaneously. The two velocity components were converted to a bed parallel and normal direction during the subsequent data-processing. The sampling frequency of the measurements was 120 Hz. The arrangement of the LDA system is shown in Figure 2. Two vertical profiles normal to the sloping bed were measured (section I-II) with measuring points distributed both below and above the surface of the armour as shown in Table 1. The exact vertical position of the measuring points differed depending on the possibilities to direct the LDA laser beams into the pores between the armour layer roughness.

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The surface elevation measurements were performed at two locations; one at an offshore location and one at the toe of the sloping bed as shown in Figure 1. The wave gauge at the toe of the breakwater was applied as reference between all experiments and corresponding video recordings. Conventional resistance type wave gauges were used in the measurements. The sampling frequency of the measurements was 120 Hz. The LDA measurements were synchronized with the surface elevation measurements at the toe of the breakwater.

In addition to the above, synchronized flow visualization were performed using a digital video recorder applying 200 frames per second. From here the detailed observations are drawn of the entire process of run-up, run-down, breaking and trailing waves.

3. Test conditions

All experiments were performed with a solitary wave. The wave height was $H = 0.14$ m. The undisturbed offshore surface elevation is given by the small-amplitude solitary wave theory as:

$$\eta = H \text{sech}^2(\omega t)$$  \hspace{1cm} (1)

where $H$ is the height of the solitary wave measured from the still water level, $t$ is time, and $\omega$ is given as:

$$\omega = \sqrt{\frac{3}{4} g H \frac{1}{h}}$$  \hspace{1cm} (2)
where $g$ is the gravitational acceleration and $h$ is the still water depth. Similar to sinusoidal waves a time scale can be defined by:

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{4}{3gH}} \frac{h}{H}$$

which can be interpreted as the time scale characterizing the width of the surface elevation time series (SUMER et al. (2011)). This quantity was $T = 2.48$ s in all experiments.

The number of realizations for each measuring point (for ensemble averaging) was 30 for the impermeable rough bed experiments and the porous bed experiments while it was 10 for the smooth bed experiments. A sensitivity analysis carried out indicated that the statistical quantities, the mean values and the standard deviations, converged to constant values for these sample sizes.

The waves were found to be reproducible. Twenty arbitrary selected time series of the surface elevation at the toe of the sloping bed were plotted together. Here it was seen that they collapsed on a single curve which confirmed the repeatability of the generated wave. This ensured that no artificial fluctuations were introduced during the ensemble averaging process.

Regarding the characteristics of the solitary wave the breaking criterion given in GRILLI et al. (1997) can be applied. Here the breaking was defined based on the slope parameter, $S_0$, defined by:

$$S_0 = 1.521 \frac{s}{\sqrt{H/h}}$$

where $s$ is the slope angle. The breaking types are characterized in GRILLI et al. (1997) as spilling ($S_0 < 0.025$), plunging ($0.025 < S_0 < 0.30$), and surging ($0.30 < S_0 < 0.37$). For the present experimental conditions the slope parameter was $S_0 = 1.71$ which was outside the defined breaking criterion thus giving a reflecting wave. This is due to the steep sloping bed. As will be seen later a breaking bore/hydraulic jump formed during the rundown.

4. Description of the run-up and run-down process

The cycle of approach, run-up and run-down is described by means of high speed video recordings. Here
the surface elevation is schematically presented for a number of relevant time steps during the process. The times given in the following were synchronised with the wave crest at the toe of the slope (wave gauge no. 2) corresponding to time $t = 0$ s. The entire cycle is divided into the following four stages, i) approach, ii) run-up, iii) run-down and, iv) secondary run-up.

Figure 3 shows the entire sequence of run-up, run-down and trailing wave with secondary run-up for the smooth bed experiments. The wave has already been characterised as being reflective i.e. no breaking takes place during the run-up. In Grilli et al. (1997) and Jensen et al. (2003) the run-up phase was divided into several flow stages depending on either the steepness of the sloping bed or the amplitude of the wave. Here it was described how the run-up was smooth with a low steepness of the surface for a reflecting solitary wave. This was also confirmed by the present experiments where the wave was seen to creep up the sloping bed. In the beginning of the run-up phase the flow was comparable to a smooth bed channel flow with large water depth. Towards the end of the run-up phase a wedge was formed at the upper part of the slope where the water depth decreased until only a thin water film reached the most upper part of the sloping bed.

Following maximum run-up the flow reversed and initiated the run-down phase. The upper part of the slope experienced a flow where the relatively low water depth was maintained. Indications of a hydraulic jump was seen as the transition between the upper part of the flow (low water depth, supercritical flow) and the lower part (higher water depth, subcritical flow). At the end of the run-down phase the downward directed flow interacted with the volume of water above the lower part of the slope which generated a breaking bore. The breaking process on the run-down was also described in Jensen et al. (2003) and was shown numerically and experimentally in Pedersen and Gjevik (1983).

In figure 4 the entire cycle for the rough bed experiments is shown. In general the same flow regimes were seen as for the smooth bed experiments, however some remarks can be made regarding the effect of the armour layer roughness elements on the bed. The front of the upper surface wedge showed a highly disturbed and turbulent flow due to the flow around the roughness elements. The upper part of the flow did not continue to the point of a thin surface film but remained with a water depth in the order of the diameter of the roughness. Furthermore the front part of the flow generated an aeration zone where a large amount of air was trapped and released from in-between the roughness elements as the surface front moved up along the slope. For the run-down phase the effect of the armour layer roughness was also seen in terms of a slightly reduced run-down level as well as larger air entrainment. The reduced run-down level and flow around the roughness elements caused the breaking bore to be less pronounced.

For the porous bed experiments the run-up and run-down are presented in Figure 5. The porous media flow is now included as seen by the surface elevation in the porous core. The run-up and run-down flow at the front...
of the breakwater was affected by the porous core as the water was allowed to flow into the structure. During run-up the delay of the water surface inside the porous core caused a higher water level outside the structure which gave a flow directed into the porous core. During the first part of the run-down process the flow was still directed into the core. Later the delay of the water surface in the porous core resulted in a higher water level inside the core which created the opposite situation where the flow was directed out of the porous core. In e.g. van der Meer and Stam (1992) this effect of the porous core was also described in the same manner in relation to the run-up level.

Due to the interaction with the porous flow in the core the run-down was reduced compared to the smooth and rough bed experiments. The breaking bore during run-down and the subsequent secondary run-up was practically removed.

5. Velocity

Further to the overall description given in Section 4 the process was investigated in details in terms of velocity measurements performed with LDA according to the layout given in Figure 2. The measurements presented in the following were synchronised with the wave crest at the toe of the slope (wave gauge no. 2) corresponding to time $t = 0$ sec. A reference signal is shown in the figures in terms of the surface elevation at the toe of the breakwater.

Figures 6 and 7 present the velocity time series for the component parallel and normal to the bed at measuring section I, respectively. All three series of experiments, smooth, rough, and porous bed, are collected in the same figures. Two measuring points; 2 mm and 57 mm (19
For the smooth bed the velocities were rather constant over the depth. At the point farthest away from the bed the entire run-up and run-down cycle was very little affected by fluctuations. Close to the bed the run-up and for the largest part of the run-down the tendency was the same in terms of very little fluctuations. At the maximum run-down level at $t \approx 1.2s$ some fluctuations were seen to be present close to the bed. This is interpreted to be due to the turbulence generated by the breaking bore during run-down as described in Section 4. It is noted that the velocity experiences a phase lead compared to the surface elevation. The measurements were taken close to the bed and as such they reflect the bed shear stress which will also have a phase lead. In Sumer et al. (2010) and Liu et al. (2007) the phase lead for solitary wave boundary layer flow in the laminar flow regime was shown to be $\approx 20^\circ$. For the present experiments the phase lead was found to be in the range of $19^\circ - 20^\circ$.

For the rough bed experiments the bed parallel velocity component in the free stream showed the same variation over time and order of magnitude as for the smooth bed experiments. However, fluctuations appeared from the beginning of the run-up phase and throughout the entire wave cycle. Close to the bed in between the spherical plastic elements the velocity was highly effected by the presence of the spheres which was also seen in terms of a high level of fluctuations throughout the wave cycle. The free stream fluctuations as well as the fluctuations around the spheres indicates one or several contributions to the turbulence during the run-up and partly the run-down phase which can be accounted for by the roughness elements.

When the porous core was added to the experimental setup the flow was directed inward and outward of the porous core. As described in Section 4 the run-down flow did not reach the same maximum run-down level as for the impermeable bed experiments and there were only very little secondary run-up. This was seen from the velocity measurements as well in terms of a reduction in the bed parallel velocities during run-down as presented in Figure 6. The bed normal velocities showed how the flow was directed slightly inward and outward of the porous core during run-up and run-down.

6. Turbulence

The turbulence between the armour layers was assumed to be generated by four sources given as:

1. Boundary layer turbulence generated in the boundary layer above the armour layer and subsequently transported down between the armour stones.
2. Wake turbulence generated behind the armour stones.
3. Wave breaking turbulence.
4. Seepage/outflow turbulence from the flow exiting the porous core.

The mechanisms for generation of turbulence can be associated to the different sequences of the wave flow cycle. At the lower part of the structure the water depth is up to several times the armour layer roughness. Here the flow resembles a rough bottom channel flow where a turbulent boundary layer will develop as described in e.g. Grass (1971) for steady current. For oscillating
A wave boundary layer will develop as described in Jensen et al. (1989) and Dixen et al. (2008). At flow reversal from run-up to run-down and during run-down the boundary layer turbulence may be transported down between the armour layer stones.

For the flow in the upper part of the run-up wedge the water depth is relatively small compared to the roughness of the armour layer and the generation of turbulence will mainly take place in-between the armour layer stones as wake generated turbulence. During run-down an additional contribution is seen in the case where a breaking bore is formed as described in Pedersen and Gjevik (1983). This may also be interpreted as a hydraulic jump. The generated turbulence from this process can be transported down to the armour layer stones as the breaking bore moves downwards the sloping bed.

Finally the outflow from the porous core will contribute to the production of turbulence in the pores between the armour layer stones. The porous flow can be of a laminar type, transitional between laminar and turbulent or fully turbulent as described in Burcharth and Andersen (1995).

The level of turbulence was analysed based on the velocity measurements. The turbulence was quantified by the root-mean-square (RMS) value of the fluctuating component of the velocity, \( u' = u - \bar{u} \) calculated by:

\[
\sqrt{u'^2(t)} = \left\{ \frac{1}{N-1} \sum_{i=1}^{N} (u_i(t) - \bar{u}(t))^2 \right\}^{1/2}
\]

where \( t \) is the time, \( N \) is the number of samples (realizations), \( u \) is the instantaneous bed-parallel velocity, and \( \bar{u} \) is the mean velocity found by ensemble averaging over \( N \) samples.

The results presented in the following were synchronised with the wave crest at the toe of the slope (wave gauge no. 2) corresponding to time \( t = 0 \) s as were also the case for the velocity and the general description given in Section 4. The time variations are presented in terms of four selected measuring points distributed over the depth. For all time series the surface elevation at the toe of the breakwater is shown as a reference signal. Also the time of maximum run-up level and maximum run-down level is indicated for each experiment.

For the smooth bed experiments Figures 8 and 9 present the RMS values for the velocity component parallel and normal to the bed, respectively, at measuring section I. Here it is clear how the breaking bore formed...
during the run-down phase generated a large peak in the turbulence at the time when the maximum run-down occurred (A in Figures 8 and 9). Hereafter the turbulence level gradually decreased. The contribution to the generation of turbulence from the run-down breaking bore was seen as a very isolated phenomenon in these measurements.

The turbulence levels for the rough bed experiments presented in Figures 10 and 11 shows the effect of the armour layer roughness. The turbulence production was initiated from the beginning of the run-up phase (B in Figures 10 and 11) and increased gradually until the point where the breaking bore during the run-down phase generated a peak increase in the turbulence level (A in Figures 10 and 11). During run-up, fluctuations were seen both in the free stream as well as in the pores. In the free stream above the roughness elements the run-up turbulence developed to a smaller level than in-between the roughness. Above the roughness (B in Figures 10 and 11) the turbulence was mainly generated as boundary layer turbulence over a rough bed. It is noted that this turbulence do not correspond to a fully developed boundary layer turbulence due to the limited distance from the toe to the measuring section. Below the top of the rough-
ness a larger turbulence level was seen (C in Figures 10 and 11). Here the effect of lee wake turbulence from the roughness elements was added to the total level of turbulence. Also it was clear how the turbulence levels drops momentarily at the point of flow reversal from run-up to run-down.

From Figures 10 and 11 it was found that the boundary layer turbulence above the armour layer roughness had a level that corresponded approximately to 25% of the turbulence produced by the run-down breaking process. The turbulence in-between the armour layer roughness reached values of the same size as for the run-down breaking process.

In Figures 12 and 13 the turbulence production for the porous bed experiments are presented. A comparison with the rough bed experiments in Figures 10 and 11 shows the effect of the porous core material. Above the armour layer roughness, during the run-up phase and partly the run-down phase, the turbulent intensity was constant at a very low value. This is in contrast to the rough bed experiments where the turbulence increased during run-up. The low turbulence during run-up for the porous bed experiments were due to the flow being directed into the core material as described in Section 4. Hereby the turbulence was also directed into the core and was not allowed to diffuse up into the free stream. At the last part of the run-down phase the flow was directed outwards from the core material and hereby also turbulence was moved up above the armour layer roughness (A in Figures 12 and 13).

Below the upper surface of the armour layer roughness the production of turbulence follows what was seen for the rough bed experiments. An increasing turbulent intensity was found during run-up caused by the lee wake turbulence from the roughness elements (B in Figures 12 and 13). At this point the turbulence reached a slightly higher level than for the rough bed experiments due to the boundary layer turbulence above the armour layer being transported down between the roughness elements. This was caused by the inward directed flow as previously described.

It was also found that the sudden drop in the turbulence level after flow reversal from run-up to run-down (C in Figures 12 and 13) was even more pronounced for the porous bed experiments than for the rough bed experiments.

Figure 10: Time variation at Section I of the RMS values of the fluctuating component of the velocity component parallel to the bed on the rough bed. The surface elevation at the toe is shown as a reference signal in the top panel. The time for maximum run-up level and maximum run-down level is shown in the top panel.

7. Shear stress and Shields parameter

The stability of sediment, gravel, and stones exposed to current and/or waves may be evaluated based on the Shields parameter defined in e.g. Fredsøe and Deigaard (1992) as:

$$\theta_c = \frac{U_{fc}^2}{(s - 1)gd}$$ (6)

where $\theta_c$ is the critical Shields parameter for movement of the stones, $U_{fc}$ is the critical friction velocity, $s$ is the relative density of the stones, and $d$ is the stone size.

The critical Shields parameter expresses the critical
value of the destabilizing fluid forces to the stabilizing forces. The destabilizing forces are expressed via the friction velocity while the gravitational acceleration acts as the stabilizing force. Eq. (6) is given for a flat bed. If the bed is sloping an additional term arises in the force balance between stabilizing and destabilizing forces. That is an additional destabilizing force due to the gravitational acceleration being projected into the bed parallel and normal direction. Furthermore, the stabilizing gravitational force is also reduced due to this projection. This gives a correction to the critical Shields parameter on a flat bed (Fredsøe and Deigaard (1992)):

$$\theta_c = \theta_{c0} \cos \gamma \left[ 1 - \frac{\tan \gamma}{\tan(\phi_s)} \right]$$

(7)

where $\theta_{c0}$ is the critical Shields parameter on a flat bed, $\gamma$ is the slope angle of the bed, and $\phi_s$ is the friction angle (angle of repose) of the stones.

The critical Shields parameter depends on the grain $Re$-number and is described by empirical data. For fully rough beds, namely for stone $Re$-numbers, $Re = dU_f/\nu$, larger than 70, the critical value is $\approx 0.06$ as shown in e.g. Fredsøe and Deigaard (1992). Hereby it is assumed that
there is a clear difference between stationary and moving sediment/stones. However, this is not the case due to variations in e.g. bed shear stress and stone size. In Breusers and Schukking (1971) experiments showed that movements of some stones could occur at \( \theta_c = 0.03 \). In CIRIA et al. (2007) it is recommended for design of armour stones and rock fill that the critical Shields parameter takes the value \( \theta_c = 0.03 - 0.035 \) at which stones first begin to move. For the flow along a sloping front of a breakwater the correction of the critical Shields parameter given in eq. (7) was applied. The friction angle is depending on the material. Assuming that there exists an interface between two materials; the armour layer and the core material, the smallest friction angle of the two materials may be used according to CIRIA et al. (2007). If the porous core is assumed to be constructed of typical gravel material the friction angle can be set to \( \phi_s = 45^\circ \). With a slope of the bed of 1:1.5 the corrected critical Shields parameter reduces to a value of \( \theta_c \approx 0.01 \).

The friction velocity, \( U_f \), was found based on the Reynolds-stresses as described in Dixen et al. (2008). In the latter study the friction velocity on a rough bed was determined based on both velocity profiles and Reynolds-stresses above the bed. The results of the different methods showed a very good agreement when compared. As such the method based on the Reynolds-stresses was adopted in the present study to evaluate the friction velocity and thereby the Shields parameter. The friction velocity was defined as:

\[
U_f = \sqrt{\frac{\tau_0}{\rho}}
\]

(8)

where \( \tau_0 \) is the bed shear stress found from the Reynolds-stresses as \( -\rho u'v' \).

Figure 14 present an example of a Reynolds-stress time series for the porous bed experiment at the measuring point at 36 mm from the bed. Two distinct events are seen with positive and negative Reynolds-stresses, respectively. During run-up the Reynolds-stresses increased to a positive level (A in Figures 14). At flow reversal the stresses dropped to zero which coincide with the corresponding drop in turbulence intensity seen in Figures 12 and 13 (point C). During run-down the stresses increases again, now to a negative level. This temporal variation also defines the sign convention for the Reynolds-stresses when compared to the velocity time series in Figures 6 and 7. During run-up when the Reynolds-stresses were positive the flow was directed upward the slope and slightly inward to the core. During run-down with negative Reynolds-stresses the flow was directed downward the slope and slightly outward from the core. With respect to destabilizing forces on the armour layer the negative Reynolds-stresses are of interest for the present case due to the outward directed flow (lift forces). However, both the run-up and run-down was dominated by the bed parallel flow for the present measuring sections and as such also the positive Reynolds-stresses are of interest in terms of drag generated destabilization.
Figures 15 and 16 presents the profile of maximum and minimum Reynolds-stresses at measuring section I. The distribution follows the results of e.g. Dixen et al. (2008) and Jensen et al. (2014) with an increasing shear stress just below the top surface of the armour layer roughness elements. In Jensen et al. (2014) the Reynolds stresses from the present experiments with the rough (impermeable) bed for the run-up phase were compared with the results of Dixen et al. (2008). Here the Reynolds stresses were found to be of the same magnitude which served as a validation of the present measurements.

From Figures 15 and 16 the maximum values were found to be $-u'v' = 106 \text{ cm}^2/\text{s}^2$ and $-u'v' = 75 \text{ cm}^2/\text{s}^2$ for the rough bed and porous bed experiments, respectively. Applying a relative density of $s = 2.65$, a stone diameter of $d = 3.8 \text{ cm}$, a density of water of $\rho = 1000 \text{ kg/m}^3$, and inserting in eq. (6) and (8) the maximum Shields parameter was found to be $\theta = 0.0172$ and $\theta = 0.0122$ for the rough bed and porous bed, respectively. Here it is noted that the Shields parameter is in the same range as the critical value for stone movement as described previously. As such the experimental conditions describes the physics for a possible destabilizing event. When the rough bed and porous bed experiment is compared it is seen that the effect of the porous bed resulted in a reduction of the Shields parameter of $\approx 30\%$.

The effect of the porous core material in terms of reduced loading on the armour stones was also noted in e.g. van der Meer (1987). In general, design formulas for breakwater stability given in e.g. CIRIA et al. (2007) predicts an increasing stability for increasing porosity of the core. This is the case for the formulas given by e.g. van der Meer (1988) and Hudson (1959). The design formulas contain a damage coefficient, $K_D$. For increasing values of $K_D$ the needed stone size for a given wave condition is decreased. In CIRIA et al. (2007) the values for $K_D$ is recommended to be $K_D = 1$ for impermeable cores and $K_D = 4$ for permeable cores corresponding to a reduction in the needed stone size (or the destabilizing loading) of $75\%$. These values were based on the results in Hudson (1959) which were given irrespective of the type of wave breaking and may as such need corrections depending on the exact breaker type. However, they give an impression of the effect of the porous core material in relation to destabilizing forces, as also seen in the present experiments. In Reedijk et al. (2008) details on the effect of the permeability of the core on armour stability was compiled. Here the effect was also reported to be in the same order of magnitude in terms of possible reduction in stone size for an permeable core compared to an impermeable core.

8. Conclusions

Three parallel experiments were performed in order to show the details of the flow and turbulence in rubble mound breakwater armour layers. In particular the effect of the porous core material was of interest. The experiments included a smooth impermeable bed, a rough
(armour layer) impermeable bed, and a porous (core material) bed. The experiments were performed with a solitary wave in order to isolate the processes of one cycle of run-up and run-down. The following conclusions were drawn.

1. The run-down phase and the breaking bore was the governing mechanism for generation of turbulence on the smooth bed. This mechanism was also present for the rough bed. For the porous bed this effect was found to be small compared to the other mechanisms of generation of turbulence (boundary layer and lee wake turbulence) due to the reduction of the run-down and secondary run-up.

2. For the rough bed experiments, wave boundary layer turbulence was generated above the armour layer roughness during run-up. It was found that the boundary layer turbulence had a level that corresponded approximately to 25% of the turbulence produced by the run-down breaking process.

3. The turbulence generated between the armour layer roughness (not from the porous core) was of the same magnitude as the contribution from the run-down breaking process.

4. With the porous core the flow was directed inward to the porous core which limited the transport of turbulence up into the boundary layer above the armour layer.

5. The porous core reduced the Shields parameter for the armour layer, found from the shear stresses, with 30% compared to the impermeable rough bed.

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The smooth bed and part of the porous bed experiments were performed by Mr. Martin Vistisen under guidance of the authors.

References

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CHAPTER 3

PRESSURE-INDUCED FORCES AND SHEAR STRESSES ON RUBBLE MOUND BREAKWATER ARMOUR LAYERS IN REGULAR WAVES

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Pressure-induced forces and shear stresses on rubble mound breakwater armour layers in regular waves

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Abstract
This paper presents the results from an experimental investigation of the pressure-induced forces in the core material below the main armour layer and shear stresses on the armour layer for a porous breakwater structure. Two parallel experiments were performed which both involved pore pressure measurements in the core material: (1) uniform core material with an idealized armour layer made out of spherical objects that also allowed for detailed velocity measurements between and above the armour, and (2) uniform core material with real rock armour stones. For both experiments, high-speed video recordings were synchronized with the pressure measurements for a detailed investigation of the coupling between the run-up and run-down flow processes and the measured pressure variations. Outward directed pressure gradients were found which exerted a lift force up to ≈ 60% of the submerged weight of the core material. These maximum outward directed pressure gradients were linked to the maximum run-down event and were in general situated at, or slightly below, the maximum run-down level. Detailed velocity and turbulence measurements showed that the large outward directed pressure gradients in general coincide, both in time and space, with the maximum bed-shear stresses on the armour layer based on the Reynolds-stresses. The bed-shear stresses were found to result in a Shields parameter in the same order of magnitude as the critical value for movement of the armour stones.

Keywords: Rubble mound breakwater, Porous flow, pressure-induced forces, Bed-shear stresses, Model scale experiments

1. Introduction
Breakwater structures are a central part of coastal protection and harbour engineering. These structures prevent coastal erosion and ensures safe and functioning harbours. As such it is of high importance that the these structures remain stable under severe wave action. In this paper the destabilizing effect, in terms of the pressure-induced lift forces on filter layers and shear stresses on armour layers, is investigated for rubble mound breakwaters.

It is a common procedure to evaluate the stability and general functioning of breakwater structures by means of physical experiments. This can be seen for example in Jensen (1984). However, there has been a great deal of focus on the overall stability of the structure while the physical background for failure mechanisms may not be seen in details. The experiments often includes a complete scale model of the breakwater structure including core material, filter layers, and armour stones. Recent examples on such experiments are Kramer et al. (2005), Burchardt et al. (2006) Andersen and Burchardt (2009), Andersen and Burchardt (2010), van Gent (2013) and van Gent and van der Werf (2014) where stability and overtopping were investigated. In addition to stability and overtopping also the flow through the porous material of the structure were studied e.g. in terms of pressure distributions through the breakwater. Recent examples of...
this are Muttray and Oumeraci (2005) and Vanneste and Troch (2012).

The stability is evaluated based on the observed damages during the experiments. The details on the failure mechanism such as the porous flow and the armour layer flow is not investigated in these types of experiments. In general the porous structure is seen as a black box where the details of the flow and loading processes is not measured and described. Examples of a more detailed approach is seen in Tørum (1994) and Moghim and Tørum (2012) where forces were measured on spherical armour units and armour stones on a sloping breakwater front in laboratory scale. In Hald (1998) forces were measured on real armour stone also in laboratory scale. These studies focused on the response as function of the incoming wave condition whereas details on velocities and turbulence in the armour layer as well as the porous flow were not investigated.

Several different physical processes contribute to a destabilisation of the breakwater structure. Failures are categorised in different failure modes depending on the type of failure and physical process involved in the failure. Details on failure modes were shown in e.g. Abbott and Price (1994) and Hald (1998). Relevant failure modes for the present investigations are rocking of armour stone, lift and drag forces in combination with rocking, sliding of filter and underlayers, sliding and/or scouring of toe protection. For the main armour stones the drag and lift forces directly caused by the flow around the stones play an important role for the destabilising forces. For sliding and suction mechanisms the filter and underlayers are critical to ensure a stable structure. One possible physical process that may contribute to suction/lifting of the filter layer is outward directed pressure gradients in the outer part of the filter layer.

In both Sumer et al. (2011) and Sumer et al. (2013) the pressure gradients in a porous sloping bed were investigated for solitary and regular waves, respectively. In these studies a mildly sloping bed was investigated and no armour stone covered the sand bed. However, it was clear how pressure gradients in the porous bed generated upward directed forces. These were found to be caused by the breaking process and particularly in the case of regular waves (Sumer et al. (2013)), subsequent generation of vortices. In Nielsen et al. (2012) the suction mechanism was investigated on sloping beds exposed to breaking waves. A steeper slope was included, up to 1:2, and armour layers were applied. It was shown that the suction mechanism may cause the underlayer material to be moved out between the armour stones. However, detailed pressure measurements were not performed to link the possible suction to the existing pressure gradient in the underlayer.

In the present work the pressure gradients were investigated along the sloping front of a breakwater for regular waves. The applied waves, characterised by e.g. the surf similarity parameter, fall within the range of those applied for the steepest bed slopes in Nielsen et al. (2012). The pressure measurements were supplemented with velocity and turbulence measurements at the armour layer and high-speed video recordings. Hereby the pressure-induced forces were linked to the physical processes of run-up and run-down.

The paper is organized as follows: Section 2 presents the experimental setup and instrumentation while the applied test conditions are given in Section 3. A general description of the run-up and run-down processes based on video recordings is given in Section 4. The results of the pressure measurements including the pressure-induced forces are presented in Section 5. Section 6 gives some results from the velocity and turbulence measurements. Section 7 describes scale effects while conclusions are given in Section 8.

2. Experimental Setup

All tests were carried out in the wave flume referred to as Flume No. 1 at the hydraulic laboratory at DTU. The flume has a length of 25 m, a width of 0.6 m, and a depth of 0.8 m. The water depth for the present experiments was fixed at 0.4 m. The flume is equipped with a piston-type wave maker in one end for generating regular as well as irregular waves. At the general testing area the sides of the flume are made of transparent glass which enables a visual observation of the experiments as well as Laser Doppler anemometry (LDA) measurements from the side. An overview of the entire flume setup is shown in figure 1. The front and rear slope of the breakwater were arranged with a slope of 1:1.5. The core material was made of gravel with a diameter of $d_{50} = 1.8$ cm. In order to maintain a stable and reproducible structure a perforated steel plate was attached on the front of the breakwater on top of the core material. The plate had a thickness of 2 mm. The perforations were made
Figure 1: Experimental setup. A) Overview of the entire flume setup, B) measuring sections for pore-water pressure, C) measuring sections for LDA velocity and turbulence.

as quadratic voids with dimension of 1 cm and a void-to-plate ratio (porosity) of 0.41. This corresponded approximately to the porosity of the underlying core material of \( n = 0.4 \). The armour layer for the first part of the experiments was made out of spherical plastic elements with a diameter of \( D = 3.8 \) cm. They were glued to the perforated steel plate in a 90° arrangement mounted on top of the core material, see Figure 2. For the second part of the experiments real rocks were applied for the armour. Here, crushed stones with a diameter of \( D_{50} = 5.0 \) cm were applied. These were also glued to the perforated steel plate. Hereby the exact same configuration of the armour could be applied for each rebuild of the structure.

