Instability of vortex pair leapfrogging

Leapfrogging is a periodic solution of the four-vortex problem with two positive and two negative point vortices all of the same absolute circulation arranged as co-axial vortex pairs. The set of co-axial motions can be parameterized by the ratio $0 < \alpha < 1$ of vortex pair sizes at the time when one pair passes through the other. Leapfrogging occurs for $\alpha > \sigma^2$, where $\sigma$ is the silver ratio. The motion is known in full analytical detail since the 1877 thesis of Gröbli and a well known 1894 paper by Love. Acheson ["Instability of vortex leapfrogging," Eur. J. Phys.21, 269-273 (2000)]10.1088/0143-0807/21/3/310 determined by numerical experiments that leapfrogging is linearly unstable for $\sigma^2 < \alpha < 0.382$, but apparently stable for larger $\alpha$. Here we derive a linear system of equations governing small perturbations of the leapfrogging motion. We show that symmetry-breaking perturbations are essentially governed by a 2D linear system with time-periodic coefficients and perform a Floquet analysis. We find transition from linearly unstable to stable leapfrogging at $\alpha = \varphi^2 \approx 0.381966$, where $\varphi$ is the golden ratio. Acheson also suggested that there was a sharp transition between a "disintegration" instability mode, where two pairs fly off to infinity, and a "walkabout" mode, where the vortices depart from leapfrogging but still remain within a finite distance of one another. We show numerically that this transition is more gradual, a result that we relate to earlier investigations of chaotic scattering of vortex pairs [L. Tophøj and H. Aref, "Chaotic scattering of two identical point vortex pairs revisited," Phys. Fluids20, 093605 (2008)]10.1063/1.2974830. Both leapfrogging and "walkabout" motions can appear as intermediate states in chaotic scattering at the same values of linear impulse and energy.

General information
Publication status: Published
Organisations: Department of Physics, Biophysics and Fluids, Virginia Tech
Contributors: Tophøj, L., Aref, H.
Pages: 014107
Publication date: 2013
Peer-reviewed: Yes

Publication information
Journal: Physics of Fluids
Volume: 25
Issue number: 1
ISSN (Print): 1070-6631
Ratings:
BFI (2013): BFI-level 1
Scopus rating (2013): SJR 1.419 SNIP 1.47
Web of Science (2013): Impact factor 2.04
ISI indexed (2013): ISI indexed yes
Web of Science (2013): Indexed yes
Original language: English
Keywords: Linear systems, Scattering, Vortex flow
Electronic versions:
DOIs:
10.1063/1.4774333

Bibliographical note
© 2013 American Institute of Physics
Source: dtu
Source ID: n:oai:DTIC-ART:compendex/380048507::26324
Research output: Contribution to journal › Journal article – Annual report year: 2013 › Research › peer-review