Container vessel stowage planning is a hard combinatorial optimization problem with both high economic and environmental impact. We have developed an approach that often is able to generate near-optimal plans for large container vessels within a few minutes. It decomposes the problem into a master planning phase that distributes the containers to bay sections and a slot planning phase that assigns containers of each bay section to slots. In this paper, we focus on the slot planning phase of this approach and present a Constraint Programming and Integer Programming model for stowing a set of containers in a single bay section. This so-called slot planning problem is NP-hard and often involves stowing several hundred containers. Using state-of-the-art constraint solvers and modeling techniques, however, we were able to solve 90% of 236 real instances from our industrial collaborator to optimality within 1 second. Thus, somewhat to our surprise, it is possible to solve most of these problems optimally within the time required for practical application.