Three types of measurements were performed: pressure measurements, velocity measurements, and surface elevation measurements. Pore pressure measurements were performed using Honeywell RS395 pressure transducers. Rigid steel pipes with a diameter of 3 mm were inserted in the core material with the opening at the position of the measuring point. The pipes were connected to the pressure transducer with transparent plastic piezometer tubes. A description of this technique was
given in Sumer et al. (2011). The sampling frequency of the measurements was 120 Hz. Profiles were measured normal to the sloping bed at 12 sections equally spaced with 5 cm. The measuring sections were placed along the sloping front as shown in Figure 1 with the first section no. 1 at a distance of 16 cm from the toe. The pore-water pressure was measured at six depth, \( y = [0; -2.0; -3.0; -5.0; -7.0; -10.0] \) cm, for all 12 measurement sections. Here, \( y \) is the distance from the core material surface (under the armour stones) with the \( y \)-axis directed upwards. At the beginning of each test series the pressure measuring pipes where placed parallel to the front slope at these depth with the openings aligned along the measuring section closest to the toe (measuring section 1). The pipes had a length so they extended up through the breakwater crest. When the first measuring section was completed the entire set of six measuring pipes where moved up to the next measuring section by slowly pulling them parallel to the front slope. All measuring pipes were fixated together in order to ensure the exact same positioning of all pipes.

After each completed test series of pressure measurements (12 measuring sections) the core material was removed at the front part of the structure in order to rearrange the pressure measuring pipes in the interior of the structure. Due the above described procedure where the pipes were moved up along the slope for each measuring section, this rearrangement was necessary in order to reset the position of the pressure measuring pipes at the first section closest to the toe. Hereafter the core and armour layer was re-build. The armour layer was reproduced in the same configuration for each test due to fixation to the perforated steel plate. The re-build of the core material between each tests introduced some variability to the physical setup. This is addressed in Section 5.3.

Measurements of velocities and turbulence were performed with LDA. A DANTEC two-component LDA system was applied in back-scatter mode where the two velocity components, horizontal and vertical, were measured simultaneously. The measurements were rotated to a bed parallel and normal direction during the subsequent data-processing. The sampling frequency of the measurements was 120 Hz. The arrangement of the LDA system is shown in figure 2. Four vertical profiles normal to the sloping bed were measured (section A-D) with measuring points distributed both below and above the surface of the armour.

Figure 2: Setup of LDA measuring. The measuring point applied for the latter analysis is indicated with the black dot at a distance from the bed (surface of core material) of \( y = 42 \) mm.

The surface elevation measurements were performed at three locations at the offshore location and at the toe of the sloping bed as shown in Figure 1. The three offshore wave gauges were applied for separating the incident and reflected waves. The wave gauge at the toe of the breakwater was applied as reference between all experiments and corresponding video recordings. Conventional resistance type wave gauges were used in the measurements. The sampling frequency of the measurements was 120 Hz. The pressure and LDA measurements were synchronized with the surface elevation measurements at the toe of the breakwater.

In addition to the above, synchronized flow visualization were performed using a digital video recorder applying 200 frames per second. From here the detailed
observations are drawn of the entire process of run-up, run-down, breaking and trailing waves.

3. Test conditions

The experiments were divided into two groups based on the type of armour layer. The first group applied spherical plastic elements in one layer as the armour while the second group used natural stones in one layer. Table 1 presents the applied test conditions where \( D \) is the armour diameter, \( d \) is the core material diameter, \( H_j \) is the incident wave height, \( T \) is the wave period, \( K_r \) is the reflection coefficient from the breakwater defined as \( K_r = H_R/H_j \) where \( H_R \) is the reflected wave height. \( \xi_0 \) is the surf similarity parameter defined in e.g. Battjes (1974) as:

\[
\xi_0 = \frac{\tan(\gamma)}{\sqrt{H_0/L_0}}
\]

where \( \gamma \) is the slope of the bed, \( H_0 \) is the deep water wave height, and \( L_0 \) is the deep water wave length.

The target wave height was \( H = 0.14 \) m. The incident, \( H_i \), and reflected, \( H_R \), waves were separated based on a three-point wave gauge method based on the method described in Manssard and Funke (1980). Hereby the incident wave condition was determined at the flat bed between the wave maker piston and the breakwater structure. This wave height, \( H_i \), is reported in Table 1 together with the reflection coefficient. Each wave condition was repeated for each of the 12 measuring sections along the slope. The presented wave heights, surf similarity parameters and reflection coefficients are given as a mean of these repetitions. The waves were generated based on first order Stokes theory. The wave generator was operated with active wave absorption control (AWACS) described in Schäffer et al. (1994) for removal of the reflected energy from the structure.

For calculating the surf similarity parameter the deep water wave height, \( H_0 \), was applied. This was calculated from the measured incident wave height at the flat bed based on linear wave theory and conservation of energy flux. This follows the method also applied in Sumer et al. (2013):

\[
H_0 = H_i \sqrt{\tanh(kh)(1 + G)}
\]

where \( k \) is the wave number, \( h \) is the water depth, and \( G = 2kh/\sinh(2kh) \). The wave number is \( k = 2\pi/L \) and was found from the linear dispersion relation as:

\[
\omega^2 = gk \tanh(kh)
\]

Table 1: Test conditions for experiments with spherical plastic elements (\( D = 38 \) mm) and stone (\( D = 50 \) mm) armour layer.

<table>
<thead>
<tr>
<th>( D ) (mm)</th>
<th>( d ) (mm)</th>
<th>( d/D )</th>
<th>( H_i ) (cm)</th>
<th>( T ) (s)</th>
<th>( \xi_0 ) (-)</th>
<th>( K_r ) (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>38/50</td>
<td>18</td>
<td>0.47/0.36</td>
<td>11.7</td>
<td>1.0</td>
<td>1.76</td>
<td>0.25</td>
</tr>
<tr>
<td>38/50</td>
<td>18</td>
<td>0.47/0.36</td>
<td>11.6</td>
<td>1.5</td>
<td>2.64</td>
<td>0.34</td>
</tr>
<tr>
<td>38/50</td>
<td>18</td>
<td>0.47/0.36</td>
<td>11.4</td>
<td>1.7</td>
<td>3.05</td>
<td>0.46</td>
</tr>
<tr>
<td>38/50</td>
<td>18</td>
<td>0.47/0.36</td>
<td>11.9</td>
<td>3.0</td>
<td>5.79</td>
<td>0.63</td>
</tr>
</tbody>
</table>

where \( \omega = 2\pi/T \) is the angular frequency.

Based on the surf similarity parameter the wave conditions can be classified according to Galvin (1968) as: spilling (\( \xi_0 < 0.5 \)), plunging (\( 0.5 \leq \xi_0 < 3.3 \)), and surging (\( \xi_0 > 3.3 \)). Comparing to Table 1 it is seen that the present experiments falls within the plunging and surging regimes. Here it is noted, that the present test conditions partly falls within the applied conditions in Nielsen et al. (2012). Here, a steep slope at 1:2 was applied which in terms of run-up and run-down resembles the present experiments. These experiments were conducted with \( \xi_0 = 5.36 \) – 5.64 i.e. in the surging regime. Also mildly sloping beds at 1:14 were applied in Nielsen et al. (2012) where \( \xi_0 \) were found in the range of \( \xi_0 = 1 \) – 3, however here the run-up and run-down process may differ from the present experiments due to the slope of the bed. In Sumer et al. (2013) the same methodology regarding assessment of the destabilising pressure gradient was applied as for the present experiments. Here the wave conditions were within the plunging regime with \( \xi_0 = 1.42 \). However, these experiments were conducted on a mildly sloping bed at 1:14.

The reflection of waves from the breakwater structure increased for an increasing surf similarity parameter. The reflection coefficient ranged from 0.25 – 0.63 which is in accordance with e.g. Zanuttigh and Van der Meer (2006) both in terms of the variation and the absolute values. As such the breakwater interacts with the waves as expected for this type of structure. Further discussion on the wave interaction with the porous breakwater in relation to a prototype structure is given in Section 7.

Each experiment had a length of 10 min which provided between 200 and 600 waves depending on the wave period. The last 100 waves were applied for data analysis providing e.g. ensemble averaged quantities.
4. Description of run-up and run-down process

The present experiments included wave conditions ranging from the plunging to the surging breaker regime. A pressure variation depending on the breaker type was found as will be described later in Section 5. In order to understand the measured pressure variations in relation to the physics of the breaking wave a general description of the run-up and run-down sequence is given in the following. The presented water-surface elevations are based on the high-speed digital video recordings.

4.1. Plunging wave

Figure 3 presents the run-up and run-down process for the plunging breaker with a wave period of $T = 1.0 \text{s}$ and $\xi_0 = 1.76$. The numbers in the figure refers to the different instants in time as given in the figure caption where the reference time $t = 0.0 \text{s}$ corresponds to zero-up crossing of the water-surface at the toe of the sloping bed. The measuring sections for pore-water pressure are indicated in the figure with measuring section 5 highlighted for reference to the later results. The run-up illustrated in Figure 3A shows how the wave initiated the run-up phase with a very steep slope of the water-surface (stage 1). Just before the wave breaking occurred the water-surface was positioned with an angle of approximately 90° to the front slope of the breakwater (stage 2). At a position just above the still water level the wave curled forward in the breaking process and impacted into the front of the breakwater (stage 3). The run-up continued hereafter which generated some splashing and air entrainment at the region of the highest run-up (stage 4). Here it may be noted, that several flow regimes are present along the slope. At the lower part of the slope the thickness of the run-up wedge was several times the roughness of the armour layer. Here the flow had similarities with a rough bottom channel flow. At the upper part of the slope the run-up wedge thickness was less than the roughness which resembled a flow around obstacles. These different flow regimes were also described in Andersen et al. (2011).

In Figure 3B the run-down process is illustrated. After flow reversal the run-down created a thin layer of water rushing down through the armour stones (stage 5). Due to the delay of the water movement inside the porous core a water level difference was seen between the core and armour layer. At the maximum run-down level the downward mowing water impacted with the successive incoming wave and hereby initiating the next run-up phase (stage 6).

4.2. Surging wave

Figure 4 presents the run-up and run-down process for the surging breaker with a wave period of $T = 3.0 \text{s}$ and $\xi_0 = 5.79$. Some differences were seen regarding both the run-up and run-down phase compared to the previous waves with a period of $T = 1.0 \text{s}$. The run-up phase was initiated with the water-surface being close to the still water level (stage 1). The long wave period resulted in a run-up where the water surface did not approach the front slope of the breakwater (stage 2). At a position just above the still water level the wave curled forward in the breaking process and impacted into the front of the breakwater (stage 3). Again, the flow is dominated by the upper part of the slope where the wedge thickness is small compared to...
the roughness and the lower part with a relatively larger wedge thickness. Figure 4B presents the run-down phase where a large downward directed velocity was reached at the armour layer (stage 4). The run-down water impacts with the main body of water which at this instant in time has not been affected by the successive incoming wave (stage 5). At the maximum run-down level a stirring zone occurred with some air entrainment. Immediately after the maximum run-down level was reached a secondary run-up was initiated as shown in Figure 4C (stage 6). This formed a breaking bore or hydraulic jump which was also described in Pedersen and Gjevik (1983) for surging waves (stage 7). The secondary run-up created some air entrainment during the breaking process (stage 7+8).

5. Pore-water pressure and pressure-gradient forces

In the following the details of the pore-water pressure and pressure gradient along the slope is given. First, the pore-water pressure time series are presented which shows the occurrence of pressure gradient near the surface of the bed. For some relevant measuring sections these results are presented in terms of profiles of the pore-water pressure below the surface of the bed. The variation of the outward directed pressure gradient is presented along the sloping bed which provides an understanding of the spatial variation in relation to the position of the water surface outside the structure.

5.1. Pore-water pressure time series

Figure 5 and 6 present the time series of the pore-water pressure for a wave period of \( T = 1.0 \) s and \( T = 3.0 \) s, respectively. Measuring section 8 and section 6 are presented. The water-surface elevation at the toe of the breakwater is presented along with the pore-water pressure time series as a reference signal. All times are given relative to the zero-up crossing of the water-surface elevation at this position.

In Figure 5 it may be seen how the pore-water pressure varied over the depth throughout the entire wave period for the short wave with a period of \( T = 1.0 \) s. Regarding the surface elevation it is noted, that the surface elevation was measured at the toe while the pressure presented in Figure 5 was measured at section 8 along the slope. As such the surface elevation does not represent the crest/trough variation at the position of the pressure measurements. Due to the small wave period, and wave length, a crest was seen at the toe (0.22 s) while the maximum run-down level occurred on the slope (stage 1 in Figure 3). Correspondingly a trough was seen at the toe (0.75 s) approximately at the same time as the maximum...
run-up level occurred on the slope (stage 5 in Figure 3).

For interpreting the pressure variations the time instant for maximum run-up and run-down levels are indicated on Figure 3. During run-up (from maximum run-down to maximum run-up) the pressure changed from being highest in the core \((y = -7 \text{ cm})\) and lowest near the surface \((y = 0 \text{ cm})\) to the opposite situation with lowest pressure in the core and highest pressure near the surface. These variations resulted in a pressure gradient directed outward (out of the core) during the first part of the run-up and inward (into the core) during the last part of the run-up. The change in direction of the pressure gradient appeared between stage 2 and stage 3 in Figure 3. During run-down the pressure gradient was directed outwards with highest pressure in the core \((y = -7 \text{ cm})\) and lowest pressure near the surface \((y = 0 \text{ cm})\) for most of the run-down stage. The maximum gradient was seen at the time where the maximum run-down occurred and the next run-up stage was initiated. In relation to instability of the armour and underlayers the outward directed pressure gradient is of interest as this will act as a lift force on the outer part of the sloping bed.

The long wave with a period of \(T = 3.0 \text{ s}\) is shown in Figure 6. Again it should be noted that the surface elevation was measured at the toe while the pressure presented in Figure 6 was measured at section 6 along the slope. As such the surface elevation does not represent the crest/trough variation at the position of the pressure measurements. However, due to the relatively longer wave the phase difference is smaller which means that crest and trough variation at the toe approximately coincided with the water level on the slope.

The pressure variation over the depth was less pronounced for a large part of the wave period than for the short wave with \(T = 1.0 \text{ s}\). However, one area exists where the outward directed pressure gradient were generated, at about \(t = 1.7 \text{ s}\). Again this corresponds to the time for the maximum run-down level. The in- and outward directed pressure gradients for all wave conditions are a result of the delay in the pore-water pressure to the run-up and run-down flow. This was shown in relation to breakwater structures in e.g. van Gent (1994) and also pointed out based on pressure measurements in both Sumer et al. (2011) and Sumer et al. (2013).

The pore-water pressure time series were compared to the details of the run-up and run-down processes in Figure 3 and 4. Here it was found that the outward directed pressure gradients in general occurred at the instant in time where the maximum run-down level was
reached. This was found to be in accordance to the observation of Sumer et al. (2011) where outward directed pressure gradients were also found during the run-down phase. Figure 7 and 8 shows the water surface elevation for that instant in time where the maximum outward directed pressure gradient was seen. The outward directed pressure gradient increased when the run-down was at its lowest level and when the run-down volume of water impacted with the main body of water at or below SWL. This mechanism was also described in relation to breakwater structures in Bruun (1989) however with no further details on the variations of the pore-water pressure.

5.2. Pore-water pressure profiles

The variation of the pressure through the core material is presented as pressure profiles normal to the bed surface in Figure 9. The pressure profiles corresponds to the data presented in Figure 5 for two instants in time; \( t = 0.23 \) s where an outward directed pressure gradient was seen, and \( t = 0.51 \) s where an inward directed pressure gradient was seen. The ensemble averaged data were used found as:

\[
\overline{p_i(y, \omega t)} = \frac{1}{N} \sum_{j=1}^{N} p_i[y, \omega(t + (j-1)T)]
\]

where \( i \) corresponds to each measurement points across the vertical and \( N \) is the number of wave periods over which the ensemble average was performed.

The pressure profiles showed the same direction of the pressure gradient as seen in the pressure time series. The measurement at the surface of the core material (\( y = 0 \) cm) is also presented in Figure 9. This measurement was performed on top of the perforated steel plate that held the core material in place and on which the armour layer was fixated. Due to uncertainties regarding the position of the pressure steel pipe (described in Section 2) on top of the perforated plate this measurement was not applied in the following analysis of the total pressure gradient. For this analysis only the pressure measurements positioned in the actual core material were applied.

5.3. Pressure gradient spatial variation

In relation to destabilising forces on the under/filter layers the outward directed pressure gradients are of interest. The variation of the outward directed pressure
gradient along the sloping front is presented in the following. Again, the two wave conditions with periods of $T = 1.0\, s$ and $T = 3.0\, s$ are presented in details as these covers the range from plunging to surging breaker types. Figure 10 presents the variation for $T = 1.0\, s$. The distance from the toe follows the definition given in Figure 1 where $x = 0$ corresponds to the position of the toe. The still water level (SWL) is indicated along with the maximum run-down level detected from the synchronised video recordings. For all the measuring sections some scattering of the data were seen, however, there was a clear trend towards increasing pressure gradients when going from the toe towards the water surface. For the lower part of the sloping bed a constant variation of the gradient was found. The maximum gradient was reached approximately at the position of the maximum run-down level. The large outward directed gradient were in general localized around this position.

For the wave period $T = 3.0\, s$ the variation of the pressure gradient is shown in Figure 11. Here the same trend was found as for the short wave with $T = 1.0\, s$. It was found that the gradient increases from the lowest part of the sloping bed where the short wave condition gave a more constant variation along the lower part. This reflects the differences in the run-up and run-down processes for the plunging and surging breakers. The maximum pressure gradient was reached at a position slightly below the maximum run-down level.

Regarding the applied armour (stones or spherical plastic elements) it was found from Figure 10 and 11 that the pressure gradients in general were not affected by the different armour types. Also the run-up and run-down processes for the spherical plastic elements armour were found to correspond to those observed for the stone armour as described in Section 4. When comparing the two repetitions with stone armour layer the effect of the re-packing of the core material were singled out as all other parameters were identical. This showed that the core material introduced some variability however the results were comparable both in terms of magnitude and variations along the sloping front.

The outward directed pressure gradients results in an outward directed force on the core material. From Figure 10 and 11 this force was found to reach values up to $-\partial(p/\gamma)/\partial y \approx 0.4$ and $-\partial(p/\gamma)/\partial y \approx 0.55$ for the wave conditions with $T = 1.0\, s$ and $T = 3.0\, s$, respectively. With the submerged weight of the core material being $(s-1)(1-n) \approx 0.9$ the outward directed forces corresponds to $\approx 40\%$ and $\approx 60\%$ of the submerged weight of the core material for the two presented wave conditions.

6. Velocities and bed shear stresses

Velocity measurements were performed along the sloping bed for the experiment with the spherical plastic element armour layer as described in Section 2. Four
measuring sections were included denoted section A-D in order to clearly distinguish the velocity measurements from the pressure measurements (denoted section 1-12). The arrangement of the measuring section is shown in Figure 2. Due to air entrainment from the breaking and/or run-down process the LDA measurements could only be performed up to a water level approximately corresponding to the maximum run-down level. For this reason, measurements are not conducted under the actual breaking point.

In relation to these measurements it is noted that the run-up and run-down process corresponded to those described in Section 4 for the stone armour layer. This was ensured by comparing video recordings for the two experiments (plastic spheres and stones) frame by frame. Furthermore, it was found that the recorded surface elevation at the toe was highly reproducible during a recorded time series. This was ensured by plotting the surface elevation from 100 succeeding waves (used for ensemble averaging) where it was seen that they practically collapsed on one curve. A high repeatability was necessary in order to make sure that the computed Reynolds-stresses only reflected the turbulence generated by the boundary layer, the wake turbulence around the armour stones, the breaking process and the seapage flow.

6.1. Velocities

The LDA measurements provided the bed parallel and normal velocity components. Figure 12 presents an example of the velocities at measuring section C for the point 42 mm above the surface of the core material (4 mm above the upper surface of the spherical plastic elements). The surging breaker with a period of \( T = 3.0 \) s is presented. The velocity measurements showed how the run-up and run-down was mostly governed by bed-parallel velocities. At the final stage of the run-down when the maximum run-down level was reached the flow was directed outwards from the core towards the main body of water generating higher bed-normal velocities, as revealed in Figure 12B. For calculation of Reynolds-stresses, the velocity measurements were applied as described in Section 6.2.

6.2. Bed shear stress

For evaluating the shear-stress on the armour layer the Reynolds-stresses \( \overline{u'v'} \) were computed from the two velocity components, \( u \) and \( v \). With the Reynolds decomposition the fluctuating part, \( u' \), was given as:

\[
u' = u - \overline{u}
\]

where \( u \) is the instantaneous velocity and \( \overline{u} \) is its mean (ensemble averaged). Here it is noted that three contributions to the generation of turbulence are expected. 1) Wave boundary layer turbulence generated above the armour layer, including vortex-shedding turbulence generated due to the flow and separation around the armour stones. 2) Turbulence from the breaking process. 3) Turbulence generated due to the seapage flow from the porous core.

Figure 13 and 14 present the Reynolds-stress time series at the point 4 mm above the armour layer for the four measuring sections with a wave period of \( T = 1.0 \) s and \( T = 3.0 \) s, respectively. The sign-convention is with the direction of the Reynolds-stress being positive upward the sloping bed. First, it is noted that a very distinct increase in the shear stress activity was observed for \( t = 0.15 \) s and \( t = 1.6 \) for the two presented wave periods. Especially for measuring section D this was very clear. When comparing to the general description of the run-up and run-down processes in Section 4 it was found that this increase in shear stress appears just before the time of maximum run-down. Secondly, when Figure 13 and 14 were compared to the corresponding pressure time-series in Figure 5 and 6 it was seen that the increase...
in shear-stresses also appears just before the generation of the outward directed pressure gradient. The velocity measurements were not performed at the exact same positions as the pressure measurements, however measuring section D for velocity approximately corresponds to the pressure measurement sections where the maximum outward directed gradient were found. This indicates that the increase in shear-stress is linked to the outward directed pressure gradients both in time and space and are caused by the run-down flow.

When Figure 13 and 14 were compared it was seen how the variation in Reynolds-stresses was more smooth for a wave period of \(T = 1.0\) s compared to the wave period of \(T = 3.0\) s. Based on the flow visualisations and the general description in Section 4 it was found that this was also linked to the run-down process. For the wave period of \(T = 1.0\) s the run-down level did not reach section D. Furthermore, the breaking process was localised slightly above SWL. At velocity section A-D the flow was oscillating upward (and slightly directed into the core) and downward (and slightly directed out of the core) without being affected by the breaking and run-down process. For the wave period of \(T = 3.0\) s the run-down level came very close to section D and therefore the Reynolds-stresses were greatly affected by the stirring-zone generated at maximum run-down. The run-down process enabled turbulence to move down along the sloping bed towards both section C and B.

In Dixen et al. (2008) measurements were performed of turbulence and Reynolds-stresses on a rough bed made out of spherical plastic elements of the same diameter as those applied for the present experiments. However, these experiments were applied on a flat bed without wave breaking. Here a production of turbulence was seen near the bottom and transported into the main body of water. This was also seen as an increase in Reynolds-stresses near the upper surface of the spherical plastic elements both during the crest and trough half period. For the present experiments the flow was also affected by the run-down processes as well as wave breaking. However, during run-up the part of the slope below SWL may be regarded as a rough bottom wave boundary layer flow. When observing the Reynolds-stresses in Figure 13 and 14 it is clear that no, or only very small, Reynolds-stresses were seen during this phase of the flow at the point at the upper surface of the armour layer. This observation deviates from the results of Dixen et al. (2008).

This can be explained by the effect of the porous core material in the present case and the converging flow along the slope during run-up as follows. Due to the porous core material the flow was allowed to pass into the core during run-up. When examining the flow direction just above the armour layer it was found that this was indeed the case. Figure 15 shows the flow direction at measuring section C above the armour layer. The flow direction was defined with \(\alpha = 0\) for bed parallel flow upward along the slope, \(0 < \alpha < 180\) corresponded to outward directed flow from the core to the main body of water, and \(-180 < \alpha < 0\) corresponded to inward directed flow from the main body of water to the core. It was found...
that an inward directed flow was generated during the run-up phase which limited the transportation of turbulence into the main body of water. At flow reversal, after the maximum run-up level, the flow was directed into the core. Immediately after this the flow turned downward the slope and at approximately $t = 1.1$ s the flow was directed outwards from the core to the main body of water.

If the above details of the flow direction are compared to the Reynolds-stresses in Figure 14 (section C) it is seen how the increase in shear-stress above the armour layer coincides with the periods of outward directed flow.

6.3. Shields parameter

As described in the above the high values of Reynolds-stresses were found at the same locations as for the large outward directed pressure gradients. At this points it is relevant to address the destabilizing forces acting on the outer armour layer in conjunction with the pressure gradient forces from the porous core. The stability of sediment, gravel, and stones exposed to current and/or waves may be evaluated based on the Shields parameter defined in e.g. Fredsøe and Deigaard (1992) as:

$$\theta_c = \frac{U_{fc}^2}{(s-1)gd}$$  \hspace{1cm} (6)

where $\theta_c$ is the critical Shields parameter for movement of the stones, $U_{fc}$ is the critical friction velocity, $s$ is the relative submerged density of the stones, and $d$ is the diameter of the stones. The critical Shields parameter expresses the critical value of the destabilizing fluid forces to the stabilizing forces. The destabilizing forces are expressed via the friction velocity while the gravitational acceleration acts as the stabilizing force. Eq. (6) is given for a flat bed. If the bed is sloping an additional term arises in the force balance between stabilizing and destabilizing forces. That is an additional destabilizing force due to the gravitational acceleration being projected into the bed parallel and normal direction. Furthermore, the stabilizing gravitational force is also reduced due to this
projection. This gives a correction to the critical Shields parameter on a flat bed (Fredsøe and Deigaard (1992)):

$$\theta_c = \theta_0 \cos \gamma \left[ 1 - \frac{\tan \gamma}{\tan \phi_s} \right]$$  \hspace{1cm} (7)

where $\theta_0$ is the critical Shields parameter on a flat bed, $\gamma$ is the slope of the bed, and $\phi_s$ is the friction angle (angle of repose) of the stones.

The critical Shields parameter is depending on the grain $Re$-number and is described by empirical data. For turbulent flow the critical value is $\approx 0.06$ as shown in e.g. Fredsøe and Deigaard (1992). Hereby it is assumed that there is a clear difference between stationary and moving sediment/stones. However, this is not the case due to variations in e.g. bed shear stress and stone size. In Breusers and Schukking (1971) experiments showed that movements of some stones could occur at $\theta_c = 0.03$. In CIRIA et al. (2007) it is recommended for design of armour stones and rock fill that the critical Shields parameter takes the value $\theta_c = 0.03 - 0.035$ at which stones first begin to move. For the flow along a sloping front of a breakwater the correction of the critical Shields parameter given in eq. (7) was applied. The friction angle is depending on the material. In this case there exists an interface between two materials; the armour layer and the core material. According to CIRIA et al. (2007) the smallest friction angle of the two materials may be used. For the present case this gives a friction angle of $\phi_s = 45^\circ$. With a slope of the bed of 1:1.5 the corrected critical Shields parameter reduces to a value of $\theta_c \approx 0.01$.

