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*Published in:*

Proceedings of the 4th International Workshop on Intelligent Information Management Systems and Technology

*Publication date:*

2011

[Link back to DTU Orbit](#)

*Citation (APA):*

Lan, T., Hu, J., Wu, G., You, S., Wang, L., & Wu, Q. (2011). Optimal Charge control of Electric Vehicles in Electricity Markets. In Proceedings of the 4th International Workshop on Intelligent Information Management Systems and Technology

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# Optimal Charge control of Electric Vehicles in Electricity Markets

Tian Lan, Junjie Hu, Guang Wu, Shi You, Lei Wang, Qidi Wu

**Abstract**—Environment constraints, petroleum scarcity, high price on fuel resources and recent advancements in battery technology have led to emergence of Electric Vehicles (EVs). As increasing numbers of EVs enter the electricity market, these extra loads may cause peak load and need to be properly controlled. In this paper, an algorithm is presented for every individual vehicles to minimize the charging cost while satisfying the vehicle owner’s requirements. The algorithm is based on a given future electricity prices and uses dynamic programming. Optimization aims to find the economically optimal solution for each vehicle.

**Index Terms**—Optimal charge plan, Electricity market, Electric vehicles, Dynamic programming.

## I. INTRODUCTION

Due to an increased societal awareness of environment issues and fuel resource limitation, electric vehicles (EVs) are becoming more popular. The emergence of EVs is also an important solution to curb CO<sub>2</sub> emission and oil dependency of current automotive technology [1], since the EVs shift petroleum consumption to electricity. Nowadays, major producers including Toyota, General Motors, Ford, and Volkswagen have plans to sell PHEVs starting in 2011 [2]. A fast growing market is expected in the near future, which leads both opportunities and challenges. Generally, functionalities of EV can be divided into two aspects. On one hand, battery of the EV can be considered as a controllable load. With optimal charging or smart charging for the battery, vehicle owners could maximize their profits by purchasing energy at the lowest possible electricity price. Moreover, charging during the off peak hours will help the load shape and avoid peak load. On the other hand, battery of the EV can also be considered as energy storage capacity which has possibility to provide V2G (Vehicle to Grid) and G2V (Grid to Vehicle) capabilities, also known as regulation service. This functionality is a bi-directional charging that offers ancillary service to grid. For instance, the concept of V2G is that power can be delivered back to the grid from EV battery during the peak hours of electrical power consumption, which is researched in series of publications [3-6].

Many researches have been done on EV optimal charging management. From the point of EV charging’s impact on grid, research can go back to 1980s. Heydt has already researched on the impact of electric vehicle in his paper [7]. He concludes

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that typical driving patterns will likely to coincide the charging with peak load periods of power system. So, methods should be developed to avoid overloading with off-peak charging. Olle presents a linear approximation based method to minimize the charging cost of electricity for EV driver, and mean while avoiding distribution grid congestion [8]. In another paper [9], Kristien researches the impact of charging PHEVs on a residential distribution grid, investigates the difference between coordinated and uncoordinated charging with respect to various penetrations of PHEVs. Some other papers research from a business perspective. In paper [2], Niklas proposes two dynamic programming-based algorithms to find the economically optimal solution for vehicle owner. The first reduces daily electricity cost substantially. The latter takes into account vehicle to grid support as a means of generating additional profits by participating in ancillary service markets. Sekyung proposes an aggregator that makes efficient use of the distributed power of electric vehicles to produce the desired grid scale power, which is V2G concept that can make revenue from providing regulation service [3]. In this paper, we only consider the EV as a controllable load, and investigate its smart charging potential. The functionality of regulation service will not be discussed here.

The purpose of this paper is to investigate a possible solution for EV smart charging under electricity market. The paper is organized as follows. Section II gives a system architecture with appropriate assumptions. Section III constructs a dynamic programming based mathematical model. In Section IV, one case is studied to investigate the optimization of EV charging cost.

