Distortional buckling modes of semi-discretized thin-walled columns

This paper presents distortional buckling solutions for semi-discretized thin-walled columns using the coupled differential equations of a generalized beam theory (GBT). In two related papers recently published by the authors a novel semi-discretization approach to GBT has been presented. The cross section is discretized and analytical solutions are sought for the variation along the beam. With this new approach the general GBT equations for identification of a full set of deformation modes corresponding to both homogeneous and non-homogeneous equations are formulated and solved. Thereby giving the (complex) deformation modes of GBT which decouple the state space equations corresponding to the reduced order differential equations. In this paper the developed semi-discretization approach to generalized beam theory (GBT) is extended to include the geometrical stiffness terms, which are needed for column buckling analysis and identification of buckling modes. The extension is based on an initial stress approach by addition of the related potential energy terms. The potential energy of a single deformation mode is formulated based on a discretization of the cross section. Through variations in the potential energy and the introduction of the constraints related to beam theory this leads to a modified set of coupled homogeneous differential equations of GBT with initial stress for identification of distortional displacement modes. In this paper we seek instability solutions using these GBT initial stress equations for simply supported columns with constrained transverse displacements at the end sections and a constant axial initial stress. Based on the known boundary conditions the reduced order differential equations are solved by using the trigonometric solution functions and solving the related eigenvalue problem. This gives the buckling mode shapes and the associated eigenvalues corresponding to the bifurcation load factors. Thus the buckling modes are found directly by the analytical solution of the coupled GBT-equations without modal decomposition. Illustrative examples showing global column buckling, distortional buckling and local buckling are given and it is shown how the novel approach may be used to develop signature curves and elastic buckling curves. In order to assess the accuracy of the method some of the results are compared to results found using the commercial FE program Abaqus as well as the conventional GBT and FSM methods using the software packages GBTUL and CUFSM.