As a measure of the bed friction in the experiments, expressed as the friction velocity, $U_f$, the Reynolds-stress was applied as described in Dixen et al. (2008). Here the friction velocity was determined based on both velocity profiles and Reynolds-stresses above the bed. The results of the different methods showed a very good agreement. As such the method based on the Reynolds-stresses was adopted in the present study to evaluate the friction velocity and hereby the Shields parameter. The friction velocity was defined as:

$$U_f = \sqrt{\frac{\tau_0}{\rho}}$$  \hspace{1cm} (8)

where $\tau_0$ is the bed shear stress found from the Reynolds-stress as $-\rho u'v'$ for the measuring point 4 mm above the upper surface of the armour layer. With this the actual Shields parameter was computed at each of the four measuring sections for the four applied wave conditions. Here, the maximum Reynolds-stress over one ensemble averaged wave period was applied. Figure 16 presents the Shields parameter for the four wave conditions. First it was noted that the variation along the distance from the toe showed a general trend of an increasing Shields parameter from the toe towards the SWL. This is clearly linked to the increase in shear stress near the run-down level due to the run-down process and outward directed flow. Secondly it was found that there was a clear difference between the flow regimes of the plunging breakers and surging breakers. For the surging breaker the Shields parameter takes a higher value which was clearly distinguished from the remaining wave conditions. The plunging breakers were grouped together with a lower Shields parameter. This was explained by the differences in the run-up process, and therefore also the run-down process, as described in Section 4. For the surging wave the run-up was long and without any severe breaking. The large run-up level and volume of water resulted in a large run-down that created higher velocities and stresses at the upper part of the slope.

![Figure 16: Shields parameter for all wave conditions. Run-down indicates the maximum run-down water level for the surging breaker with a wave period of $T = 3.0$ s.](image)

Regarding the values of the Shields parameter the surging wave resulted in a value of $\theta = 0.009$. The critical Shields parameter for movements of some stones corrected for the sloping bed was found to be $\theta_c \approx 0.01$ according to CIRIA et al. (2007). This indicated that the destabilizing forces on the outer armour layer were in the same order of magnitude as those required to initialize movements of the stones. This was the case for the measuring sections included in the present study. If the
Shields parameters presented in Figure 16 was extrapolated to a larger distance from the toe it was expected that even higher values may occur. It should be noted that the present evaluation of the possible movement of the stones do not take into account the contact forces and inter-locking forces between the individual armour stones. These will act as stabilizing and may as such prevent the stones from moving.

6.4. Verification of Reynolds-stress measurements

In order to verify the measurement technique for Reynolds-stresses the data from our earlier study, Jensen et al. (2012), were applied. The experiments presented in Jensen et al. (2012) were part of another study performed with a solitary wave on a rough impermeable sloping bed. The setup was identical to the present experiments except for the impermeable bed in Jensen et al. (2012). The same LDA system as described in Section 2 was applied. Details on the experimental setup were shown in Jensen et al. (2012) where also some results were presented. The data were re-analysed as part of the present study in order to compute the Reynolds-stresses for comparison with the data of Dixen et al. (2008).

The impermeable bed experiments in Jensen et al. (2012) were applied for three reasons: 1) The porous core applied in the present study enabled the flow to be directed into the structure and thereby not generating the boundary layer turbulence as expected for an impermeable bed. By applying the impermeable bed experiments this effect was removed from the system. 2) The solitary wave enabled a flow which was not affected by the breaking process during the run-up stage (the wave was surging) nor the breaking and run-down stage from the previous wave. The experiments of Dixen et al. (2008) were conducted with no wave breaking and as such the solitary wave experiments offers a more direct comparison. 3) The experiments in Dixen et al. (2008) were conducted with the same spherical plastic elements and comparable wave conditions. The orbital amplitude to diameter ratio was $a/D = 1.58$. The impermeable bed experiments of Jensen et al. (2012) applied an orbital amplitude to diameter ratio of $a/D = 3.95$.

It is noted, that the turbulence generated due to the wave boundary layer was not represented correctly with the solitary wave experiments. However, the order of magnitude of the Reynolds-stresses were expected to be comparable. In Jensen et al. (1989) results were presented for turbulent oscillatory boundary layer with regular waves and in Sumer et al. (2010) results were presented for turbulent boundary layer with a solitary wave. Here the maximum turbulent quantities during the crest half period in the fully-developed turbulent boundary layer stage were found to be $\frac{\overline{u''^2}}{U_f} \approx 4$ for both the regular and solitary waves.

Figure 17 presents the comparison of the vertical distribution of Reynolds-stresses obtained from the solitary wave experiments of Jensen et al. (2012), and the regular wave experiments of Dixen et al. (2008). In Jensen et al. (2012) measurements were conducted both above and below the center line of the spherical plastic elements. In Dixen et al. (2008) measurements were only performed down to the theoretical bed (0.23D below the surface of the spheres). The data for the solitary wave experiments were taken for the last part of the run-up stage ($\omega t \approx 100^\circ$). For Dixen et al. (2008) the data for the last time instant presented for the crest half period ($\omega t = 100^\circ$) was applied. The depth was non-dimensionalized as $y/k_e$ where $k_e$ is the equivalent sand roughness taken as $2.5D$ as found in Bayazit (1976) and Dixen et al. (2008) for spheres. The Reynolds-stresses were non-dimensionalized as $-\overline{\nu'v'}/U_f^2$ where the friction velocity was found as described in Section 6.3.

It was found that the non-dimensionalized Reynolds-stresses were comparable for the two experiments. The same order of magnitude were seen for the maximum Reynolds-stress that occurred slightly below the upper surface of the spherical plastic elements. The general distribution across the vertical direction was comparable in terms of a decreasing level above the spheres. In Dixen et al. (2008) the stresses decrease rapidly below the point where the maximum values were found while the data from the solitary wave experiments in Jensen et al. (2012) showed a distribution of higher Reynolds-stresses further down between the spheres. It should be noted, that the present solitary wave experiments were conducted on a steep sloping bed which created a converging flow during run-up. Hereby the flow was more likely to be directed down between the spheres.

With the agreement between the data of Dixen et al. (2008) and the experiments of Jensen et al. (2012) (performed with the same experimental setup as the present study) the measurements of the Reynolds-stresses were verified.
7. Remarks about scale effects

In the following a short note is given regarding scale effects in relation to applying model scale dimensions. In general, scale effects related to model scale experiments of breakwater structures are caused by different flow regimes experienced in prototype and model scale. An overview of scale effects for hydraulic model experiments was given in Heller (2011). The problem of scale effects related to porous breakwater structures was treated in e.g. Pérez-Romero et al. (2009) which investigated reflection and transmission of breakwaters. In Andersen et al. (2011) the impact of the scale effects on the overtopping rate was investigated. Several studies have proposed methods to minimize the scale effects e.g. Jensen and Klinting (1983), Martin et al. (2002) and Vanneste and Troch (2012).

For the present study the experiments were not designed to be an exact scaled copy of a prototype structure. Here, the scale model was applied to investigate some physical processes. As such it was important to ensure similarity between model and prototype for those processes in order to extrapolate the results to prototype scale. This was done by identifying the non-dimensional parameters which are governing for the physical processes.

The scope of the present study was the pressure-induced lift forces on the core material and the drag forces on the armour layers. As it was found in Sections 5 and 6 these forces are related to the run-up and run-down flow, in particular the maximum run-down level, and the delay of the porous flow in the core to the fluid loading.

7.1. Surf-similarity parameter, $\xi_0$

The surf-similarity parameter, $\xi_0$, governs the breaker type and thereby the run-up and run-down flow. The relation between the run-up level and $\xi_0$ was shown in e.g. van der Meer and Stam (1992). $\xi_0$ was defined as shown in eq. (1) and takes the values presented in Table 1. If Froude model law is applied for scaling to prototype the same values will be achieved.

7.2. Armour layer roughness, $a/k_s$

In van der Meer and Stam (1992) the dependency on the run-up level of the surface roughness was investigated. There was a clear effect on the maximum run-up level when a smooth and rough surface were compared. The reduced run-up level for a rough surface will also influence the maximum run-down level. The roughness may be expressed in non-dimensional terms as the ratio between the amplitude of the oscillatory motion and the equivalent roughness, $a/k_s$. This number may also be regarded as the Keulegan-Carpenter ($KC$) number which expresses the ratio of the stroke of the oscillating motion to the size of the roughness. It is noted that also the forces on an object exposed to an oscillatory flow is governed primarily by the $KC$-number.

For a Froude model law scaling the $a/k_s$-number will be identical in model and prototype scale.

7.3. Armour stone Reynolds-number, $Re_D$

For evaluating the stability of armour stones the processes accounting for the destabilising forces must be included correctly. The governing parameters for the forces on stones exposed to an oscillatory flow are the $KC$-number and the $Re$-number. The $KC$-number is identical in model scale and prototype as shown in Section 7.2. The $Re$-number found in prototype can not be maintained in model scale for a Froude model law scaling. The $Re$-number governs the flow regimes in terms of laminar, transitional or turbulent boundary layer flow around the stone. This affects the total drag force due to variations in the position of flow separation points.

![Figure 17: Reynolds-stresses on a rough impermeable sloping bed from Jensen et al. (2012) (surging solitary wave) and a rough impermeable plane bed from Dixen et al. (2008) (regular wave on plane bed). $y$ is measured from the physical impermeable bed.](image-url)
For circular cross sections this effect may lead to an increase in drag coefficient, $C_D$, of 50% to 100% when the $Re$-number is decreased from $5 \cdot 10^5$ to $1 \cdot 10^4$, see data compiled in e.g. Sumer and Fredsøe (2006). For cross sections with sharper edges this effect is less pronounced due to fixation of the flow separation at the sharp edges.

For evaluating scale effects due to the above dependency of the $Re$-number the armour stone Reynolds number, $Re_D$, is often used as shown in e.g. Andersen and Burchart (2010) and Andersen et al. (2011):

$$Re_D = \frac{\sqrt{gHD_{n50}}}{\nu}$$  \hspace{1cm} (9)

where $D_{n50} = Volume^{1/3}$ is the equivalent cube length exceeded by 50% of the armour stones.

In Dai and Kamel (1969) experiments were performed with regular waves where it was found that the critical armour stone Reynolds number for evaluating stability was $3 \cdot 10^4$. For the present experiments the armour stone Reynolds numbers take the value $Re_D = 4.4 \cdot 10^4$ and $Re_D = 3.3 \cdot 10^4$ for the stone and spherical plastic element armour layers, respectively. These are seen to be above the critical value given by Dai and Kamel (1969) and as such scale effects are expected to be minimal.

7.4. Pore Reynolds-number, $Re_p$

For the porous core the scale effects are related to whether the flow is in the laminar/Forchheimer, transitional or turbulent flow regimes. For prototype measures the flow will often be in the turbulent regime while in model scale the laminar regime is often found. The flow in a porous medium is often described by the Darcy-Forchheimer equation which includes both a linear term (laminar) and a non-linear term (turbulent). Depending on the flow regime these terms will have different importance and thereby the effective resistance of the porous medium will differ, see the discussion on porous medium terms in e.g. Jensen et al. (2014). The flow in a porous medium with relation to the Darcy-Forchheimer equation was investigated in e.g. Engelund (1954) and Burchart and Andersen (1995). The flow regimes can be classified by the pore Reynolds number, $Re_p$, given as:

$$Re_p = \frac{UD_{n50}}{n \nu}$$  \hspace{1cm} (10)

where $U$ is a characteristic filter velocity and $n$ is the porosity. In general the fully turbulent flow regime is reached for $Re_p > 300$. In Jensen and Klinting (1983) a method was proposed for minimizing the scale effects. This was done by ensuring the same hydraulic gradient through the structure in prototype and model scale. The hydraulic gradient was described based on the Darcy-Forchheimer equation which resulted in a correction factor for the core material dimension. Here a characteristic velocity is needed which for the present study was found from the Darcy-Forchheimer equation applying the maximum pressure gradient across the structure based on linear wave theory for an undisturbed wave on the sea-side and still water level on the shore-side. It should be noted, that discussions on the method for determining the velocity in the porous media was given in e.g. Martin et al. (2002) and Vanneste and Troch (2012). The pore Reynolds-number was found to be $Re_p \approx 320$. For prototype structures, $Re_p$ may take values in the order of $Re_p \approx 10^5$. This shows how the prototype flow in the porous core is in the fully turbulent regime while in model scale it approached the transitional regime from laminar to turbulent. When relating the model scale results to prototype dimensions this deviation in $Re_p$ should be taken into account. For typical model scale tests the scaling factor for the core material will be reduced by $15 - 25$% by the method of Jensen and Klinting (1983).

8. Conclusions

The present study investigated the generation of destabilizing pressure gradients in the core/filter material below the main armour layer of a sloping porous breakwater. The pressure gradients were measured together with velocities and turbulence. High-speed video recordings enabled a visual connection between the measured quantities and the physical processes. The following conclusions were drawn:

- Outward directed pressure gradients at the surface layer of the core material were found for all investigated wave conditions.
- The temporal variation of the pressure gradient showed a connection between the maximum outward directed pressure gradient and the maximum run-down event.
• The spatial variation along the sloping front of the breakwater showed that the maximum pressure gradients appears at or just below the position of the maximum run-down water level.

• Outward directed pressure gradient exerted a lift force up to ≈ 60% of the submerged weight of the core material for surging breakers.

• On the lower part of the breakwater slope the turbulence produced near the armour layer was found not to be moved up into the main body of water during the run-up phase due to the flow being directed into the porous core.

• During run-down the flow was directed outward from the porous core and turbulence was produced and moved up above the armour layer. At the time of maximum run-down the turbulence and the resulting Reynolds-stresses experienced a large increase.

• The bed shear stress determined based on the Reynolds-stresses gave a Shields parameter close to the critical value for initiation of movements of the stones.

• The high shear stress levels coincide in time and space with the generation of the large outward directed pressure gradients.

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CHAPTER 4

INVESTIGATIONS ON THE POROUS MEDIA EQUATIONS AND RESISTANCE COEFFICIENTS FOR COASTAL STRUCTURES

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Investigations on the porous media equations and resistance coefficients for coastal structures

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Abstract

This paper considers the flow in porous media that occurs in coastal and offshore engineering problems. Over the past decades numerous formulations of flow equations for porous media have been presented. The present work re-examines the porous media equations of the most recent form and corrects some shortcomings which were identified. The applied type of porosity models relies on empirical resistance coefficients which often needs to be measured or calibrated. Only few examples of calibration for numerical models are present in the literature which often applied the same experimental results. In this study new calibration cases were introduced to the calibration procedure in order to achieve a better understanding of the variation of the resistance coefficients. Hereby the coefficients were determined with a better description over the entire parameter space for the resistance coefficients than previously found in the literature. Constant values for the resistance coefficients for a broad range of flow conditions were recommended based on the new calibrations. The model was validated for the main physical processes that occur in wave-structure interaction in coastal structures including three-dimensional wave-structure interaction, run-up, run-down and pressure damping, regular and irregular wave conditions and evaluation of overtopping. Simple two and three dimensional uniform caisson structures and breakwater layouts were investigated. The model was implemented in the open source CFD library OpenFOAM\textsuperscript{R} and has been made publicly available to the engineering community as part of the wave generation framework waves2Foam.

Keywords: Wave-structure interaction, Breakwaters, Porous media, Resistance coefficients, Navier-Stokes equations, VOF

1. Introduction

Porous structures are often encountered in coastal engineering. One example is the breakwater structures, which form a key element in coastal and harbour engineering. A typical breakwater structure consists of a porous core material such as sand or gravel. The core is protected against erosion by means of one or several filter layers which are themselves protected by an armour layer. The porous breakwater allows for a flow through the structure. In order to ensure a correct and safe functioning of the breakwater it is essential that it remains stable during wave action.

A common procedure during the design process of a breakwater is to perform model scale experiments. In such experiments the entire profile of the breakwater is constructed in a hydraulic laboratory, including core material, filter, and armour layers. The structure is exposed to the design wave conditions and the response of the structure is observed.

In recent years, numerical methods and computational resources have developed to a level at which consulting and design engineers apply numerical simulations as an integrated part of the design and development pro-
cess. However, a complete resolution of the porous structure in breakwaters is not yet feasible. A method for simulating the effect of a porous medium without resolving the actual pores in the porous material must be applied. In van Gent (1995) a model based on the Navier-Stokes equations was developed, where the effect of the porous media was included via resistance source terms. The porous media, consisting of a rigid skeleton and pores, is treated as one continuum which exerts forces on the fluid due to drag, friction, and acceleration. The resistance was described with the extended Darcy-Forchheimer equation where the unknown resistance coefficients were determined from physical experiments. In Burcharth and Andersen (1995) the flow in porous media was also investigated and several references to the resistance coefficients were considered. The same method for including the porous effect in the solution of the Navier-Stokes equations was shown in Liu et al. (1999) where the formulation in van Gent (1995) was adopted. In Hsu (2002) the same numerical model was applied and extended to handle turbulence in the porous media. In both cases the empirical coefficients in the extended Darcy-Forchheimer equation were determined based on the recommendations in van Gent (1995) and a limited number of test cases were selected for the current applications. The same numerical model was applied in Garcia et al. (2004), where low-crested breakwaters were investigated and in Lara et al. (2006) who included irregular waves in the investigations. Karim et al. (2003) developed a two-phase model (fluid and air) based on the volume of fluid (VOF) method, that was extended to include interaction with porous media. This was also achieved by the inclusion of resistance terms in the Navier-Stokes equations. The resistance terms were based on a drag and inertia coefficient which needed calibration. Furthermore, the drag resistance terms included a dependency on the resolution of the computational mesh which makes it of little use in practical applications. Their development was further described in Karim and Tanimoto (2006), Karim and Tingsanchali (2006) and Hieu and Tanimoto (2006). In the latter, breaking wave interaction with porous structures was investigated. Further attention was given to wave transformation in the porous structure in Karim et al. (2009) with the same model development. The model presented in Liu et al. (1999) was further developed in Losada et al. (2008) where overtopping of rubble mound breakwaters was investigated and in Guanche et al. (2009) that focused on wave loads on impermeable caisson structures placed on a porous rubble-mound. In Lara et al. (2010) a new model development was presented, which was also based on the volume averaged Navier-Stokes equations in order to describe the porous media by using resistance terms. Here the resistance was formulated as it was originally in Engelund (1954), who proposed a relation between the Darcy-Forchheimer coefficients and physical parameters such as grain diameter and porosity. A thorough description of this model was later given in del Jesus et al. (2012) and Lara et al. (2012b) for model formulation and validation, respectively. The resistance terms were calibrated based on the same experimental results originally presented in Liu et al. (1999).

As described above, the resistance-type porosity models have been developed and applied several times. However, the exact formulations of the porous media equations are still under debate with proposed changes and modifications for each new model development. The latest contribution towards a general set of porous media equations was shown in del Jesus et al. (2012). Here it was also pointed out that some discrepancies were seen compared to the earlier work by Hsu (2002) and de Lemos (2006). As will be seen in the following some further changes to the formulation in del Jesus et al. (2012) are introduced in the present work related to the formulation of continuity and momentum which re-introduces the formulations in Hsu (2002).

The open source CFD code OpenFOAM is gaining popularity not only within the academic community but also among consulting engineers. A framework for the generation and absorption of free surface waves was recently presented in Jacobsen et al. (2012) and has contributed to the availability of OpenFOAM for coastal engineering topics. Examples of the use of OpenFOAM for considering both flow in connection with breakwaters, modelling of scour around structures, and cross-shore morphodynamics were recently seen in for example Lara et al. (2012a), Matsumoto et al. (2012), El Safti et al. (2012), Stahlmann and Schlurmann (2012), and Jacobsen and Fredsøe (in press). Recently, a numerical model for coastal engineering problems was presented in Higuera et al. (2013a) and Higuera et al. (2013b), in which OpenFoam was also applied.

This rises a problem as the porosity model in the official OpenFoam releases (presently up to v. 2.2) does
not conserve mass for free surface flows in porous media (further details are given in Section 2.4), leading in many cases to errors. Furthermore, the porosity was not included in the momentum equation according to the derivation presented in this work (see Appendix A).

The present work proposes changes to the mathematical formulations regarding continuity, momentum, and free surface modelling. The volume averaging process was described in details for deriving the volume averaged equations for porous media flow based on the averaging procedures given in e.g. Whitaker (1966), Gray (1975) and Whitaker (1986a), and also summarised in Slattery (1999). The current implementation was re-implemented in a physically correct manner. The resistance coefficients were described based on the formulations of van Gent (1995), which express the coefficients as a function of porosity, stone diameter and KC number. Only two parameters are unknown in the present formulation, and these were calibrated as part of the calibration procedure. The calibration was performed with focus on the relevant flow regimes and the entire resistance coefficient parameter space.

The main objectives of the present paper are as follows:

- Revise and re-implement the porous media equations for a correct handling of mass continuity and momentum.
- Implement a mass-conserving formulation of the VOF model in the porous media.
- Re-implement the porous media description in OpenFoam based on physical parameters such as porosity, stone diameter and KC number.
- Perform a calibration of the resistance coefficients, which covers a detailed investigation of the resistance coefficients for the relevant flow regimes.
- Validate the recommended resistance coefficients for coastal structure applications.

2. Model description

The model was based on the Navier-Stokes equations, that were transformed to the Volume Averaged Navier-Stokes equations for including the effect of the porosity. The numerical method was based on a finite volume discretisation on a collocated grid arrangement. The present paper used a version of the Navier-Stokes equations where the eddy viscosity is not taken into account; for a formulation of turbulence closures in porous media, the reader is referred to the literature, e.g. Nakayama and Kuwahara (1999). Further discussion on turbulence modelling is given in Section 2.5. The starting point was the general form of the incompressible Navier-Stokes equations formulated with the continuity equation:

\[ \frac{\partial u_i}{\partial x_i} = 0 \]  \hspace{1cm} (1)

and the momentum equation:

\[ \frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + g_j \delta_{ij} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]  \hspace{1cm} (2)

where \( \rho \) is the density of the fluid, \( u_i \) is the Cartesian velocity vector \( (u, v, w) \), \( p \) is the excess pressure, \( g_j \) is the \( j^{th} \) component of the gravitational vector, \( \mu \) is the dynamical viscosity, \( t \) is the time, and \( x_i \) are the Cartesian coordinates.

If Eq. (1) and (2) are to be solved in the porous media, knowledge is required about the geometry of the pores forming the porous media. Furthermore, a very finely resolved computational mesh would be required in order to capture this geometry, which in most cases is not feasible. To overcome this problem the Navier-Stokes equations were averaged over a volume that was assumed to be larger than the length scale of the pores constituting the porous media. A detailed description of the derivation of the volume averaged equations for porous media is presented in Appendix A.

In the following the continuity and momentum equations for porous media flow applied in the present work are presented. Discussions on the physical interpretation related to previous work are included as well.

2.1. Continuity equation

In del Jesus et al. (2012) the continuity equation was first presented based on the pore (intrinsic) velocities and later rearranged based on filter (superficial) velocities by means of the relation \( \langle \overline{v} \rangle = n \langle \overline{v} \rangle' \), where \( \langle \rangle \) denotes the superficial volume average over the entire control volume including the solids, \( \langle \rangle' \) denotes the intrinsic volume average over the pore volume only, and \( n \) is the porosity given as the ratio of the pore volume to the total volume. Here, the continuity equation was given as the divergence of the pore velocity being zero \( (\nabla \cdot \langle \overline{v} \rangle' = 0) \).
Below it is argued, that a physical correct representation will be the divergence of the filter velocity being zero \((\nabla \cdot \langle \mathbf{u} \rangle = 0)\) which was also the definition applied in the previous works, e.g. Hsu (2002).

Figure 1 presents a sketch of a one-dimensional system with a flow through a porous media confined by impermeable walls parallel to the flow direction. The porosity is decreasing in the flow direction i.e. a gradient in the porosity is present. The flux through the system is stationary which leads to an increasing pore velocity in the flow direction as well. Applying the local volume averaging of the pore velocity over the pore volume in the continuity equation yields a non-zero divergence of the velocity field.

If the filter velocity is considered then it is found to be constant in the flow direction. Applying the volume averaging over the entire volume in the continuity equation thus provides a divergence free velocity field. This corresponds to the volume averaging procedure by Gray (1975), Whitaker (1986a), and Whitaker (1996) which was also followed in the present work (see Appendix A). The continuity equation was hereby expressed as:

\[
\frac{\partial \langle \mathbf{u} \rangle}{\partial x_i} = 0
\]

where \(\langle \mathbf{u} \rangle\) is the volume averaged ensemble averaged velocity over the total control volume including the solids of the porous media.

2.2. Momentum equation

For the momentum equation del Jesus et al. (2012) also used pore velocities for the initial volume averaged formulation. This formulation is physically correct as the correct amount of momentum is maintained when pore velocities are used. Correspondingly the formulation was rearranged based on filter velocities divided by the porosity which then again ensures the correct momentum contribution although filter velocities are used instead of pore velocities. The same formulation is adopted in the following according to the derivation shown in Appendix A, where the momentum equation for the porous media becomes:

\[
(1 + C_m) \frac{\partial \rho \langle \mathbf{u} \rangle}{\partial t} + \frac{1}{n} \frac{\partial \rho \langle \mathbf{u} \rangle}{\partial x_j} n = - \frac{\partial \langle p \rangle}{\partial x_i} + g_j x_j + \mu \left( \frac{\partial \langle \mathbf{u} \rangle}{\partial x_j} + \frac{\partial \langle \mathbf{u} \rangle}{\partial x_i} \right) + F_i
\]

where \(C_m\) is the added mass coefficient to take the transient interaction between grains and water into account. Furthermore, an additional term on the right hand side, \(F_i\), was included to take account of the resistance force due to the presence of the porous media. The details of the derivation of the resistance force term are presented in Appendix A. The term, \(F_i\), will also be discussed further in Section 2.3.

The definition of the pressure gradient term is given some further attention in the following. Figure 2A presents a simple case with stationary water in a domain with a clear fluid region and a porous media region. The hydrostatic pressure distribution in both regions will be linear and identical outside and inside the porous media as shown in Figure 2A. Hence the pressure in points C and D will be identical and no pressure gradient exists in the horizontal direction. This also shows that the pore pressure in the porous media is equal to the pressure in the clear fluid region.

In del Jesus et al. (2012) the momentum equation was
defined as the superficial pressure given as the average over the total control volume. This pressure was divided by the porosity in the pressure gradient term. This implies that the pressure gradient term will provide different values inside and outside the porous media. Returning to the simple case of stationary water the formulation given in del Jesus et al. (2012) provides a pressure distribution according to Figure 2B where the pressure gradient is affected by the porosity. Hereby, a pressure gradient is introduced from point C to point D which is not physically correct.

Based on the above it is noted that Eq. (4) deviates from the latest formulation in del Jesus et al. (2012) in terms of the pressure gradient. In the present formulation the pressure was defined as being the pore pressure directly in the momentum equation as described in details in Appendix A. This procedure also follows the volume averaging of the pressure gradient term presented in Whitaker (1986a). With this implementation the simple case in Figure 2A will yield a pressure distribution that is easy to interpret both inside and outside the porous media.

2.3. Porous media resistance forces

When the momentum equation was volume averaged in the porous media, two terms arose representing frictional forces from the porous media and pressure forces (form drag) from the individual grains. The derivation of these terms is shown in Appendix A Eq. (A.37). Also Hsu (2002) and del Jesus et al. (2012) presented these terms. A closure model must be applied to describe the contributions from these terms as they cannot be resolved due to the volume averaging of the porous media. Here the extended Darcy-Forchheimer equation was applied, that includes linear and nonlinear forces as well as inertia forces to account for accelerations. The linear and nonlinear resistance forces were expressed as:

$$F_i = -a \rho \langle u_i \rangle - b \rho \sqrt{\langle u_i \rangle \langle u_j \rangle}$$  \hspace{1cm} (5)

where \( a \) and \( b \) are resistance coefficients. These coefficients must be determined. Engelund (1954) formulated a relation between the resistance coefficients and the porosity, viscosity, and grain diameter for steady state flow, as also later included in Burcharth and Andersen (1995). These relations were included in the model presented by del Jesus et al. (2012). A similar expression to Engelund (1954) was formulated in van Gent (1995), where the effect of oscillatory flows was added to the expressions in terms of the KC-number. The latter formulations were applied in the model presented by Liu et al. (1999) and were also adopted and implemented as part of the present study. The resistance coefficients were formulated as:

$$a = \alpha \frac{(1 - n)^2}{n^3} \frac{\nu}{\rho d_{50}^2}$$ \hspace{1cm} (6)

$$b = \beta \left( 1 + \frac{7.5}{KC} \right) \frac{1 - n}{n^3} \frac{1}{d_{50}}$$ \hspace{1cm} (7)

where \( d_{50} \) is the grain diameter and \( KC = u_m T/(n d_{50}) \), where \( u_m \) is the maximum oscillating velocity and \( T \) is the period of the oscillation. \( \alpha \) and \( \beta \) are empirical coefficients set to 1000 and 1.1 by van Gent (1995). These coefficients are further discussed in Section 3. However, it should be noted, that the literature provides different values depending on how the coefficients have been determined or calibrated and how the resistance coefficients,

![Figure 2: Sketch of definition of pressure gradients (for total pressure, \( P \)) for porous media. A. Correct hydrostatic pressure distribution. B. Non-physical hydrostatic pressure distribution due to porosity effect in the pressure gradient term.](image)
\[ a \text{ and } b \text{ were formulated. The formulation by Engelund (1954) yields } \alpha \text{ and } \beta \text{ values that differ from those obtained in the formulation by van Gent (1995).} \]

Finally, the inertia term in the extended Darcy-Forchheimer equation was included in Eq. (4) through \( C_m \), which van Gent (1995) gave as:
\[ C_m = \gamma_p \frac{1 - n}{n} \]  
where \( \gamma_p \) is an empirical coefficient, which takes the value 0.34.

