



## Design of MgB<sub>2</sub> superconducting dipole magnet for particle beam transport in accelerators

Abrahamsen, Asger Bech; Zangenberg, N.; Baurichter, A.; Grivel, Jean-Claude; Andersen, Niels Hessel

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Risø-R-1578(EN)

# Design of MgB<sub>2</sub> superconducting dipole magnet for particle beam transport in accelerators

A.B. Abrahamsen, N. Zangenberg, A. Baurichter, J-C. Grivel  
and N.H. Andersen

**Author:** A.B. Abrahamsen<sup>1</sup>, N. Zangenberg<sup>2</sup>, A. Baurichter<sup>2</sup>, J-C. Grivel<sup>1</sup> and N.H. Andersen<sup>1</sup>

**Title:** Design of MgB<sub>2</sub> superconducting dipole magnet for particle beam transport in accelerators

**Department:** Materials Research Department

<sup>1</sup>Materials Research Department, Risø

<sup>2</sup>Danfysik A/S, Møllehaven 31, Jyllinge, Denmark

**Abstract (max. 2000 char.):**

A comprehensive analysis of the innovation potential of superconductivity at Risø was performed in February 2004 by the main author of this report [1]. Several suggestions for new products and new markets were formulated by the superconductivity group and examined by the innovation staff at Risø. The existing markets of superconducting technology is within highly specialized scientific areas such as magnetic confinement in fusion energy, sample environment in neutron scattering and large scale accelerators such as the Large Hadron Collider(LHC) at Cern, or in the nuclear magnetic resonance (NMR) community using MR-imaging scanners in medicine and phase identification in organic chemistry. Only the NMR applications can be categorized as a highly profitable and commercial market today. The superconductivity group of Risø formulated and presented the gearless superconducting wind turbine multipole generator as the most promising new concept [2], but further initiatives were stopped due to unclear patent possibilities. The experience of the innovation review was used in the STVF framework program "New superconductors: mechanisms, processes and products"[3] to identify potential new product for the collaborating company Danfysik A/S, which has a strong tradition in building resistive magnets for particle accelerators[4]. A technology transfer project was formulated at the end of 2005 with the purpose to collect the knowledge about the MgB<sub>2</sub> superconductor gained in the STVF program and in the European Framework Program 6 project HIPERMAG[5]. It was presented at the Risø innovation seminar January 2006, and recently a collaboration between Risø and Danfysik A/S was initialized. The present report aims to outline a potential superconducting product within the STVF program.

The use of the MgB<sub>2</sub> superconductors in a dipole magnet for guiding particle beams in a small scale accelerator is examined with the purpose to build lighter and smaller than the present resistive magnets. Here the critical current density of primarily MgB<sub>2</sub> will be compared with current density determined by specifications similar to the Tevatron accelerator,  $B = 4.4$  Tesla and coil aperture  $D = 76$  mm [6], which has been identified by Danfysik A/S as interesting.

It is concluded that MgB<sub>2</sub> is useful for the dipole application and construction of a small test coil of one half of the magnet is planned in 2007.

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Risø National Laboratory  
Information Service Department  
P.O.Box 49  
DK-4000 Roskilde  
Denmark  
Telephone +45 46774004  
[bibl@risoe.dk](mailto:bibl@risoe.dk)  
Fax +45 46774013  
[www.risoe.dk](http://www.risoe.dk)

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# 1 Dipole magnets for small scale particle beam accelerators

An increasing number of small scale accelerators are being produced and a range of interesting field and homogeneity limits of conventional resistive dipole magnets for particle beam transport were identified in a collaboration with Danfysik A/S[4].

The question is if the introduction of superconductor wire in the dipoles will make it possible to reach field strengths, which are considerably higher than the saturation field of soft iron used in normal resistive magnets. To clarify this a calculation of the needed current density of the superconductor in the coil will be performed and compared to the present performance of the  $MgB_2$  superconducting wires or tapes and other commercially available superconductors.

