The Diagonal Compression Field Method using Circular Fans

In a concrete beam with transverse stirrups the shear forces are carried by inclined compression in the concrete. Along the tensile zone and the compression zone of the beam the transverse components of the inclined compressions are transferred to the stirrups, which are thus subjected to tension. Since the eighties the diagonal compression field method has been used to design transverse shear reinforcement in concrete beams. The method is based on the lower-bound theorem of the theory of plasticity, and it has been adopted in Eurocode 2. The paper presents a new design method, which is a modification of the traditional method, the modification consisting of the introduction of circular fan stress fields. To ensure proper behaviour for the service load the $\psi$-value ($\psi = \cot \theta$, where $\theta$ is the angle relative to the beam axis of the uniaxial concrete compression) chosen should not be too large. As a necessary condition it must be required that the stirrups do not yield for the service load. It has been suggested that for conventional beams $\psi$ has to satisfy the condition $1.0 \leq \psi \leq 2.5$ which is also adopted in Eurocode 2. Fully prestressed beams are designed to have no cracks for the service load; hence there should be no limits in this case (contrary to Eurocode 2). The traditional method does not allow changes of the concrete compression direction throughout a given beam if equilibrium is strictly required. This is conservative, since it is not possible fully to utilize the concrete strength in regions with low shear stresses. The larger inclination (the smaller $\psi$-value) of the uniaxial concrete stress the more transverse shear reinforcement is needed; hence it would be optimal if the $\psi$-value for a given beam could be set to a low value in regions with high shear stresses and thereafter increased in regions with low shear stresses. Thus the shear reinforcement would be reduced and the concrete strength would be utilized in a better way. In the paper it is shown how circular fan stress fields may be used whenever changes in the concrete compression direction are desired. Between two homogeneous stress regions a circular fan stress field may be used to change the inclination of the concrete compression. Unfortunately, the circular fan stress field becomes strongly inhomogeneous if the angle between the two inclined faces limiting the fan becomes large. Thus normally more than one circular fan must be applied. A new design method based on the utilization of circular fans is presented in the paper. It is explained how to determine the optimal subdivision in homogeneous stress regions and circular fans, and thereby how to determine the amount of transverse shear reinforcement, required in a given concrete beam. To illustrate the new design method, a specific example of a prestressed concrete beam is calculated. In the example it is shown, that the traditional method with constant $\psi$-values requires 23% more shear reinforcement than calculated by the new method using circular fan solutions.