2.4. Free surface VOF modelling

The present study also revised the original formulation in OpenFoam in terms of the tracking of the free surface interface in the porous media. The interface tracking was conducted using a volume of fluid approach (VOF), where the specific details are given in Berberović et al. (2009). The tracking was resolved by the solution to:
\[ \frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial x_i} \left( \frac{1}{n} \langle \alpha \rangle \right) + \frac{\partial}{\partial x_i} \left( \frac{1}{n} \langle \alpha \rangle \alpha (1 - \alpha) \right) = 0 \]  
where \( \alpha \) is 1 for water and 0 for air. \( \langle \alpha \rangle = \langle \alpha' \rangle \) is a relative velocity between the fluid and the air as described in Berberović et al. (2009). The last term is denoted the compression term and handles the compression of the interface between fluid and air. This term vanishes in the fluid (\( \alpha = 1 \)) and in the air (\( \alpha = 0 \)), and is only active in the interface region of the free surface. Note that the \( 1/n \) terms have to be included to account for the fact that a given volume is filled/emptied faster, when a sediment grain takes up part of the volume.

2.5. Turbulence modelling

The present implementation used a version of the Navier-Stokes equations in which no turbulence closure was introduced i.e. the eddy viscosity was not taken into account. This is considered to be a valid approximation in many engineering applications. As will be seen in the validation cases described below, the results do not suffer from the lack of a turbulence closure model. This was also noted in Higuera et al. (2013a) where validation tests were performed applying a \( \kappa - \epsilon \)-turbulence closure model and in Higuera et al. (2013b) where also a \( \kappa - \omega \)-SST closure model was applied. Here the negligible effect of including a turbulence closure model was attributed to the fact that no or only little wave breaking occurred in the test cases. This is also the case for the simulations presented throughout the present paper.

The above concerns the flow outside the porous media. If the actual level of turbulence kinetic energy is of interest inside the porous media this must be modelled. It was shown in e.g. Hsu (2002), del Jesus et al. (2012) and Lara et al. (2012b) how a \( \kappa - \omega \)-SST model can be applied in the porous media based on Nakayama and Kuwahara (1999). However, the overall effect on the results of applying a turbulence model in the porous media was not investigated. In the following it is argued that when the actual turbulence levels are of minor interest the effect of the turbulence can be included via the Darcy-Forchheimer equation.

The Darcy-Forchheimer equation was introduced to the Navier-Stokes equations as a closure model for handling the porous media resistance force which cannot be resolved directly in the model. This also corresponds to the concept of a closure model for turbulence modelling. If the resistance coefficients, \( \alpha \) and \( \beta \), are found from measurements they already includes the effect of turbulence. If these coefficients are used in the numerical model the contribution from turbulence is therefore included via the coefficients.

If the numerical model, without a turbulence model, is applied for calibration of the resistance coefficients the same effect is achieved in terms of describing the effect of the turbulence via the coefficients. In that case the calibrated coefficients may also be compared to experimental values. If a turbulence model is applied in the numerical model an extra contribution to the resistance is included. In this case the coefficients found from experiments may no longer be applicable. Furthermore, coefficients found from a numerical calibration including a turbulence model may also not be comparable with coefficients found from experiments.

3. Calibration of porous media resistance coefficients

As shown in Section 2.3 the formulation of the Darcy-Forchheimer equation includes two resistance coefficients, \( \alpha \) and \( \beta \), which must be determined empirically. The actual variation of the resistance coefficients are not investigated in-depth at present although the coefficients have previously been investigated by means of physical experiments and theoretical and numerical consider-
ations. In order to provide a general overview a brief summary is given in the following of previous findings from experiments and numerical calibrations. Following this a calibration is performed related to the present implementation.

3.1. Flow-regimes

The resistance coefficients represent the contribution from the linear and non-linear terms in the Darcy-Forchheimer equation. These terms will have different importance depending on the flow regime. For a very low Reynolds number the linear term will dominate the resistance and the non-linear term will not influence the total resistance to a very high degree. The opposite is the case for a high Reynolds number flow. To achieve a valid calibration with a broad applicability it is important to include investigations that covers all flow regimes.

A general description of the flow regimes in a porous media was summarised by Burcharth and Andersen (1995). Here, the regimes were defined based on the pore Reynolds-number given as:

$$Re_p = \frac{(\bar{u})D_50}{\nu}$$

(10)

where $D_50$ is the median grain size, and $\nu$ is the kinematic viscosity. The Darcy (laminar) flow regime was defined for $Re_p < 1$. At $1 < Re_p < 10$ the boundary layer around the solid grains becomes more pronounced and for $Re_p > 10$ a non-linear relationship appears between the resistance and flow rate. This non-linear laminar flow regime is referred to as the Forchheimer flow regime and is present for $Re_p$-number up to $\approx 150$. For $150 < Re_p < 300$ an unsteady laminar flow regime occurs, which is a transitional regime between the Forchheimer and the fully turbulent flow regime. For $Re_p > 300$ a fully turbulent flow regime has developed.

To summarise, the following flow regimes are given attention in the present work:

1. The Forchheimer flow regime, $10 < Re_p < 150$
2. The transitional flow regime, $150 < Re_p < 300$
3. The fully turbulent flow regime, $Re_p > 300$

3.2. Previous investigations on resistance coefficients

The existing knowledge about the variation of the resistance coefficients originates from theoretical considerations, physical experiments and numerical calibrations. Engelund (1954) presented a formulation for the resistance terms in the Darcy-Forchheimer equation with recommendations for the resistance coefficients for irregular sand grains. The values were proposed up to $\alpha = 1500$ and $\beta = 3.6$ (reformulated into the formulations by van Gent (1995) in Eq. (6)-(7) the coefficients take the values $\alpha = 360$ and $\beta = 3.6$, assuming a porosity of $n = 0.4$). The work by Engelund (1954) was based on sand materials and was in general not in the Re-number ranges typically found in coastal engineering applications. In van Gent (1995) an experimental investigation was performed with focus on coastal structures. Here a new formulation of the terms in the Darcy-Forchheimer equation was proposed which incorporated the effect of an oscillating flow via the $KC$-number as shown in Eq. (7). The resistance coefficients were proposed to take the values $\alpha = 1000$ and $\beta = 1.1$. Burcharth and Andersen (1995) examined the porous media flow equations and also based on the formulations in Engelund (1954) recommended values for the resistance coefficients.

The two formulations of the resistance terms given in Engelund (1954) and van Gent (1995) have frequently been used for numerical modelling of fluid interaction with porous media. However, in terms of calibration of these coefficients when applied in a numerical model the background is more weak. In Liu et al. (1999) a dam break experiment was applied to calibrate the resistance coefficients. Here the starting point was the values recommended in van Gent (1995). Based on comparisons with the experimental results the $\beta$-coefficient was maintained at a value at 1.1 while the $\alpha$-coefficient was reduced to a value at 200. This was for an experiment with glass beads and low $Re$-numbers where the viscous effects had greater importance. For a corresponding experiments with crushed rocks Liu et al. (1999) applied the original values from van Gent (1995) as $\alpha = 1000$ and $\beta = 1.1$. No further investigations of the variation over the parameter space were performed. In Hsu (2002) the coefficients were adopted with the same values as given in Liu et al. (1999) also with no further investigations of the parameter space.

del Jesus et al. (2012) returned to the dam break experiments given in Liu et al. (1999) and performed a more detailed investigation. Here the resistance coefficients were calibrated by testing a range of both $\alpha$ and $\beta$. Three values of $\alpha$ were selected as $\alpha = [5000, 10000, 20000]$, while $\beta$ remained constant at $\beta = 3.0$. Here the best
comparison with the experimental data was found for \( \alpha = 10000 \). Hereafter \( \beta \) was tested with the values \( \beta = [1.0, 3.0, 6.0] \), while \( \alpha \) remained constant at \( \alpha = 10000 \) (corresponding to the best fit from the first part of the calibration). Here the best comparison was found for \( \beta = 3.0 \). It is not surprising to arrive at a value of \( \beta = 3.0 \) as this was the only value applied when calibrating \( \alpha \). In del Jesus et al. (2012) the formulation of resistance parameters by Engelund (1954) was applied. Reformulated into the formulations by van Gent (1995) in Eq. (6)-(7) the coefficients take the values \( \alpha = 2500 \) and \( \beta = 3.6 \), assuming a porosity of \( n = 0.49 \).

In Wu and Hsiao (2013) the model formulation previously showed in Hsu (2002) was applied. Here three combinations of resistance coefficients were investigated. The two first were based on Liu et al. (1999) (\( \alpha = 200 \) and \( \beta = 1.1 \)) and van Gent (1995) (\( \alpha = 1000 \) and \( \beta = 1.1 \)). The third combination was based on Lara et al. (2011) which presented formulas for calculating the resistance coefficients (\( \alpha = 724.57 \) and \( \beta = 8.15 \)). As also noted in Wu and Hsiao (2013) the latter set of coefficients were out of range of what has previously been referred to in the literature regarding the \( \beta \)-coefficient and did also provide the largest discrepancies compared to experimental results. The two remaining set of coefficients were selected with the same value for the \( \beta \)-coefficient. The investigated flows were in the fully turbulent regime with \( Re_p = 1.5 \times 10^5 \). As such the non-linear coefficient, \( \beta \), is expected to be dominating while the total friction will only have little effect of the linear coefficient, \( \alpha \). Based in this it is seen that the parameter space has been very poorly investigated. Especially as the dominating non-linear term was maintained at the same value for two of the parameter combinations.

### 3.3. Calibration procedure

The calibration presented in the following was set up in order to close some of the gaps presently found in the literature. One major concern is the use of only one calibration case for the calibration of two parameters. In the calibration procedure applied in the present study three cases were included; 1) Forchheimer flow with \( Re_p = 62 \) where the linear coefficient, \( \alpha \), was dominating, 2) Transitional (Forchheimer)/turbulent flow with \( Re_p = 325 \) where \( \alpha \) was dominating while also the non-linear coefficient, \( \beta \), had some effect on the flow, and 3) Fully turbulent flow with \( Re_p = 2750 \) where the non-linear parameter, \( \beta \), was dominating. Hereby the effect of both parameters was investigated and a better understanding of the entire parameter space was achieved.

In the following the three calibration cases are presented and the results in terms of the error compared to experimental results are included. The error was computed as:

\[
\epsilon = \frac{1}{M} \sum_{j=1}^{M} \left( \frac{1}{X} \sum_{i=1}^{N} \left( \eta_{exp,i} - \eta_{num,i} \right) \Delta x_i \right)
\]

where \( M \) is the total number of time steps (equidistantly spaced) for which comparisons are made, \( X \) is the total horizontal distance over which comparisons are made, \( N \) is the number of data points over the distance \( X \), \( \eta \) is the quantity being compared, \( \Delta x \) is the sub-distance corresponding to each data point.

Following the three cases a common set of resistance coefficients are found and the results of all three cases are finally presented based on these recommended coefficients.

#### 3.4. Forchheimer flow - \( Re_p = 62 \)

In the following a calibration was performed including a flow where the linear parameter, \( \alpha \), was most dominating. The case is a stationary flow through a dam where the flow was established by separating two water bodies by a porous rectangular dam. A head difference between the front and back of the dam was enforced which drove the flow through the porous material. The Reynolds number was found to be \( Re_p = 62 \) which corresponds to the Forchheimer flow regime. The position of the free surface through the porous material was compared to experimentally obtained results in Billstein et al. (1999). The setup is shown in Figure 3.

The test was conducted with the length of the porous dam being \( L = 0.215 \) m. The water depth on left hand side was set to \( H = 0.522 \) m, while the water depth on the right hand side was \( h = 0.015 \) m. The porous dam was made out of spherical glass beads in the experiments. The porosity was \( n = 0.34 \) and the grain diameter was \( d = 2 \) mm.

The numerical model was setup as a two-dimensional domain with a length of 1.0 m and a height of 0.5 m. This gave a length of 0.39 m on the upstream and downstream sides of the porous dam for inlet and outlet conditions. Uniform grid resolution was applied in the entire domain.
with a cell size of 0.4 cm in all directions. A total of 31,250 grid cells were applied. The total simulation time was 30 s which was found to provide a stationary solution. Each simulation was performed on one processor core and was completed in approximately 1 h.

The calibration was achieved by completing a simulation matrix, where the two coefficients were varied as \( \alpha = [100, 500, 1000, 2000, 3000] \) and \( \beta = [0.5, 1.0, 2.0, 3.0, 4.0] \). This yields a total of 25 simulations. The error between simulation and experiment was computed as the difference between the measured and simulated water surface according to Eq. (11). Figure 4 shows the error between the simulated and experimental surface elevation as contours over the parameter space. It was found that not one unique combination of \( \alpha \) and \( \beta \) provided the best fit between simulated and experimental results. An area in the parameter space was found to provide low errors as indicated with white colour in Figure 4. This area is mostly depending on the \( \alpha \)-value.

Figure 3: Sketch of the setup for stationary dam break through a porous dam following Billstein et al. (1999).

3.5. Forchheimer/turbulent flow - \( \text{Re}_p = 325 \)

The second calibration case considered a simple dam break through a porous media with a Reynolds number of \( \text{Re}_p = 325 \). This corresponds to the flow regime at the transition between the Forchheimer and the turbulent flow regime. Here a stronger dependency on the non-linear coefficient, \( \beta \), was expected. The simulated results were compared to the experimental results given in Liu et al. (1999).

Figure 5 presents the experimental setup in terms of geometry and dimensions. The porous structure consisting of gravel was considered with a porosity of \( n = 0.49 \) and a diameter of \( d_{50} = 1.59 \) cm. The water was initially separated from the porous media by a vertical gate on the left hand side. When the experiment was started the gate was rapidly removed and the water was allowed to flow through the porous media. At the initial stage the flow experienced a high acceleration into the porous media, while later the flow resembles a more stationary flow through the porous media.

Figure 4: Contours of the error between simulated and experimental surface elevation for stationary dam flow with \( \text{Re}_p = 62 \). Black dots corresponds to simulations.

Figure 5: Setup of porous dam break following Liu et al. (1999).

The numerical model was setup as a two-dimensional domain with the exact dimensions as the physical model experiment. This gave a length of the domain of 0.892 m
and a height of 0.5 m. The porous dam was placed at the centre of the domain. Uniform grid resolution was applied in the entire domain with a cell size of 0.4 cm in all directions. A total of 27,875 grid cells were applied. The total simulation time was 4 s for comparisons with the experimental data. Each simulation was performed on one processor core and was completed in approximately 20 min.

The calibration was achieved, as for the stationary dam flow, by completing a simulation matrix, where the two coefficients were varied as $\alpha = [100, 500, 1000, 2000, 3000]$ and $\beta = [0.5, 1.0, 2.0, 3.0, 4.0]$. The error between simulation and experiment was computed as the difference between the measured and simulated water surfaces according to Eq. (11). Figure 6 shows the error between the simulated and experimental surface elevation as contours over the parameter space. A large dependency on $\beta$ was found as also reported in Liu et al. (1999). This is seen as the contours are mostly aligned along the axis representing $\alpha$ which means that only little effect of $\alpha$ is seen. Furthermore it was found that not one unique combination of $\alpha$ and $\beta$ would give a minimum error. Rather a band in the parameter space was found to provide low errors. This is indicated in Figure 6 with the white contour.

This simulation was the only one in this work where water neither entered nor left the computational domain. Therefore it was well suited for validating the implementation of Eq. (9). The fluid fraction in the domain was found to be constant throughout the simulation as seen in Figure 7 in terms of the curve with porosity correction. For comparison, the original formulation in OpenFoam where the $1/n$ terms were not included is also shown in Figure 7 as the curve without porosity correction. Here it is seen how the mass was decreasing as the fluid entered the porous media.

![Figure 6: Contours of the error between simulated and experimental surface elevation for the dam break dam flow with $Re_p = 325$. Black dots corresponds to simulations.](image1)

![Figure 7: Fluid phase fraction in the computational domain. With porosity correction refers to the new implementation according to Eq. (9). Without porosity correction refers to the original OpenFoam implementation.](image2)

### 3.6. Turbulent flow - $Re_p = 2750$

The third calibration case was a stationary dam flow with a Reynolds number at $Re_p = 2750$. This corresponds to the fully turbulent flow regime. The setup corresponded to the stationary flow through a dam shown in Section 3.4 where the flow was established by separating two water bodies by a porous dam. In this case the test was conducted with the length of the porous dam being $L = 0.5$ m. The water depth on the left hand side was set to $H = 0.292$ m, while the water depth on the right hand side was $h = 0.118$ m. The porous dam was made out of spherical glass beads, the porosity was $n = 0.41$ and the grain diameter was $d = 25$ mm.

The numerical model was setup as a two-dimensional domain with a length of 1.0 m and a height of 0.5 m. This gave a length of 0.25 m on the upstream and downstream sides of the porous dam for inlet and outlet conditions.
Uniform grid resolution was applied in the entire domain with a cell size of 0.4 cm in all directions. A total of 31,250 grid cells were applied. The total simulation time was 30 s which was found to provide a stationary solution. Each simulation was performed on one processor core and was completed in approximately 2 h.

The same simulation matrix as for the two previous calibration cases was completed with \( \alpha = [100, 500, 1000, 2000, 3000] \) and \( \beta = [0.5, 1.0, 2.0, 3.0, 4.0] \). The error between the simulated and measured surface elevation is presented in Figure 8 as contours over the parameter space. Here the effect of the fully turbulent flow was seen in terms of the non-linear coefficient, \( \beta \), having the most dominating effect on the results. Practically no effect of the linear coefficient, \( \alpha \), was seen. The error was found to be minimized for a rather large area in the parameter space indicated with the white contour in Figure 8.

![Figure 8: Contours of the error between simulated and experimental surface elevation for stationary dam flow with \( Re_p = 2750 \). Black dots corresponds to simulations.](image)

3.7. Recommended coefficients and calibration results

Based on the parameter investigation presented in Section 3.4, 3.5, and 3.6 it was found that a common area in the parameter space would provide an optimised solution for all three tested flow regimes. For the Forchheimer flow regime a strong dependency on the linear coefficient, \( \alpha \), was found with practically no effect on the solution from the non-linear coefficient, \( \beta \). The optimal solution was found for relative low values for \( \alpha \) at 100 to 500. For the fully turbulent flow regime the strong dependency was found on the non-linear coefficient, \( \beta \). Here the optimal solution was found for \( \beta \) taking values between 2 and 3. For the transitional flow regime between Forchheimer and turbulent flow some dependency was seen on both \( \alpha \) and \( \beta \) however the most dominating was the non-linear coefficient, \( \beta \). The \( Re_p \)-number for this case was just above the upper limit for the transitional regime given in Burchart and Andersen (1995), hence a stronger dependency on \( \beta \) was expected. The optimal solution was found for 1) low values of \( \beta \) at around 1 and corresponding high values of \( \alpha \) at around 2500 or, 2) higher values of \( \beta \) at around 2 and corresponding lower values of \( \alpha \) at around 500.

When the results of all three parameter investigations were taken into account the optimal values were proposed to be \( \alpha = 500 \) and \( \beta = 2 \). The results of the three calibration cases with these coefficients are presented in the following.

The results in terms of the free surface position through the porous dam during stationary flow conditions are presented in Figure 9. A good agreement with the experimental data was seen for both the Forchheimer regime (Figure 9A) and the turbulent regime (Figure 9B). It should also be noted that the vertical jump in the water surface at the downstream interface of the porous media for the Forchheimer regime was well predicted by the model as it may be seen in Figure 9A.

The results for the non-stationary porous dam break case are shown in Figure 10. The first time steps in the comparison showed some deviations with the experimental results. The same deviations were seen in Liu et al. (1999) and del Jesus et al. (2012) and may be attributed to the different initiations of the dam break in the physical experiment and the numerical model. In the physical experiment the gate separating the water from the porous structure was removed by pulling it upwards. Hereby the water was allowed to flow towards the porous structure in the bottom first. In the numerical model the entire volume of water was released at the same time. After about 0.6 s the experimental and numerical results coincide. This was the case both inside the porous media and outside in the free fluid where reflections from the end walls of the tank were captured as well.

The calibration results showed that a constant set of
resistance parameters may represent the flow for both Forchheimer, transitional, and turbulent flow regimes. As such it was found that the proposed resistance coefficients may also be applied for different parts of a porous structure with different stone diameters and porosity, and therefore different flow regimes.

### 3.8. Scale effects

In the following a brief account is given on scale effects between model and prototype scales. Hydraulic model experiments with porous structures are often conducted in a scale where the flow in some parts of the structure is not fully turbulent. If the corresponding flow in prototype scale is turbulent it may lead to scale effects.

In Jensen and Klinting (1983) an analysis was presented of the scale effects for model experiments with breakwater structures. Here a method was proposed for compensating for the scale effect by adjusting the stone diameter in the model experiment. The adjustment was based on the fact that for complete similarity the hydraulic gradient must be identical in model and prototype. Expressing the hydraulic gradient in terms of the Darcy-Forchheimer equation (Eq. (5)) for both model and prototype scale the stone diameter for model scale can be found as a function of the stone diameter in prototype, velocity, and porosity. This can also be seen as a function of the Reynolds number. In Pérez-Romero et al.
(2009) the scale effect in porous flow was also investigated and the same conclusions as in Jensen and Klinting (1983) were given in terms of the method of adjusting the stone diameter in model scale.

The possible scale effects in model scale experiments as referred above should be taken into consideration when applying resistance coefficients found from experiments or numerical calibrations. The main reason for the scale effect related to the flow in the porous part of a structure is due to the different flow regimes in model and prototype. As such the flow regime expressed by the Reynolds number should be evaluated for the experiments in which the calibration is based on. In the previous work related to numerical applications of the Darcy-Forchheimer equation the resistance coefficients were often evaluated for one specific application i.e. only one calibration test case. See for example Hsu (2002), Garcia et al. (2004), Hieu and Tanimoto (2006), Karim et al. (2009), and del Jesus et al. (2012). This provides coefficients for the given case however, a general understanding of the variation of the resistance coefficients over several flow regimes may not be found. In the present work the calibration procedure was designed in order to provide information on the resistance coefficients for several flow regimes. Based on the three calibration cases for three different flow regimes the resistance coefficients were proposed with values covering a broader applicability.

The calibration case for the fully turbulent flow was based on a Reynolds number of $Re_p = 2750$. Although this corresponds to a turbulent flow there will still be a large deviation from the largest Reynolds numbers seen in prototype scale. For the armour layers exposed to run-up and run-down the Reynolds number may be 2-3 orders of magnitude higher than for the present test case. Based on this it is noted, that further investigations for higher Reynolds numbers may provide more information about possible scale effects within the fully turbulent flow regime.

4. Validation cases

Validation cases were carried out to demonstrate the ability of the model to reproduce the physical conditions involved in coastal applications. Three cases were selected with the focus on three-dimensional wave-structure interaction, wave run-up, run-down, and pressure damping, and finally overtopping events on breakwater profiles.

4.1. Three-dimensional wave interaction with caisson

The first validation case was based on experimental data presented in del Jesus et al. (2012) for wave interaction with a porous caisson structure. The structure was rectangular and was placed in a wave flume with an opening on one side of the caisson. Hereby a three-dimensional flow was seen both outside and inside the porous structure. The experimental data consisted of measurements of the surface elevation around the structure.

Details on the experimental setup are given in del Jesus et al. (2012). The wave flume had a length from the wave maker mean position to the end of the flume of 20.595 m. The width was $d = 0.585$ m and total depth was $0.78$ m. The porous caisson structure was positioned with the centre of the structure at a distance of 11.519 m from the wave maker. The arrangement of the caisson structure is shown in Figure 11. The structure had a length, $L$, and width, $W$, at 0.24 m and was placed along one side of the flume. A block of impermeable Plexiglass with a thickness of 0.06 m was placed between the structure and the side of the flume. The porous structure was made out of crushed stones with a mean diameter at $D_{50} = 0.083$ m. The final structure had a porosity at $n = 0.48$.

The experiment was conducted with regular waves based on cnoidal theory with a wave height at $H = 0.06$ m and a period at $T = 2$ s. The mean water depth was fixed at $h = 0.25$ m. The experiment had a total time of 20 s which allowed about 6 waves to pass the structure. Measurements were performed of the surface elevations at positions around the structure. The position of the gauges used in the present validation are shown in Figure 12.

The numerical model was setup with a domain length corresponding to the physical flume at 20.595 m and a width at 0.585 m. The height of the domain was 0.45 m with a mean water level at $h = 0.25$ m. The porous caisson was placed according to the experiments at a distance of 11.519 m from the inlet. The resistance coefficients applied in the numerical model were $\alpha = 500$ and $\beta = 2.0$ according to the calibration results in Section 3.7. The overall grid size was set to 4.0 cm in all directions. A refinement zone was applied in a band around the free surface at $-0.05m < z < 0.08m$ where $z$ is the
vertical coordinate. The grid cells were refined in all di-
rections to a size of 1.0 cm in this area. A relaxation
zone for wave generation and absorption was applied at
the inlet and outlet with a length at 6.0 m and 3.0 m, re-
spectively. A total of 2.1M grid cells were applied. The
computational time was 7 h on 24 cores for simulating
20 s.

Figure 11: Setup of experimental test for three-dimensional wave-
structure interaction as given in Lara et al. (2012b).

Figure 12: Wave gauge arrangement around the porous caisson fol-
lowing the setup given in Lara et al. (2012b).

Figure 13 shows the free surface elevation in the ap-
plied wave gauge positions. The results are compared to
the experimental data from Lara et al. (2012b). In gen-
eral a good agreement was found both in terms of phase
and amplitude. Some discrepancies were observed in the
beginning and the end of the time series. At the begin-
ing the numerical and experimental wave generations
are not identical. It appears that the experimental wave
generation has gained up the input signal over 1-2 peri-
ods which is not the case for the numerical wave gen-
eration. At the end of the time series the experimental
results are affected by reflected waves from the end of
the flume, which returns to the model testing area. This
is especially seen in wave gauges no. 7 where the am-
plitude is suddenly increased during the last 1-2 wave
periods. This effect was also noted in Lara et al. (2012b).
The numerical simulations applied a relaxations zone at
the outlet boundary which absorbed the majority of the
wave energy. Therefore the effect of wave reflection was
not seen in the numerical results thus causing some devi-
ations at the end of the time series. With the above men-
tioned causes for the observed discrepancies the model
was found to provide a good representation of the three-
dimensional flow around the porous caisson structure.

The three-dimensionality of the flow field around and
inside the porous caisson is shown in Figure 14 in terms
of a contour plot of the surface elevation at one instance
in time. Here it can be seen how the surface elevation
was affected by the presence of the porous caisson which
causes the flow to pass partly around the structure and
partly through the porous structure.

4.2. Run-up and run-down velocities, and pressure in
rubble mound breakwater

An experimental test series was performed as part of
the present study, in order to evaluate the external and
internal flows for a rubble mound breakwater structure.
In these experiments, velocity measurements were made
with Laser Doppler Anemometry (LDA) of the detailed
flow in the armour layer and the internal flow in the
porous core. Pressure measurements were performed
across the porous core and detailed profiles were ob-
tained below the armour layer. In the following the re-
results obtained for the flow velocities on the front slope
(run-up and run-down) and the pressure variation across
the structure will be used. The experimental setup and
procedures are described in Vistisen (2012) while a brief
description of the experiments is given in the following.