As a starting point we will analyze the specifications of the Nb-Ti based dipoles of the Tevatron accelerator build at Fermilab in 1983 [6]:

- Field strength :  $B_{max} = 4.4$  Tesla
- Diameter of homogenous field :  $D_{homogen} = 0.076$  m

## 2 $\cos(\theta)$ dipole geometry

A homogenous dipole field can be generated in an infinitely long circular tube by realizing a current distribution at the surface given by a cosine function [6]:

$$I = I_0 \cos(\theta) \quad (1)$$

where  $I_0$  is the surface current at  $\theta = 0$  as indicated by the blue arrows on figure 1. The total current is  $I_{tot} = 2I_0$ .

This causes a homogenous analytical magnetic field in the interior of the tube

$$B(\theta) = \frac{\mu_0 I_0}{2r_0} \cos(\theta) \quad (2)$$

which in the xy coordinate system becomes

$$B_x = 0 \quad (3)$$

$$B_y = \frac{\mu_0 I_0}{2r_0} \quad (4)$$

where the vacuum permeability  $\mu_0 = 4\pi \cdot 10^{-7} NA^{-2}$  and  $r_0$  is the diameter of current tube.

### 2.1 Current calculation of $\cos \theta$ geometry

By inserting numbers one can calculate the maximum current  $I_0$  at the surface needed to obtain the target field:

$$\begin{aligned} I_0 &= \frac{2r_0}{\mu_0} B \\ &= \frac{D_{homogen}}{\mu_0} B_{max} \\ &= \frac{0.076m}{4\pi \cdot 10^{-7} NA^{-2}} 4.4 NA^{-1} m^{-1} \\ &= 2.66 \cdot 10^5 A \end{aligned} \quad (5)$$

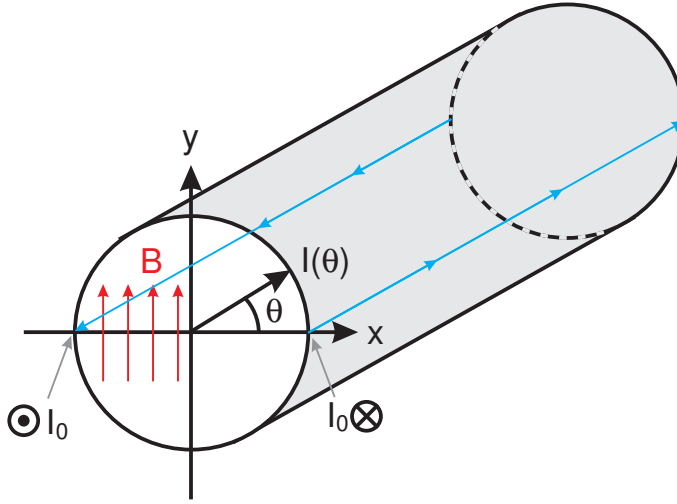


Figure 1. A homogenous dipole magnetic field  $B$  can be created inside a tube by realizing a current distribution at the surface given as  $I(\theta) = I_0 \cos(\theta)$ . The blue arrows indicates the path of  $I_0$  at both sides of the tube.

Note that  $[B] = \text{Tesla} = \text{NA}^{-1}\text{m}^{-1}$  and the the vacuum permeability is given by  $\mu_0 = 4\pi \cdot 10^{-7} \text{NA}^{-2}$ . The above formula does not take into account the dimensions of the current carrying region and one might violate the assumption that  $I_0$  is distributed as a surface current. Thus a more realistic description is needed.

### 3 Intersecting ellipses geometry

An alternative analytical solution that creates a homogenous field inside the beam tube is realized by intersecting two elliptically shaped conductors as illustrated on figure 2 [6, 7, 8]. The field distribution of one ellipse is:

$$\begin{aligned} \mathbf{B} &= B_x \hat{x} + B_y \hat{y} \\ &= -\frac{\mu_0 J}{a+b} a y \hat{x} + \frac{\mu_0 J}{a+b} b x \hat{y} \end{aligned} \quad (6)$$

where  $a$  and  $b$  are the major and minor axis of the ellipse centered at the origin,  $\mu_0$  is the vacuum permeability,  $J$  is the current density pointing out of the paper,  $x$  and  $y$  are the coordinate inside the conductor and  $\hat{x}$  and  $\hat{y}$  are unit vectors of the coordinate system. One can confirm the known result for a circular conductor by setting  $a = b$ .

