The primary catalyst of this PhD project has been an ambition to design advanced materials and structural systems including, and possibly even exploiting, nonlinear phenomena such as nonlinear modal interaction leading to energy conversion between modes. An important prerequisite for efficient design is accurate and somewhat simple analysis tools, as well as a fundamental understanding of the physical phenomena responsible for the relevant effects. The emphasis of this work lies primarily in the investigation of various advanced material models, developing the necessary analytical tools to reveal the fundamental dynamic characteristics and thus the relevant design parameters. The thesis is built around the characterization of two one-dimensional, periodic material systems. The first is a nonlinear mass-spring chain with periodically varying material properties, representing a simple but general model of inhomogeneous structural materials with nonlinear material characteristics. The second material system is an “engineered” material in the sense that a classical structural element, a linear elastic and homogeneous rod, is “enhanced” by applying a mechanism on its surface, amplifying the inertia of the system, thus creating a new “material” with improved properties for wave attenuation. Both materials are investigated by Floquet-Bloch analysis, a powerful tool for determining the dynamic material characteristics of periodic materials. An asymptotic approach is utilized for the direct application of the Floquet-Bloch analysis on the nonlinear chain. The wave-characteristics are determined, exploring the effect of nonlinear modal interaction on the band structure as well as the potential for energy conversion through higher harmonic generation. While modal interaction definitely affects the band structure, the potential for energy conversion appears to be limited. The material with amplified inertia however, shows great promise for low-frequency and broadband wave attenuation, requiring a significantly lower mass than what is needed in comparable systems to obtain a similar effect. This is demonstrated both analytically, numerically and experimentally. This makes the material a strong candidate for mechanical filtering for sound and vibration isolation purposes, not least for systems with varying natural frequencies such as nonlinear structures or structures with variable mass due to, e.g., human loading.