## II. SYSTEM ARCHITECTURE AND ASSUMPTIONS

In this paper, charging of electric vehicles come up through purchases in the electricity spot market is presumed. Electric vehicles may directly access to this market or indirectly access through an interface between EV and energy market. And vehicle would be plugged in every time when the driving finished. It is assumed that this market with day-ahead and spot market pricing, which is well suited for the application of smart charging control. Since the V2G is not considered here, a price for ancillary services is not necessary available. Besides the predicted spot price signal, another important piece of information is the future driving pattern. In order to have a successful charging plan, a representative driving pattern is essential. Normally, intra city or short term driving patterns are largely predictable due to fixed working hours and fixed business schedules and routes. Therefore, a future

driving pattern is assumed to be obtained by estimating data of past trips or established driving plans. Moreover, electricity demand of every trip is also needed to be assumed based upon driving pattern.

Centralized control architecture is presumed, in which a single entity (aggregator) directly controls the charging strategy of every vehicle to facilitate smart charging [10], and each vehicle indirectly access to electricity market through this aggregator, which is a smart interface between EV fleets and market to play a role of coordinating charge and discharging operation of multiple vehicles. We assume the aggregator doesn't have a sufficiently large market share to affect electricity price. Therefore, not only vehicle owner, but also aggregator is price taker. In this case, it is expected that most of charging occur at night-time, when given the lower prices. With an automated communication technology, all information can be immediately communicated to aggregator, which then returns a charging plan for an individual EV for the following day.

After gathering all information, the aggregator is fed with following data for charging plan making: predicted electricity price, future driving pattern, energy requirement during every trip and EV status data, such as state of charge of EV battery.

### III. CONTROL TASK FORMULATION

The following notation will be used throughout this paper. Since the market with day-ahead pricing is assumed, the charging plan covers an entire day. For this short-term planning, the time horizon  $[0, N]$  of a day is discretized in to equidistant time intervals  $[k, k+1]$  with  $k=0, \dots, N-1$ . It is assumed that the time interval is  $\Delta t$ .

This problem is addressed by considering the following discrete system which describes the battery:

$$x_{k+1} = T(x_k, u_k, k) \quad (3.1)$$

State variable  $x_k$  represents the state of charge (SOC) of the battery at time  $k$ .  $x_k$  is not only discrete in time (index  $k$ ) but also in value. Any value has to be included in the predefined set  $X$ . It is defined as follows:

$$x_k = \frac{E_k}{E_{max}} \times 100\% \quad (3.2)$$

The control variable  $u_k$  is a dimensionless and discrete representation of  $P_k$ , which is the charge power when plug-in. In order to obtain  $P_k$ ,  $u_k$  is multiplied with the maximum available charge power ( $P_{max-plug}$ ) when plug-in. The electric vehicle discussed in this paper is purely electric propulsion system, which is characterized by an electric energy conversion chain upstream of the drive train, roughly consisting of a battery (or another electricity storage system) and an electric motor with its controller [11]. EV doesn't have an internal combustion engine (ICE) to provide power for propulsion. Battery must be charged from an external electric network. Due to this fact, the values of  $u_k$  are fixed at 0 when driving, while these values range from 0 to 1 when plug-in. If  $U_{plug}$  is set that covers all possible values of  $u_k$ , its discretization may be described as follows:

$$u_k = \begin{cases} u_k \in U_{plug}, & k \in K_{plug} \\ u_k = 0, & k \in K_{driv} \end{cases} \quad (3.3)$$

$K_{plug}$  denotes the set of indices  $k$  within the time periods when the vehicle is plugged in, while  $K_{driv}$  refers to the driving intervals. The summation of the number of elements in  $K_{plug}$  and  $K_{driv}$  is  $N$ , which denotes the total number of time intervals. Any index  $k$  in  $K_{plug}$  or  $K_{driv}$  has to be element of the predefined set  $K$ .