The field distribution of two intersecting ellipses can be calculated from vector addition of equation 6 giving :

$$\begin{aligned} \mathbf{B} &= \mathbf{B}_{\text{left}} + \mathbf{B}_{\text{right}} \\ &= -\frac{\mu_0 J}{a+b} a y \hat{x} + \frac{\mu_0 J}{a+b} b(x+s) \hat{y} \\ &\quad + \left( -\frac{\mu_0(-J)}{a+b} a y \hat{x} + \frac{\mu_0(-J)}{a+b} b(x-s) \hat{y} \right) \\ &= 0 \hat{x} + \mu_0 J \frac{2bs}{a+b} \hat{y} \end{aligned} \quad (7)$$

where  $s$  denotes the center position at  $x = \pm s$  of the two ellipses and all other parameters are defined as above.

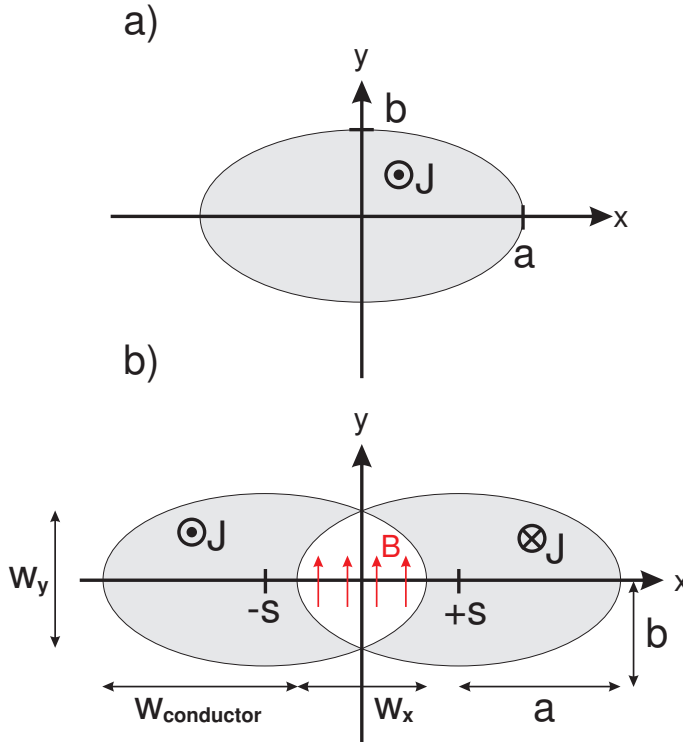


Figure 2. a) A homogenous current density  $J$  inside an elliptical conductor with major and minor axis given by  $a$  and  $b$  causes a field distribution given by equation 6. b) A homogenous dipole field  $B$  along the  $y$ -axis is created in between two intersecting elliptical conductors separated by  $\pm s$  and with opposite current direction. The opening along the  $x$ - and  $y$ -axis inside the dipole are denoted  $w_x$  and  $w_y$  and the thickness of the conductor region along the  $x$ -axis is  $w_{conductor}$ .

## 4 Current densities of superconductor in dipole magnet

In order to calculate the current density needed to fulfil the field strength one should first work out the approximative dimensions. The opening inside the two ellipses along the  $x$ -axis is given by

$$\begin{aligned} w_x &= -s + a - (s - a) \\ &= 2(a - s) \end{aligned} \quad (8)$$

The opening  $w_y$  along the  $y$ -axis is most easily found from the parameter description of an ellipse

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 = 1 \quad (9)$$

where the center of the ellipse is at  $(x_0, y_0)$  and the major and minor axis are  $a$  and  $b$ .  $w_y$  is found by inserting  $(x_0, y_0) = (s, 0)$  and  $x = 0$ , which gives

$$w_y = 2y = 2b\sqrt{1 - \left(\frac{s}{a}\right)^2} \quad (10)$$

The conductor thickness along the x-axis is given by

$$\begin{aligned} w_{conductor} &= s + a - (-s + a) \\ &= 2s \end{aligned} \quad (11)$$

which is just the distance between the two ellipse centers.