All tests were carried out in a wave flume at the hy-
draulic laboratory at the Technical University of Den-
mark. The flume has a length of 25 m, a width of 0.6 m,
and a depth of 0.8 m. The water depth for the present ex-
periments was fixed at $h = 0.4$ m. The flume is equipped
with a piston-type wave maker at one end for generating
regular or irregular wave conditions.
The breakwater model had a slope of 1:1.5 on both front and rearward sides. The crest height was \( H_c = 0.68 \) m which was sufficiently high to prevent overtopping. The crest width was \( w = 0.2 \) m. The porous interior of the structure was composed of randomly packed spherical plastic elements with a diameter of 38 mm. A cage was constructed with a perforated steel plate to hold the interior plastic spheres in place. The porosity of the randomly packed spheres was measured to be \( n = 0.4 \). An armour layer was placed on the front slope, which was composed of the same spherical plastic elements as the internal porous structure. These were glued to the surface of the perforated steel plate in a structured 90 degree pattern following the definition given in Dixen et al. (2008) in their Figure 4.

Three types of measurements were performed: velocity measurements, pressure measurements and surface elevation measurements. Measurements of velocities and turbulence were performed with LDA. A DANTEC two-component LDA system was applied in backscatter mode, where two velocity components (horizontal and vertical) were measured simultaneously. Velocities were measured above the armour layer on the front slope as shown in Figure 15 (points v1 and v2).

Pressure measurements were performed at seven locations inside the porous structure as shown in Figure 15 (points p1-p7). Rigid tubes were inserted in the structure with the opening at the position of the measurement points. The tubes were connected by plastic tubes to the pressure transducers.

The measurements of the surface elevation were performed at two locations in the offshore area (WG1) and at the toe of the sloping bed (WG2). Conventional resistance type wave gauges were used in the measurements. The LDA and pressure measurements were synchronised with the surface elevation measurements at WG2.

Regular waves were generated with Stokes 1st order theory. The wave height was \( H = 0.14 \) m and the period was \( T = 3 \text{ s} \). The wave maker was operated with active absorption (DHI AWACS) which removes the energy re-
flected from the structure back to the wave maker. The experiments had a duration of 6 min, where the last 1 min where applied for ensemble averaging.

The numerical model was set up to correspond to the experiments with a distance from the inlet boundary to the toe of the breakwater at 13.55 m. The distance from the leeward side of the breakwater to the outlet was 6.0 m. The height of the domain was 0.8 m. Relaxation zones were added at the inlet and outlet for wave generation and absorption with a length at 8.0 m and 5.0 m respectively. A uniform grid size of 0.02 m was used in the domain. Grid refinement was applied around and inside the breakwater structure with a grid size at 0.01 m in all directions. A total of 80,000 grid cells were applied. The total simulation time was 6 min and each simulation was completed on one processor core in approximately 28 h. The porosity module was applied with resistance coefficients set to $\alpha = 500$ and $\beta = 2.0$ according to the calibration results in Section 3.

The measured velocities at points v1 and v2 are compared to the numerical results in Figures 16 and 17. The horizontal and vertical velocity components are projected onto the slope of the breakwater in order to represent a parallel, $u$, and a normal, $v$, velocity component. The results are presented in terms of ensemble averaged values over 20 wave periods. A reasonable comparison with the experiments was found, in which the general variation was captured as well as the maximum and minimum values over one wave period. However, some deviations may be seen, especially during the last part of the wave period (from about $t/T = 0.5$). This is found to be due to corresponding deviations in the representation of the free surface which also suffers from minor deviations from the experimental results at this point. This is linked to the fact that 1st order wave theory was applied, which causes higher order harmonics to be formed as the wave travels down through the wave flume, see Chapalain et al. (1992). The interaction of these harmonics will affect the free surface profile, which is therefore very sensitive to the exact position of the wave generation relative to the structure. Further optimization of the numerical setup in terms of the wave generation layout may improve the prediction of the free surface profile and is also expected to improve the predicted velocities.

The pressure measurements are compared to the numerical results in Figure 18. The maximum and minimum pressure over one wave period is shown. At the first measuring point, p1, the pressure was slightly overestimated for the maximum pressure. At this point the pressure reading is mostly a representation of the water depth, as the position is close to the edge of the front slope. These deviations are therefore linked to the representation of the free surface, which also exhibits minor deviations from the experimental results. This is discussed above in connection with the velocity results. A good representation of the pressure envelope was found from point p2 and onwards through the structure.

It should be noted that the present breakwater case was simulated in a two-dimensional model, which in the case...
of wave breaking and air entrapment may have a limitation. The numerical model was formulated as incompressible and with a two-dimensional setup the entrapped air phase cannot escape in the transversal direction. In a case with possible wave breaking a three-dimensional setup may be more accurate. However, in the case presented here there were no or only little wave breaking and as such a two-dimensional model was found to be appropriate.

4.3. Overtopping events

A quantity which must be evaluated for a given breakwater design is the amount of overtopping i.e. how much water passes above the structure towards the shoreward side. Breakwaters are designed to prevent overtopping or to allow a certain amount of overtopping depending on the specific use of the structure. In the following, two overtopping tests are presented. The results were compared to the experimental results and empirical relations reported in Bruce et al. (2009). These experiments are also part of the CLASH database reported in Steendam (2004). Here more than 10,000 test results concerning wave overtopping are collected in one single data base.

Based on results from model experiments the overtopping may be estimated by empirical relations. For the present configuration of the breakwater structure the amount of overtopping, \( q \), may be estimated by Bruce et al. (2009):

\[
q = \frac{1}{\sqrt{gH_{m0}^3}} \left(-2.6 \frac{R_c}{H_{m0} \gamma_f}\right)
\]

(12)

where \( H_{m0} \) is the incident significant wave height, \( R_c \) is the crest freeboard and \( \gamma_f \) is the roughness influence factor. The roughness influence factor includes the effect of the armour roughness and porosity and was determined in Bruce et al. (2009) for the applied armour configurations to take the value \( \gamma_f = 0.42 \). For a smooth impermeable surface the value is \( \gamma_f = 1.0 \).

Details on the entire experimental series can be found in Bruce et al. (2009). In the following a review of the experiments used in the present investigations is given. The case with an armour layer composed of natural rocks was selected. The general layout of the structure is presented in Figure 19. The breakwater had a slope at 1:1.5. The breakwater was composed of three materials; core, filter layer and armour layer. The thickness of the layers was related to the diameter of the applied armour units in the experiments. For the selected case the armour stone had a diameter, \( d_{50} = 0.03 \) m. The corresponding grain diameters for the filter and core material are 0.014 m and 0.007 m based on the characteristics given in Bruce et al. (2009). The resistance parameters in the porosity model, \( \alpha \) and \( \beta \), were set to the calibrated parameters from Section 3.

According to the experiments two water depths were
used, given as \( h = 0.185 \) m and \( h = 0.222 \) m. An armour crest level of 0.2812 m yields a free board of \( R_c = 0.0962 \) m and \( R_c = 0.0592 \) m respectively. An irregular wave condition was applied based on a JONSWAP spectrum with a peak enhancement factor of \( \gamma = 3.3 \). Three different significant wave heights and peak wave periods were applied as \( H_{\text{ref}} = [0.111, 0.074, 0.0555] \) m and \( T_p = [1.56, 1.16, 0.97] \) s. A total of 12 simulations were conducted as the two smallest wave heights were not simulated for the highest crest free board.

The numerical model was setup with a total domain length of 16.0 m and height of 0.5 m. The porous breakwater was placed at a distance of 10 m from the inlet. An overall uniform grid resolution was applied with a grid cell size of 5 cm. Refinement was applied around the free surface and breakwater structure with three refinement levels. This gave a grid cell size of 0.625 cm around the free surface and the porous breakwater. This gave a total of 63,000 grid cells. The total simulation time was set to 1024 s following the experiments, which included between 700 and 1300 waves depending on the wave period. Each simulation was simulated in parallel on eight processor cores and was completed in 96 h. The waves were generated according to the target wave conditions reported for the experiments. The actual incident and reflected wave characteristics were determined based on the method described in Zelt and Skjelbreia (1992) for separating incident and reflected wave fields.

First, a benchmark test was performed with a smooth impermeable structure. The impermeable slope was simulated as a solid boundary i.e. the porosity model was not applied in this case. From this it is seen how the overtopping rate is predicted for the extreme case with maximum overtopping.

Figure 19: Layout of the breakwater structure following the experimental setup given in Bruce et al. (2009).

The overtopping rate was measured at the back edge of the armour crest and is represented as the dimensionless overtopping as given in Eq. (12). Figure 20 presents the overtopping rate for the smooth surface compared to experimental data (Bruce et al. (2009)) and the empirical relation (Eq. (12)). The model was found to provide a good agreement between the simulated and experimental results. From this it may be seen that the run-up and overtopping process was described correctly without any interaction with the porous structure.

Figure 20: Comparison between the empirical overtopping, experimental data by Bruce et al. (2009) and simulated values for the smooth impermeable slope with \( \gamma_f = 1.0 \).

Figure 21 presents the simulated overtopping for the porous breakwater structure compared to the empirical relation (Eq. (12)) and the experimental results (Bruce et al. (2009)). The model was found to give a good estimation of the overtopping. The simulated results were distributed around the empirical relation in the same manner as the experimental results. No general over- or underestimation was seen.

The incident and reflected spectra were separated in order to evaluate the reflection coefficient of the present structure. The incident and reflected wave height, \( H_I \) and \( H_R \) were determined based on the integral of the spectra, \( m_0 \), as \( H = 4 \sqrt{m_0} \). The spectra were truncated at 0.05 Hz and 10.0 Hz. The results for all simulated cases are shown in Figure 22 as a function of the breaker (surf-similarity) parameter, \( \zeta_0 = \tan \alpha / \sqrt{S_0} \), where \( S_0 \) is the wave steepness based on the spectral period \( T_{m \rightarrow 0} = m_{-1} / m_0 \) and the significant wave height at the toe of the breakwater. The results were compared to the empirical relation given in Zanuttigh and Van der Meer (2006). It is noted, that the reflection coefficients for the experiments given in Bruce et al. (2009) were dis-
tributed around the empirical relation with the same accuracy as seen for the numerical predictions.

1. The OpenFoam porosity model was revised and implemented using resistance parameters based on the physical parameters porosity, stone diameter and KC number.
2. The VOF model for simulating free surface flows was implemented for the porous media in a mass conserving form.
3. The resistance coefficients for the linear and nonlinear contributions were investigated in details over the parameter space for both Forchheimer, transitional, and turbulent flow regimes.
4. Based on the parameter investigation the coefficients were recommended to take the values \( \alpha = 500 \) and \( \beta = 2.0 \).
5. The model was found to correctly predict interaction with both two and three dimensional structures.

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DHI provided the AWACS system applied for the generation and active absorption of waves during the physical experiments described in section 4.2.

Appendix A. Derivation of volume averaged equations

The Navier-Stokes equations were ensemble averaged to obtain the Reynolds Averaged Navier-Stokes equations (RANS) following the general temporal averaging procedure described in e.g. Ferziger and Peric (2002). The volume averaging procedure was applied on the RANS-equations to obtain the Volume Averaged/Reynolds Averaged Navier-Stokes equations (VARANS). The method applied in the following refers to the procedures described in Whitaker (1966), Gray (1975), Whitaker (1986a), and Whitaker (1986b). The same methodology was later applied in Ochoa-tapia and Whitaker (1995) and Whitaker (1996). In Slattery (1999) a general overview of the method was summarised.

A porous media was defined as illustrated in Figure A.23. The surface, \( S \), creates the averaging volume, \( V \), which may include both the solid phase and the fluid phase. In this example the surface, \( S \), is defined by a
circle with the radius, $r_0$. The actual volume of the fluid phase, $V_f$, may variate over the porous media depending on the position of the averaging volume while the total volume, $V$, is constant. The macroscopic length scale, $L$, and pore length scale, $l$, are defined as shown in the figure. The volume averaging procedure is applied with the length scale constraint given as $l \ll r_0 \ll L$.

For a gradient, $\nabla B$, the corresponding volume averaging operator is defined according to Howes and Whitaker (1984) as:

$$
\langle \nabla B \rangle \equiv \frac{1}{V} \int_{V_f} \nabla B dV = \nabla \left( \frac{1}{V} \int_{V} B dV \right) + \frac{1}{V} \int_{S_w} B \mathbf{n} dA
$$

(A.5)

where $\langle \nabla B \rangle$ is the volume averaged gradient, $S_w$ is the surface area of the solids, $\mathbf{n}$ is the normal vector to the surface of the solids. This volume averaging operator is referred to as the theorem for the volume average of a gradient.

For the ensemble averaging procedure the velocity in a point is assumed to be composed of an ensemble averaged value and a temporal fluctuations as:

$$
u_i = \overline{\nu}_i + u_i'$$

(A.6)

where $\overline{\nu}_i$ is the ensemble averaged value and $u_i'$ is the temporal fluctuation.

When the volume averaging procedure is applied to the ensemble averaged value, it is convenient to introduce the velocity decomposition given by Gray (1975) as:

$$
\overline{\nu}_i = \langle \overline{\nu}_i \rangle_f + \overline{\nu}_i''
$$

(A.7)

where $\langle \overline{\nu}_i \rangle_f$ is the intrinsic volume averaged ensemble averaged value and $\overline{\nu}_i''$ is the spatial fluctuation.

### Appendix A.1. Continuity equation

The starting point is the Reynolds averaged continuity equation written as:

$$
\frac{\partial \overline{\nu}_i}{\partial x_i} = 0
$$

(A.8)

where $\overline{\nu}_i$ is the ensemble averaged Cartesian velocity vector $\overline{\nu}_i = (\overline{\nu}, \overline{\nu}, \overline{\nu})$ and $x_i$ are the Cartesian coordinates.

Applying the averaging theorem in Eq. (A.5) the volume averaged ensemble averaged continuity equation becomes:

$$
\frac{\partial \overline{\nu}_i}{\partial x_i} = \frac{\partial (\overline{\nu})}{\partial x_i} + \frac{1}{V} \int_{S_w} \overline{\nu}_i \cdot \mathbf{n} dA = 0
$$

(A.9)

where $n$ is the porosity given by:

$$
n = \frac{V_f}{V}
$$

(A.4)
The second term on the right hand side integrates over the surface of the solids (pore walls) which are closed surfaces. This, together with the assumption of the velocities on the surface of the solids being zero, reduces Eq. (A.9) to:

\[
\frac{\partial \langle \mathbf{u} \rangle}{\partial x_i} = 0 \tag{A.10}
\]

where \(\langle \mathbf{u} \rangle\) is the volume averaged ensemble averaged velocity over the total volume including the solids of the porous media. This velocity is also referred to as the filter velocity or the Darcy velocity.

**Appendix A.2. Momentum equation**

The starting point is the Reynolds averaged momentum equations given as:

\[
\frac{\partial \rho \mathbf{u}_i}{\partial t} + \frac{\partial \rho \mathbf{u}_i \mathbf{u}_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + g_i + \frac{\partial}{\partial x_i} \mu \left( \frac{\partial \mathbf{u}_i}{\partial x_i} + \frac{\partial \mathbf{u}_j}{\partial x_j} \right) \tag{A.11}
\]

where \(\rho\) is the density of the fluid, \(p\) is the excess pressure, \(g_j\) is the \(j\)th component of the gravitational vector, \(\mu\) is the dynamical viscosity, and \(t\) is the time.

The first term on the left hand side was volume averaged according to Eq. (A.1). This gave the following:

\[
\left\langle \frac{\partial \rho \mathbf{u}_i}{\partial t} \right\rangle = \frac{1}{V} \int \frac{\partial \rho \mathbf{u}_i}{\partial t} \mathrm{d}V = \frac{\partial}{\partial t} \left\langle \frac{1}{V} \int \rho \mathbf{u}_i \mathrm{d}V \right\rangle = \frac{\partial \rho \langle \mathbf{u}_i \rangle}{\partial t} \tag{A.12}
\]

The second term on the left hand side of the momentum equation, Eq. (A.11) was treated as follows. First, the ensemble averaging decomposes the term into a mean and fluctuating part as in Eq. (A.6):

\[
\frac{\partial \rho \mathbf{u}_i \mathbf{u}_j}{\partial x_j} = \frac{\partial \rho \langle \mathbf{u}_i \mathbf{u}_j \rangle}{\partial x_j} + \frac{\partial \rho \langle \mathbf{u}_i \mathbf{u}_j \rangle}{\partial x_j} \tag{A.13}
\]

Hereafter, the volume averaging theorem is applied on the first term on the right hand side of Eq. (A.13):

\[
\left\langle \frac{\partial \rho \mathbf{u}_i \mathbf{u}_j}{\partial x_j} \right\rangle = \frac{\partial \rho \langle \mathbf{u}_i \mathbf{u}_j \rangle}{\partial x_j} + \frac{1}{V} \int_{S_w} \mathbf{u}_i \mathbf{u}_j \mathrm{d}A \tag{A.14}
\]

Assuming the velocities on the surface of the solid are zero this reduces to:

\[
\left\langle \frac{\partial \rho \mathbf{u}_i \mathbf{u}_j}{\partial x_j} \right\rangle = \frac{\partial \rho \langle \mathbf{u}_i \mathbf{u}_j \rangle}{\partial x_j} \tag{A.15}
\]

The right hand side of Eq. (A.15) expresses an average of a product. Here the velocity decomposition given by Gray (1975) and shown in Eq. (A.7) was applied. The velocity decomposition is shown without the tensor notation in the following for reasons of space:

\[
\langle \mathbf{u}_i \mathbf{u}_j \rangle = \langle \mathbf{u}_i \mathbf{u}_j \rangle^f + \langle \mathbf{u}_i \mathbf{u}_j \rangle^f + \langle \mathbf{u}_i \mathbf{u}_j \rangle^f \tag{A.16}
\]

Following Gray (1975) the relation between superficial and intrinsic volume average given in Eq. (A.3) was applied which gave:

\[
\langle \mathbf{u}_i \mathbf{u}_j \rangle = n \langle \mathbf{u}_i \mathbf{u}_j \rangle^f + n \langle \mathbf{u}_i \mathbf{u}_j \rangle^f + n \langle \mathbf{u}_i \mathbf{u}_j \rangle^f \tag{A.17}
\]

Assuming that the intrinsic volume averages are constant within their averaging volumes as given in Gray (1975) and Whitaker (1996), the following was seen:

\[
\langle \mathbf{u}_i \mathbf{u}_j \rangle^f = \langle \mathbf{u}_i \mathbf{u}_j \rangle^f = 0 \tag{A.18}
\]

\[
\langle \mathbf{u}_i \mathbf{u}_j \rangle^f = \langle \mathbf{u}_i \mathbf{u}_j \rangle^f = 0 \tag{A.19}
\]

and

\[
\langle \mathbf{u}_i \mathbf{u}_j \rangle^f = \langle \mathbf{u}_i \mathbf{u}_j \rangle^f \tag{A.20}
\]

These relations reduced Eq. (A.17) to:

\[
\langle \mathbf{u}_i \mathbf{u}_j \rangle = n \langle \mathbf{u}_i \mathbf{u}_j \rangle^f + n \langle \mathbf{u}_i \mathbf{u}_j \rangle^f \tag{A.21}
\]

Returning to Eq. (A.13), the volume average of the first term on the right hand side can now be written as:

\[
\left\langle \frac{\partial \rho \mathbf{u}_i \mathbf{u}_j}{\partial x_j} \right\rangle = \frac{\partial \rho \langle \mathbf{u}_i \mathbf{u}_j \rangle}{\partial x_j} + \frac{\partial \rho \langle \mathbf{u}_i \mathbf{u}_j \rangle}{\partial x_j} \tag{A.22}
\]

Reformulating from intrinsic (pore) velocities to superficial (filter) velocities by Eq. (A.3) gave:

\[
\left\langle \frac{\partial \rho \mathbf{u}_i \mathbf{u}_j}{\partial x_j} \right\rangle = \frac{\partial \rho \langle \mathbf{u}_i \mathbf{u}_j \rangle}{\partial x_j} + \frac{\partial \rho \langle \mathbf{u}_i \mathbf{u}_j \rangle}{\partial x_j} \tag{A.23}
\]

The first term on the right hand side of the momentum equation in Eq. (A.11) describes the pressure gradient. This term was analysed in details in Whitaker (1986a) and Quintard and Whitaker (1993) regarding the volume averaging procedure. In the following the pressure gradient was volume averaged according to Eq. (A.5). This gave:

\[
\left\langle -\frac{\partial p}{\partial x_i} \right\rangle = -\frac{\partial \langle p \rangle}{\partial x_i} + \frac{1}{V} \int_{S_w} \mathbf{p} \cdot \mathbf{n} \mathrm{d}A \tag{A.24}
\]
Here, the last term on the right hand side describes the pressure induced forces acting on the surface of the solids. When analysing flow in porous media the superficial (filter) velocity is generally preferred. However, for the pressure the intrinsic volume average is preferred as this corresponds to the actual measured pore pressure. Therefore the relation between superficial (filter) quantities and intrinsic quantities given by Eq. (A.3) is applied to Eq. (A.24) along with the velocity decomposition by Gray (1975) given by Eq. (A.7). This follows the derivation by Whitaker (1986a) and gave:

\[-\frac{\partial \langle \rho \rangle}{\partial x_i} + \frac{1}{V} \int_{S_w} \bar{p} \cdot n \, dA = -n \frac{\partial \langle \rho \rangle}{\partial x_i} - \langle \rho \rangle \frac{\partial n}{\partial x_i} - \frac{1}{V} \int_{S_w} \bar{p}' \cdot n \, dA \quad (A.25)\]

In Whitaker (1986a) it is assumed that the intrinsic (pore) pressure, \(\langle \rho \rangle\), is a constant with respect to integration over the surface area, \(S_w\), and further that the relation:

\[\frac{1}{V} \int_{S_w} \langle \rho \rangle \cdot n \, dA = \frac{1}{V} \left\{ \int_{S_w} n \, dA \right\} \langle \rho \rangle \quad (A.26)\]

is satisfied when the radius of the averaging volume is much smaller than the macroscopic length scale. Using the volume average theorem it can be shown that:

\[\frac{1}{V} \int_{S_w} n \, dA = -\frac{\partial n}{\partial x_i} \quad (A.27)\]

Using this with Eq. (A.26) and substituting into Eq. (A.25) gives:

\[-\frac{\partial \langle \rho \rangle}{\partial x_i} + \frac{1}{V} \int_{S_w} \bar{p} \cdot n \, dA = -n \frac{\partial \langle \rho \rangle}{\partial x_i} - \frac{1}{V} \int_{S_w} \bar{p}' \cdot n \, dA \quad (A.28)\]

The gravitational term in the momentum equation was treated as:

\[g_j x_i \frac{\partial \rho}{\partial x_j} = n g_j x_i \frac{\partial \rho}{\partial x_j} \quad (A.29)\]

where the volume average was included via the porosity based on Eq. (A.3).

The last term on the right hand side of the momentum equation, Eq. (A.11) includes the viscous stress tensor. Again, the volume average of a gradient was applied according to Eq. (A.5) on the ensemble averaged values.

This gave:

\[\left\langle \frac{\partial}{\partial x_j} \left( \frac{\partial \langle \rho \rangle}{\partial x_j} + \frac{\partial \langle \rho \rangle}{\partial x_i} \right) \right\rangle + \left( \frac{1}{V} \int_{S_w} \frac{\partial \langle \rho \rangle}{\partial x_j} \right) \right\rangle \cdot n \, dA \quad (A.30)\]

Applying the volume averaging theorem a second time gave:

\[\left\langle \frac{\partial \langle \rho \rangle}{\partial x_j} + \frac{\partial \langle \rho \rangle}{\partial x_i} \right\rangle = \left( \frac{\partial \langle \rho \rangle}{\partial x_j} + \frac{\partial \langle \rho \rangle}{\partial x_i} \right) + \left( \frac{1}{V} \int_{S_w} \frac{\partial \langle \rho \rangle}{\partial x_j} \right) \cdot n \, dA \quad (A.31)\]

Here the assumption of the velocity being zero at the surface of the solids removes the second term on the right hand side. Using this relation in Eq. (A.30) expresses the viscous term as:

\[\left\langle \frac{\partial \langle \rho \rangle}{\partial x_j} \mu \left( \frac{\partial \langle \rho \rangle}{\partial x_j} + \frac{\partial \langle \rho \rangle}{\partial x_i} \right) \right\rangle = \left( \frac{\partial \langle \rho \rangle}{\partial x_j} \mu \left( \frac{\partial \langle \rho \rangle}{\partial x_j} + \frac{\partial \langle \rho \rangle}{\partial x_i} \right) \right) + \left( \frac{1}{V} \int_{S_w} \frac{\partial \langle \rho \rangle}{\partial x_j} \right) \cdot n \, dA \quad (A.32)\]

Using the decomposition from Gray (1975) on the last term in Eq. (A.32) gave:

\[\frac{1}{V} \int_{S_w} \mu \left( \frac{\partial \langle \rho \rangle}{\partial x_j} + \frac{\partial \langle \rho \rangle}{\partial x_i} \right) \cdot n \, dA \]

Substituting Eq. (A.33) into Eq. (A.32) and reformulating from intrinsic (pore) velocities to superficial (filter) velocities by Eq. (A.3) gave the volume averaged viscous term as:

\[\left\langle \frac{\partial \langle \rho \rangle}{\partial x_j} \mu \left( \frac{\partial \langle \rho \rangle}{\partial x_j} + \frac{\partial \langle \rho \rangle}{\partial x_i} \right) \right\rangle = \left( \frac{\partial \langle \rho \rangle}{\partial x_j} \mu \left( \frac{\partial \langle \rho \rangle}{\partial x_j} + \frac{\partial \langle \rho \rangle}{\partial x_i} \right) \right) + \left( \frac{1}{V} \int_{S_w} \frac{\partial \langle \rho \rangle}{\partial x_j} \right) \cdot n \, dA \quad (A.34)\]

Here, the last term on the right hand side corresponds to the viscous stress forces acting on the surface of the solids.
Substituting Eq. (A.12), (A.23), (A.28), (A.29) and (A.34) into Eq. (A.11) the momentum equation reads:

\[
\frac{\partial (\rho \vec{u})}{\partial t} + \frac{\partial (\rho \vec{u} \cdot \vec{u}')}{\partial x_j} + \frac{\partial (\mu \vec{u}')}{\partial x_j} = -n \frac{\partial (\rho \vec{f} / \partial x_j)}{\partial x_j} + \frac{1}{V} \int_{S_u} \vec{p}' \cdot \text{nA} + g_j \frac{\partial \rho}{\partial x_i} + \frac{1}{n} \frac{\partial (\mu \vec{u}')}{\partial x_j} + \frac{1}{n} \frac{\partial (\mu \vec{u}')}{\partial x_j}
\]

Dividing by the porosity, \( n \), gave the final form of the momentum equation:

\[
\frac{\partial (\rho \vec{u})}{\partial t} + \frac{1}{n} \frac{\partial (\rho \vec{u} \cdot \vec{u}')}{\partial x_j} + \frac{1}{n} \frac{\partial (\mu \vec{u}')}{\partial x_j} + \frac{1}{n} \frac{\partial (\mu \vec{u}')}{\partial x_j} = -\frac{\partial (\rho \vec{f} / \partial x_j)}{\partial x_j} + \frac{1}{n} \frac{\partial (\mu \vec{u}')}{\partial x_j} + \frac{1}{n} \frac{\partial (\mu \vec{u}')}{\partial x_j} + \frac{1}{n} \frac{\partial \rho}{\partial x_i} \mu \left( \frac{\partial \vec{u}}{\partial x_j} + \frac{\partial \vec{u}}{\partial x_j} \right)
\]

\[
(A.35)
\]

The third term on the left hand side describes the spatial velocity variations within the averaging volume. The second term on the right hand side describes the pressure induced forces acting on the surface of the porous media solids. The last term on the right hand side includes the viscous forces on the surface of the porous media solids. These three terms originate from the volume averaging procedure. They cannot be simulated directly and must be determined by a closure model. The three terms are collected in one term as:

\[
F_i = -\frac{1}{n} \frac{\partial (\rho \vec{u} \cdot \vec{u}')}{\partial x_j} - \frac{1}{n V} \int_{S_u} \vec{p}' \cdot \text{nA} + \frac{1}{n} \frac{\partial (\mu \vec{u}')}{\partial x_j} + \frac{1}{n} \frac{\partial (\mu \vec{u}')}{\partial x_j} + \frac{1}{n} \frac{\partial \rho}{\partial x_i} \mu \left( \frac{\partial \vec{u}}{\partial x_j} + \frac{\partial \vec{u}}{\partial x_j} \right) \]

\[
(A.36)
\]

Introducing this into the momentum equation gave:

\[
(1 + C_m) \frac{\partial \rho \vec{u}}{\partial t} + \frac{1}{n} \frac{\partial \rho \vec{u} \cdot \vec{u}'}{\partial x_j} + \frac{1}{n} \frac{\partial \mu \vec{u}'}{\partial x_j} = -\frac{\partial (\rho \vec{f} / \partial x_j)}{\partial x_j} + \frac{1}{n} \frac{\partial \rho}{\partial x_i} \mu \left( \frac{\partial \vec{u}}{\partial x_j} + \frac{\partial \vec{u}}{\partial x_j} \right) + F_i
\]

The last term on the left hand side describes the turbulent fluctuations in the porous media. These cannot be resolved directly and must be modelled by a closure model. This is typically handled by introducing an extra contribution to the viscosity denoted the eddy-viscosity. Examples on turbulent closure models are the \( k - \epsilon \)-model and \( k - \omega \)-model described in e.g. Wilcox (2006). These closure models may also be modified in order to account for the porous media. An example on a turbulent closure model for porous media was given in Nakayama and Kuwahara (1999).