$$k \in K = \{K_{plug}, K_{driv}\} \quad (3.4)$$

A specific control strategy is denoted by

$$u = \{u_0, u_1, u_2, \dots, u_{N-1}\} \quad (3.5)$$

Any value of  $u_k$  has to be element of a predefined set  $U$ , which known as set of admissible decision. The total cost of a sequence,  $f_0^U$ , is given by the cost of the final step,  $f_N(x_N)$ , plus the cost for all other steps,  $v_k(x_k, u_k, k)$ :

$$f_0^U(x_0) = f_N(x_N) + \sum_{k=1}^{N-1} v_k(x_k, u_k, k) \quad (3.6)$$

To minimize the objective function (3.6), the optimal control strategy  $u^* = \{u_0^*, u_1^*, u_2^*, \dots, u_{N-1}^*\}$  has to be obtained. This is a classic dynamic programming formulation and can be solved as described by literature [12, 13]. The optimal trajectory is calculated starting with the cost of the last step and going backwards through time until the first state's optimal cost  $f_0^*(x_0)$  is given by the algorithm. The recursive equation is listed as follows:

$$\begin{aligned} f_k(x_k, u_k) &= \min\{v_k(x_k, u_k) + f_{k+1}(x_{k+1})\} \\ &= \min\{v_k(x_k, u_k) + f_{k+1}(T(x_k, u_k, k))\} \end{aligned} \quad (3.7)$$

$$u_k^* = \operatorname{argmin}(f_k(x_k, u_k)) \quad (3.8)$$

According to dynamic programming, some special terms are given as follows:

$k$ : step

$X$ : set of admissible state

$U$ : set of admissible decision

Due to the discretization of  $x_k$  and  $u_k$ , admissible state set  $X$  must be defined appropriately, because  $T(x_k, u_k, k)$  may not be any of the elements of  $X$  where  $f_{k+1}$  is known if set  $X$  is not defined good enough, which means  $T(x_k, u_k, k)$  may not equal with  $x_{k+1}$  as described in equation (3.1). Actually in practice operation, errors are hardly avoided that  $T(x_k, u_k, k)$  will usually not be on a grid point no matter how the set  $X$  is defined. Therefore, an approximation is needed. Normally,  $x_{k+1}$  is defined as a range by plus a margin of 10% to cover the possible  $T(x_k, u_k, k)$ .

### A. Cost of final step

The objective function is to ensure that the battery is fully charged before the first trip of the following morning. Step N is the moment right before the next day's departure. Therefore, the value of  $x_N$  is 100%, and no charging operation happens at step N. The cost of final step  $f_N(x_N)$  should be defined as.

$$f_N(x_N) = 0 \quad (3.9)$$

### B. Cost of other step

For EV, a purely electric propulsion system, it is important to acknowledge different step function  $v_k$  for driving mode and plug in charging mode. The most general case is given by introducing the following discontinuity:

$$v_k(x_k, u_k, k) = \begin{cases} v_{plug}(x_k, u_k, k), & k \in K_{plug} \\ v_{driv}(k), & k \in K_{driv} \end{cases} \quad (3.10)$$

where

$$v_{plug}(x_k, u_k, k) = \eta_k \cdot u_k \cdot P_{max-plug} \cdot C_{el}(k) \cdot \Delta t \quad (3.11)$$

and

$$v_{driv}(k) = 0 \quad (3.12)$$

$\eta_k$  denotes the efficiency parameter between step  $k$  and  $k+1$ .  $C_{el}$  is the price of electricity per unit of energy.  $\Delta t$  is the time interval between step  $k$  and  $k+1$ . Since EV is a purely electric propulsion system, charging cannot take place during driving period as hybrid electric vehicle did, and the charging cost is set to 0.