## 4.1 Circular cross section example

A circular cross section will be assumed in the following example to simplify the calculations,  $a = b$ . By making the following assumption:

- $w_x = 0.076$  m
- $a = 0.0475$  m

we can calculate the displacement of the circle

$$s = \frac{2a - w_x}{2} = \frac{0.095m - 0.076m}{2} = 0.0095m \quad (12)$$

and the opening along the y-axis

$$w_y = 2b\sqrt{1 - \left(\frac{s}{a}\right)^2} = 2 \cdot 0.0475m\sqrt{1 - \left(\frac{0.0095m}{0.0475m}\right)^2} = 0.093m \quad (13)$$

The conductor thickness along the x-axis becomes

$$w_{conductor} = 2s = 2 \cdot 0.0095m = 0.019m \quad (14)$$

Now the current density can easily be calculated by inverting equation 7

$$\begin{aligned} B &= \mu_0 J \frac{2a}{2a} s \implies \\ J &= \frac{B}{\mu_0 s} \\ &= \frac{4.4 \text{ Tesla}}{4\pi \cdot 10^{-7} \text{ NA}^{-2} \cdot 0.0095m} \\ &= 3.7 \cdot 10^4 \text{ Acm}^{-2} \end{aligned} \quad (15)$$

The example calculation above is illustrated on figure 3.

# 5 Current densities of commercial superconductors

In the following the critical current density of various commercial superconductors are presented. It should be noted some of the superconductors such as the  $MgB_2$  and coated conductors are still not developed to fulfil their full potential and both price and availability must be examined in more detail.