In the present work it is argued, that the effect of the turbulence in terms of an additional resistance, may be described via the resistance term, \( F_i \), given in Eq. (A.37). Further discussion on this topic is given in Section 2.5. Hereby the resistance term is extended to include the turbulent fluctuations as:

\[
F_i = -\frac{1}{n} \frac{\partial (\rho \vec{u} \cdot \vec{u}')}{\partial x_j} - \frac{1}{n V} \int_{S_u} \vec{p}' \cdot \text{nA} + \frac{1}{n} \frac{\partial (\mu \vec{u}')}{\partial x_j} + \frac{1}{n} \frac{\partial \rho}{\partial x_i} \mu \left( \frac{\partial \vec{u}}{\partial x_j} + \frac{\partial \vec{u}}{\partial x_j} \right)
\]

\[
(A.37)
\]

This reduces the momentum equation to:

\[
(1 + C_m) \frac{\partial \rho \vec{u}}{\partial t} + \frac{1}{n} \frac{\partial \rho \vec{u} \cdot \vec{u}'}{\partial x_j} + \frac{1}{n} \frac{\partial \mu \vec{u}'}{\partial x_j} = -\frac{\partial (\rho \vec{f} / \partial x_j)}{\partial x_j} + \frac{1}{n} \frac{\partial \rho}{\partial x_i} \mu \left( \frac{\partial \vec{u}}{\partial x_j} + \frac{\partial \vec{u}}{\partial x_j} \right) + F_i
\]

For modelling the resistance terms given in Eq. (A.39) the extended Darcy-Forchheimer equation was applied, which includes linear and nonlinear forces as well as inertial forces to account for accelerations. The linear and nonlinear resistance forces were expressed as:

\[
F_i = -\rho \vec{u} - \rho \sqrt{\vec{u} \cdot \vec{u}'}/(\vec{u} \cdot \vec{u}') \]

\[
(A.41)
\]
where \(a\) and \(b\) are resistance coefficients. These coefficients must be determined. Engelund (1954) formulated a relation between the resistance coefficients and the porosity, viscosity, and grain diameter for steady state flow, as also later included in Burcharth and Andersen (1995). These relations were included in the model presented by del Jesus et al. (2012). A similar expression to Engelund (1954) was formulated in van Gent (1995), where the effect of oscillatory flows was added to the expressions in terms of the KC-number. The latter formulations were applied in the model presented by Liu et al. (1999) and were also adopted and implemented as part of the present study. The resistance coefficients were formulated as:

\[
a = a \left(1 - n\right)^2 \frac{\nu}{n^5 \rho d_{50}^2}
\]

\[
b = \beta \left(1 + \frac{7.5}{KC} \frac{1 - n}{n^5} \frac{1}{d_{50}}\right)
\]

where \(d_{50}\) is the grain diameter and \(KC = u_m T/(n d_{50})\), where \(u_m\) is the maximum oscillating velocity and \(T\) is the period of the oscillation. \(a\) and \(\beta\) are empirical coefficients set to 1000 and 1.1 by van Gent (1995). These coefficients are further discussed in Section 3.

Finally, the inertia term in the extended Darcy-Forchheimer equation was included on the time derivative in Eq. (A.38) through \(C_{m}\), which van Gent (1995) gave as:

\[
C_{m} = \gamma_p \frac{1 - n}{n}
\]

where \(\gamma_p\) is an empirical coefficient, which takes the value 0.34.

References


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CHAPTER 5

POROUS MEDIA AND IMMERSED BOUNDARY HYBRID MODELLING FOR COASTAL STRUCTURES

In preparation as:
Porous media and immersed boundary hybrid modelling for coastal structures

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Abstract

This paper presents a new numerical modelling approach for coastal and marine applications where a porous media conceptual model was combined with an immersed boundary method (IBM). The work is an extension of the porous media modelling presented in Jensen et al. (2014). The starting point was the Navier-Stokes equations solved by a finite-volume method. This model was re-formulated to handle porous media flow based on the volume averaged Reynolds averaged Navier-Stokes equations (VARANS) and coupled to an immersed boundary model. The immersed boundary model covers the method of describing a solid object in a simple computational mesh without the need to resolve the object with a conventional body-fitted mesh. A breakwater structure consists of several geometrical scales e.g. large armour layer stones on the front and core material based on more fine materials. With the new hybrid model it is possible to resolve some parts of the structure e.g. the armour layer while the core material is modelled with the conceptual porosity model. The model was furthermore combined with the Volume of Fluid (VOF) method for simulating free surface water waves. The implementation of the coupled model in the open source CFD library OpenFOAM® is presented together with a number of validation cases that proves the capabilities of the new implementation. A rubble mound breakwater rock toe was investigated with the new hybrid modelling approach.

Keywords: Wave-structure interaction, Immersed boundary method, Porous media, Breakwaters, Navier-Stokes equations

1. Introduction

Breakwater structures have been investigated intensively by means of physical model scale experiments. This has resulted in empirical relations describing armour stability, front slope recession, overtopping, reflection etc. Some of the most applied stability formulas were described in van der Meer (1987), van der Meer (1988), and van der Meer (1992). More recently both reflection, transmission and stability were further described in Zanuttigh and Van der Meer (2006), Vanneste and Troch (2010), Andersen and Burcharth (2010), and Vanneste and Troch (2012). Common for these methods is that they relate on an observation of the effect of the wave-structure interaction e.g. level of damage to the armour layer, transmission, and overtopping rate. However, some details are not treated in these experiments. Topics which may be of interest are: forces on individual armour stones, effect of filter and core porosity on forces, hydrodynamics in the armour layer, turbulence between armour stones, details of suction mechanisms etc.

Some scale experiments have focused on the above topics. In Tørum (1994) the forces on a single armour stone on a rubble mound breakwater were measured. Hald (1998) followed the same procedure and recently Moghim and Tørum (2012) repeated the same type of measurements on a reshaping rubble mound breakwater. The results were not completely conclusive on all mat-
terms. For example, in Tørum (1994) the force measurements for lift forces normal to the sloping bed could not provide a conclusive result with respect to force coefficients. Also the details on the flow field, turbulence, and interaction with the filter layers were not described.

In recent years, numerical methods and computational resources have developed to a level at which consulting and design engineers also apply numerical simulations as an integrated part of the design and development process. However, a complete resolution of the porous structure in breakwaters is not yet feasible. Therefore a common procedure is to apply resistance-type models where the effect of the porous breakwater structure is added as a source term in the momentum equation. Examples of this was seen in van Gent (1995) where a model based on the Navier-Stokes equations was developed. The porous media, consisting of a rigid skeleton and pores, was treated as one continuum which exerts forces on the fluid due to drag, friction, and acceleration. Several developments have been presented applying the same methodology for two-dimensional domains e.g. Liu et al. (1999), Hsu (2002), Garcia et al. (2004), and Lara et al. (2006). Del Jesus et al. (2012) and Lara et al. (2012) presented a model where also three-dimensional flows were considered. Recently Jensen et al. (2014) presented a detailed investigation of the theoretical background and resistance coefficients. These models provide results at the same level of details as corresponding physical experiments such as reflection and transmission, overtopping and front slope velocities for stability evaluation etc.

If the actual armour stones could be resolved directly in the numerical model a higher degree of details could be achieved. This would correspond to measuring the forces directly at an armour stone as in Moghim and Tørum (2012). However, the complex geometrical layout of multiple armour stones on the front of a breakwater gives a complicated computational mesh. A traditional way of including solid structures in a computational model is by a body-fitted mesh where the grid cells follow the surface of the structure. For randomly placed armour stones with complicated shapes this may be an impossible or at least very time consuming task. Here the immersed boundary method provides an efficient way of including complex shapes in a simple computational mesh.

The immersed boundary method was originally developed to simulate blood flow. Here the entire simulation was performed on a simple Cartesian grid, which did not conform to the shape of e.g. the heart. Since this first introduction of the method several modifications and new developments have been presented. In Mittal and Iaccarino (2005) a general overview of the different methods was given. The general idea of the method is to impose the effect of the solid structure into the computational mesh without actually including the structure. Two methods generally referred in the literature are given as the continuous forcing method and discrete forcing method. For the continuous forcing method, a forcing term is added to the Navier-Stokes equations before they are discretized which imposes a forcing in the entire computational domain. The distinguishing between the solid structure and the free fluid is handled by resistance parameters, damping functions etc. For the discrete forcing method the forcing is applied in the discretized Navier-Stokes equations at the position of the immersed boundary.

Ye et al. (1999) presented a finite-volume model with the immersed boundary method implemented as the discrete forcing method. However a cut-cell approach was included in order to form new control volumes around the boundary which followed the shape of the solid surface. Fadlun et al. (2000) also presented an immersed boundary method based on a discrete forcing where the forcing at the boundary was calculated by the Navier-Stokes equations. This ghost cell approach was also adopted in Ghias et al. (2007), Mittal et al. (2008), and Nasr-Azadani and Meiburg (2011). In the latter it was also noted that the best results were obtained when the forcing term was ap-
plied in the first cell inside the solid, e.g. the ghost cell compared to the first cell outside the solid (in the fluid region).

Regarding the application of the immersed boundary method for coastal engineering some previous examples are seen. Petit et al. (1994) presented a model where virtual velocities were applied to simulate an impermeable sloping bed. In Peng et al. (2012) an immersed boundary method was applied for simulating the interaction between free surface water waves and submerged objects. Ha et al. (2014) applied the immersed boundary method for simulating solitary waves where the sloping bed was resolved by the immersed method for simulating the run-up processes of solitary objects. The combination of the immersed boundary with a porous media model was shown in van Gent et al. (1994) where the model by ? was extended to include porous media. However, applications involving both impermeable structures (IBM) and porous media were not presented. In Nielsen et al. (2013) the IBM was combined with a porous media model for simulating scour protection around offshore wind turbine foundations. The combination of IBM and porous media modelling including free surface water waves for detailed simulation of flow around armour stones on breakwaters has, to the author’s knowledge, not previously been presented.

The remainder of this paper is organised as follows. In Section 2 the numerical model is described with the modifications for handling the combined porous media flow and immersed boundary method. The model is validated in Section 3 with special emphasis on the immersed boundary implementation. Section 4 presents an application with a breakwater rock toe. Final conclusions are given in Section 5.

2. Model description

The numerical model was based on the Navier-Stokes equations with an additional body force term for including the immersed boundary method. The numerical method was based on a finite volume discretisation on a collocated grid arrangement. The general form of the Navier-Stokes equations was formulated as the continuity equation:

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p^*}{\partial x_i} + g_j \frac{\partial \rho}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right) + F_i
\]

and the momentum equation:

\[
\frac{\partial u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p^*}{\partial x_i} + g_j \frac{\partial \rho}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right) + F_i
\]

where \( \rho \) is the density of the fluid, \( u_i \) is the Cartesian velocity vector \( u_i = (u, v, w) \), \( p^* \) is the excess pressure, \( g_j \) is the \( j \)th component of the gravitational vector, \( \mu \) is the dynamical viscosity, \( t \) is the time, and \( x_i \) are the Cartesian coordinates. The Navier-Stokes equations were modified to include the immersed boundary method as described in the following.

2.1. Porous media model

The first modification of the Navier-Stokes equations is introduced in order to handle the flow in porous media where a direct resolution of the grains and pores are not possible. Here, a volume averaging of the equations is performed in order to derive the VARANS equations. A detailed account on the derivation and implementation of the model was given in Jensen et al. (2014). In the following the final equations are presented as the continuity equation:

\[
\frac{\partial \langle \rho \rangle}{\partial x_i} = 0
\]

where \( \langle \rangle \) is the volume averaged ensemble averaged velocity over the total control volume including the solids of the porous media. And the momentum equation:

\[
(1 + C_m) \frac{\partial \langle \rho \rangle}{\partial t} + \frac{1}{n} \frac{\partial }{\partial x_j} \left( n \langle \rho \langle u_j \rangle \rangle \right) = -\frac{\partial \langle \rho \rangle}{\partial x_i} + g_j \frac{\partial \rho}{\partial x_i} + \frac{1}{n} \frac{\partial }{\partial x_j} \left( \mu \left( \frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) + \langle \rho \rangle F_i \right)
\]

where \( C_m \) is the added mass coefficient to take the transient interaction between grains and water into account, \( \langle \rangle \) denoted the volume average over the pore volume, in this case applied for the pore pressure. Furthermore, an additional term on the right hand side, \( F_i \), was included to take account of the resistance force due to the presence of the porous media. The details of the derivation of the resistance force term is presented in Jensen et al. (2014). The extended Darcy-Forchheimer equation was applied to represent the resistance force, that includes linear and nonlinear forces as well as inertia forces to account for...
accelerations. The linear and nonlinear resistance forces were expressed as:

\[ F_i = -a \rho \langle \mathbf{u} \rangle - b \rho \sqrt{\langle \mathbf{u} \rangle \langle \mathbf{u} \rangle} \tag{5} \]

where \( a \) and \( b \) are resistance coefficients. These coefficients must be determined. Engelund (1954) formulated a relation between the resistance coefficients and the porosity, viscosity, and grain diameter for steady state flow, as also later included in Burchartha and Andersen (1995). These relations were included in the model presented by del Jesus et al. (2012). A similar expression for coastal structures presented in Jensen et al. (1999) and were also adopted and implemented as part of the present study. The resistance coefficients were formulated as:

\[ a = \alpha \frac{(1 - n)^2}{n^3} \rho \mu_{f0} \tag{6} \]

\[ b = \beta \left[ 1 + \frac{7.5}{K} \right] \left( 1 - n \right) \frac{d_{s0}}{n^2} \tag{7} \]

where \( d_{s0} \) is the grain diameter and \( K \) is the period of the oscillation. \( \alpha \) and \( \beta \) are empirical coefficients which are set to 500 and 2.0 according the investigations for coastal structures presented in Jensen et al. (2014). It should be noted, that the literature provides different values depending on how the coefficients have been determined or calibrated and how the resistance coefficients, \( a \) and \( b \) were formulated. The formulation by Engelund (1954) yields \( \alpha \) and \( \beta \) values that differ from those obtained in the formulation by van Gent (1995).

Finally, the inertia term in the extended Darcy-Forchheimer equation was included in eq. (4) through \( C_m \), which van Gent (1995) gave as:

\[ C_m = \gamma_p \frac{1 - n}{n} \tag{8} \]

where \( \gamma_p \) is an empirical coefficient, which takes the value 0.34.

### 2.2. Immersed Boundary Method

The Navier-Stokes equations were transformed to the VARANS equations as presented in Section 2.1 in order to handle a porous media without resolving the actual porous structure. These equations were further modified for a coupling with the immersed boundary method for resolving some parts of the porous structure. The present IBM-model was based on a discrete forcing approach applying ghost cells. The method was described in e.g. Ghias et al. (2007), Mittal et al. (2008), and Nasr-Azadani and Meiburg (2011) and was also applied in Liu (2013). The solid boundary was imposed via ghost cells by a body force term included on the right hand side of the momentum equation (eq. (2)).

The general description of the method and the enforcement of boundary conditions is given in the following based on the velocity field. The method can with relative ease be extended to any quantity. The assumption of a no-slip boundary condition at the immersed boundary (IB) was applied, i.e. \( u = (0, 0, 0) \) m/s. Looking at a point at the solid surface and following the notations in Figure 1 the following approach was applied. First, the grid cell with cell center inside the solid surface was identified and denoted a Ghost Cell (GC). The normal distance from the GC centre to the surface of the solid, \( d \), was computed. An equivalent distance from the solid surface and into the fluid region determined the position of a point denoted an Image Point (IP). This point may not necessarily correspond to a cell centre. The velocity vector in the IP was determined based on interpolation from the surrounding fluid cells based on a certain search radius. This IP-velocity was mirrored into the GC and subsequently the direction was reversed which gives the desired velocity, \( V_i \). Hereby the actual velocity in the IP and the desired velocity in the GC cancels out each other at the location of the immersed boundary (IB).

The desired velocity, \( V_i \), was enforced into the flow field by including a body force in the momentum equation as:

\[
(1 + C_m) \frac{\partial \rho \langle \mathbf{u} \rangle}{\partial t} \frac{1}{n} + \frac{1}{n} \frac{\partial}{\partial x_j} \frac{\rho \langle \mathbf{u} \rangle \langle \mathbf{u} \rangle}{n} = -\frac{\partial \rho \langle \mathbf{p} \rangle}{\partial x_i} + g_{x_j} \frac{\partial \rho}{\partial x_i} + \frac{1}{n} \frac{\partial}{\partial x_j} \left( \frac{\partial \langle \mathbf{u} \rangle}{\partial x_j} + \frac{\partial \langle \mathbf{u} \rangle}{\partial x_j} \right) + F_i + B_i \tag{9}
\]
\[(1 + C_m) \rho \langle u_{n+1} \rangle - \rho \langle u_n \rangle \cdot n \Delta t + 1 \frac{\partial}{\partial x_j} \rho \langle u_n \rangle \langle u_j \rangle \cdot n \Delta t = -\frac{\partial \langle p_n \rangle}{\partial x_i} + g_j x_j \frac{\partial \rho}{\partial x_i} + \mu \left( \frac{\partial \langle u_n \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) - F_i^n \]

where $\Delta t$ is the time-step increment and superscript $n$ and $n + 1$ indicates the current and next time step. To satisfy the desired velocity, $V_i$, at the current time step the momentum forcing term was hereby approximated as:

\[
B_i^n = (1 + C_m) \rho V_i - \rho \langle u_i \rangle \frac{n \Delta t}{n \Delta t} + 1 \frac{\partial}{\partial x_j} \rho \langle u_j \rangle \langle u_i \rangle \cdot n \Delta t + \frac{\partial \langle p \rangle}{\partial x_i} - g_j x_j \frac{\partial \rho}{\partial x_i} - \mu \left( \frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) - F_i^n \]

Here it is noted, that the body force was determined as an explicit solution only based on known values from the current time step. The body force thereby lags one time step behind which puts some limitations on the size of the time step increment. The body force was only enforced in cells inside the solid objects which was controlled by a cell marker with the value 1 inside and 0 outside the solid object.

2.3. Enforcement of boundary conditions

The above section outlines the general approach regarding the computation of the momentum source term in order to impose the force field in the flow corresponding to the effect of the solid structure. As it appears from eq. (11) the body force term is found based on the desired velocity, $V_i$, in the ghost cells. The description so far is based on the assumption of a no-slip velocity condition on the immersed boundary. In the following the general expressions are derived for imposing Dirichlet (fixed value) and Neumann (gradient) boundary conditions.

2.3.1. Dirichlet boundary condition

The velocity at the IB was expressed by linear interpolation along the normal probe between the image point and ghost cell. This gave:

\[
u_{IB} = \frac{n u_{IP} + u_{GC}}{2} \quad (12)
\]

where $u$ is the Cartesian velocity vector, subscript $IB$ is the velocity at the immersed boundary, subscript $IP$ is the velocity at the image point, and subscript $GC$ is the velocity at the ghost cell. Hereby the ghost cell velocity was expressed as:

\[
u_{GC} = \frac{2}{n} u_{IB} - u_{IP} \quad (13)
\]

The velocity at the image point, $u_{IP}$, was found based on interpolation from the surrounding velocity field. The velocity at the immersed boundary, $u_{IB}$, was set according to the desired boundary condition. For the case of a no-slip boundary condition i.e. $u_{IB} = 0$, eq. (13) reduced to:

\[
u_{GC} = -u_{IP} \quad (14)
\]

2.3.2. Neumann boundary condition

The Neumann boundary condition at the immersed boundary had the generic form of:

\[
(\nabla u) \cdot \vec{n} = q \quad (15)
\]
where $\vec{n}$ is the normal vector pointing outwards from the solid surface. It was discretized on the immersed boundary along the normal vector direction as:

$$\frac{u_{ip} - u_{GC}}{\Delta} = q$$  \hspace{1cm} (16)

where $\Delta$ is the distance between the ghost cell center and its image point. Hereby the value at the ghost cell center was given as:

$$v_{GC} = v_{ip} - q\Delta$$  \hspace{1cm} (17)

For the simple case of a zero gradient, i.e. $q = 0$, the ghost cell center had the same variable value as the image point.

2.4. Interpolation

As described in Section 2.2 the immersed boundary method was based on interpolation of quantities near the immersed boundary into the image points. The accuracy of the immersed boundary method is related to the applied interpolation method. In this work, two types of interpolation schemes were implemented, namely Shepard interpolation and radial basis function interpolation.

Details on the radial basis function can be found in Buhmann (2003) and Press et al. (2007). The radial basis function interpolation is based on the assumption that any known points influences its surrounding points according to a functional form, $\phi(r)$, referred to as the radial basis function. This function depends only on the radial distance, $r = |x - x_i|$ from the point. The interpolated value was hereby given by a linear combination of the radial basis functions as:

$$V(x) = \sum_{i=0}^{N-1} \omega_i \phi(|x - x_i|)$$  \hspace{1cm} (18)

where $N$ is the number of interpolation cells, $\omega_i$ is a set of unknown weights and $\phi(r)$ is the radial basis function which only depends on the radial distance $r = |x - x_i|$. The weights, $\omega_i$, are determined by requiring the interpolation to be exact at all the neighbour cell centres. Each surrounding point with known values included in the interpolation routine is given as:

$$V_j = \sum_{i=0}^{N-1} \omega_i \phi(|x_j - x_i|)$$  \hspace{1cm} (19)

This will result in a set of $N$ linear equations in $N$ unknowns for the weights, $\omega_i$. By solving this linear system the weights are found. Applied in eq. (18) the interpolated value can be found.

There are several choices for the radial basis function $\phi(r)$. In Press et al. (2007) a description was given on multiquadric, inverse multiquadric, thin-plate, and Gaussian. Comparison has been made in the literature among these functions and no consensus has been reach on which one is optimal. In this work the above mentioned methods have been implemented. According to Press et al. (2007) the most commonly used is the multiquadric given as:

$$\phi(r) = (r^2 + r_0^2)^{1/2}$$  \hspace{1cm} (20)

where $r_0$ is a scaling factor whose value is problem dependent and should be properly chosen. Specifically, $r_0$ should be larger than the minimum separating distance between the neighbour cells and smaller than the size of the interpolating cell cloud, $R$. Several orders of magnitude difference between the interpolation accuracy with different choices of $r_0$ have been reported. Thus, some trial and error on $r_0$ is needed in the simulation. Details on all implemented radial basis function can be found in Press et al. (2007).

For the Shepard interpolation scheme, the values at the image point can be estimated as:

$$V_i = \frac{\sum_{j=0}^{N-1} v_j |x - x_j|^{-p}}{\sum_{j=0}^{N-1} |x - x_j|^{-p}}$$  \hspace{1cm} (21)

where $p$ is a parameter which usually ranges in $1 < p < 3$. It is noted that the Shepard scheme is a special case of the more general radial basis function interpolation where the weights, $\omega_i$, are equal to the respective function values at the neighbour cell centres. The Shepard interpolation is simple and fast since no linear system is solved. However, its accuracy rarely exceeds those of the general radial basis functions.

2.5. Free surface VOF modelling

The free water surface was modelled by the volume of fluid approach (VOF), where the specific details are given in Berberović et al. (2009). The method was corrected to handle free surface flow in porous media as described in Jensen et al. (2014). The tracking was resolved by the solution to:

$$\frac{\partial \alpha}{\partial t} + \frac{1}{n} \frac{\partial}{\partial x_i} ((\overline{u_i}) \alpha) + \frac{1}{n} \frac{\partial}{\partial x_i} ((\overline{p_i}) \alpha (1 - \alpha)) + S = 0$$  \hspace{1cm} (22)
where α is 1 for water and 0 for air. \( \langle u' \rangle = \langle u - u' \rangle \) is a relative velocity between the fluid and the air as described in Berberović et al. (2009). The last term is denoted the compression term and handles the compression of the interface between fluid and air. This term vanishes in the fluid (α = 1) and in the air (α = 0), and is only active in the interface region of the free surface.

The source term, S, was introduced to enforce a Neumann boundary condition for the α-scalar field on the immersed boundary. The term was based on the same methodology as described in Section 2.2 and 2.3 for the desired velocity in the ghost cells. Similarly the α-field was determined in the image-points based on interpolation and subsequent applied in the ghost cells according to the Neumann boundary condition given in Section 2.3.2.

2.6. Large Eddy Simulation turbulence model

For high Reynolds number flows the turbulent fluctuations may not be resolved directly by the computational grid. Therefore a turbulence model must be introduced to account for the effects of the turbulent fluctuations. For the present simulations a Large Eddy Simulation (LES) model has been applied. This allows for a direct simulation of the large scale turbulent fluctuations while the second term is moved to the right hand side of eq. (23) and must be modelled. This term is also referred to as the sub-grid scale Reynolds stress:

\[
\tau_{ij} = -\rho \left( \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j \right)
\]

Eq. (25) is the closure problem for which a model must be applied. This model will be referred to as a sub-grid scale (SGS) model. For the present simulations with high Reynolds numbers the Smagorinsky SGS model is applied. The sub-grid scale stresses given in eq. (25) are modelled as:

\[
\tau_{ij}^* = 1/3 \tau_{kk} \delta_{ij} = 2\mu_S \delta_{ij},
\]

\[
\mu_i = \rho (C_s \Delta)^{2/3} \bar{S}
\]

where \( \Delta \) is the filter length scale and \( \bar{S} = (2S_{ij}S_{ij})^{1/2} \). \( C_s \) is the Smagorinsky constant that is generally in the order of 0.065 to 0.2. For the present simulations it is set to 0.1 however the optimal value can vary from case to case.

3. Validation of immersed boundary method

The numerical model was validated by a number of test cases. The flow past a circular cylinder was simulated for low Reynolds numbers where the boundary layer and wake is laminar hence no turbulence closure is applied. The flow over a sphere for high Reynolds numbers was simulated including the LES turbulence closure model. The interaction between the free water surface i.e. not the in the porous media. A discussion on the turbulence modelling in the porous media is given in Jensen et al. (2014) where it is argued that the effect of the turbulence can be included via the resistance terms based on the Darcy-Forchheimer equation. In eq. (23) the second term on the left hand side is split up into two terms as:

\[
\frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} + \left( \frac{\partial \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j}{\partial x_j} \right)
\]

The first term on the right hand side is simulated directly while the second term is moved to the right hand side of eq. (23) and must be modelled. The interaction between the free water surface and wake is laminar hence no turbulence closure is applied. The flow over a sphere for high Reynolds numbers where the boundary layer and wake is laminar hence no turbulence closure is applied. The flow over a sphere for high Reynolds numbers was simulated including the LES turbulence closure model. The interaction between the free water surface
and the immersed boundary was validated by simulating the run-up of regular waves around a vertical cylinder. Grid refinement and convergence tests were performed to evaluate the needed resolution of the computational mesh and the accuracy of the solver.