### C. Equation of state transition

The performance measure is defined as shown in equation (3.7), where  $x_k$  is an actual SOC at the step  $k$ ,  $x_{k+1}$  is a desired SOC at step  $k+1$ . The equation of state transition  $T(x_k, u_k, k)$  is specified as follows:

$$\begin{aligned} x_{k+1} &= T(x_k, u_k, k) = x_k + \Delta x \\ &= x_k + \frac{\Delta E_k}{E_{max}} \times 100\% \end{aligned} \quad (3.13)$$

Where

$$\begin{aligned} \Delta E_k &= \\ \begin{cases} \Delta E_{k-plug} = \eta_k \cdot u_k \cdot P_{max-plug} \cdot \Delta t, k \in K_{plug} \\ \Delta E_{k-driv} = -P_{dr} \cdot \Delta t, k \in K_{driv} \end{cases} \end{aligned} \quad (3.14)$$

$\Delta E_{k-plug}$  is the energy increase of the battery from step  $k$  to step  $k+1$  when plug in. Similarly,  $\Delta E_{k-driv}$  is the energy decrease of the battery from step  $k$  to step  $k+1$  when driving.  $P_{dr}(k)$  denotes power requirement during driving cycle. It is obvious that power requirement of every step when driving is hardly predicted. Instead, it is possible to predict energy requirement throughout a whole driving

trip, which can be a replacement of power requirement. This solution is described as follows:

$$E_{dr}^i = \sum_{j=1}^m \Delta E_{dr}^i(j) = - \sum_{j=1}^m P_{dr}^i(j) \cdot \Delta t \quad (3.16)$$

Here  $E_{dr}^i$  is the energy requirement of the  $i^{th}$  trip during a day.  $m$  is number of time intervals of the trip, where as  $P_{dr}^i$  is the corresponding power requirement. Substitute equation (3.16) into equation (3.13), we have the state transition equation of  $i^{th}$  trip when driving as follows:

$$\begin{aligned} x_{k+m} &= x_k - \frac{\sum_{j=1}^m P_{dr}^i(j) \cdot \Delta t}{E_{max}} \times 100\% \\ &= x_k + \frac{E_{dr}^i}{E_{max}} \times 100\% \end{aligned} \quad (3.17)$$

Similarly, the state transition equation of other trips can be obtained in the same way. The drawback of equation (3.17) is that intermediate states  $\{x_{k+1}, x_{k+2}, \dots, x_{k+m-1}\}$  are difficult to be calculated precisely due to the impossible prediction of  $P_{dr}$  in practice. Fortunately, the imprecision of these states has no influence on the accuracy of final optimal control strategy  $u^*$ . The reason is that no charging happens when driving, and  $u_k$  is determined by battery state of charge  $x_k$  and  $x_{k+1}$ , where  $k \in K_{plug}$ . This relationship is illustrated by following equation (3.18).

Substitute equation (3.14) into equation (3.13), we have the state transition equation when plug in as follows:

$$\begin{aligned} x_{k+1} &= x_k + \frac{\eta_k \cdot u_k \cdot P_{max-plug} \cdot \Delta t}{E_{max}} \times 100\% \\ &= x_k + \eta_k u_k \cdot \frac{P_{max-plug} \cdot \Delta t}{E_{max}} \times 100\% \end{aligned} \quad (3.18)$$

From equation (3.18), it is clear that  $P_{max-plug}$  and  $E_{max}$  are constant for a specific vehicle battery, efficiency parameter  $\eta_k$  and time interval  $\Delta t$  are also assumed to be known. Therefore,  $u_k$  is a function of  $x_k$  and  $x_{k+1}$ .

These state transition models (equation (3.17) and (3.18)) are general enough to account for the possibility of parallel processing among the various control strategies, as well as for redundancy in the database. Once the concept of state transition has been properly defined, dynamic programming can be used to find the state containing the answer to the query that has the minimum cost and to find the optimal trajectory to that state (i.e., optimal sequence of processing operations) [14].

## IV. CASE STUDY

In this section, a case is studied and the goal of optimization is to present a charging schedule for every individual vehicle to minimize the cost of electricity while satisfying the vehicle owner's requirements. A comparison is made between the results of an EV with a fast charging scheme and those of the dynamic programming based method.

