A vapor compression heat pump absorbs heat from the environment at a low temperature level and rejects heat at a high temperature level. The bigger the difference between the two temperature levels the more challenging is it to gain high energy efficiency with a basic cycle layout as found in most small capacity heat pump applications today. Many of the applicable refrigerants also reach their technical limits regarding low vapor pressure for very low source temperatures and high discharge temperatures for high sink temperatures. These issues are especially manifest for air-water heat pumps. Many alternative cycle setups and refrigerants are known to improve the energy efficiency of a vapor compression cycle and reduce discharge temperatures. However not all of them are feasible for small capacity heat pumps from a cost and complexity point of view. This paper presents a novel numerical approach to evaluate and compare different cycle layouts and different refrigerants in regard of multiple objectives. The emphasis is on the objective of energy efficiency which is defined as seasonal coefficient of performance. We discuss reasonable assumptions to allow a generic cycle comparison without restricting the evaluation to a single set of components. A special focus is on the heat exchanger sizes as a dominant influence on the cycle efficiency. Varying sizes are taken into account using a design of experiment factorization to determine a quadratic regression model for the energy efficiency. In combination with component cost correlations a constrained non-linear optimization problem is formulated and solved. Strengths and weaknesses of the proposed procedure are discussed in regard of its ability to help in systematically identifying promising combinations of cycle layout and refrigerant.

1. INTRODUCTION

Due to changes in legislation, costs and consumer demands various technologies known to offer a potential for improving energy efficiency of vapor compression systems become interesting also for small scale systems like residential heat pumps. Some examples are variable speed compressors, special expansion solutions, advanced control algorithms, advanced cycle layouts and alternative refrigerants. A review of recent trends is given by Chua et al. (2010). However not every possible combination of technologies is equally adequate or feasible, nor equally costly or equally complex. Any solution must be evaluated regarding several, often opposing, objectives. Hence finding a ‘good’ design of a vapor compression system states a multi objective optimization problem. The ‘best’ design does not exist but only solutions which are optimal compromises, graphically represented in the Pareto front. Most literature dealing with multi objective optimization of heat pumps chooses the two objective functions energy efficiency and costs. Many of these contributions focus on the optimization of individual system components (e.g. Gholap and Khan (2007) or Schiffmann and Favrat (2010)) or the optimization of component selection and control of a given cycle layout (e.g. Sayyaadi et al. (2009) or Sanaye and Nirooand (2011)). Evolutionary algorithms are used in these studies to solve the optimization problem. This optimization method is applied by Zehnder (2004) on the cycle layout level. He creates a generic cycle model which consists of different modules representing individual components and refrigerants. The modules can be activated or deactivated by the optimization algorithm. Thus Zehnder (2004) includes various cycle layouts in this superstructure and a mixed integer optimization problem is formed. This model is used to optimize for a single objective
function, the coefficient of performance. Bertsch and Groll (2008) compare different cycle layouts in terms of energy efficiency. Cost and complexity considerations are mentioned but not quantified. In contrast Hwang et al. (2004) compares energy efficiency taking investment cost into account by trading off increased costs for flammable refrigerant systems with a reduced condenser size for a single cycle layout. The result of such a trade-off strongly depends on the baseline at which the comparison is made. Effects of changing heat exchanger size on energy efficiency will be different whether the area of a small baseline heat exchanger is increased or the area of a big heat exchanger. Also an optimal ratio between condenser and evaporator heat exchanger size exists, changing the size of one without the other is not beneficial. Hence it is difficult to draw a general conclusion based on the comparison of two different condenser sizes alone. The described interdependence of heat exchanger sizes also shows that a trade-off between system and component costs is not restricted to one component; it could be done also for the evaporator or the compressor technology. Which option to choose poses an optimization problem in itself.

In the open literature for small scale vapor compression systems a gap exists between the comparative studies of various cycle layouts and refrigerants quantifying a single objective only and the multiobjective optimization of singular aspects of the problem. Filling this gap is obviously not straightforward since the number of possibilities for refrigerant choice, system layout and component design forms a vast solution space. Zehnder’s approach (2004) of using a superstructure is explicitly designed to find optimal solutions on a global level and can be extended to account for multiple objectives. However it has several disadvantages like high modeling effort, problems with numerical stability, high computation time and open questions regarding the effect of uncertainties in modeling assumptions.

In this paper an alternative method is presented to compare different cycle layouts and refrigerants in regard of two objective functions by combining simulation, data regression and optimization tools. The method is demonstrated for residential heat pumps with a small selection of cycle layouts and refrigerants. Challenges which mainly concern modeling assumptions are discussed.

