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COMPARISON OF THE FFT/MATRIX INVERSION AND SYSTEM MATRIX TECHNIQUES FOR HIGHER-ORDER PROBE CORRECTION IN SPHERICAL NEAR-FIELD ANTENNA MEASUREMENTS

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ABSTRACT

Two higher-order probe-correction techniques for spherical near-field antenna measurements are compared in details for the accuracy they provide and their computational cost. The investigated techniques are the FFT/matrix inversion and the system matrix inversion. Each of these techniques allows correction of general high-order probes, including non-symmetric dual-polarized antennas with independent ports. The investigation was carried out by processing with each technique the same measurement data for a challenging case with an antenna under test significantly offset from the center of rotation and a higher-order probe.

1. INTRODUCTION

Probe-corrected spherical near-field antenna measurements represent the most accurate method for characterization of the radiated fields of antennas. The traditional spherical near-field antenna measurement technique requires a first-order ($\mu = \pm 1$) probe, i.e., a probe with only first-order azimuthal modes in the spherical vector wave expansion of the probe field, since this provides an efficient and robust probe-correction in the near-field to far-field transformation [1]. This traditional technique yields supreme accuracy and it has matured into a well-established technique that forms the basis for many existing antenna measurement facilities. However, the first-order requirement significantly limits the types of antennas that can be used as probes; one of a very few high-accuracy, practical probes being the conical horn fed through a circular waveguide operating in the fundamental TE_{11}-mode. This antenna has certain disadvantages, since it provides only some 15% bandwidth and becomes unmanageable large and heavy at frequencies below 1 GHz.

Recently, higher-order probe correction techniques have gained a lot of interest in the antenna measurement research community [2-4]. In the application of such techniques, in addition to the azimuthal modes with index $\mu = \pm 1$, also the azimuthal modes with indices $\mu \neq \pm 1$ are included in the modeling and correction of the probe pattern. The use of higher-order probe correction techniques allows a greater flexibility in choosing a probe that leads to an optimal compromise between the desired properties of the probe, for example, the bandwidth, the weight, the size, and the cost. In particular, the higher-order probe correction facilitates the use of very wideband antennas as probes [5].

In this paper two higher-order probe-correction techniques are compared in details for the accuracy they provide and their computational cost. The investigated techniques are the FFT/matrix inversion [3] and the system matrix inversion [4]. Each of these techniques allows correction of general higher-order probes, including non-symmetric dual-polarized antennas with independent ports. The investigation was carried out by processing with each technique the same measurement data for a challenging case with an antenna under test (AUT) significantly offset from the center of rotation. The higher-order probe used in the measurement is a dual-polarized wideband antenna with noticeable contents of the $\mu \neq \pm 1$ modes. Reference data were also obtained for the AUT with a high-quality first-order probe and traditional first-order probe correction. The far-field AUT pattern obtained from the measurement with the higher-order probe and processed with the two higher-order probe correction techniques were then compared versus each other and versus the reference pattern.

2. MEASUREMENTS

The measurement of the AUT and the steps of the probe calibration are described in this section.

2.1. Measurement of AUT

The AUT is a log-periodic 1-18 GHz antenna measured in an offset configuration. The offset is 1.6 m from the *z*-axis along the *x*-axis, i.e. the coordinates of the AUT in the measurement coordinate system is (–1.6m, 0, 0), see Fig. 1.

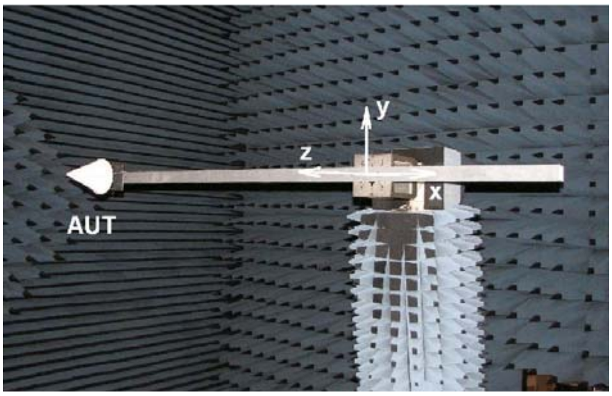

Figure 1. Offset log-periodic antenna used as AUT.

The AUT measurement was carried out only for the x-polarized port at 2.9 GHz and 3.0 GHz. The choice of the AUT configuration and frequency was driven by the intention to use the most difficult case for which the influence of the high-order modes of the probe is the largest. For an AUT centred in the measurement coordinate system the probe receives mainly through its on-axis pattern where the first-order modes are dominant, but for an offset AUT the probe receives through a larger part of its pattern where the higher-order modes are more significant.