3.1. Unidirectional flow past a cylinder

The flow past a circular cylinder is a basic test case which has been widely used for testing the validity of numerical solvers. For the immersed boundary method this example has also been used previously, see for example Sun et al. (2010), Chiu et al. (2010), Mittal et al. (2008), and Choi et al. (2007). For low Reynolds numbers below 200 the flow can be treated as two-dimensional and laminar. Hereby the models general ability to reproduce flow and pressure around a solid object can be investigated without the complexity of turbulence modelling. For \( Re < 40 \) the flow is symmetric while at \( Re > 40 \) vortex shedding occurs. Simulations were performed for \( Re = [40; 60; 80; 100; 120; 160] \).

The cylinder had a diameter of \( D = 0.4 \) m. The model domain was set up with a transverse and longitudinal dimension at \( 50D \). The upstream distance from the center of the cylinder to the inlet boundary was \( 10D \). The mesh aspect ratio was 1 in the entire domain. Mesh refinement was applied in areas around the cylinder. The largest grid cell dimension was 0.1 m while the smallest dimension around the cylinder was 0.00625 m corresponding to 64 grid cells per diameter, \( D \). Regarding the spatial resolution further details are given in Section 3.2 where a grid convergence test for the present test case is presented. A total of 68,000 grid cells were applied. The mesh is presented in Figure 2 and 3 with a close-up of the local mesh refinement around the cylinder. Figure 4 shows the grid near the cylinder surface with the detected ghost cells marked with black.

At the inlet boundary a Dirichlet condition was applied for the velocity while a Neumann condition was applied for the pressure. At the sides parallel to the flow direction a slip condition was applied for the velocity while the pressure was given as a Neumann condition. At the outlet boundary the velocity was given as a Neumann condition while a Dirichlet condition was applied for the pressure.

For \( Re = [40; 100] \) comparisons were made with data for the Strouhal number with experimental data given in Williamson (1989) and the base suction pressure coefficient, \(-C_{pB}\), at the trailing edge with data given in Williamson and Roshko (1990).

Figure 5 shows the pressure coefficient, \( C_p \), along the cylinder half periphery where \( 0^o \) degrees corresponds to the upstream leading edge and \( 180^o \) the downstream trailing edge. For \( Re = 40 \) the flow was stationary with two symmetrical separation zones downstream the cylinder. For \( Re = 100 \) laminar vortex shedding was seen with a vortex street downstream the cylinder. In both cases the pressure distribution along the surface of the cylinder was estimated with good agreement with the results given in Park et al. (1998), Kim et al. (2001), and Dennis and Chang (1970).

Figure 6 presents the base pressure coefficient at the trailing edge corresponding to \( 180^o \) in Figure 5. Here several \( Re \)-numbers from 40 to 160 were included. At
a $Re$-number at 200 the flow becomes three-dimensional hence a two-dimensional model will not be sufficient to describe the flow at these regimes. For the $Re$-number range where the flow can be treated as two-dimensional the model was found to give a good estimation of the base pressure coefficient.

At a $Re$-number above 50, vortex shedding takes place which was also seen for the present simulations. This introduces an oscillating flow as the vortex shedding alternates between the two sides of the cylinder. The vortex shedding frequency is expressed with the Strouhal number, $St$, defined as $St = fD/U_0$, where $f$ is the frequency of the vortex shedding, $D$ is the cylinder diameter, and $U_0$ is the free stream velocity. Figure 7 presents the $St$-number compared to experimental data given in Williamson (1989).

In Figure 8 the velocity vectors are shown near the immersed boundary. The ghost cells where the forcing term was applied are marked with a grey colour. It is noted that the velocity in the ghost cells is reversed compared to the velocity just outside the immersed boundary. This follows from the enforcement of a dirichlet boundary condition with the velocity being zero at the immersed boundary according to Section 2.3.1.

### 3.2. Grid convergence and accuracy

With the finite-volume method each term in the momentum equation is integrated over a cell volume. The spatial derivatives are converted to integrals over the cell surfaces by the Gauss theorem. In order to obtain the surface values of a given field an interpolation scheme were selected for each term. Linear second order schemes were selected for the convective and diffusive terms as well as gradients which should provide a solution with second order accuracy. The immersed boundary method introduces the interpolation of variables near the surface in order to impose the immersed boundary conditions. Here a lower order accuracy may be introduced into the domain.

A convergence test was performed to document the accuracy of the current IBM implementation. The test case was the flow around a cylinder at $Re = U_0D/\nu = 40$ where $U_0$ is the free stream velocity, $D$ is the cylinder diameter, and $\nu$ is the kinematic viscosity. The domain was $4d \times 4d$ with a uniformly distributed hexahedral grid. As a reference solution for computing the error a highly resolved solution was obtained with a resolution of $1260 \times 1260$ grid cells. The time step was $1 \times 10^{-4}D/U_0$ and the
solution was obtained after a total of $1 \times 10^4$ time steps. The same flow was computed with a grid resolution of $420 \times 420, 252 \times 252, 180 \times 180$, and $84 \times 84$. The same time step size and total number of time steps were applied for all simulations. The $L_2$ and $L_\infty$ norms of the error for each solution was found as:

$$
\varepsilon_2 = \left( \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} (\phi_{i,j}^{N \times N} - \phi_{i,j}^{1260 \times 1260})^2 \right)^{1/2} \tag{28}
$$

$$
\varepsilon_\infty = \max |\phi_{i,j}^{N \times N} - \phi_{i,j}^{1260 \times 1260}| \tag{29}
$$

The $L_2$-norm expresses the global accuracy while the $L_\infty$-norm captures the local errors near the immersed boundary. Figure 9 shows the variation of the error norms for the two velocity components, $u$ and $v$, plotted against $1/N$ where $N$ is the number of grid cells in one direction. The line with a slope of 2 denotes second order accuracy while the line with a slope of 1 denotes first order accuracy. It was found that the global accuracy was close to and slightly better than first order. The local accuracy was close to first order which is introduced due to the immersed boundary interpolation. Depending on the overall domain size this also affects the global accuracy. For the present case the domain is small relative to the size of the immersed boundary which gives a global accuracy also close to first order.

With respect to the temporal accuracy the time derivatives are all discretized with a first order Euler scheme that results in a temporal accuracy at first order.

Regarding the spatial discretisation, a grid refinement test was performed in order to estimate the necessary grid resolution for the immersed boundary method. The test case for flow past a cylinder given in Section 3.1 was applied. The overall setup in terms of domain size and boundary conditions were as described in Section 3.1. Three different refinement levels were applied around the cylinder corresponding to grid size, $\Delta$, of $D/16, D/32$, and $D/64$ where $D$ is the cylinder diameter. In Figure 10 the pressure coefficient around the cylinder is compared to experimental data. For the two coarsest resolutions the pressure was found be purely estimated at the cylinder surface. Especially for the coarsest resolution of $\Delta = D/16$ some spatial oscillations were seen along the downstream edge from $\omega = 100^\circ - 180^\circ$. For the third
refinement level of $\Delta = D/64$ the pressure converged towards the experimental data. For the following simulations the relative resolution were selected to be of the same size as found for the present grid refinement test.

![Figure 9: $L_2$ and $L_\infty$-norms of the error for grid convergence.](image)

For high Reynolds numbers the flow becomes turbulent and may therefore not be treated as two dimensional. For validation of the immersed boundary method for three dimensional high Reynolds numbers flow a sphere was simulated at high Reynolds numbers. The Smagorinsky LES model was applied for the turbulent closure. In order to address the effect of including the turbulent closure a corresponding simulations was performed without the LES model. The simulations were performed for $Re = 1.62 \cdot 10^5$ corresponding to the experimental data presented in Achenbach (1972).

The sphere had a diameter of $D = 1.0 \text{ m}$. The model domain was set up with a transverse and longitudinal dimension of $40D$. The upstream distance from the center of the sphere to the inlet boundary was $10D$. The mesh aspect ratio was 1 in the entire domain. Mesh refinement was applied in areas around the sphere. The largest grid cell dimension was $0.3 \text{ m}$ while the smallest dimension around the sphere was $0.019 \text{ m}$. A total of 493,000 grid cells were applied.

At the inlet boundary the velocity was specified as a Dirichlet condition while a Neumann condition was applied for the pressure. At the sides parallel to the flow direction a slip condition was applied for the velocity while the pressure was given as a Neumann condition. The outlet boundary was specified with a Neumann condition on the velocity and a Dirichlet condition on the pressure.

Figure 11 presents the distribution of the pressure coefficient compared with data given in Achenbach (1972). The model was found to provide a good representation of the pressure distribution around the sphere. Especially it is noted that the pressure on the downstream side of the sphere and the base pressure at $180^\circ$ is well predicted. This implies that the flow separation is estimated correctly as the downstream and base pressure is sensible to the position of the separation point.

For the results shown in Figure 11 the LES model was applied. The effect of the LES model was addressed by a repetition of the simulation where no turbulence closure was applied. Figure 12 presents the results where it is seen how the pressure from about $60^\circ$ and further downstream was higher than the experimental measurements. The pressure drop is caused by the accelerating flow from $0^\circ$-90°. When the flow separates from the surface of the sphere the negative pressure at this point continues around the sphere. In this case where the pressure was too high it indicates that the separation point was positioned upstream compared to the position in the measurements. Hereby the pressure has not decreased sufficiently before the separation occurs.

The comparison between the solution with and with-
out the LES turbulence closure shows that a better solution was obtained by applying the LES model. It should be noted, that a detailed verification of the LES model in relation to the IBM implementation is recommended. This is part of the future work planned for the present model development.

3.4. Free surface water waves around vertical cylinder

The coupling between the immersed boundary method and the VOF model for simulating free surface flows was validated by simulating regular wave interaction with a vertical circular cylinder. The cylinder was included in a structured hexahedral mesh with immersed boundary method as for the case with flow over a circular cylinder. The surface elevation along the cylinder surface was compared to experimental data for wave run-up given in Kriebel (1992).

Figure 13 presents a definition sketch of the run-up around the cylinder. The incident wave is magnified as it interacts with the cylinder. Here the run-up, \( R \), is defined as the distance from the mean sea level (MSL) to the position of the free surface on the cylinder at any given time and angular position around the cylinder. The run-up envelope is defined as shown in Figure 13 as the maximum free surface position that has occurred during one wave period around the cylinder.

A rectangular model domain was setup with dimensions corresponding to the experiments reported in Kriebel (1992). A sketch of the layout is shown in Figure 14. The cylinder had a diameter at \( D = 0.32 \) m and the water depth was \( h = 0.45 \) m for all experiments. Two cases were simulated with regular Stokes 2nd order waves given as \( kH = 0.215 \) (\( H = 0.13 \) m and \( T = 1.95 \) s) and \( kH = 0.402 \) (\( H = 0.17 \) m and \( T = 1.5 \) s). At the inlet and outlet boundary a relaxation zone with a length of approximately one wave length was used for generating and absorbing the waves, respectively. Wave generation and relaxation was based on Jacobsen et al. (2012). A hexahedral base mesh was setup with an overall dimension of (length \( \times \) width \( \times \) height) \( (20.0 \times 2.5 \times 1.0) \) m.
A uniform grid resolution of (length × width × height) (0.08 × 0.04 × 0.02) m was applied in the entire domain. A refinement zone was applied in a band around the position of the free surface which gave a vertical resolution of 0.01 m. Two zones were applied around the cylinder with refinement in the horizontal direction as well as one local zone following the surface of the cylinder. This gave a horizontal resolution near the cylinder of 0.01 m. A total of approximately 1,100,000 computational cells were applied. The simulation time was 30 s. Each simulation was performed in parallel on 16 processor cores and was completed in approximately 20 h.

Figure 14: Model setup according to the experiments in Kriebel (1992).

Figure 15 presents the results of the two cases with \( kH = 0.215 \) and \( kH = 0.402 \) respectively. The run-up, \( R \), was normalized with \( H/2 \) in order to follow the notation in Kriebel (1992). The results are shown as time stamps of the surface elevation around the cylinder during one wave period. Experiments are shown as the envelope of the surface elevation i.e. the maximum position of the water surface at any angular position around the cylinder.

4. Application: Breakwater rock toe

A rubble mound breakwater rock toe was simulated with the hybrid model. The breakwater core, filter and main armour layers were simulated with the porosity model while the rock toe was resolved with the immersed boundary model. The stability of the toe structure of a breakwater is of general interest regarding the overall stability of the structure. The toe acts as support for the main armour layer as well as protection against scour.

Some recent examples of investigations of rock toe stability were presented in Muttray (2013) and van Gent and van der Werf (2014). Stability formulas were derived based on model experiments taken into account the effect of e.g. toe dimension, wave steepness and water depth. This example shows how the numerical model can be applied for investigating the toe stability as function of the surf similarity parameter. The actual forces on the stones were evaluated as well as the flow in and above the toe.

It is noted, that a general investigation of stability would apply irregular sea states based on a defined wave spectrum. This is also apparent from the design formulas in e.g. CIRIA et al. (2007), Muttray (2013), and van Gent and van der Werf (2014) where spectral quantities for wave height and period are applied. In the following it is demonstrated how some details on the physical processes can be investigated by means of the presented model. For this, shorter time series of regular waves were applied. For a practical design application it is proposed to combine a 2-dimensional screening process with the 3-dimensional immersed boundary hybrid model as follows. First, a 2-dimensional model is applied for an irregular time series representing the defined design condition. Here the breakwater is included only by the porous media resistance model. This simulation provides an overall estimation of the stability based on velocities near the structure. From these results the most severe events are picked out. These are reproduced as short time series in a 3-dimensional model where the im-
mersed boundary hybrid model is applied. Hereby the
details on the loading process and stability is found for
the events which may cause damages to the structure.

Figure 16 presents the general layout of the breakwater
including the toe structure. The breakwater consisted of
a core, a filter layer and an outer armour layer. Both the
front and rear side had a slope of 1:1.5. The crest height
was 30 m and the crest width was 6 m.

4.1. Main breakwater setup

The main part of the breakwater consisting of the core,
filter and armour layers was included with the porosity
model. The characteristics of the individual components
of the main breakwater are given in Figure 16. The resis-
tance parameters, \( \alpha \) and \( \beta \), for the porosity model were
set to \( \alpha = 500 \) and \( \beta = 2.0 \) according to Jensen et al.
(2014).

The computational domain had a dimension of
(length \( \times \) width \( \times \) height) 800 m \( \times \) 10.5 m \( \times \) 40 m with
the breakwater toe positioned at a distance of 600 m
from the inlet boundary. A hexahedral mesh was ap-
plied with a uniform grid spacing in all directions of 0.5
m. Grid refinement was applied in a band around the
free water surface with one refinement level providing a
grid spacing of 0.25 m corresponding to 20 points per
wave height. The grid around the main breakwater struc-
ture was also refined with one refinement level where the
porosity model was applied. Further refinement was ap-
plied at the toe structure as described in Section 4.2. In-
cluding the toe refinement the total number of grid cells
was 2,400,000.

4.2. Rock toe setup

Each individual stone in the toe section was resolved in
the model. For this a detailed representation of the
stones were needed. A number of natural stones were
mapped in a 3D scanner in order to obtain a digital
representation of the surface of the stones. The stones
were selected from the hydraulic laboratory at DTU and
the scanning was performed at Texas University at San
Antonio (UTSA). Figure 17 presents the digital repre-
sentation of one stone as obtained from the 3D scan-
ning. To achieve a random packing of the stones a six-degree-of-freedom rigid body motion solver was ap-
plied to arrange the stones. The libraries by Bullet
Physics (http://www.bulletphysics.com) were used. Here
the stones were dropped along an impermeable sloping
bed which gave a natural random placement at the toe of
the breakwater. Figure 18 shows the final placement of
the stones next to the main part of the breakwater that
was modelled with the porosity resistance model. The
final placement of the stones was exported to one final
digital surface that was applied for the immersed bound-
ary method.

The overall grid was setup as described in Section
4.1 with refinement levels around the free water surface
and the porous breakwater structure. An additional three
refinement levels were applied around the toe structure
providing a minimum grid size near the stone surfaces
of 3.1 cm corresponding to 64 grid cells per stone diam-
eter, \( d_{50} \). This corresponded to the same relative reso-
nution as for the validation case for unidirectional flow
past a cylinder in Section 3.2. The computational mesh
is shown for a vertical section through the toe structure
in Figure 19. It is noted that the grid refinement was
optimized with a refinement procedure that detected the
solid surface of the stones and applied refinement levels
around this surface. As such the number of grid cells
were optimized as the refinement for the smallest grid
cells around the stones were only present where it was
needed for the immersed boundary.

4.3. Wave conditions

Regular waves were applied for investigating the
forces and loading process. Four wave conditions were
simulated with constant wave height and varying wave
period as shown in Table 1. The surf similarity para-
meter, \( \xi_0 \), was defined in e.g. Battjes (1974) as:

\[
\xi_0 = \frac{\tan(\gamma)}{\sqrt{H_0/L_0}}
\]  

(30)

where \( \gamma \) is the slope of the bed, \( H_0 \) is the deep water wave
height, and \( L_0 \) is the deep water wave length.

A water depth of \( h = 18 \) m was applied for all sim-
ulations. The waves were generated as Stokes 5th order
waves by means of relaxation zones according to Jacob-
son et al. (2012).

4.4. Forces on and stability of toe

The stones in the toe structure were placed in approx-
imately two layers as shown in Figure 18. The forces
were extracted for stones placed in the top and bottom
layer. Figure 20 and 21 presents the lift force time series
for the two wave conditions with a period of \( T = 6.7 \) s
and \( T = 20.1 \) s, respectively. The water surface elevation above the toe is presented as well.

The effect of having two layers is apparent for both wave periods. In the bottom layer the stones are sheltered and experiences a low lift force. At the top layer the stone is exposed to the flow acceleration above the toe that creates a large lift force. For stability the top layer is of interest in order to ensure that the destabilizing forces due not exceed the stabilizing forces.

The stones experienced similar loading cycle for the different wave lengths. For the stones in the top layer the loading cycle was oscillating in a regular manner with two positive lift events during one wave period. This was caused by the low pressure above the stone due to flow acceleration both during run-up and run-down. The positive lift force was larger during run-up than run-down. For the stones in the bottom layer only one positive lift event was found for each wave period. This was associated with the run-up stage while the flow was dampened sufficiently to prevent positive lift during run-down.

The effect of the surf similarity parameter was investigated by four wave conditions ranging from the plunging to the surging breaker regime. Figure 22 presents the maximum lift force over one wave period. First it is noted that the lift force increased for increasing surf similarity parameter. This is a result of larger orbital velocities at the bed for larger wave lengths as well as the differences in the run-up and run-down process for the different wave lengths. For the stones in the top layer the loading cycle was oscillating in a regular manner with two positive lift events during one wave period. This was caused by the low pressure above the stone due to flow acceleration both during run-up and run-down. The positive lift force was larger during run-up than run-down. For the stones in the bottom layer only one positive lift event was found for each wave period. This was associated with the run-up stage while the flow was dampened sufficiently to prevent positive lift during run-down.

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Table 1: Wave conditions.

<table>
<thead>
<tr>
<th>Wave No.</th>
<th>( H ) (m)</th>
<th>( T ) (s)</th>
<th>( \xi_0 )</th>
<th>Breaker type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.13</td>
<td>6.7</td>
<td>2.31</td>
<td>Plunging breaker</td>
</tr>
<tr>
<td>2</td>
<td>5.13</td>
<td>10.1</td>
<td>3.07</td>
<td>Plunging breaker</td>
</tr>
<tr>
<td>3</td>
<td>5.13</td>
<td>16.8</td>
<td>4.39</td>
<td>Surging breaker</td>
</tr>
<tr>
<td>4</td>
<td>5.13</td>
<td>20.1</td>
<td>5.11</td>
<td>Surging breaker</td>
</tr>
</tbody>
</table>
different surf similarity parameter.

Regarding stability, the submerged weight of the stones is indicated in Figure 22 based on the diameter, $D_{50}$. The simulated wave conditions did not cause lift forces that exceeded the submerged weight of the stones. It is noted, that based on the most recent rock toe stability formulas in Muttray (2013) and van Gent and van der Werf (2014) the applied wave conditions should not cause instabilities.

5. Conclusions

The present paper presented a hybrid modelling approach for coastal engineering problems. The new model combines a standard Navier-Stokes solver with a porous media resistance model and an immersed boundary model for solid structures. The following conclusions can be summarised:

1. A discrete forcing procedure was selected as the immersed boundary method.
2. The method was implemented in the OpenFoam CFD model based on an additional forcing term in the momentum equation.
3. The flow around a 2D cylinder and 3D sphere was represented with good accuracy compared to experimental data.
4. Free surface water waves interaction with a vertical circular cylinder was reproduced with good accuracy.

5. The immersed boundary model was coupled with the porous media resistance model for simulation of solid structures near sand and gravel materials.
6. The possible extend of the model was exemplified by simulation of a breakwater rock toe.

Further work is recommended on this topic. Currently, future work is planned to include a detailed validation of the LES turbulence closure in relation to the IBM method, inclusion of validation cases for stones placed on/in porous materials, and rigid-body-motion solvers for movements of armour stones.

Acknowledgements

The support of the Danish Ministry of Science, Technology and Innovation through the GTS grant: Fremtiden Marine Konstruktioner (Marine Structures of the Future), is acknowledged.

The 3D-scanning of the rock stones was performed by students at UTSA.
Figure 21: Lift force time series for wave period $T = 20.1$ s for stones in the bottom and top layer.

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CHAPTER 6

WAVE INTERACTION WITH LARGE ROUGHNESS ELEMENTS ON AN IMPERMEABLE SLOPING BED

WAVE INTERACTION WITH LARGE ROUGHNESS ELEMENTS
ON AN IMPERMEABLE SLOPING BED

Bjarne Jensen¹, Erik Damgaard Christensen¹ and B. Mutlu Sumer¹

The present paper presents the results of an experimental and numerical investigation of the flow between large roughness elements on a steep sloping impermeable bed during wave action. The setup is designed to resemble a breakwater structure. The work is part of a study where the focus is on the details in the porous core flow and the armour layer flow i.e. the interaction between the two flow domains and the effect on the armour layer stability. In order to isolate the processes involved with the flow in the porous core the investigations are first carried out with a completely impermeable bed and successively repeated with a porous bed. In this paper the focus is on the impermeable bed. Results are obtained experimentally for flow and turbulence between the roughness elements on the sloping bed. Numerical simulations have reproduced the experimental results with good agreements and can hereby add more details to the understanding of the fluid-structure interaction.

Keywords: Fluid-structure interaction; Breakwaters; Model scale experiments; Numerical modelling

INTRODUCTION

The breakwater structure is widely used within coastal and harbour engineering for providing sheltering against offshore wave action. Often the breakwater structure is designed as a porous structure which allows a water flow through the structure while the wave energy is removed. The internal porous core can be made of sand and gravel materials with specific gradations. To protect these rather fine grained internal layers from being eroded by the waves one or several layers of larger stones are placed on top of the core material. These are referred to as cover or armour stones.

The scope of the work presented here is to study the stability of the breakwater armour layers with special focus on the interaction between the armour and the porous core material. The flow in and out of the porous core contributes to the stability and/or instability of the armour layers. In general the flow can be divided into two domains: (i) porous flow in the core material and (ii) armour layer flow just above, and in between, the armour layers. The theoretical background for the porous flow was described in Engelund (1954) for both laminar and turbulent flows. In Burchart and Andersen (1995) the porous flow was investigated with reference to breakwater structures. The armour layer flow has typically been included via model scale experiments where the entire structure including the porous core is constructed in laboratory scale and exposed to wave action. Recent examples on such experiments are Andersen et al. (2011) and Burchart et al. (2006) where stability and overtopping is investigated. In addition to stability and overtopping also the flow through the porous material of the structure is studied e.g. in terms of pressure distributions through the breakwater. Recent examples of this are Muttray and Oumeraci (2005) and Vanneste and Troch (2012).

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The stability is evaluated based on the observed damages during the experiments; however the details on the failure mechanism such as the porous flow and the armour layer flow is not investigated in these types of experiments. Examples of a more detailed approach are seen in Tørum (1994) where forces have been measured on spherical armour units on a sloping breakwater front in laboratory scale. In Hald (1998) forces were measured on real armour stone also in laboratory scale. These studies focused on the response as function of the incoming wave condition whereas details on velocities and turbulence in the armour layer as well as the porous flow were not investigated.

Numerically the entire system can be further investigated which is the case in for example Losada et al. (2005) and recently in del Jesus et al. (2012) and Lara et al. (2012); however due to limitations in computational resources it is still difficult to fully resolve the details of the flow between the armour stones and in the porous core. In Lai et al. (2010) the flow between actual spherical stones were resolved in a numerical model however on a mild sloping beach.

The present work is focused on the details in the porous core flow and the armour layer flow i.e. the interaction between the two flow domains and the effect on the armour layer stability. In order to isolate the processes involved with the flow in the porous core the experiments are first carried out with a completely impermeable bed and successively repeated with a porous bed. In this paper the focus is on the impermeable bed experiments with one layer of roughness elements. With this methodology the complexity of the structure is increased step by step as the experiments are progressing. When adding the different structural parts to the experiments one at a time it is possible to see the effect of the physical processes accounted for by these structural elements.

EXPERIMENTAL SETUP AND CONDITIONS

Facility and Setup

All experiments were carried out in a wave flume at the hydraulic laboratory at DTU. The flume has a length at 25m, a width at 0.6 m, and a depth at 0.8m. The water depth for the present experiments was fixed at 0.4m. The flume is equipped with a piston-type wave maker in one end for generating regular as well as irregular wave conditions. In the opposite end the flume is equipped with a parabolic shaped wave absorber. At the general testing area the sides of the flume is made out of transparent glass which enables a visual observation of the experiments as well as laser LDA measurements from the side. An overview of the entire flume setup is shown in figure 1.

The slope used for the rough bed experiments was arranged with an inclination of 1:1.5. The bed was made out of a plastic PVC plate with a thickness at 20mm and a length at 1.5m. The width corresponds to the width of the flume at 0.6m. The plate was supported on the top of the flume by a steel profile spanning over the flume in the transverse direction. Furthermore the plate was fixated at the bottom of the flume to ensure that there was no movements of the bed during wave action. The interface between the flume walls and the sides of the sloping bed was sealed with silicon filler to ensure that no water exchange took place between the front and the back of the sloping bed. Water where pumped to the rear side of the sloping bed before start of the experiments. A general sketch of the rigid bed
model is shown in figure 2a. The sloping bed was covered with an idealized armour layer consisting of spherical plastic elements with a diameter of $D = 38\,\text{mm}$ glued to the bed in a 90 degree arrangement, see figure 2b. The plastic spheres were applied in one layer.

In addition to the above described rough bed experiments a series of smooth bed experiments has been conducted as well. These will only be included briefly in the present paper. The setup for the smooth bed experiments was identical to the rough bed experiments only the bed being smooth. Hereby the hydrodynamics accounted for by the sloping bed only can be investigated without the effect of the roughness elements.

**Test Conditions**

The experiments were performed with a solitary wave which allows for an idealized investigation of the dynamics within one wave cycle including approach, run-up, and run-down. Other studies have previously applied this methodology in order to study the run-up and run-down processes. Examples are Grilli et al. (1994) where solitary wave breaking induced by a breakwater was investigated, Sumer et al. (2011) who studied flow and sediment transport due to a plunging solitary wave, and Lara et al. (2012) who applied a solitary wave
for investigating wave interaction with a breakwater both experimentally and numerically.

The offshore water depth was $h = 40$ cm for all experiments and the height of the solitary wave was $H = 14$ cm. The undisturbed offshore surface elevation is given by the small-amplitude solitary wave theory as:

$$\eta = H \text{sech}^2(\omega t)$$  \hspace{1cm} (1)

where $H$ is the height of the solitary wave measured from the still water level, $t$ is time, and $\omega$ is given as:

$$\omega = \sqrt{\frac{3}{4gH^2}} \frac{1}{h}$$  \hspace{1cm} (2)

where $g$ is the acceleration due to gravity. Similar to sinusoidal waves a time scale can be defined by:

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{4}{3gh}}$$  \hspace{1cm} (3)

which can be interpreted as the time scale characterizing the width of the surface elevation time series as described in Sumer et al. (2011). This quantity was $T = 2.48$ s in all experiments. The experiments were performed at a Keulegan-Carpenter number at $KC = 45(= U_m T_w / D)$ where $U_m$ is the maximum bed parallel velocity, and a corresponding Reynolds number at $Re = 4 \times 10^4(= DU_m / \nu)$.