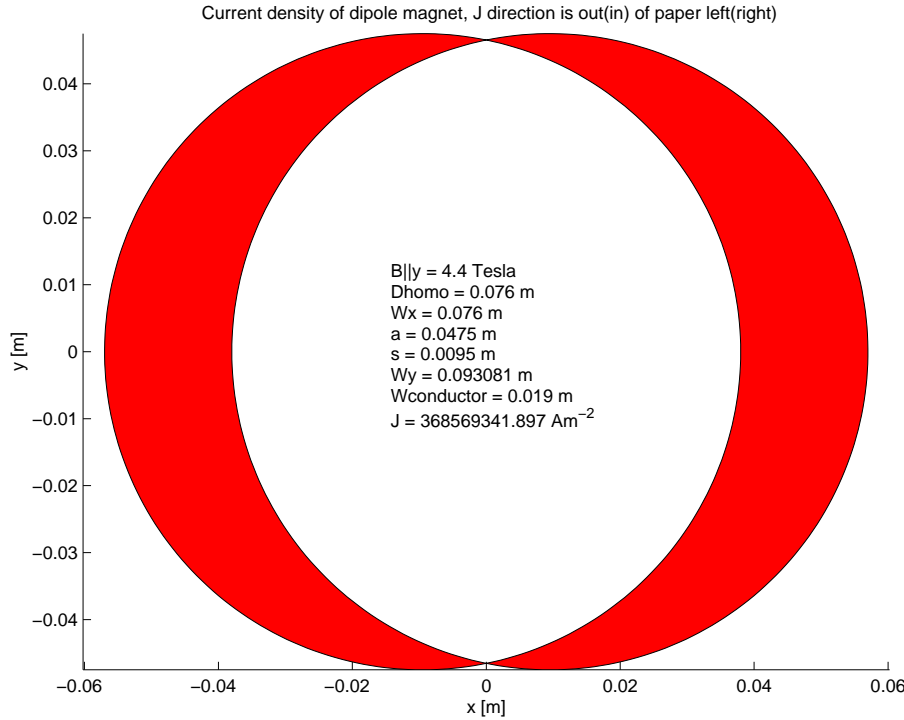


Figure 3. Example of intersecting circle geometry to create  $B = 4.4$  Tesla by having a current density of  $J = 3.7 \cdot 10^4 \text{ Acm}^{-2}$  flowing in the colored regions. The circles have a radius of  $a = 0.0475 \text{ m}$ . Given the opening along the  $x$ -axis  $w_x = 0.076 \text{ m}$  one can determine the center displacement of the circles to  $s = 0.0095 \text{ m}$  and  $w_y = 0.093 \text{ m}$ . The thickness of the conductor region becomes  $W_{\text{conductor}} = 0.019 \text{ m}$ . The unit of the figure scale is  $\text{m}$ .

## 5.1 Magnesium diboride $MgB_2$

Figure 4 shows a plot of the critical current density  $J_c$  of the  $MgB_2$  superconductor as function of field. The superconductor is produced by the company Columbus Superconductors and the values corresponding to step 3 will be implemented at the end of the year 2006 [9].  $J_c$  is expected to reach  $J_c = 1.5 \cdot 10^5 \text{ Acm}^{-2}$  at  $T = 4.2 \text{ K}$  and  $B = 4 \text{ Tesla}$ . Thus the critical current density of the  $MgB_2$  wire exceeds the target current density by approximately a factor of 4 at  $B = 4 \text{ Tesla}$  and this indicate that  $MgB_2$  might be quite useful for the dipole application. However the most important information is the engineering critical current density  $J_e$ , which includes the non superconducting matrix around the  $MgB_2$  in the calculation of the critical current density. According to G. Grasso then Columbus will be able to alter the geometry of the wires to obtain the needed  $J_e$  [9]. Figure 5 shows a cross section of a prototype multifilament wire with a filling factor of  $f = 30\%$  and the engineering critical current density is expected to be

$$\begin{aligned}
 J_e &= J_c \cdot f \\
 &= 1.5 \cdot 10^5 \text{ Acm}^{-2} \cdot 0.3 \\
 &= 4.5 \cdot 10^4 \text{ Acm}^{-2}
 \end{aligned} \tag{16}$$

whereby the target current density will be at 82 % of the limit of the wire at  $T = 4.2 \text{ K}$  and  $B = 4.4 \text{ Tesla}$ . Usually the loadline is positioned at 90 % of the critical current density. A reduction of 30 % of the critical current density is expected if the operation temperature is increased to  $T = 10 \text{ K}$  instead of  $T = 4.2 \text{ K}$ .

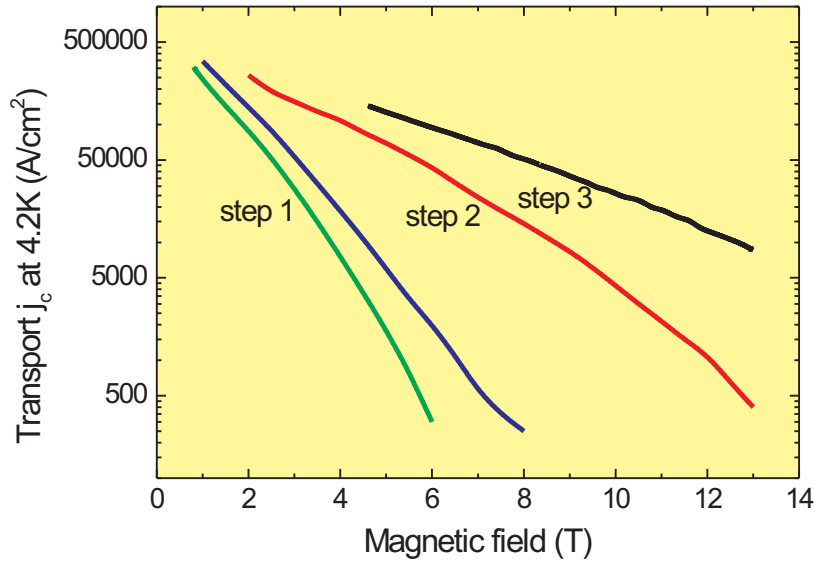


Figure 4. Critical current density of  $MgB_2$  tapes from Columbus Superconductors. Step 3 values will be implemented in the production line at the end of 2006 [9].

