Available fluid codes for turbulence study

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Available fluid codes for turbulence study

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Periodic codes (spectral)

- Probably the simplest codes to make
- Easy and fast to develop, a master study
- Easy, fast and compact to run, a bachelor study
- Can use fairly high number of modes on a single CPU: 2048x2048
- Can reach fairly high Reynolds numbers: \( \text{Re}(2D) = \frac{UL}{\mu} < 20.000 \)
- Can fairly easy be parallelized using MPI: linear speedup using 100 CPUs on a 2048x2048 grid
- Note that the domain is infinite with a periodic restriction!
Periodic codes (spectral)

- Solutions are expanded into Fourier modes (global)

\[
\begin{pmatrix}
\omega(x, y, t) \\
\psi(x, y, t)
\end{pmatrix} = \sum_{m} \sum_{n} \begin{pmatrix}
\omega_{mn}(t) \\
\psi_{mn}(t)
\end{pmatrix} \exp \left( \frac{2\pi imx}{L_x} \right) \exp \left( \frac{2\pi iny}{L_y} \right)
\]

- Vorticity equation

\[
\frac{\partial \omega}{\partial t} + J(\psi, \omega) = \nu \nabla^2 \omega \Rightarrow \forall (mn) : \frac{\partial \omega_{mn}}{\partial t} + \left[ \psi, \omega \right]_{mn} = \nu \nabla^2 \omega_{mn}
\]

\[
\left[ \psi, \omega \right]_{mn} = FT \left\{ \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} \right\}_{mn}
\]

- Fast Fourier Transformation will take 75% of computational time
- De-aliasing scheme, zero pad the largest 1/3 of the modes
- The Poisson equation is trivial: \( k^2 \psi_k = \omega_k \)
The Risøs Euler code

\[ E(k) = \sum_{m,n} u_{mn}^2(t) + v_{mn}^2(t) \]

Inverse cascade :

\[ \Omega_k = k^2 E_k \Rightarrow \begin{cases} E \rightarrow \text{small } k \\ \Omega \rightarrow \text{large } k \end{cases} \]

- 1024x1024 points
- 512x512 modes
- de-aliased removes upper 1/3
- \( K_{\text{max}} = 340 \)
- Taken account for viscosity leave us with approximately 2 decades!
Solid boundaries

Poisson equation
\[ \nabla^2 \phi = \omega \Rightarrow \]
\[ \forall k : \frac{\partial^2 \phi_k}{\partial x^2} - k^2 \phi_k = \omega_k \Rightarrow \]
\[ \forall k : A_k \phi_k = \omega_k + BC \]

- Finite difference in x, Fourier expansion in y
- Multiplication simple, derivatives complex

\[
\frac{\partial f_i}{\partial x} \frac{f_{i+1} \Delta f_{i-1}}{2dx} \\
\frac{\partial^2 f}{\partial x^2} \frac{f_{i+1} - 2f_i + f_{i-1}}{(\Delta x)^2}
\]

Banded matrix, solved by Gauss elimination

\[
A_k = \frac{1}{\Delta x^2} \begin{pmatrix}
-1 & 1 & & & \\
1 & -2 & 1 & & \\
& 1 & -2 & 1 & \\
& & & \ddots & \\
& & & 1 & -2 & 1 \\
& & & & 1 & -2 & 1 \\
& & & & & 1 & -1
\end{pmatrix}
\]
Diffusion equation
\[
\frac{\partial \omega}{\partial t} = \nabla \cdot (D(\vec{x},t)\nabla \omega) + \cdots \Rightarrow \\
(1 - \Delta t \nabla \cdot (D(\vec{x},t)\nabla))\omega(t + \Delta t) = \omega(t) + \cdots
\]
Generally gives a complicated matrix, has to be solved by iteration (Petsc)

\[D = D_0: \quad \text{Helmholtz equation}\]
\[(1 - \Delta t D_0 \nabla^2)\omega(t + \Delta t) = \omega(t)\]
banded matrix, Gauss elimination possible
Finite difference, ESEL

