An Equivalent Source Method for Modelling the Global Lithospheric Magnetic Field

Kother, Livia Kathleen; Hammer, Magnus Danel; Finlay, Chris; Olsen, Nils

Publication date: 2014

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
An Equivalent Source Method for Modelling the Global Lithospheric Magnetic Field

Livia Kother¹, Magnus D. Hammer², Christopher C. Finlay¹, Nils Olsen¹

¹ Division of Geomagnetism, DTU Space, Technical University of Denmark
² Copenhagen, Denmark

Summary
We produce a new model of the global lithospheric magnetic field based on 3-component vector field observations at all latitudes from the CHAMP satellite using an equivalent source technique.

Method
A regularized iteratively reweighted least squares algorithm is applied. Data error covariance matrices are implemented, including both the latitude dependence of data error variances $\sigma^2$ (Fig.1) and covariances $\mathbf{C}$ between the vector field components due to unmodelled sources. The regularization norm $\mathbf{R}$ is defined to be the Euclidean length of the model solution. Our scheme iteratively minimizes:

$\Theta(\mathbf{m}) = \mathbf{d} - \mathbf{Gm} + \lambda \mathbf{R}(\mathbf{m})$

$\mathbf{W}_k = \mathbf{C}^{-1/2} \mathbf{H}_k \mathbf{C}^{-1/2}$

Huber weighting ensures a robust solution in the presence of non-Gaussian data errors

$\mathbf{H}_{k+1} = \min_{\lambda} \left( \lambda + \frac{1.5}{\text{dev}} \right)$

An initial unregularized ($\lambda = 0$) model is derived using 10 iterations. The final model is obtained with 5 further iterations using quadratic regularization and $\lambda = 3E-13$.

Equivalent Source Method
The equivalent potential field sources $\mathbf{m}$ (monopoles) are arranged in an icosahedron grid (Fig.2), consisting of $K = 30722$ vertices and midpoints, placed at a depth of 100km below the Earth’s surface. The derived model can be transformed into a spherical harmonic representation by:

$g_l^m = \sum_{k=1}^{K} m_k \mathbf{P}_l^m(\cos \theta_k) \cos (\phi_k)$

$h_l^m = \sum_{k=1}^{K} m_k \mathbf{P}_l^m(\cos \theta_k) \sin (\phi_k)$

Results and Outlook
The presented model has a power spectrum that compares well to CHAOS-4, MF7 and CMS (cf. Poster EGU2014-6883) models to degree $n = 100$ (Fig.4). Ongoing investigations concern non-quadratic regularization using maximum entropy. Looking forward, we plan to explore local grid refinement options in order to incorporate aeromagnetic survey data.

Fig 3: Left: Final model degree correlation with CHAOS-4, MF7, GRIM-L120 and CMS. Right: Sensitivity matrix between final model and CHAOS-4. The scale saturates at $100 \text{nT}$.

Fig 4: Left: Modelled lithospheric radial magnetic field at the Earth’s surface for degree $n = 16$-180. The scale saturates at $200 \text{nT}$. Right: Power spectrum for MF7, CMS and CHAOS-4 models in comparison to model results with different regularization damping values. The chosen model is represented by the red line.