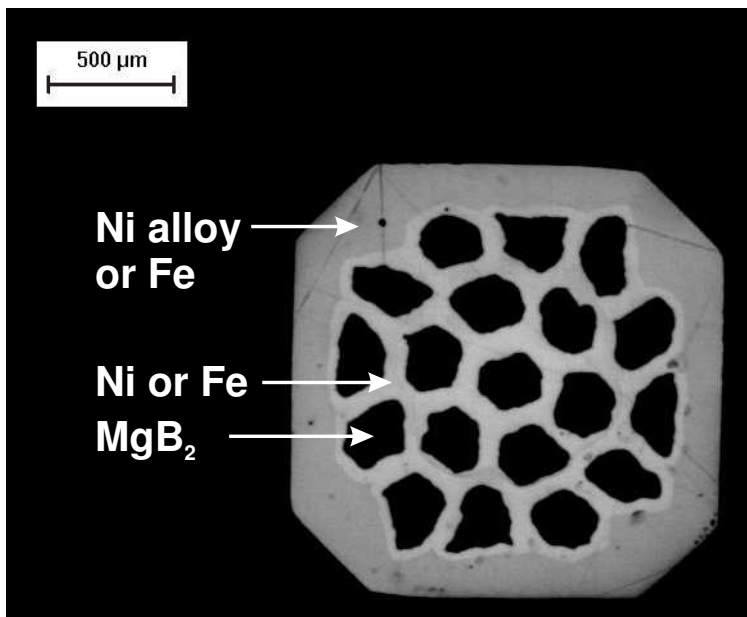


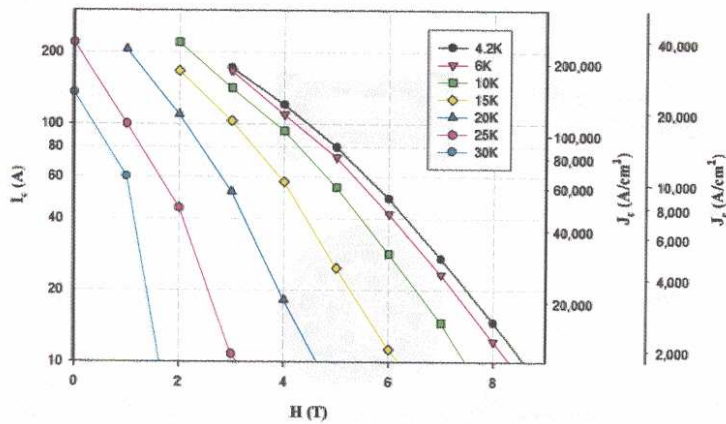
Figure 5. Cross section of prototype multifilament  $MgB_2$  wire from Columbus superconductors with filling factor of 30 %. The dark  $MgB_2$  filaments are surrounded by Ni or Fe and the outer wall is made of Ni alloy or Fe [9].

Figure 6 shows the specifications of the  $MgB_2$  wire from Hyper Tech Research in Ohio, USA. It is seen that the critical current density exceeds that target value from the calculations above  $J_c = 10^5 Acm^{-2}$  at  $T = 10 K$  and  $B = 4$  Tesla, but the engineering critical current density is only  $J_e = 2.2 \cdot 10^4 Acm^{-2}$  even at  $T = 4.2 K$  and  $B = 4$  Tesla. Thus an alternative geometry of the wires is needed to obtain sufficient  $J_e$ .