The charging schedule is divided into time intervals for a 24-h based period. The period starts from the first second when

the vehicle owners begin their first trip and ends right before the next day's departure, which has 288 intervals of 5-minutes each. As mentioned previously, the objective function is to ensure that the battery is fully charged before the first trip of the following morning, which means  $x_0=100\%$  and  $x_N=100\%$ . The vehicle used to study in this paper is Nissan LEAF Electric Car, which is a purely electricity propulsion system. The basic battery information [15] is shown in Table I. Typically, the battery operation is limited to a given state of charge operating range. We assume Nissan LEAF battery has a minimum state of charge of 10%. Hence, here the SOC is limited to [0.1, 1.0]. Moreover, it is important to know a driving behavior of a vehicle, which includes the departure time, return time and energy requirements of every trip. Based on the vehicle parameters, a driving map includes three trips during a day is given in table II.

Another important piece of information is electricity prices, which are based upon a typical work day of 04.05.2011 from Nordpool Spot market area Denmark West [16].

TABLE I  
SIMULATION PARAMETERS

<i>Discretization</i>	<i>Parameters</i>
U	11
X	721
$\Delta t$	300s
<i>Battery</i>	
<i>Value</i>	
Total capacity	24KWh
Maximum plug power	4KW
Maximum driving distance	160km
Energy consumption	150Wh/km

TABLE II  
DRIVING BEHAVIOR

<i>Trip</i>	<i>DepartureTime</i>	<i>ReturnTime</i>	<i>EnergyRequirement</i>
1	8:00	9:00	13.5KWh
2	15:00	16:00	9KWh
3	20:00	21:00	13.5KWh

### A. Fast Charging

Fast charging is a kind of uncoordinated charging. It assumes that vehicles are charged instantaneously when they are plugged in, and the batteries will be fully recharged as fast as possible without considering the daily electricity price. Nissan LEAF vehicles have their own definition for battery fast charging. The LEAF's battery is intended to accept several rapid charging scenarios including a 50KW "fast charge" which gives 80% charge in thirty minutes, or a five minute fast-charge which delivers an additional 31 miles of range. These rapid recharge modes will require a special three-phase charger, which is most likely to be owned by commercial or governmental entities in distributed charging

stations [15]. This fast charging offers most flexibility to driver. However, it is not the fast charging we discussed here, because homeowners don't have a spare 50KW charging power, but prefer to have a common, single-phase 220V with maximum 4KW charging power. The profiles with fast charging are given by Fig.1. From Fig.1, it is clear that every time when the vehicle finishes a trip, the battery will be charged immediately without the considering the electricity prices. The battery is fully charged as fast as possible once it is plugged in. As a result, the electricity costs will be high, which for the profiles amount to 4.5995EU.

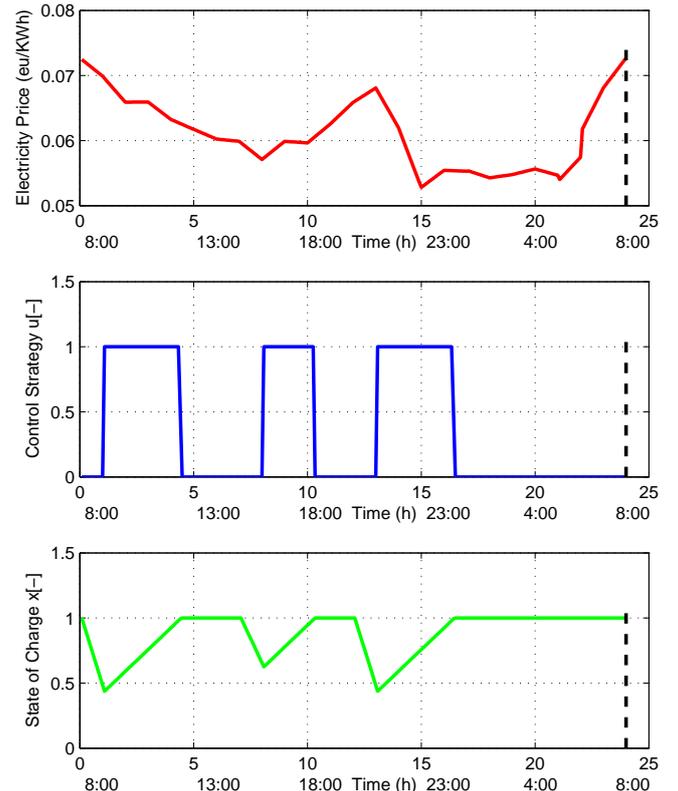


Fig. 1. Profile with fast charging

### B. Smart Charging

The idea of the smart charging is to achieve optimal charging to minimize the charging cost. The optimal control strategy will be obtained and sent to the individual vehicle as control signals for charging power. The results of the dynamic programming based method are shown in Fig.2. The simulation parameters are given in Table I.