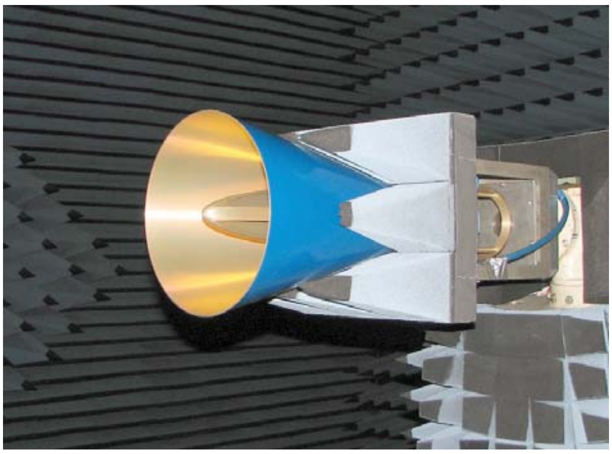

Figure 2. Wideband dual-polarized higher-order probe on the antenna tower during pattern calibration.

The reference measurement of the AUT was carried out with a high-quality dual-polarized first-order probe called S2 for which the level of spherical higher-order μ-modes, $|\mu| \neq 1$, is below –45 dB. The measurement was done as phi-scan and theta-step with the angular intervals both in theta and phi of 1.2°. Next, the test measurement of the AUT was carried out with a dual-polarized high-order probe (HOP) SP800 (see Fig. 2) with the same scanning scheme and the same angular intervals. The μ-mode spectrum of the HOP, in addition to indices $\mu = \pm 1$, also contain power in indices $\mu = \pm 3$ at the level of about –16 dB and also in indices $\mu = 0$ and $\mu = \pm 2$ at the level of about –25 dB, see Fig. 3.

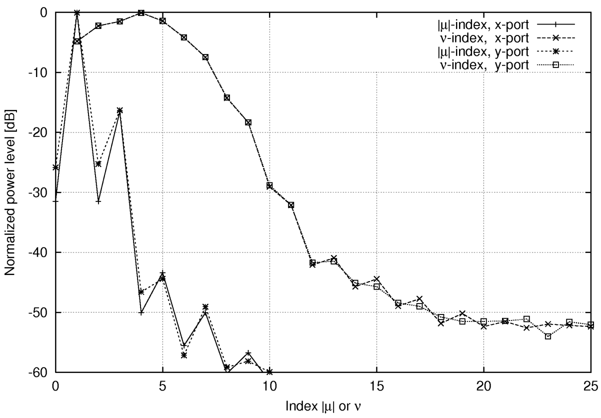

Figure 3. Spherical mode spectrum of the higher-order wideband probe SP800 at 3.0 GHz.

2.2. Probe calibration

For each probe the necessary data were measured according to the corresponding probe correction technique. For the first-order probe, two co-polar pattern cuts were measured for the x-port (for the y-port pattern is assumed the same, but rotated 90°) as well as the polarization characteristics for each port and the channel balance with the probe connected to the polarization switch of the receiver. For the higher-order probe, for each port, the full-sphere data was measured with a polarization calibrated auxiliary antenna, from which the spherical wave coefficients of the probe were then calculated. The channel balance was also measured with the probe connected to the polarization switch of the receiver.

3. HIGHER-ORDER PROBE-CORRECTION TECHNIQUES

The considered higher-order probe-correction techniques, FFT/matrix inversion (FMI) and the system matrix inversion (SMI), are described in details in [3, 4]. Thus only the main steps of the techniques are presented here.

3.1. FFT/matrix inversion technique

In this technique the inversion of the transmission formula is done in two steps. First, the Inverse Discrete

Fourier Transform of the measured signal at the probe ports along the ϕ-coordinate is calculated. Second, for each index m a system of linear equations is set up for the unknown AUT spherical wave coefficients Q_{smn}. The least-square solution to the over-determined system of equations is then found using the pseudo-inverse operation [3].

The computational complexity of this technique is estimated to be of the order of $O(N^4)$, where N is the electrical size of the AUT such that the total number of the AUT coefficients is $2N(N + 2)$. The computation time is mainly determined by the time required to set up and invert the system of equations. With the software implemented in FORTRAN90, for the considered experimental data with maximum probe indices $\mu_{max} = 3$ and $\nu_{max} = 18$ and for the maximum AUT index $N = 150$, it is about 8 min on a usual modern desktop PC.

3.2. System matrix inversion technique

This technique is based on a renormalized least-squares approach. A system of linear equations is set up for the unknown AUT spherical wave coefficients and the measured probe signals. The obtained non-square matrix equation is then renormalized to obtain a normal equation in which the normal matrix almost equals the identity matrix, when most of the energy in the higher-order probe pattern is confined to the first-order modes. The last condition is usually satisfied by symmetric antennas with the z-axis coinciding with the symmetry axis and with the main beam oriented along z-axis.

The computational complexity of the technique is estimated to be $O(N^3)$. With the software implemented in Matlab, for the considered experimental data with the same parameters as described above, the computation time is between 8 and 27 sec on a usual modern PC, depending on the approach used for the matrix inversion.