- Global model with self-consistent profiles
- Simulation domain include both edge, SOL and limiter shadow regions
- 2D approximation; parallel loss mechanism modeled by a parameterize loss term
- Input; basic plasma parameters
- Test on TCV, JET, ASDEX with reasonable results
- Collisional diffusion coefficients and parallel loss terms from first principal
- It is a very simple 2D model!
Finite difference, ESEL

Interchange model

\[
\frac{dn}{dt} + n\frac{\partial}{\partial t}(\phi) - \frac{\partial}{\partial x}(Tn) = \Lambda_n
\]

\[
\frac{dT}{dt} + \frac{2}{3} T\frac{\partial}{\partial t}(\phi) - \frac{7}{3} T\frac{\partial}{\partial x}(T) + \frac{2}{3} n\frac{\partial}{\partial x}(n) = \Lambda_T
\]

\[
\frac{d\omega}{dt} + \omega = \Lambda_\omega
\]

\[
\left(\frac{B}{B} = \frac{\partial}{\partial t} + \frac{1}{B} b \times \nabla \phi \cdot \nabla
\right)
\]

\[
\frac{1}{B} = 1 + \frac{r_0 + \rho_s \chi}{R_0}
\]

\[
\Delta (B, f) = \frac{\partial B}{\partial x} \frac{\partial f}{\partial y} - \frac{\partial B}{\partial y} \frac{\partial f}{\partial x}
\]

\[
\Lambda_\alpha = -\frac{\alpha}{\tau_{\alpha}} + D_{\perp,\alpha} \nabla^2 \alpha
\]

Subsonic advection:

\[
\tau_{\alpha,n} \tau_{\alpha,o} \frac{M_e c_s}{L_p}, M_\alpha = 0.5
\]

Spitzer-Harm diffusion:

\[
\tau_{\alpha,T} \frac{3L_p^2}{2\chi_e}
\]

Collisional SOL limit

\[
10 < \nu_e^* = \frac{L}{\lambda_e} < 80
\]
TCV-ESEL Comparison

Conditionally averaged density blob structure

PDF of particle density flux

Direct comparison with experimental results from the TCV-Tokamak, Lausanne: excellent quantitative agreement

TCV-ESEL Comparison

Density profile and relative fluctuations

Particle flux profiles

Good agreement between experiment and turbulence simulations

Passive particles

\[ x(t) = x(0) + \int_0^t v(x, t) dt \]

Summer student S. Boudaux, (2005)
Solid boundaries - Disk

- Radial points are cosine distributed
- Chebyshev polynomials calculated via cosine transformation
- The Poisson equation decouples in $\theta$, a series of banded 1D problem to be solved in $r$
  - $r=0$ should be a regular point
  - As $r \to 0$ the grid spacing in $\theta$ decreases: $1/(2\pi r)$
  - Even thought these scales are well below the viscosity scale they are extremely unstable and have to be removed manually (zero pad).

\[ \omega(r, \theta, t) \]
\[ \bar{u}(r, \theta, t) = \sum_m \sum_n \left( \omega_{mn}(t) \right) T_m(r) \exp(-in\theta) \]
\[ \psi(r, \theta, t) = \sum_m \sum_n \left( \psi_{mn}(t) \right) T_m(r) \exp(-in\theta) \]
\[ r \in [-1 : 1], \theta \in [0 : 2\pi] \]

\[ T_n(x) = \cos(n \cos^{-1}(x)) = \cos(nx) \]
\[ T_0(x) = 1, \quad T_i(x) = x, \quad T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \]
\[ T_n(-1) = (-1)^n, \quad T_n(1) = 1 \]

Poisson equation
\[ \nabla^2 \phi = \omega \Rightarrow \]
\[ \forall k: \frac{\partial^2 \phi_k}{\partial r^2} + r \frac{\partial \phi_k}{\partial r} - k^2 \phi_k = r^2 \omega_k \Rightarrow \]
\[ \forall k: A_k \phi_k = \omega_k + BC \]

Energy Spectrym: \[ E(r, n) = \frac{r}{2} \sum_n u_n^2(r, t) + v_n^2(r, t) \]
Spectral versus finite difference

Fig. 5.4. Time evolution of the vorticity field for the interaction of a Lamb-dipole with a no-slip wall. The spectral scheme has been used with $M = N = 1024$ and $Re = 2,000$. Notice that only a part of the computational domain is displayed.