## Typical Wire Specifications

(1 to 4 kilometer lengths are available  
with 7 and 19 filaments)

Diameter	0.7 mm to 0.9 mm (others available)
Number of Filaments	7 and 19 (higher available experimentally)
Condition	Reacted and un-reacted and insulated with S-glass
Heat Treatment Temperature/Time	700°C / 20 minutes (Nominal)
$J_c$ @ 20 K – 2T	175,000 A/cm <sup>2</sup> (Nominal)
Maximum Allowable Axial Strain	0.35%
Continuous Piece Length	1-4 km
Superconductor fraction	13 – 18% (Under development - increase to 30%)



Typical  $I_c$ ,  $J_c$  and  $J_c$  versus  $H$  at various temperatures for  
0.84 millimeter diameter 19 filament wire

Figure 6. Critical current density of  $MgB_2$  wire from HyperTech Research Inc. [10]

### 5.2 $YBa_2Cu_3O_{6+x}$ coated conductor

Figure 7 shows a plot of the upper critical field of the second generation(2G) High Temperature Superconductor(HTS) tape produced by SuperPower. These new tapes will have  $J_c = 10^5 Acm^{-2}$  at  $H = 4$  tesla and  $T = 77$  K, which sound extremely promising, but the delivery and price might be a problem at present. August 2006 prices of coated conductor are around 100 US\$ per meter.

### 5.3 $Bi_2Sr_2Ca_2Cu_3O_{10+x}$ (Bi2223) first generation HTS tape

Figure 8 and 9 shows the critical current density of the Bi2223 tape from American Superconductors with both the field perpendicular and parallel to the Cu-O planes of the superconductor. It is important to note that Bi2223 is from a fundamental

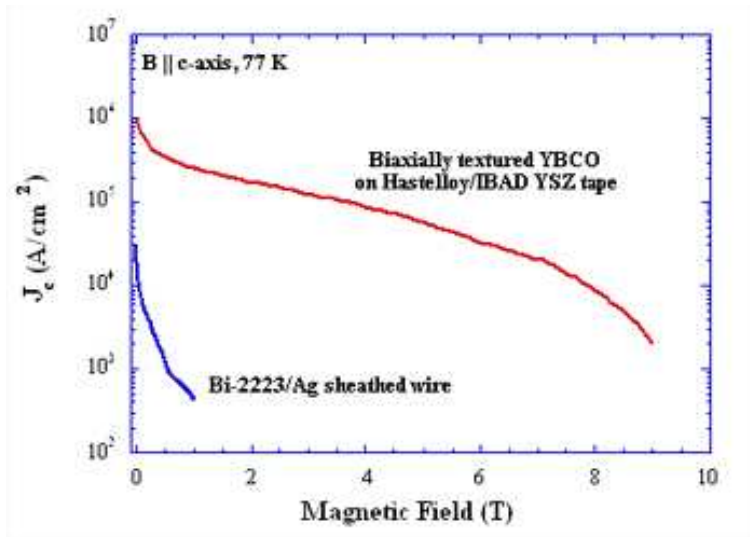


Figure 7. Critical current density of second generation(2G) High Temperature Superconductor (HTS)  $YBa_2Cu_3O_{6+x}$  coated conductor tape from SuperPower [11].

point of view quite limited in the operation in magnetic field at  $T = 77$  K. This is seen in the engineering critical current densities  $J_e = 2.4 \cdot 10^4 \text{ Acm}^{-2}$  when  $B \parallel c = 4$  Tesla and  $J_e = 3.9 \cdot 10^4 \text{ Acm}^{-2}$  when  $B \parallel ab = 4$  Tesla. Thus Bi2223 must be cooled to  $T = 10$  K to support the current density of the dipole magnet specified in this note.

It should be noted that Bi2212 wires from Nexans [13] ( $J_e = 4.5 \cdot 10^4 \text{ Acm}^{-2}$  when  $B = 20$  Tesla and  $J_e = 9.1 \cdot 10^4 \text{ Acm}^{-2}$  when  $B = 0$  Tesla both when  $T = 4.2$  K) and Bi2223 from Sumitomo [14] should be checked before final conclusion can be made on the Bi-family conductors.