2. METHOD DESCRIPTION AND EXEMPLARY RESULTS

Comparing different cycle layouts and refrigerants both in a generic and meaningful way is challenging. The goal should be to compare cycles without actually defining the components in much detail because the answer “cycle A is better than cycle B” should be valid independent of whether a scroll or piston compressor is used or an evaporator with two or three tube rows is incorporated. Hence the thermodynamic models should aim at reflecting the dominating physical behavior without requiring detailed geometrical information. For this purpose some components, namely the heat exchangers, can be better defined by their size and others like compressor or expansion device by their technology. Different approaches are used to take this difference into account in the method. However there is a limit to the concept of generality and idealizing assumptions must be made as discussed below. Details regarding the correlations used to calculate the two objective functions and the thermophysical component models and can be found in Mader (2011).

2.1 Objective functions

From the viewpoint of a heat pump manufacturer one objective is to maximize the energy efficiency of the system as defined by standards and regulations. For this study the seasonal coefficient of performance (SCOP) as defined in the preliminary European standard prEN14825 is selected. The operating conditions (building demand capacity curve, water outlet temperature, water flow rate and air inlet temperature) at which measurements of the coefficients of performance are to be performed are defined in this standard. The same conditions are used for the energy efficiency simulations. Also the calculation procedure to derive the SCOP from these measurements respective simulations follows closely the standard.

Minimizing costs is the second objective chosen. A special focus here is on the heat exchanger costs which directly depend on the heat exchanger size. Linear correlations between heat exchanger weight and cost are derived for different types of heat exchangers based on market data. Weight and heat exchange area are then correlated assuming a fin and tube geometry for the evaporator, brazed plates for the condenser and a tube in tube geometry for the suction line heat exchanger. For all geometries typical values for the heat transfer coefficients are used to finally relate $UA$ values with the heat exchange area and hence with the costs. Costs of compressor, fan and expansion device are assumed to be dominated by the used technology and independent of the chosen heat exchanger sizes. Because the calculations performed here serve mainly the
purpose of demonstrating the method a further simplification is that costs for the technology dominated components also are equal for different refrigerants, neglecting sizing effects e.g. due to varying specific volumes. System costs and costs related to safety, risks and uncertainties are also not included in the objective function derived here. These assumptions can easily be improved and adapted to gain more realistic cycle costs.

2.2 Assumptions for the thermophysical model

Each cycle model consists of several partial models representing the individual components and can be calculated for different refrigerants. Pressure drop and heat transfer in pipes and valves connecting the main components are neglected. For the expansion an isenthalpic process is assumed. The coefficient of performance is calculated as the ratio of capacity delivered at the condenser to power input at compressor and fan. The pumping power of the central heating system is neglected.

Optimal control

The control of the heat pump has a strong influence on the energy efficiency that can be reached for a given cycle layout and refrigerant, especially for air water heat pumps where operating conditions vary strongly. The cycle comparisons will be made for optimal operation of the heat pump. Hence each cycle is modeled assuming ideal control of capacity, air flow rate, subcooling, superheat and, if applicable, ideal intermediate pressure level for two stage cycles and ideal phase separation in separators. With these assumptions the simulation results represent the maximally possible system efficiency that can be gained for the given component selection.

For an ideal capacity control the heat pump delivers for each operating condition exactly the capacity demanded by the building. The required adjustment of the mass flow rate is done by a variable speed compressor. This implies that no electric backup heater is required and no on/off cycling is performed. An ideal control of the air flow rate requires finding the optimal trade-off between air flow rate and fan power consumption for each operating condition. To model this trade-off the system curve of the evaporator must be defined. This curve relates air flow rate to air side pressure drop or directly to fan power consumption. In general a cubic relation between fan power consumption and air volume flow rate can be assumed. A single design parameter in this cubic relation accounts for fan efficiency and system curve of a given evaporator-fan combination (Cai (2007)). In the model this design parameter is adjusted such that in a baseline case the fan power consumption accounts for 7% of the total power consumption. This fit is based on the findings of Miara et al. (2011) who show in field tests that power consumption of the fans is always below 7% of total power consumption. For all simulations this design parameter is kept constant which implies that for all cycle layouts, refrigerants, operating conditions and heat exchanger sizes the fan efficiency and the system curve of the evaporator are constant. Under this assumption the air flow rate is optimized for each operating condition to gain maximum system efficiency, the model hence represent an idealized variable speed fan.

