Pseudospherical surfaces with singularities

We study a generalization of constant Gauss curvature $-1$ surfaces in Euclidean 3-space, based on Lorentzian harmonic maps, that we call pseudospherical frontals. We analyse the singularities of these surfaces, dividing them into those of characteristic and non-characteristic type. We give methods for constructing all non-degenerate singularities of both types, as well as many degenerate singularities. We also give a method for solving the singular geometric Cauchy problem: construct a pseudospherical frontal containing a given regular space curve as a non-degenerate singular curve. The solution is unique for most curves, but for some curves there are infinitely many solutions, and this is encoded in the curvature and torsion of the curve.