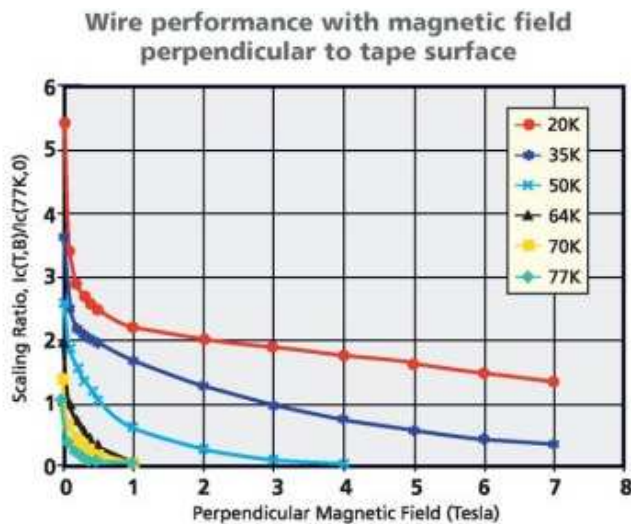


Figure 8. Engineering critical current density of first generation HTS Bi2223 tape from American Superconductors when  $H \parallel c$ . The reference is  $J_e = 1.07 - 1.33 \cdot 10^4 \text{ Acm}^{-2}$  at  $T = 77$  K and  $H = 0$  Tesla [12].

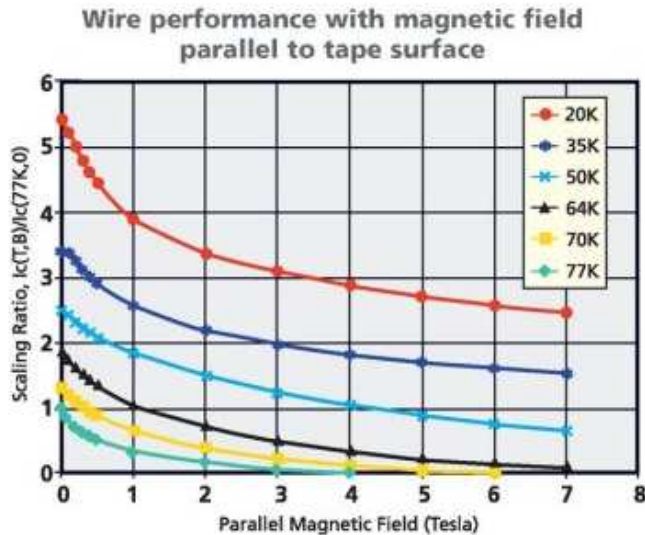


Figure 9. Engineering critical current density of first generation HTS Bi2223 tape from American Superconductors when  $H \parallel ab$ . The reference is  $J_e = 1.07 - 1.33 \cdot 10^4 \text{ Acm}^{-2}$  at  $T = 77 \text{ K}$  and  $H = 0 \text{ Tesla}$  [12].

#### 5.4 Nb – Ti Low Temperature Superconductor

A Nb-Ti Superconductor was found on the home-page of Luvata [15] and a Nb-Ti wire with 552 filaments will have a critical current density of  $J_c = 3.3 \cdot 10^5 \text{ Acm}^{-2}$  at  $T = 4.2 \text{ K}$  and in a field of  $H = 5 \text{ Tesla}$ .

## 6 Conclusion

It is concluded that  $MgB_2$  is a promising candidate as superconductor material for dipole accelerator magnets, since the critical current density exceeds the target current density at  $T = 4.2 \text{ K}$  outlines in this report. However Nb-Ti is much better at  $T = 4.2 \text{ K}$ , but the critical temperature of Nb-Ti  $T_c = 9.5 \text{ K}$  is exceeded at  $10 \text{ K}$ , which is the really interesting operation temperature, because it can be obtained by modern cryo-coolers. The construction of a small  $MgB_2$  superconducting test coil of one half of the magnet seems quite possible and this will be done in 2007 as part of the fulfilment of the STFV framework program "Nye superledere: mekanismer, prosesser og produkter". The second generation YBCO coated conductors may be very useful for future dipole magnets operating at  $T = 77 \text{ K}$ , but most like at a higher production cost than the  $MgB_2$  based ones.

## Acknowledgements

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