The superheat at the evaporator outlet should be as low as possible but higher than zero to prevent liquid entering the compressor. Hence a constant value of 2K is assumed, idealizing an electronic superheat controller. The subcooling in the condenser is typically not a control variable. It depends on the charge in the system and the inner volumes of the heat exchangers, accumulators, separators and so on. However as Pöttker and Hrnjak (2011) show COP variations with subcooling are not negligible. To avoid charge modeling which would require knowledge about the inner volumes in the system here the subcooling is introduced as a design parameter. This parameter is not kept constant but, following the idea of optimal control, it is assumed that for each operating condition it can be adjusted to gain maximum efficiency. While in practice such a control would be extremely complex to implement, this assumption allows to hold up the interpretation of the simulation result as maximal possible system efficiency for the given component selection.

Compressor model

The power used by the compressor is calculated as the product of the mass flow rate and the enthalpy difference at suction and discharge port. The isentropic discharge enthalpy is calculated based on the suction enthalpy. The enthalpy at discharge pressure is then adjusted with the isentropic compressor efficiency. This efficiency is calculated as a function of the suction and discharge pressure as described by Granryd (2005) for reciprocating compressors. Heat losses are neglected.
Heat exchanger models

The heat exchangers are modeled with the lumped parameter method. For the different zones (liquid, gas, two-phase) averaged values for fluid properties and heat transfer are used. The condenser is modeled as an ideal counterflow heat exchanger while the evaporator is described as cross flow heat exchanger. Adequate ε-NTU correlations are implemented to describe the heat transfer in the individual zones of both heat exchangers. Pressure drop in the heat exchangers and heat transfer due to condensing air humidity on the evaporator is neglected. Input variables for the condenser model are the capacity, the water flow rate, the water outlet temperature and the subcooling. For the evaporator model the superheat, the air flow rate and the air inlet temperature are given as input parameters. The overall UA value of the heat exchanger is chosen as the design parameter for all heat exchangers.

2.3 Quadratic regression model

The size respective UA value of a heat exchanger not only determines its cost but also strongly influence the system energy efficiency because it directly affects the pressure difference between high and low pressure side. Since the increase of cycle efficiency is not linear with UA but approaches asymptotically the thermodynamic maximum efficiency, the choice of the heat exchanger UA values at which to compare different cycle layouts regarding SCOP and component cost is crucial. This is not only true for the condenser and evaporator but also for other internal heat exchangers used in various cycle layouts. Instead of comparing cycles only for one given set of UA values a comparison for a whole range of sizes of the heat exchangers would be desirable. To describe such a range a multi-dimensional space must be covered with one dimension for the evaporator size, one for the condenser size and, if applicable, one for each additional heat exchanger in the system. This would however dramatically increase the simulation effort and complicate the interpretation of the results.

Figure 1. Simulation nodes for a 3-D quadratic regression model (a) and distribution of UA_{evaporator} (b)

Klepmann (2003) describes a Central Composite Design (CCD) as sketched for three dimensions in Fig. 1a) that reduces the number of experiments required to derive a quadratic regression model for a target parameter (here SCOP) within the design variable space (here UA values). This approach can be applied to gain information about a whole range of UA values with a reduced simulation effort. Each node in Fig. 1a) represents a SCOP simulation for a given set of values of the design variables UA. The center node in this plan has the coordinate 0 in all dimensions, while the nodes of the hyper-cube are located at 1 or -1. The scale factor σ and -σ of the outer nodes is determined for an orthogonal CCD by Eq. 1 with the number of all simulation nodes N and the number of nodes in the hyper-cube N_c:

\[ \sigma = \left(0.5 \left(0.5^N - N_c \right)\right)^{0.5} \]  

(1)

It is known a priori that effects on efficiency change strongly at low UA values but changes are small at high values, therefore nodes along each dimension are distributed according to Fig. 1b). For the condenser and evaporator the upper UA value at σ is chosen such that a heat exchanger effectiveness of 0.92 is reached for the highest temperature difference indoor/outdoor defined in prEN14825, the lower boundary at -σ is at an effectiveness of 0.75 at lowest temperature difference. After simulating SCOP for all nodes the coefficients β of a quadratic model of n dimensions as given in Eq. 2 are determined by quadratic regression. Tests show that the deviation between simulation and quadratic model is smaller than 0.1% for two dimensions and smaller than 1.5% for three dimensions.
\[
SCOP(\overline{UA}) = \beta_0 + \sum_{i=1}^{n} \beta_i UA_i + \sum_{j=1}^{n} \beta_{i+j} UA_j \tag{2}
\]

Using the same approach additional quadratic models can be derived for variables presenting physical limits of operation, for example high temperatures or low pressures.

