Using ab initio calculations we investigate the energy level alignment at the graphene/MoS$_2$ heterostructure and the use of electron doping as a strategy to lower the Schottky barrier and achieve a low-resistance Ohmic contact. For the neutral heterostructure, density functional theory (DFT) with a generalized gradient approximation predicts a Schottky barrier height of 0.18 eV, whereas the G$_0$W$_0$ method increases this value to 0.60 eV. While the DFT band gap of MoS$_2$ does not change when the heterostructure is formed, the G$_0$W$_0$ gap is reduced by 0.30 eV as a result of the enhanced screening by the graphene layer. In contrast to the case of metal substrates, where the band alignment is governed by Pauli repulsion-induced interface dipoles, the graphene/MoS$_2$ heterostructure shows only a negligible interface dipole. As a consequence, the band alignment at the neutral heterostructure is not changed when the two layers are brought into contact. We systematically follow the band alignment as a function of doping concentration and find that the Fermi level of the graphene crosses the MoS$_2$ conduction band at a doping concentration of around $10^{12}$ cm$^{-2}$. The variation of the energy levels with doping concentration is shown to be mainly governed by the electrostatic potential resulting from the doping charge.