Unstructured nodal DG-FEM solution of high-order Boussinesq-type equations - DTU Orbit (16/10/2019)

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The main objective of the present study has been to develop a numerical model and investigate solution techniques for solving the recently derived high-order Boussinesq equations of \cite{MBL02} in irregular domains in one and two horizontal dimensions. The Boussinesq-type methods are the simplest alternative to solving full three-dimensional wave problems by e.g. Navier-Stokes equations, which can capture all the important wave phenomena such as diffraction, refraction, nonlinear wave-wave interactions and interaction with structures.

The main goal can be reached by using multi-domain methods with support for a spatial discretization based on unstructured grids. In the current work, a standard method of lines approach has been adapted, and the method of choice for the spatial discretization is the nodal Discontinuous Galerkin Finite element method (DG-FEM), which provides a highly flexible basis for the model. This method is combined with an explicit Runge-Kutta method for the temporal discretization. The resulting discrete set of equations enables us to simulate water waves accurately in complex geometric settings and possibly employ local adaption techniques to optimize the computational effort.

%As of today, the high-order Boussinesq equations represent the most advanced set of Boussinesq-type equations capable of modelling nonlinear and dispersive waves from shallow to deep water without the practical limitations of classical Boussinesq-type equations.

The high-order Boussinesq equations constitute a highly complex system of coupled equations which put any numerical method to the test. The main problems that need to be overcome to solve the equations are the treatment of strongly nonlinear convection-type terms and spatially varying coefficient terms; efficient and robust solution of the resultant time-dependent linear system; and the numerical treatment of high-order and cross-differential derivatives. The suggested solution strategy of the current work is based on a collocation approach where the DG-FEM is used to approximate spatial derivatives and the boundary conditions are imposed weakly using a symmetry technique. Since collocation methods are prone to aliasing errors, various anti-aliasing strategies are applied for the stabilization of the models. A practical and relatively straightforward discretization is applied, which is based on a simple treatment of slip boundary conditions at wall surfaces.

A linear Fourier analysis has been applied to obtain generic analytical results which can be used for validating the discrete implementation and provide the basis for choosing stable discretization parameters as well as giving new insight into the properties of the high-order Boussinesq equations. Remarkably, it is demonstrated that the linear eigenspectra of the linearized semi-discrete equation system is bounded and hence the stable time increment is not dictated by the spatial discretization. This is a favorable property for explicit time-integration schemes as the stable time increment is not subject to severe restrictions which can affect the performance of the scheme. It is demonstrated that the discrete properties of both DG-FEM and finite difference methods can be discretized to mimic the analytical properties.

It is investigated mathematically and demonstrated numerically how the relaxation method of \cite{LD83} can be applied in spectral/$hp$ multi-domain methods for both accurate internal wave generation of arbitrary wave fields and efficient absorption near domain boundaries. The method is considered to be particular attractive for wave generation purposes for use with high-order Boussinesq models as it alleviates the need for specifying consistent boundary conditions, and importantly, it is a very straightforward and flexible method.

The DG-FEM models have been applied to a number of tests in both one and two horizontal dimensions with the objective of both validating the setup against known analytical and experimental test results, and at the same time demonstrating the attractive properties of the method. It has been demonstrated that difficult nonlinear and dispersive wave problems can be solved accurately in one horizontal dimension. In two horizontal dimensions it has been demonstrated that the model can solve problems in both regular and irregular geometries and by comparison with analytical results it is shown that the results are in general in excellent agreement.

Thus, it has been established that the DG-FEM can be used to solve this relatively complicated system of equations. The computational efficiency of the method has yet to be demonstrated.

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