The modern economy relies on cheap and reliable transportation of goods all around the globe. Liner shipping networks represent an important link in the global supply chain, as they connect countries and continents over long distances at comparatively low transportation costs. In large liner shipping networks, several hundred container vessels operate more than a hundred shipping routes. The individual routes are linked through ports, where containers can be loaded, unloaded but also be transshipped between shipping routes. The resulting networks constitute inherently complex systems. In this thesis we present mathematical modeling and optimization tools that help decision makers in the liner shipping industry to find solutions to complex decision problems. The decision problems we address involve questions like: Which ports shall be covered by the network? How shall each single shipping route be designed to achieve a well-connected but cost-efficient network? How shall port calls be scheduled and synchronized between shipping routes to offer the most economical and fastest transportation between ports? On which route shall containers be transported, if multiple options exist? The articles in this thesis contribute to the field of Operations Research with application in maritime optimization. More particularly, the first two articles present models and solution methods to (re-)design and (re-)schedule large liner shipping networks. The articles combine and substantially extend modeling features of previous contributions and narrow the gap to the economic and operational reality of liner shipping. The results obtained from solving the models shed light on previously unexamined issues. The developed solution algorithms cannot only handle the increased complexity inherent to the models, but improve over existing methods proposed in the literature. The third article addresses a strategic infrastructure and tanker fleet sizing problem as part of an industrial case study with a large liner shipping company. The case study is motivated by recent changes in environmental regulations that may substantially change the way liner vessels are operated in the future. The article addresses the establishment of a large-scale liquefied natural gas supply chain along a major trade lane. It analyzes the interaction between long-term investment and operational costs, derives basic decision rules and evaluates the robustness of the solutions.