2.4 Deriving Pareto fronts

Fig. 2 gives a graphical overview of the procedure to derive Pareto fronts of optimal component combinations for each cycle layout and refrigerant. For each node of the CCD plan all operating conditions required by prEN14825 are simulated. For each of these operating conditions an optimization of airflow rate and subcooling is performed. COP as well as the discharge temperature at compressor outlet is determined. Based on these then \(SCOP\) is calculated and the maximum discharge temperature within a season is determined for each node. Afterwards both for \(SCOP\) and the discharge temperature quadratic models are derived according to Eq. 2.

As a result now \(SCOP\) and the discharge temperature are expressed as functions of the \(UA\) values of the heat exchangers in the system. Above it is described how heat exchanger costs can be correlated with the \(UA\) values; \(UA\)-independent costs for other system components are added to derive the final cost functions for each cycle. With these functions several optimization problems can be solved to find the Pareto front for each cycle and refrigerant as indicated in the lower right corner of Fig. 2. The lower left end of the curve shows the lowest cost a feasible component selection would cause and the maximum \(SCOP\) this selection would give. To determine this point following optimization problem is solved:

\[
\begin{align*}
\min & \quad \text{cost}(\overline{UA}) \\
\text{subject to} & \quad T_{\text{dis,h}}(\overline{UA}) \leq T_{\text{max}} \\
& \quad \overline{UA}_{\sigma} \leq \overline{UA} \leq \overline{UA}_{\sigma}
\end{align*}
\tag{3}
\]

An analogous optimization problem is solved to find the upper right end, representing the feasible solution which gives the maximum \(SCOP\) and the minimal costs at which this can be achieved. The points in between are determined by maximizing \(SCOP\) for several given cost values between the two extremes. The resulting Pareto front depicts for a given cycle layout the set of optimal, also called non-dominated, combination of heat exchanger sizes.

2.5 Results

To demonstrate the method three cycle layouts as depicted in Fig. 3 are compared using the simulation, regression and optimization tools described above. The basic reversed Rankine cycle as in Fig. 3a) is today’s most common layout in commercially available residential heat pumps, called “Basic” in the following. The “Slhx” cycle of Fig 3b) includes a suction line heat exchanger, the “Flash” cycle of Fig 3c) is a two stage cycle; gas from a flash tank on an intermediate pressure level is mixed with gas of the outlet port of the low stage compressor.
The assumptions for the control parameters for the Basic and the Slhx cycle are identical and described above. For the Flash cycle an ideal separation of gas and liquid in the flash tank is assumed additionally. As intermediate pressure level the geometric mean of evaporation and condensing pressure is chosen since previous investigations (Mader (2011)) showed this to be the close to the optimal level when subcooling is controlled optimally. For the Basic and Flash cycle two dimensional quadratic models are derived, a third dimension is added for the Slhx cycle. Simulations are performed for the refrigerants R410A and R290.

Figure 3. Compared cycle layouts: a) Basic – b) Slhx – c) Flash

As described above \( SCOP \) and heat exchanger cost are described as functions of the \( UA \) value. In the cost function an exemplarily chosen constant value of 100 € for each compressor and 20 € for each expansion device in a cycle is included. Since the resulting cost correlation can so far mainly be used for demonstration purposes but do not reflect real system cost, they are indicated as cost* in Fig. 4. Additionally a limit of 140°C is set for the maximum discharge temperature occurring during a heating season. Fig. 4 shows the results for a medium water outlet temperature of 45°C and a cold climate profile as defined in prEN14825.

Figure 4. Pareto fronts of optimal solutions for different cycle layouts and refrigerants

With the given assumptions the comparison of R410A and R290 for the Basic cycle shows a slight improvement in \( SCOP_{45c} \) at equal component costs for the natural refrigerant. At the lower left end of the two curves it can be seen that the costs for the R410A system are slightly higher. This results from the constraint on the discharge temperature: The sizes of evaporator and condenser must be increased above \(-\sigma\) to avoid too high temperatures. This results in higher costs. The discharge temperature is also a limiting factor for the Slhx cycle with R410A both at the lower end and the higher end. For the Slhx cycle with R290 on the other hand the temperature constraint does not play a role – the full size range \([\sigma, -\sigma]\) is theoretically feasible. However assuming a tube-in-tube geometry the higher \( UA \) value bound would require a large suction line heat exchanger which is reflected in the high component costs and might be infeasible from a practical point of view. For the Flash cycle only R290 is considered since a decomposition of the mixture refrigerant R410A in the flash tank makes this solution infeasible. A comparison between the Flash and the Basic cycle with R290 shows only a small improvement potential in \( SCOP_{45c} \) at increased component costs due to a second compressor and expansion valve. For the given climate profile the high improvement in \( COP \) of up to 25% at low air temperatures seen in previous studies is not reflected in the \( SCOP_{45c} \) because
operating hours at low temperatures are little. This picture might change however with higher pressure differences and hence for the $SCOP_{SSC}$.