The sampling frequency of the measurements was 120 Hz. The number of runs for each measuring point (for ensemble averaging) was 30 for the impermeable rough bed experiments. A sensitivity analysis carried out indicated that the statistical quantities, the mean values and the standard deviations, converged to constant values for these sample sizes as shown in figure 3. Here the maximum ensemble averaged RMS value of the fluctuating component of the velocity is depicted against the number of repetitions applied for the ensemble averaging. The left panel shows the error in terms of the absolute RMS value normalised by the mean value of the ensemble averages for 25-30 repetitions. The right panel shows the absolute RMS values.

The RMS values shown in figure 3 is also applied to quantify the turbulence level in terms of the RMS value of the fluctuating component of the velocity, $u' = u - \bar{u}$. This is found as:

$$\sqrt{\overline{u'^2(t)}} = \left\{ \frac{1}{N-1} \sum_{i=1}^{N} [(u(t)]_i - \bar{u}(t))^2 \right\}^{1/2}$$  \hspace{1cm} (4)

where $N$ is number of repetitions.

The waves were found to be reproducible. Twenty arbitrary selected time series of the surface elevation at the toe of the sloping bed were plotted together. Here it was seen that they collapsed on a single curve, which confirmed the repeatability of the generated wave.
Figure 3: Sensitivity analysis for convergence of RMS values as function of sample size.

Regarding the characteristics of the solitary wave the breaking criterion given in Grilli et al. (1997) can be applied. Here the breaking is defined based on the slope parameter, \( S_0 \), defined by:

\[
S_0 = 1.521 \frac{s}{\sqrt{H/h}}
\]

(5)

The breaking types are characterized in Grilli et al. (1997) as spilling \((S_0 < 0.025)\), plunging \((0.025 < S_0 < 0.30)\), and surging \((0.30 < S_0 < 0.37)\). For the present experimental conditions the slope parameter takes the value \( S_0 = 1.71 \) which falls outside the defined breaking criterion thus giving a reflecting wave. This is due to the steep sloping bed. As will be seen later a breaking bore/hydraulic jump forms during the run-down.

It is noted that two processes in ordinary oscillating waves are missing in the present idealized solitary wave case, namely the process controlling the wave setup and the process controlling the water table in the porous core of the structure. The latter is of no importance for the present experiments as the bed is impermeable.

**Instrumentation and Measurements**

Two types of measurements were performed: velocity measurements and surface elevation measurements. Measurements of velocities and turbulence were performed with Laser Doppler Anemometry (LDA). A DANTEC two-component LDA system was applied in back-scatter mode where the two velocity components, horizontal and vertical, were measured simultaneously. The arrangement of the LDA system is shown in figure 4 for the rigid bed setup with one layer of spherical elements. The velocity is measured in the pore between the flume wall and the two neighbouring spheres across the vertical. Hence the velocity does not represent that measured in a regular pore. However, the effect of the wall has been investigated by repeating a reference experiment with measurement at a larger distance from the wall. This showed no wall effect compared to the pore measurements near the wall. The wall pore is chosen as it gives a greater flexibility in term of positioning
the LDA measuring point.

Figure 4: Arrangement of LDA measurements for the rigid bed model with one layer of spherical plastic balls. Measuring points are indicated with dots.

The surface elevation measurements were performed at two locations: at the offshore location and at the toe of the sloping bed as shown in figure 1, WG1 and WG2 respectively. Conventional resistance type wave gauges were used in the measurements. The LDA measurements and the surface elevation measurements were synchronized. In addition to the above, synchronized flow visualization were performed using a digital video recorder applying 250 fps. From here the detailed observations are drawn of the entire process of run-up, run-down, breaking and trailing waves.

EXPERIMENTAL RESULTS

The experiments cover three types of measurements: LDA velocity measurements, surface elevation measurements and video visualization. In the following the main results of the experimental investigations are presented. First, a description of the run-up and run-down cycle is given based on visualisations of the experiments. Here the different flow regimes are outlined. Second, the details of the flow is investigated in terms of velocities and turbulence based on the LDA measurements.

Wave Run-up and Run-down Cycle

The complete cycle of run-up and run-down is visually depicted and described based on high-speed video recordings. Here the surface elevation is schematically presented for a number of relevant time steps during the process. The entire cycle is divided into the following four regimes. i) approach, ii) run-up, iii) run-down and, iv) secondary run-up.

Figure 5 shows the entire sequence of run-up, run-down and trailing wave with secondary run-up. The wave has already been characterised as being reflective i.e. no breaking takes place during the run-up. In Grilli et al. (1997) and Jensen et al. (2003) the run-up phase is divided into several types of flow regimes depending on either the steepness of the sloping bed or the amplitude of the wave. At the lower part of the slope the thickness of the run-up wedge may be several times the roughness of the armour layer. Here the flow has similarities with a rough bottom channel flow. At the upper part of the slope the run-up wedge thickness is less than the roughness, which may resemble flow around obstacles as
also described in Andersen et al. (2011). The effect of the roughness elements on the bed is clearly seen at the front of the upper surface wedge. This shows a highly disturbed and turbulent flow due to the flow around the roughness elements. Furthermore the front part of the flow generates an aeration zone where a large amount of air is trapped and released from in-between the roughness elements as the surface front moves up along the slope.

![Diagram](image)

**Figure 5:** Visualisation of surface elevation on rough bed during A) run-up, B) run-down, and C) trailing wave.

Following maximum run-up the flow reverses and initiates the run-down phase. The upper part of the slope experiences a flow where the relatively low water depth is maintained. A hydraulic jump is seen at the transition between the upper part of the flow (low water depth with supercritical flow) and the lower part (higher water depth with subcritical flow). At the end of the run-down phase the downwards directed flow interacts with the volume of water above the lower part of the slope and generates a breaking bore. The breaking process on the run-down is also described in Jensen et al. (2003) and is shown numerically and experimentally in Pedersen and Gjevik (1983).
Velocity and Turbulence

Figure 7 shows the bed-parallel-velocity time series. The flow direction above the spheres is calculated based on the measured parallel and normal velocities, $u$ and $v$ and are shown in the top panel of figure 7. Here the flow reversal from run-up to run-down is clearly seen. The definition of the flow direction is given in figure 6. Fluctuations appear in the run-up phase (B in figure 7) and throughout the entire wave cycle as a result of locally generated turbulence around the spheres. Close to the bed just above and in between the spherical plastic elements the velocity is highly effected by the presence of the spheres which is also seen in terms of a high level of fluctuations throughout the wave cycle. The free stream fluctuations as well as the fluctuations around the spheres indicates one or several contributions to the turbulence during the run-up and partly the run-down phase which can be accounted for by the roughness elements. However the additional turbulence due to the breaking bore during run-down is also seen (A in figure 7). Also turbulence generated due to the turbulent boundary layer on the impermeable bed will be present however this may be difficult to distinguish from the turbulence generated due to the roughness elements. Later in this paper a comparison is given to smooth bed experiments where the effect of the roughness is removed.

The turbulence levels presented in figure 8 further show the effect of the roughness elements as already mentioned. Again, the flow direction is shown in the top panel and follow the definition in figure 6. The turbulence production is initiated from the beginning of the run-up phase and grows gradually until the point where the breaking bore during the run-down phase generates a peak increase in the turbulence level. During run-up fluctuations are seen both in the free stream as well as in the pores. These may originate from two different processes namely the boundary layer turbulence above the roughness elements and wake turbulence formed locally in-between the roughness elements.

Above the roughness elements the run-up turbulence develops to a smaller level than in-between the roughness. At this point (B in figure 8) the turbulence is mainly generated by boundary layer turbulence above the roughness elements as well as lee wake turbulence from in-between the roughness elements which is diffused up into the upper layers. A large increase is seen when the run-down breaking occurs (A in figure 8). Above the roughness
elements this process is seen earlier than in-between the roughness. Below the top of the roughness but above the center of the roughness a larger turbulence level is seen (C in figure 8). Here the effect of lee wake turbulence from the roughness elements is seen. Also it is clear how the turbulence levels drop momentarily at the point of flow reversal from run-up to run-down.

Figure 7: Time series for velocities parallel to the bed for solitary wave at measuring section I on rough bed with one layer of spheres. The flow direction in the free stream point above the spheres is shown as a reference signal in the top panel. Notation A, B and C refers to the discussion in the text.
Figure 8: Time variation of the RMS values of the fluctuating component of the velocity component parallel to the bed at measuring section I on rough bed with one layer of spheres. The flow direction in the free stream point above the spheres is shown as a reference signal in the top panel. Notation A, B, C and D refers to the discussion in the text.

Finally a brief comparison is given with the corresponding data performed with the smooth sloping bed setup. When the results from the rough bed experiments are compared to identical smooth bed experiments the effect of the surface roughness can be seen. The comparison is summarised in figure 9. Here it is seen how the bed parallel velocities experiences very little fluctuations during run-up and most of the run-down phase for the smooth
At the end of the run-down phase fluctuations are initiated first due to boundary layer turbulence and immediately after further increased due to the breaking bore which is very pronounced for the smooth bed experiments. Compared to the rough bed experiments it is seen how the turbulent fluctuations starts to develop during the run-up phase caused by locally generated turbulence in-between the spherical elements. Again a large peak in the fluctuations are found at the end of the run-down phase where the breaking bore transports turbulence down to the measuring section. Furthermore it is seen how the flow reverses from run-down to secondary run-up at an earlier stage for the rough bed compared to smooth bed.

![Time series for velocities and RMS values parallel to the bed for solitary wave at measuring section I. Comparison of smooth bed and rough bed with one layer of spheres. The flow direction in the free stream point above the spheres is shown as a reference signal in the top panel.](image)

**Figure 9:** Time series for velocities and RMS values parallel to the bed for solitary wave at measuring section I. Comparison of smooth bed and rough bed with one layer of spheres. The flow direction in the free stream point above the spheres is shown as a reference signal in the top panel.

**NUMERICAL MODEL AND SETUP**

The experimental setup has been further investigated by numerical simulations. The open source CFD library OpenFOAM® has been applied including Large Eddy Simulation (LES) turbulence modelling. A detailed model has been setup where the roughness elements on the sloping bed are resolved directly in a periodic domain. The measured free
stream flow from the physical experiments is applied as boundary conditions hereby enabling a direct comparison of the simulated and measured results. The numerical model and the model setup is described in the following.

**Numerical Model**

The numerical model is based on a finite volume discretisation of the Navier-Stokes equations on a collocated grid arrangement. The Navier-Stokes equations consists of the continuity and momentum equation as follows:

**Continuity equation:**

\[
\frac{\partial u_i}{\partial x_i} = 0
\]  

**Momentum equations:**

\[
\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

where \(\rho\) is the density of the fluid, \(u_i\) is the velocity vector \(u = (u, v, w)\), \(p\) is the pressure, \(\mu\) is the dynamical viscosity, \(t\) is the time, and \(x\) is the spatial variable.

For high Reynolds number flows the turbulent fluctuations may not be resolved directly by the computational grid. Therefore a turbulence model must be introduced to account for the effects of the turbulent fluctuations. For the present simulations a Large Eddy Simulation (LES) model has been applied that allows for a direct simulation of the large scale turbulent fluctuations while the LES model adds the effect of the small scale turbulent fluctuations. LES modelling includes the direct simulation of turbulent fluctuations larger than the selected filter scale however this also sets some strict requirements to the grid resolution. In general a finer grid resolution is required compared to RANS turbulence models however some applications of LES within coastal engineering has been seen e.g. Christensen and Deigaard (2001) and Christensen (2006) applied LES modelling for investigation of spilling and plunging breakers.

The LES model is based on a spatial filtering of the Navier-Stokes equations. A top-hat filter is applied where the computational grid is used as the filter. The filtered Navier-Stokes equation reads:

\[
\rho \frac{\partial \overline{u_i}}{\partial t} + \rho u_j \frac{\partial \overline{u_i}}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \mu \left( \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right)
\]

where overbar denotes a filtered quantity. In equation 8 the second term on the left hand side is split up into two terms as:

\[
\frac{\partial \overline{u_i} u_j}{\partial x_j} = \frac{\partial \overline{u_i} \bar{u}_j}{\partial x_j} + \frac{\partial}{\partial x_j} \left( \overline{u_i} \bar{u}_j - \overline{u_i \bar{u}_j} \right)
\]

The first term on the right hand side is simulated directly while the second term is moved to the right hand side of equation 8 and must be modelled. This term is also referred to as the sub-grid scale Reynolds stress:
Equation 10 is called the closure problem for which a model must be applied. This model will be referred to as a sub-grid scale model (sgs model). For the present simulations the Smagorinsky sgs model is applied. The sub-grid scale stresses given in equation 10 are modelled as:

\[
\tau_{ij,s} = \frac{1}{3} \tau_{kk} \delta_{ij} = 2 \mu_t \overline{S}_{ij},
\]

\[
\overline{S}_{ij} = \frac{1}{2} \left( \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right)
\]

Here \( \mu_t \) is the eddy viscosity which is found as:

\[
\mu_t = \rho (C_s \Delta)^2 |\overline{S}|
\]

where \( \Delta \) is the filter length scale and \( |\overline{S}| = (2 \overline{S}_{ij} \overline{S}_{ij})^{1/2} \). \( C_s \) is the Smagorinsky constant that is generally in the order of 0.065 to 0.2. For the present simulations it is set to 0.1 however the optimal value can vary from case to case.

Model Setup

The model has been setup to reproduce the experimental results obtained for one layer of spherical roughness elements on the sloping impermeable bed. Currently the focus has been on the detailed flow around the spherical elements. Therefore a model has been setup that includes the local area around the spheres. Only one spherical elements has been resolved and periodic boundary conditions has been applied in order to add the effect of multiple elements. With this approach the oscillating flow around the roughness element is modelled; however the free surface is not included. This leaves out some effects such as wave breaking as will be seen later.

Figure 10 presents the setup in terms of geometry and applied boundary conditions. The domain has horizontal dimensions corresponding to the diameter of one sphere, \( D = 38 \text{mm} \) and a height at \( 3D \).

At the bottom below the sphere a wall boundary condition is applied where \( u = 0 \) at the boundary. The surface of the sphere is modelled with a wall boundary condition as well. A slip boundary condition is applied at the top of the model while a periodic boundary condition is applied on the vertical sides of the model. Hereby any quantity transported out of the domain e.g. via the right hand boundary will at the same time be moved into the domain via the left hand boundary. This will create the effect of an infinite number of spheres placed next to each other.

The flow is driven by a forcing term in the momentum equation that is based on the experimental measurements. Here the velocity measurement from the free stream region above the spheres (one run-up and run-down cycle) are used as input. The momentum equation (7) is extended as:
\[ \rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right) + f_i \]  

(13)

where the forcing term, \( f_i \), is included as:

\[ f_i = \rho \frac{\partial u_{ei}}{\partial t} \]  

(14)

where \( u_{ei} \) is the experimentally measured velocity vector \( u_{ei} = (u_e, v_e, w_e) \).

The computational grid is setup with the smallest grid cell being 0.3x0.3x0.3 mm (at the surface of the sphere) and the largest grid cell being 2x2x4 mm (at the top of the model). A total of 800,000 grid cells were applied. The computational grid is presented in figure 11. Parallel processing was applied where the model domain was decomposed into 6 domains.

**NUMERICAL RESULTS**

The numerical results have been compared to the experimental measurements previously presented. The bed parallel velocity is presented in figure 12 as ensemble averaged vertical velocity profiles. Time steps are selected to cover the entire cycle of run-up, run-down and secondary run-up. Measurements (black line with circles) are compared to the numerical results (red line).

The model is seen to capture the measurements with good agreement during the run-up and most of the run-down phase. Both the flow above the spheres as well as in the pores in-between the spheres are well described. During the last part of the run-down and especially at the secondary reflected run-up some deviations are seen. This might be due to the fact that the free surface is not included in the model and thereby the breaking bore during the run-down is not simulated. As seen for the experimental results in figure 7 and figure 8 the
flow is clearly affected by the run-down breaking in terms of velocities and fluctuations. This effect is not represented in this numerical model.

Figure 13 shows an iso-surface plot of vorticity around the longitudinal direction (in-line with the flow direction) and the transversal direction. On the left hand panel it is seen how the flow separates on the surface of the sphere and four symmetrical vortices are formed which travels downstream with the flow. These vortices have similarities with horse-shoe vortices which are well known from e.g. flow around vertical cylinders. On the right hand panel it can be seen how a boundary layer develops both on the smooth bed below the sphere as well as above the sphere. On the down stream side of the spheres a lee zone is apparent where the boundary layer does not develop.
Figure 12: Ensemble averaged velocity profiles for velocities parallel to the bed for solitary wave at measuring section I. Comparison of measurements (black line with circles) and simulated results (red line) for rough bed with one layer of spheres. The horizontal dashed line shows the top surface of the spheres. \( t = -1 - 0.2 \) s corresponds to the run-up phase and \( t = 0.4 - 1.2 \) s corresponds to the run-down phase.

CONCLUSION

Experiments have been conducted with an impermeable sloping bed with a structured layer of spherical roughness elements. The setup resembles a simplified breakwater structure. The experiments are part of a series of several experimental investigations where the individual physical processes are described. This is achieved by starting out with a very simple setup where the different structural parts are added; one at a time. Hereby the physical processes can be seen and distinguished from each other.
Figure 13: Iso-surface of vorticity at a phase angle of $\omega t = 30$ deg. Left panel: vorticity around the longitudinal direction (X-axis). Right panel: vorticity around the transversal direction (Y-axis).

The experiments showed the effect of the roughness elements in terms of the turbulence generation during run-up both locally between the roughness elements and above the roughness as boundary layer turbulence. The breaking mechanism on run-down showed a transport of turbulence below the surface in-between the roughness elements. This process is very clear for the smooth bed experiments however it is also found for the rough bed experiments.

The experiments have been followed by numerical simulations which have been setup to reproduce the experimental results. A periodic domain was applied to simulate the oscillating flow around one spherical roughness element however including the effect of several spheres placed next to each other. Good agreement was found between the simulated and measured ensemble averaged velocities. Some deviations were seen at the very last part of the run-down phase which is explained by the fact that the free surface is not included in the present simulations and thereby the breaking mechanism during the run-down is not described. This effect will be included in future simulations where the free surface is simulated. The numerical results showed a level of details which can be used for in-depth analysis of e.g. erosion mechanisms and armour stone forces.

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REFERENCES


NOMENCLATURE

Roman letters

\( B_i \)  Immersed boundary method body force
\( \langle B \rangle \)  Superficial (filter) volume averaged quantity
\( \langle B \rangle^f \)  Intrinsic (pore) volume averaged quantity
\( a \)  Porosity model resistance parameter
\( b \)  Porosity model resistance parameter
\( C_{pB} \)  Base suction pressure coefficient
\( C_p \)  Pressure coefficient
\( C_s \)  Smagorinsky constant
\( D \)  Diameter
\( d \)  Core material diameter
\( D_{n50} \)  Equivalent cube length
\( D_{85}/D_{15} \)  Stone grading
\( F_i \)  Porosity model resistance force term
\( f \)  Frequency
\( g \)  Gravitational acceleration
\( H \)  Wave height
\( H_0 \)  Deep water wave height
\( H_m0 \)  Significant wave height
\( H_I \)  Incident/incoming wave height
\( H_R \)  Reflected wave height
\( h \)  Water depth
\( I \)  Hydraulic gradient
\( K \)  Empirical coefficient for stability formula
\( KC \)  Keulegan-Carpenter number
\( k \)  Wave number
\( k_s \)  Equivalent roughness height
\( K_D \)  Empirical coefficient for stability formula
\( K_R \)  Reflection coefficient
\( L \)  Wave length
\( L_0 \)  Deep water wave length
\( N \)  Counter for e.g. number of realisations, number of grid points etc.
\( N_z \)  Number of waves
\( n \)  Porosity
\( n \)  Normal vector
$P$ Permeability factor in Chapter 1, Eq. (1.4)
$p$ Pore water pressure
$\bar{p}$ Ensemble averaged pore water pressure
$\langle \bar{p} \rangle$ Volume averaged ensemble averaged pore water pressure
$\langle \bar{p} \rangle_f$ Intrinsic (pore) volume averaged ensemble averaged pore water pressure
$\bar{p}''$ Spatial fluctuating part of the pore water pressure
$R$ Run-up on circular cylinder
$R_c$ Crest free board
$R_{cf}$ Crest free board
$Re$ Reynolds number
$Re_{A}$ Armour stone Reynolds number
$Re_p$ Pore Reynolds number
$r$ Radial distance for RBF interpolation
$r_0$ Scaling factor for RBF interpolation
$S$ Damage level
$S$ Source term in Chapter 5, Eq. (22)
$S_0$ Slope parameter for solitary wave breaking
$St$ Strouhal number
$s$ Relative density of stones
$s$ Slope angle in Chapter 2, Eq. (4)
$T$ Wave period
$t$ Time
$U_0$ Free stream velocity
$U_f$ Friction velocity
$U_{fc}$ Critical friction velocity
$u_i$ Cartesian velocity vector
$\langle u_i \rangle$ Superficial (filter) volume averaged ensemble averaged velocity vector
$\langle u_i \rangle_f$ Intrinsic (pore) volume averaged ensemble averaged velocity vector
$u'$ Fluctuating component of the velocity
$u''$ Spatial fluctuating component of the velocity
$V_i$ Control volume for volume averaging
$V_i$ Desired velocity vector for immersed boundary method
$x_i$ Cartesian coordinate vector

Greek letters

$\alpha$ Porosity model resistance coefficient
$\alpha$ Free water surface tracer (VOF) in Chapter 4, Eq. (9)
$\beta$ Porosity model resistance coefficient
$\gamma$ Slope angle of bed
$\gamma_f$ Surface roughness factor
$\gamma_p$ Added mass coefficient for porosity model
$\Delta$ Relative submerged stone density
$\Delta t$ Time step increment
$\epsilon$ Error between measured/analytical and simulation
$\eta$ Water surface elevation
\( \theta_c \) Critical Shields parameter
\( \theta_{c0} \) Critical Shields parameter on flat bed
\( \mu \) Dynamical viscosity
\( \mu_t \) Eddy viscosity
\( \nu \) Kinematic viscosity
\( \xi_0 \) Surf similarity parameter
\( \rho_s \) Stone density
\( \rho_w \) Water density
\( \tau_0 \) Bed shear stress
\( \tau_{sij} \) Sub-grid scale Reynolds stresses
\( \phi_s \) Friction angle for stone and gravel
\( \phi(r) \) Radial basis function (RBF) for interpolation
\( \phi_{N,i,j} \) Numerical solution on a \( N \) grid
\( \omega \) Cyclic frequency
\( \phi_{1260 \times 1260} \) Numerical solution on a \( 1260 \times 1260 \) grid
\( \omega_i \) Weights for RBF interpolation
APPENDIX A

DISCRETISATION SCHEMES FOR NUMERICAL SIMULATIONS

In the following an overview is given on the applied discretisation schemes for the numerical simulations. The choice of discretisation, e.g. first order, second order etc., can influence the accuracy of the numerical model. The details of the individual models regarding domain size, computational mesh, boundary conditions etc. are given in the appropriate chapters.

The applied discretisation is given in the following for each terms in the equations. The momentum equation is used to exemplify the different terms:

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + g_j x_j \frac{\partial \rho}{\partial x_i} + \frac{\partial}{\partial x_j} \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]  

(A.1)

where \( \rho \) is the density of the fluid, \( u_i \) is the Cartesian velocity vector \( u_i = (u,v,w) \), \( p \) is the excess pressure, \( g_j \) is the \( j \)th component of the gravitational vector, \( \mu \) is the dynamical viscosity, \( t \) is the time, and \( x_i \) are the Cartesian coordinates.

The first term on the left hand side is the temporal term. The second term on the left hand side is the convective term which corresponds to a divergence term. The two first terms on the right hand side are gradient terms of pressure and density, respectively. The third term on the right hand side is the diffusive term which corresponds to a Laplacian term. The Laplacian term can also be seen as the divergence of a gradient, i.e. a combination of the divergence term and gradient term. The details on how each of these terms were treated in the numerical simulations are given in the following.

The finite volume (FV) discretisation of each term was done by integrating over a cell volume, \( V \). The Gauss integration was applied for all terms where the spatial derivatives
were converted to integrals over the cell surface, $S$, that enclosed the cell volume by the Gauss theorem. Figure A.1 presents the definitions for the discretisation where $P$ is the centre point of the cell of interest, $N$ is the centre point of the neighbouring cell, $\mathbf{d}$ is the length vector between $P$ and $N$, $f$ is the face plane shared between the two cells, and $S_f$ is the surface area vector to the face plane, $f$.

The following descriptions are shown for a general version of each term for the quantity, $\phi$.

![Figure A.1: Definitions for finite volume discretisation.](image)

### A.1 Gradient terms

Gradient terms are integrated over the control volume and linearised as:

$$
\int_V \frac{\partial \phi}{\partial x_i} dV = \int_S dS \phi = \sum_f S_f \phi_f
$$

(A.2)

where $V$ is the volume of the cell, $S$ is the surface area of the cell. The field value at the face of the control volume, $\phi_f$, was evaluated by linear differencing also referred to as central differencing (CD):

$$
\phi_f = \frac{|\mathbf{d}_{f,N}|}{|\mathbf{d}_{P,N}|} \phi_P + \left(1 - \frac{|\mathbf{d}_{f,N}|}{|\mathbf{d}_{P,N}|}\right) \phi_N
$$

(A.3)

where $|\mathbf{d}_{f,N}|$ is the distance between the face plane $f$ and the centre point $N$, and $|\mathbf{d}_{P,N}|$ is the distance between the center point $P$ and the centre point, $N$. The linear CD-scheme is unbounded and second order accurate.
A.2 Divergence terms

The divergence terms, e.g. the convective term in the momentum equation, are integrated over the control volume and linearised as:

\[
\int_V \frac{\partial \rho u_i \phi}{\partial x_j} dV = \int_S dS \cdot (\rho u_i \phi) = \sum_f \mathbf{S}_f \cdot (\rho u_i)_f \phi_f \quad (A.4)
\]

The field value at the face of the control volume, \(\phi_f\), is determined by the linear upwind differencing (LUD) scheme:

\[
\phi_f = \phi_P + \frac{\phi_P - \phi_N}{|\mathbf{d}|} |\mathbf{d}_{P,f}| \quad (A.5)
\]

The LUD-scheme is based on the upwind differencing (UD) scheme but includes a correction term based on the upstream gradient. Only values from upstream nodes are applied which ensures that the scheme is bounded. The LUD-scheme is second order accurate.

A.3 Laplacian terms

The Laplacian terms, e.g. the diffusive term in the momentum equation, are integrated over the control volume and linearised as:

\[
\int_V \frac{\partial}{\partial x_j} \Gamma \left( \frac{\partial \phi}{\partial x_j} \right) dV = \int_S dS \cdot \Gamma \left( \frac{\partial \phi}{\partial x_j} \right) = \sum_f \Gamma_f \mathbf{S}_f \cdot \left( \frac{\partial \phi}{\partial x_j} \right)_f \quad (A.6)
\]

where \(\Gamma\) is the diffusion coefficient. The value of the diffusion coefficient, which corresponds to \(\mu\) in the case of the momentum equation, at the cell face is obtained by linear differencing by the CD-scheme as given in Eq. (A.2). For the surface normal gradient a linear corrected scheme is applied. The correction accounts for mesh non-orthogonality in the case where the length vector \(\mathbf{d}\) between the centre of cell \(P\) and the neighbouring cell \(N\) is not orthogonal to the face plane, \(f\), between the two cells. For the orthogonal case the linear differencing of the surface normal gradient is:

\[
\mathbf{S}_f \cdot \left( \frac{\partial \phi}{\partial x_j} \right)_f = |\mathbf{S}_f| \frac{\phi_N - \phi_P}{|\mathbf{d}|} \quad (A.7)
\]
A.4 Temporal terms

The temporal term is integrated over a control volume and discretised by differencing in time using the implicit Euler scheme:

$$\frac{\partial}{\partial t} \int_V \rho \phi dV = \frac{(\rho_P \phi_P V_P)^{n+1} - (\rho_P \phi_P V_P)^n}{\Delta t}$$ (A.8)

where $n + 1$ is the time step the equations are solved for (one step ahead in time), $n$ is the current time step with known values, and $\Delta t$ is the time step increment. The implicit Euler is first order accurate in time.