4. COMPARISON

The far-field AUT patterns obtained after processing with the two higher-order probe correction techniques the measured data from the higher-order probe were then compared versus the reference pattern and versus each other. The reference AUT pattern was obtained applying the SNIFTD software [6] to the measured data from the first-order probe.

The far-field co-polar and cross-polar patterns in $\phi = 0°$ and in $\phi = 90°$ planes are shown in Fig. 4. It is seen that the agreement between all three patterns is excellent. Minor difference can only be noted in the cross-polar pattern. It is also noted that the results from the two higher-order techniques are almost on top of each other, but differ slightly from the cross-polar reference pattern in few places.

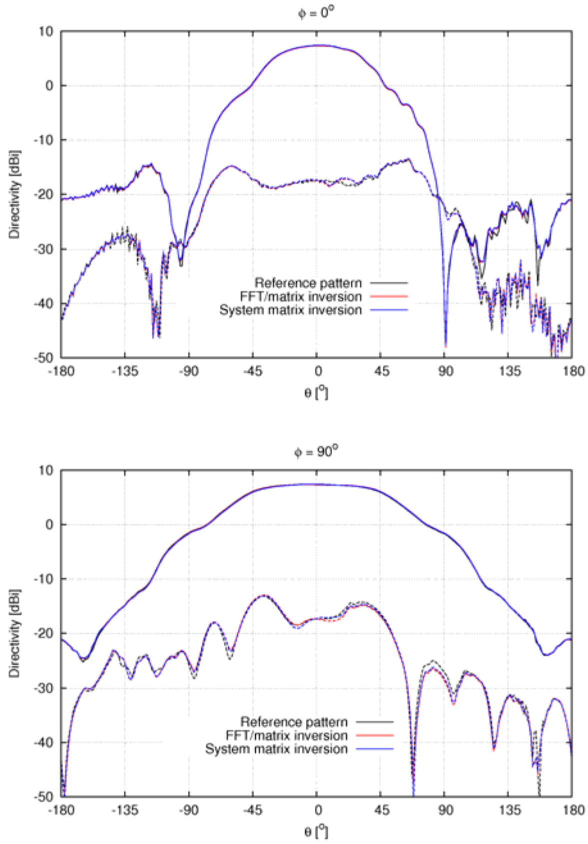

Figure 4. Comparison of the AUT radiation pattern obtained with two high-order probe correction techniques versus reference pattern: $\phi = 0°$ plane (top) and $\phi = 90°$ plane (bottom).

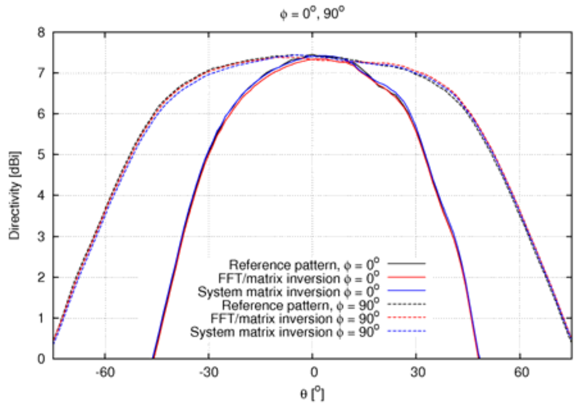

Figure 5. Zoomed view of the co-polar pattern in the main beam region in $\phi = 0°$ plane and $\phi = 90°$ plane.

Fig. 5 shows a zoomed view of the main beam region. It is seen that the co-polar patterns agree to within few hundreds of a dB. In order to quantify the difference, the statistics for the pattern difference in logarithmic scale was calculated. The standard deviation of the difference between the co-polar patterns in logarithmic scale calculated within $\theta = [0, 35°]$ (–3 dB from the main beam peak) is shown in Table 1. It is seen that the standard deviation does not exceed some 0.06 dB, which indicates a very high accuracy of all three results.

Table 1: The standard deviation of the difference between the co-polar patterns in logarithmic scale calculated within θ = [0, 35°].

STD	SMI vs REF	FMI vs REF	SMI vs FMI
dB	0.053	0.057	0.055

It should be noted that the uncertainty of the reference pattern is estimated to be of the order of 0.05 dB (1σ).

The difference between the results obtained from the two higher-order probe correction techniques applied to the same input data can be explained by the following. First, the probe is modeled differently in the two techniques, and second, the inversion of the transmission formula is carried out differently.

5. CONCLUSIONS

Two higher-order probe-correction techniques, the FFT/matrix inversion technique and the system matrix inversion technique, were compared in details for the accuracy they provide and their computational cost. Each of these techniques provides very accurate far-field pattern results. The main difference is in the computational time: for the investigated AUT with the electrical dimension of about 35 wavelengths, the FFT/matrix inversion technique performs calculations in 8 min, while the system matrix inversion technique provides the result within less than a half-minute.

6. ACKNOWLEDGEMENT

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