Quantum-Atom Optics and Dynamical Simulations of Fermionic Many-Body Systems

Abstract The measurement and characterisation of correlation functions has recently become one of the central themes in the studies of ultracold quantum gases. By probing complex many-body states of these interacting systems, correlation functions facilitate the most in-depth understanding of the underlying physics, not accessible through simple density profile measurements. In this thesis, we study the dynamics of second-order correlation functions for atoms created in pair-production processes such as molecular dissociation and atomic four-wave mixing. More specifically, in Chapter 3 we analyse the dynamics of correlated atom pairs created via controlled dissociation of a Bose-Einstein condensate of diatomic molecules. The atomic constituents can be either a pair of bosonic or fermionic atoms. The process provides a matter-wave analog of parametric down-conversion with photons, which was pivotal in the advancement of quantum optics by enabling experimental demonstrations of the famous Einstein-Podolsky-Rosen paradox and violations of a classical Bell inequality. The atom-optics analogues of parametric down-conversion make it possible to envisage similar demonstrations using ensembles of massive particles in quantum gas regimes. In the case of fermionic constituents, molecular dissociation offers the possibility of a new paradigm in fermionic quantum-atom optics. By aiming at experimentally realistic modelling, we examine the role of spatial inhomogeneity of harmonically trapped molecular condensates on the strength and the shape of atom-atom correlations. Using the undepleted molecular-field approximation, we obtain explicit analytic results for the short-time asymptotic behaviour of the second-order correlation functions and for relative number squeezing between the dissociated atoms in one, two, and three spatial dimensions. Comparisons with the numerical results, which incorporate the molecular depletion and s-wave scattering interactions, show that the analytic approaches employed here capture the main underlying physics and provide useful insights into the dynamics of dissociation for conversion efficiencies up to 10%. The results show explicitly how the strength of atom-atom correlations and relative number squeezing degrade with the reduction of the size of the molecular condensate. In Chapter 4, we study highly anisotropic molecular condensates in order to quantify the effects of bosonic stimulation and fermionic Pauli blocking on the dissociation dynamics. We show that the difference in quantum statistics of the atomic constituent is manifested as complementary geometric structures in the density profiles of the dissociated atoms. Atomic bosons are preferentially emitted along the long axis of the molecular condensate due to bosonic stimulation, while atomic fermions are preferentially emitted along the short axis due to Pauli blocking. This anisotropy potentially simplifies the measurement of correlations between the atoms through relative number squeezing. In Chapter 5 we apply our analytic approaches to a related problem of atomic four-wave mixing and analyze atom-atom correlations in the s-wave scattering halo of two colliding condensates. The results in the short-time limit are in agreement with the first-principles simulations using the positive-P representation and provide analytic insights into the experimental observations of Perrin et al. [Phys. Rev. Lett. 99, 150405 2007]. The final chapter of the thesis, Chapter 6, is devoted to the first application of the Gaussian fermionic phase-space method to a dynamical simulation of a multimode fermion-boson system. As a first-principles approach, the method provides benchmarking of approximate approaches and can be used to validate dynamical probes for characterizing strongly correlated phases of fermionic systems. We have applied the Gaussian phase-space method to the problem of molecular dissociation in the simplest uniform case and demonstrated a successful treatment of large molecular condensates containing up to $10^4$ molecules and $10^3$ distinct atomic modes. Simulating a system in such a large Hilbert space would be impossible using a more traditional exact numerical method based on the number-state representation. Our simulations reveal, in the long-time limit, significant deviations of atom-molecule and molecule-molecule correlations from the predictions of approximate pair mean-field theories, as well as significant departures of atom-atom correlations from Wick's factorisation scheme. We have examined the accuracy and limitations of the fermionic phase-space results by comparison with the number-state calculations for small-size systems and via the analysis of conserved quantities. Future extensions of this work will have to incorporate spatial inhomogeneity and s-wave scattering interactions in order to enable first-principles dynamical simulations of a broader class of fermionic systems of current experimental interest. To this end, we have implemented a phase-space treatment of the Fermi-Hubbard model and present some exploratory simulations of Hubbard dynamics.

Keywords: Fermionic dynamics, Stochastic methods, Correlation functions, Degenerated Fermi gases, Sub-Poissonian statistics and number squeezing, Ultracold atoms and molecules, Quantum many-body theory, Phase-space representation theory, Bose-Einstein condensates, Quantum-atom optics