In the current work the reconstruction of the far-field region of the turbulent axi-symmetric jet is performed in order to investigate the modal turbulence kinetic energy production contributions. The reconstruction of the field statistics is based on a semi-analytical Lumley Decomposition (LD) of the PIV sampled field using stretched amplitude decaying Fourier modes (SADFM), derived in Hodžić et al. 2019, along the streamwise coordinate. It is shown that, a wide range of modes obtain a significant amount of energy directly from the mean flow, and are therefore not exclusively dependent on a Richardson-like energy cascade even in the $\kappa$-range in which the energy spectra exhibit the $-5/3$-slope. It is observed that the $-7/3$-range in the cross-spectra is fully reconstructed using a single mode in regions of high mean shear, and that shear-stresses are nearly fully reconstructed using the first two modes. These results indicate that most of the energy production related to shear-stresses is related to the first LD mode.