From Fig.2, we have a general idea that electric vehicle charging is done when the price for electricity is lowest. The SOC of battery shows that the battery doesn't have to be fully charged before next trip. Instead, it would be sufficient if the SOC is charged enough to support the energy consumption for the next trip. This leads to a electricity cost of 2.0393EU for

a whole day, which is much cheaper than the fast charging cost. Smart charging cannot offer flexibility for driver as fast charging does. Consequently, sometimes when drivers drive away their vehicles before the preannounced departure time, the battery may not be enough charged to meet the energy requirement for the next trip, which the drivers have to accept.

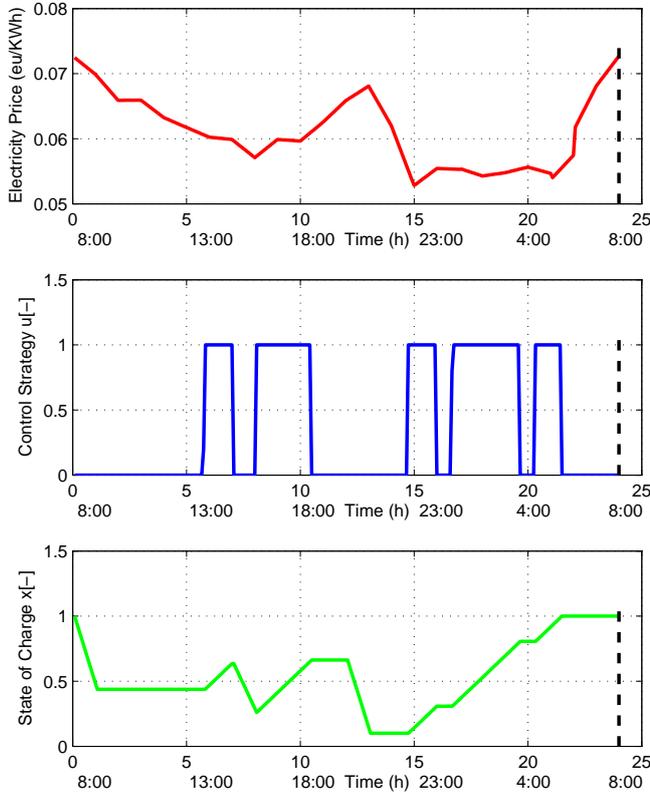


Fig. 2. Profile with smart charging

### C. Computing Time

The code optimizes a 24-h interval in Matlab takes 77.79s on 2.8-GHz CPU with 3.12GB of RAM.

## V. CONCLUSIONS

This paper presents a mathematical formulation and a dynamic programming based algorithm for optimizing EV charging given electricity prices and driving pattern. Smart charging without provision of regulation service reduces daily electricity costs for driving from 4.5995EU to 2.0393EU compare with fast charging. With smart charging, EV is recharged during the lowest electricity price period, where is also the off peak hours. It naturally drops the possibility of grid overload during the peak load hours.

Future work is needed with respect to several possible extensions. The optimization model should be extended to account for providing of regulation service and given different

type of electric drive vehicles as well as various driving patterns. Furthermore, in considering the possibility that EV charging may impact the electricity price, electricity price forecasting models should be properly developed.

## VI. ACKNOWLEDGEMENT

This work is supported in part by the National Science Foundation of China (grant no. 61005090, 61034004, 61075064, 70871091), Ph.D. Programs Foundation of Ministry of Education of China (grant no. 20100072110038), Program for New Century Excellent Talents in University of Ministry of Education of China and Foundation of the Ministry of Education of China for Returned Scholars.

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