Spectral versus finite difference

Solve the vorticity equation with solid boundaries in annulus geometry

\[ \frac{\partial \omega}{\partial t} + [\omega, \psi] = v \nabla^2 \omega, \quad \nabla^2 \psi = -\omega, \quad \bar{u} \big|_{\partial D} = 0 \]

We used a spectral code (Chebyshev-Fourier expansion) and finite difference code (cosine distributed radial points). A Lamb dipole

\[ \omega = \begin{cases} \frac{2\lambda U}{J_0(\lambda R)} J_1(\lambda r) \cos(\theta) & , r \leq R \\ 0 & , r > R \end{cases} \]

was used as initial condition and let it interact with the outer wall for different Reynolds numbers, Re=UL/\nu.

Conclusion:
• Spectral schemes are more accurate than FD using the same resolution BUT
• Using the same computer power we can obtain similar results for the two different schemes

DIESEL

• Global version of “ESEL”
• Covers the full toroidal domain

\[
\frac{\partial n^i}{\partial t} + \{\phi, n^i\} = \mu_n \nabla_\perp^2 n^i + c_s \nabla_\perp n
\]

\[
\frac{\partial \omega^i}{\partial t} + \{\omega^i, \omega^i\} = \left\{ \frac{1}{B}, n^i \right\} + \mu_\omega \nabla_\perp^2 \omega^i + V_A \nabla \omega
\]

\[
\omega = \frac{1}{B} \nabla_\perp^2 \phi \quad , \quad B(r, \theta) = \frac{B_0}{1 + \frac{x}{R} \cos \theta}
\]

\[
c_s \nabla_\perp n^i = \begin{cases}
\frac{c_s}{L} (n^{i+1}(r, \theta + \Delta \theta) - n^i(r, \theta)) & \text{if } n^i \geq n^{i+1} \\
\frac{c_s}{L} (n^{i-1}(r, \theta - \Delta \theta) - n^i(r, \theta)) & \text{otherwise}
\end{cases}
\]

\[
c_s \nabla_\perp \omega^i = \begin{cases}
\frac{c_s}{L} (\omega^{i+1}(r, \theta + \Delta \theta) - \omega^i(r, \theta)) & \text{if } |\omega^i| \geq |\omega^{i+1}| \\
\frac{c_s}{L} (\omega^{i-1}(r, \theta - \Delta \theta) - \omega^i(r, \theta)) & \text{otherwise}
\end{cases}
\]

\[
\Delta \theta = \frac{2\pi}{q n_{\text{drift}}}
\]

□ Normalisation$[4]$, space and time $x \rightarrow \frac{x}{a}, t \rightarrow \gamma t$

□ Interchange growth rate: $\gamma = \sqrt{\frac{2}{aR} c_s}$

□ $c_s$ in normalized unites: $c_s \rightarrow \frac{R}{\sqrt{2a}}$
DIESEL

- Global model using full toroidal geometry on closed magnetic field lines
- Model, at present, based on a simple interchange model, see e.g. [1,2]
- $n_{\text{drift}}$ 2-D drift planes each covering the full cross section of the torus
- In the above equations $i \in [1; n_{\text{drift}}]$ and denotes the particular drift plane
- Parallel numerical code, based on spectral expansion of the solutions
- Scale linearly at least up to 100 CPU using 1024x2048 pr. drift plane
- Estimated maximum number of CPU is above 1,000!? (to be tested)
- The drift planes are separated toroidally by $L_i = \frac{2\pi R}{n_{\text{drift}}}$
- Parallel velocities are parameterized using $c_s$ and $V_A$
- $q$ enters in the two parallel terms: