Theory of pressure acoustics with viscous boundary layers and streaming in curved elastic cavities

The acoustic fields and streaming in a confined fluid depend strongly on the acoustic boundary layer forming near the wall. The width of this layer is typically much smaller than the bulk length scale set by the geometry or the acoustic wavelength, which makes direct numerical simulations challenging. Based on this separation in length scales, we extend the classical theory of pressure acoustics by deriving a boundary condition for the acoustic pressure that takes boundary-layer effects fully into account. Using the same length-scale separation for the steady second-order streaming, and combining it with time-averaged short-range products of first-order fields, we replace the usual limiting-velocity theory with an analytical slip-velocity condition on the long-range streaming field at the wall. The derived boundary conditions are valid for oscillating cavities of arbitrary shape and wall motion as long as the wall curvature and displacement amplitude are both sufficiently small. Finally, we validate our theory by comparison with direct numerical simulation in two examples of two-dimensional water-filled cavities: The well-studied rectangular cavity with prescribed wall actuation, and the more generic elliptical cavity embedded in an externally actuated rectangular elastic glass block.