2.6 Discussion

The proposed approach to evaluate cycle layouts and refrigerants in regard of two objectives allows taking different heat exchanger sizes and operating limits into account. The resulting Pareto front of each cycle and refrigerant combination also implicitly gives for each $SCOP$ or cost level the optimal ratio of heat exchanger sizes within the system. Hence the amount of available information is much increased in contrast to comparisons of cycles for a single set of heat exchanger sizes. At the same time the simulation effort is kept low, especially in contrast to evolutionary algorithms which are often used to solve multiobjective optimization problems. An advantage is also that thermodynamic simulation and optimization are strictly separated. This allows changing cost correlations and repeating the computationally inexpensive optimization with a new objective function for costs without having to repeat the time consuming thermophysical simulations. On the other hand different component technologies cannot easily be accounted for. Investigating e.g. the impact of a different compressor technology requires the recalculation of the Pareto front. Here the evolutionary algorithm has the advantage that it makes no distinction between continuous and integer parameters; the optimal solution of cycle layout and component selection is found in a single step.

Independent of the chosen optimization method it is required to keep the component models as simple and generic as possible when comparing cycles and refrigerants which requires to make several simplifications. The assumptions made here for comparing air/water heat pumps are discussed in the following.

In prEN14825 the building capacity demand curve is assumed to be linearly dependent on the outdoor temperature. It is required to fix a design temperature and capacity determining the size of the heat pump. The model as described above allows comparing cycles and refrigerants for a given building demand curve only. For other inclinations of the building demand curve results might differ. Even if upper and lower bounds for the $UA$ values are fixed using the same heat exchanger effectiveness, the problem is not scalable. A sensitivity analysis would be required to quantify differences.

The power consumption of the pump is neglected; since no pressure drop in the condenser is calculated, no corrections are made on $SCOP$ as required in prEN14825. However the water flow rate at equal operating conditions is equal for all cycle layouts, therefore no differences in pumping power exist between them. A possible improvement would hence be to introduce an empirical constant for this consumption and include it in the calculation of the $COP$ of each operating condition. For calculating the power consumption of the fan for each operating condition an optimization procedure is performed using a cubic correlation between power and flow rate. An empirical factor is introduced in this correlation which is kept constant for all heat exchanger sizes. The validity of this assumption and the influence of this factor should be investigated in a sensitivity study.

Both for compressor and fan an ideal variable speed control is assumed. A justification to compare cycles using variable speed fan and compressor is given by the fact that component manufacturers put high effort in the development of this technology so that it is assumed to be state of the art soon. Regarding the capacity control Köpke (2011) shows that a very big part of the demand curve can actually be covered by speed control. Only few operating conditions at the low and high end of the outdoor temperature range require either electric heating or on/off cycling. Solely the latter might influence the $SCOP$ for the warm climate zone with a higher number of operating hours under this condition, hence assuming an ideal control for the whole operating range seems acceptable. The compressor efficiency is assumed to vary with pressure difference which should allow a reasonable comparison between single stage and two stage cycles. However no corrections for varying compressor speed are made.

The influence of varying air flow rates on the air side heat transfer as well as other variations in heat transfer coefficients due to changing mass flow rates or refrigerants is neglected. The reason is that otherwise a detailed definition of the heat exchanger geometry would be required to use correlations for heat transfer coefficients. This would be inconsistent with the aim to keep the comparisons as generic as possible. Here a sensitivity study should investigate the impact of this assumption. A possible concept to improve the model without detailing heat exchanger geometries might be the introduction of figures of merit, proposed by Palm (2008). These numbers quantify generically heat transfer and pressure drop for different refrigerants.
Several other effects which might influence the operation of heat pumps are neglected so far, namely maldistribution in heat exchangers and the frosting and defrosting of the evaporator.

### 3. CONCLUSIONS

A method is proposed to evaluate in an energy efficiency/component cost space the potential of different cycle layouts and refrigerants. The method is applied to air/water heat pumps and exemplary results are presented comparing R410A and R290. The method is supposed to be used as a screening method to compare competing solutions. As such it fulfills several main goals: Computation time is reduced so that a high number of cycle-refrigerant combinations can be simulated. Assumptions are made such that comparisons are as generic as possible without requiring detailed component information. The separation of component costs and system effects as well as the use of basic correlations for all objectives make the evaluation process transparent. The approach is applicable not only for heat pumps but also for other vapor compression applications.

### REFERENCES


